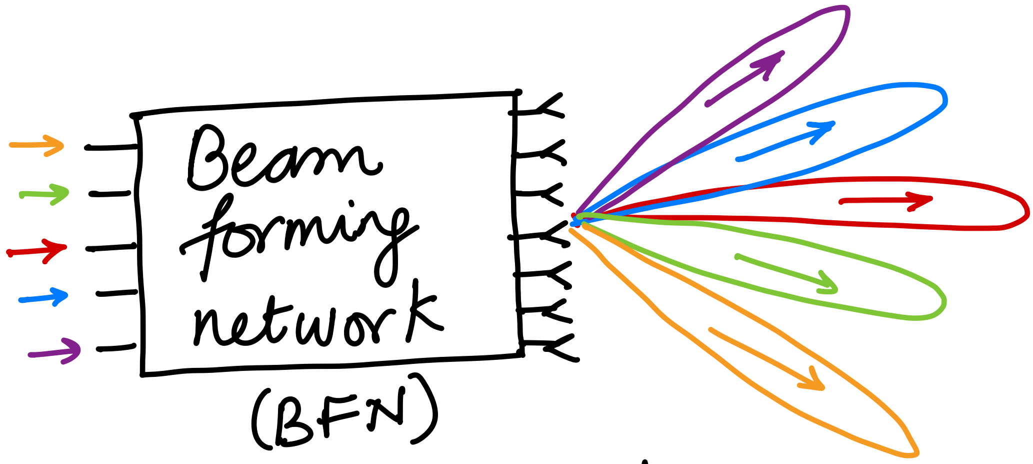
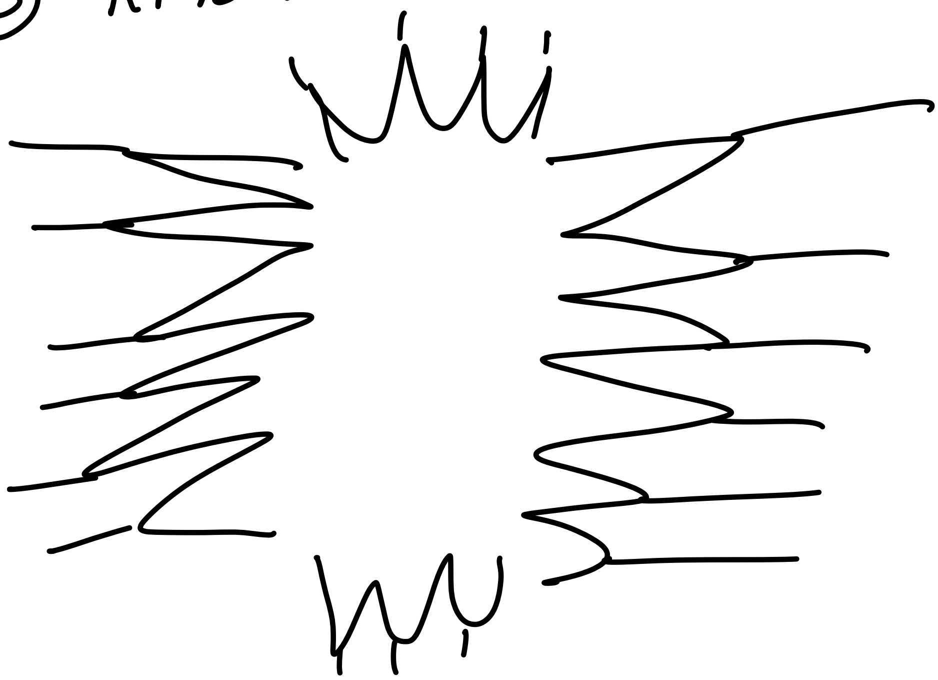
Rotman Lens - Introduction & Theory



> Rotman Lens is a BFN.

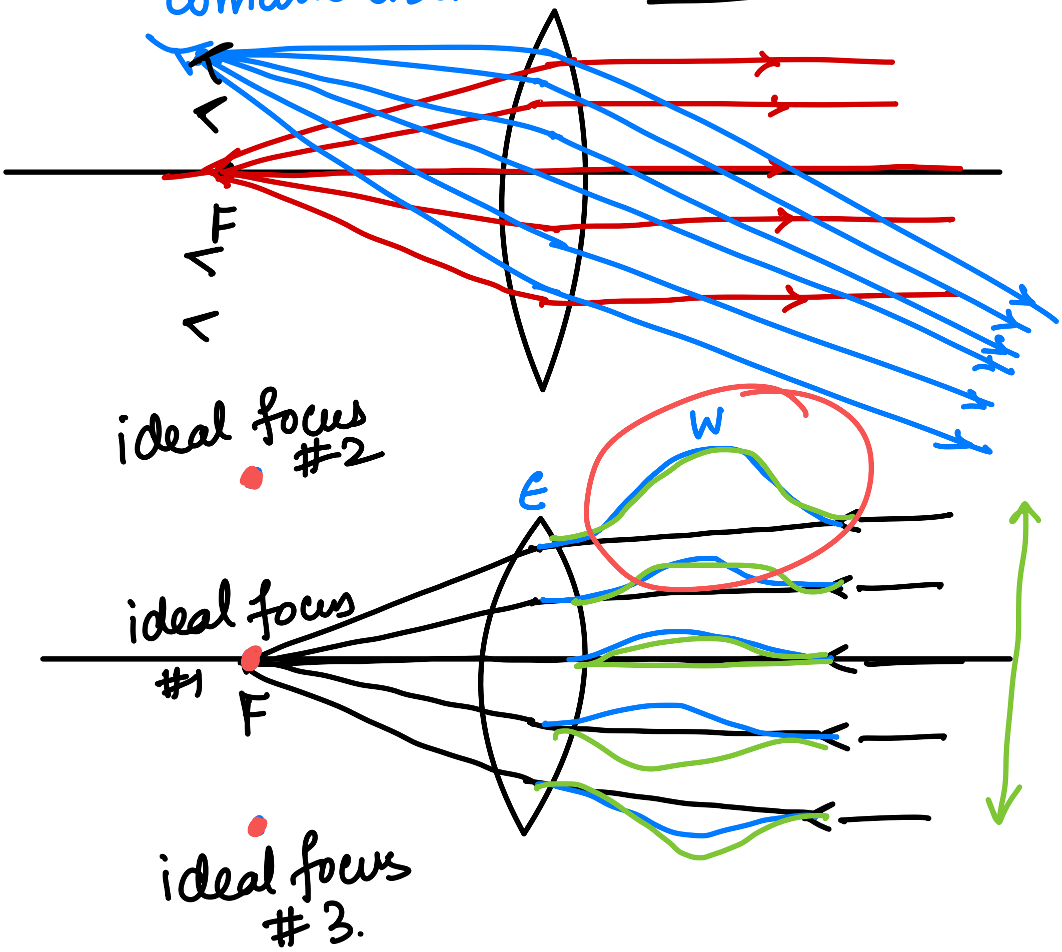
Adv.
1) Wide-band (Relies on True-time delay).
2) Wide-angle
3) Low-profile & easy to fabricate.

Appl.
1) Radar (automotive).
2) Comm. (UWB, 5G/6G)
3) RFID & IoT (5G wireless power transfer).



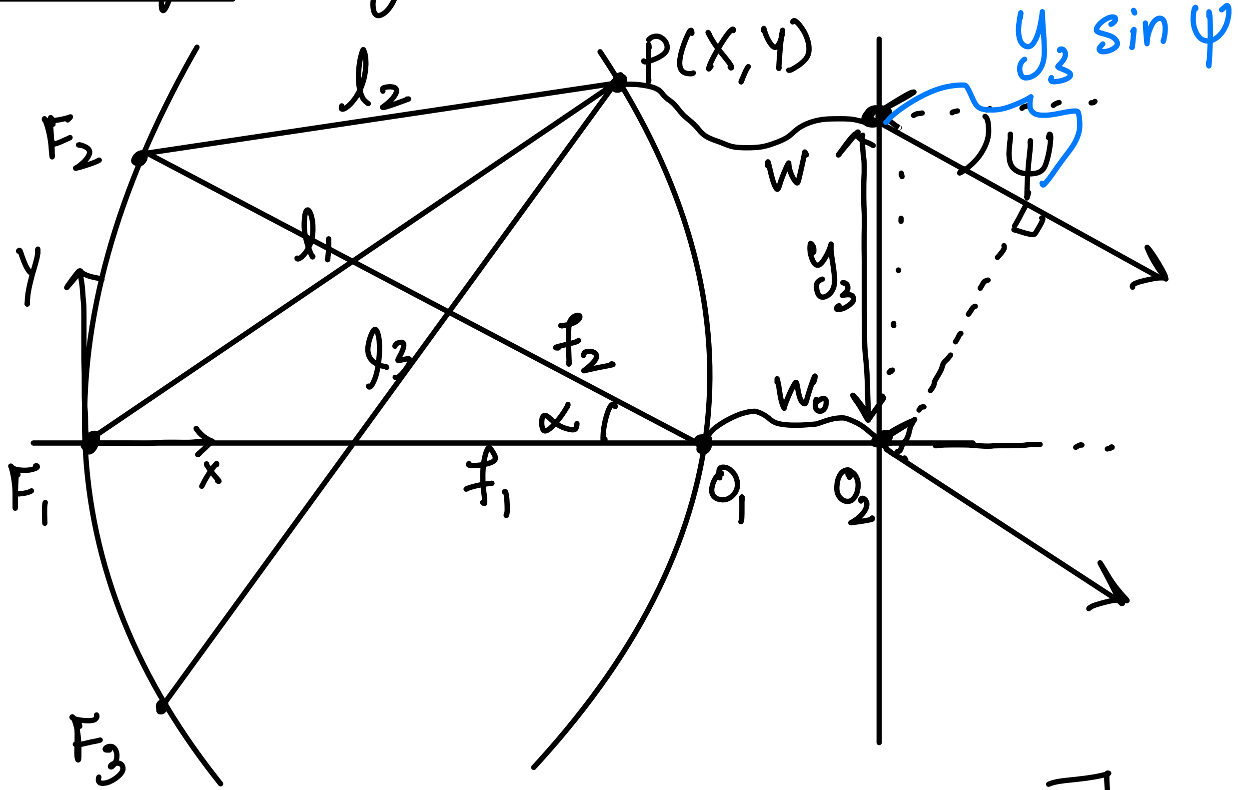
Lens Design (Intuition)

Comatic aberration (coma)



IDEA: Adding "w" gives an extra DOF s.t we can form a second (& third) ideal focus off axis.

Lens Design (Geometrical Optics)



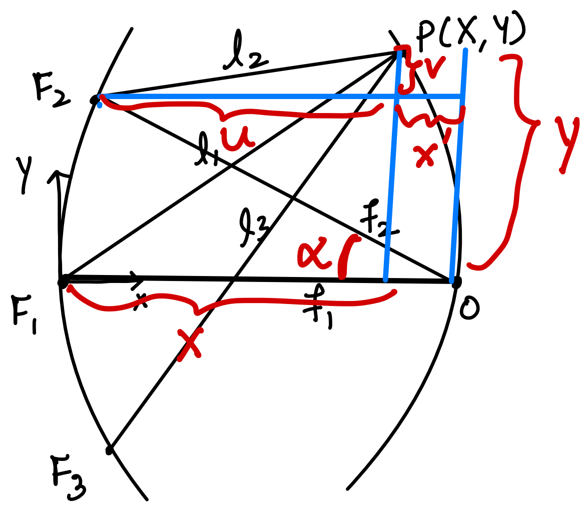
$$\left.\begin{aligned} l_2 + W + y_3 \sin\psi &= f_2 + W_0 \\ l_3 + W - y_3 \sin\psi &= f_2 + W_0 \\ l_1 + W &= f_1 + W_0 \end{aligned}\right] \quad (1)$$

$$f_2 \cos\alpha = u + X'$$

$$f_2 \sin\alpha = Y - v$$

$$l_2^2 = u^2 + v^2$$

$$= f_2^2 + X'^2 + Y^2 - 2 f_2 X' \cos\alpha - 2 f_2 Y \sin\alpha$$



$$
(2)\begin{cases}
l_2^2 = f_2^2 + X'^2 + Y^2 - 2 f_2 X' \cos\alpha - 2 f_2 Y \sin\alpha \\
l_3^2 = f_2^2 + X'^2 + Y^2 - 2 f X' \cos\alpha + 2 f_2 Y \sin\alpha \\
l_1^2 = (f_1 - X')^2 + Y^2
\end{cases}
$$

<u>From (1) & (2)</u>

$$X'^2 + Y^2 + f_2^2 - 2 f_2 X' \cos\alpha - 2 f_2 Y \sin\alpha = (f_2 + W_0 - W - Y_3 \sin\varphi)^2$$

$$X'^2 + Y^2 + f_2^2 - 2 f_2 X' \cos\alpha + 2 f_2 Y \sin\alpha = (f_2 + W_0 - W + Y_3 \sin\varphi)^2$$

$$(-X' + f_1)^2 + Y^2 = (f_1 + W_0 - W)^2$$

<u>Substitutions</u>

$$x' = \frac{X'}{f_1}\ ; \quad y = \frac{Y}{f_1}\ ; \quad w = \frac{W - W_0}{f_1}\ ; \quad \beta = \frac{f_2}{f_1}\ ; \quad \gamma = \frac{\sin\varphi}{\sin\alpha}$$

$$\zeta = \frac{y_3 \gamma}{f_1} = \frac{Y_3 \sin\varphi}{f_1 \sin\alpha}\ ; \quad x = 1 - x' \quad \text{since } X - f_1 = -X'.$$

The above equations now are:

Solve for X, Y, W

$$
\begin{cases}
(3)\ x'^2 + y^2 + \beta^2 - 2\beta x' \cos\alpha - 2\beta y \sin\alpha = (\beta - w - \zeta \sin\alpha)^2 \\
(4)\ x'^2 + y^2 + \beta^2 - 2\beta x' \cos\alpha + 2\beta y \sin\alpha = (\beta - w - \zeta \sin\alpha)^2 \\
(5)\ (-x' + 1)^2 + y^2 = (1 - w)^2
\end{cases}
$$

$$④-③\quad 4\beta y \sin\alpha = 4\zeta(\beta - w)\sin\alpha$$

$$\Rightarrow \boxed{y = \zeta\left(1 - \frac{w}{\beta}\right)} \quad 6a$$

$$③+④ \Rightarrow x'^2 + y^2 + \cancel{\beta^2} - 2\beta x'\cos\alpha = w^2 + \cancel{\beta^2} - 2\beta w + \zeta^2\sin^2\alpha$$

$$⑤\left\{ x'^2 + y^2 - 2x' + \cancel{1} = w^2 + \cancel{1} - 2w \right.$$

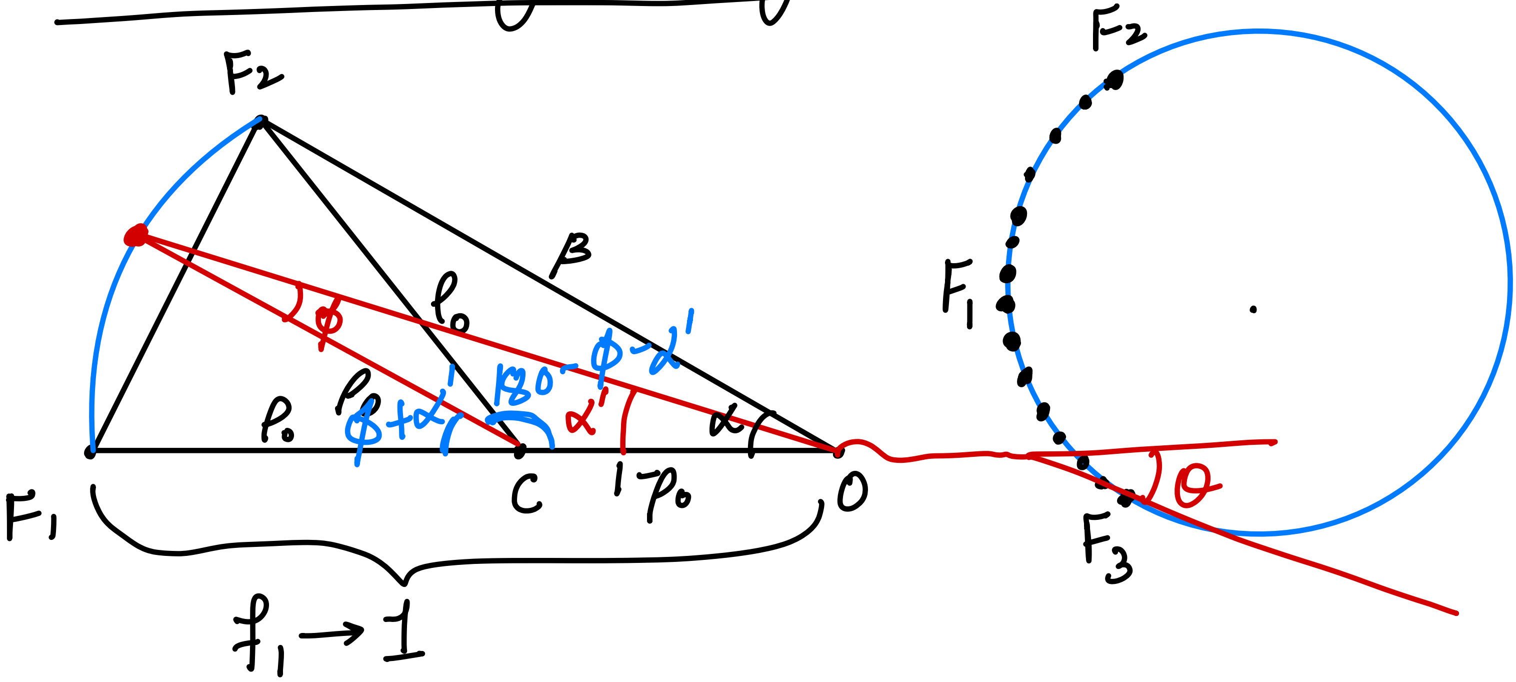
$$x' = -\frac{-2w + 2\beta w - \zeta^2\sin^2\alpha}{2(1-\beta\cos\alpha)}$$

$$6b \quad \boxed{x = 1 - \left[\frac{\frac{\zeta^2}{2}\sin^2\alpha + (1-\beta)w}{(1-\beta\cos\alpha)}\right]}$$

Subs. x & y back into ⑤ & solving gives

$$⑦\left[\begin{aligned}
&a w^2 + bw + c = 0, \quad \text{where} \\
&a = 1 - \frac{(1-\beta)^2}{(1-\beta\cos\alpha)^2} - \frac{\zeta^2}{\beta^2} \\
&b = -2 + \frac{2\zeta^2}{\beta} + \frac{2(1-\beta)}{1-\beta\cos\alpha} - \frac{\zeta^2\sin^2\alpha\,(1-\beta)}{(1-\beta\cos\alpha)^2} \\
&c = -\zeta^2 + \frac{\zeta^2\sin^2\alpha}{1-\beta\cos\alpha} - \frac{\zeta^4\sin^4\alpha}{4(1-\beta\cos\alpha)^2}
\end{aligned}\right.$$

Beam Port Geometry



$$\rho_0^2 = (1-\rho_0)^2 + \beta^2 - 2\beta(1-\rho_0)\cos\alpha$$

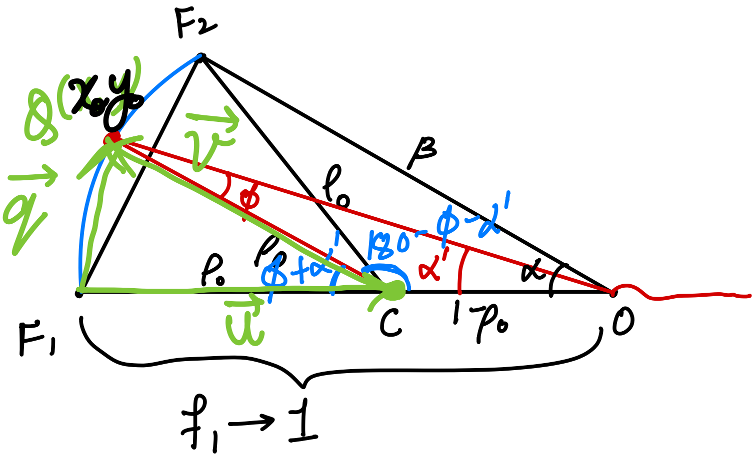
$$\rho_0 = \frac{1+\beta^2-2\beta\cos\alpha}{2(1-\beta\cos\alpha)} = \boxed{1 - \frac{1-\beta^2}{2(1-\beta\cos\alpha)}}$$

$$\gamma = \frac{\sin\psi}{\sin\alpha} = \frac{\sin\theta}{\sin\alpha'} \Rightarrow \boxed{\alpha' = \sin^{-1}\left(\frac{\sin\theta}{\gamma}\right)}$$

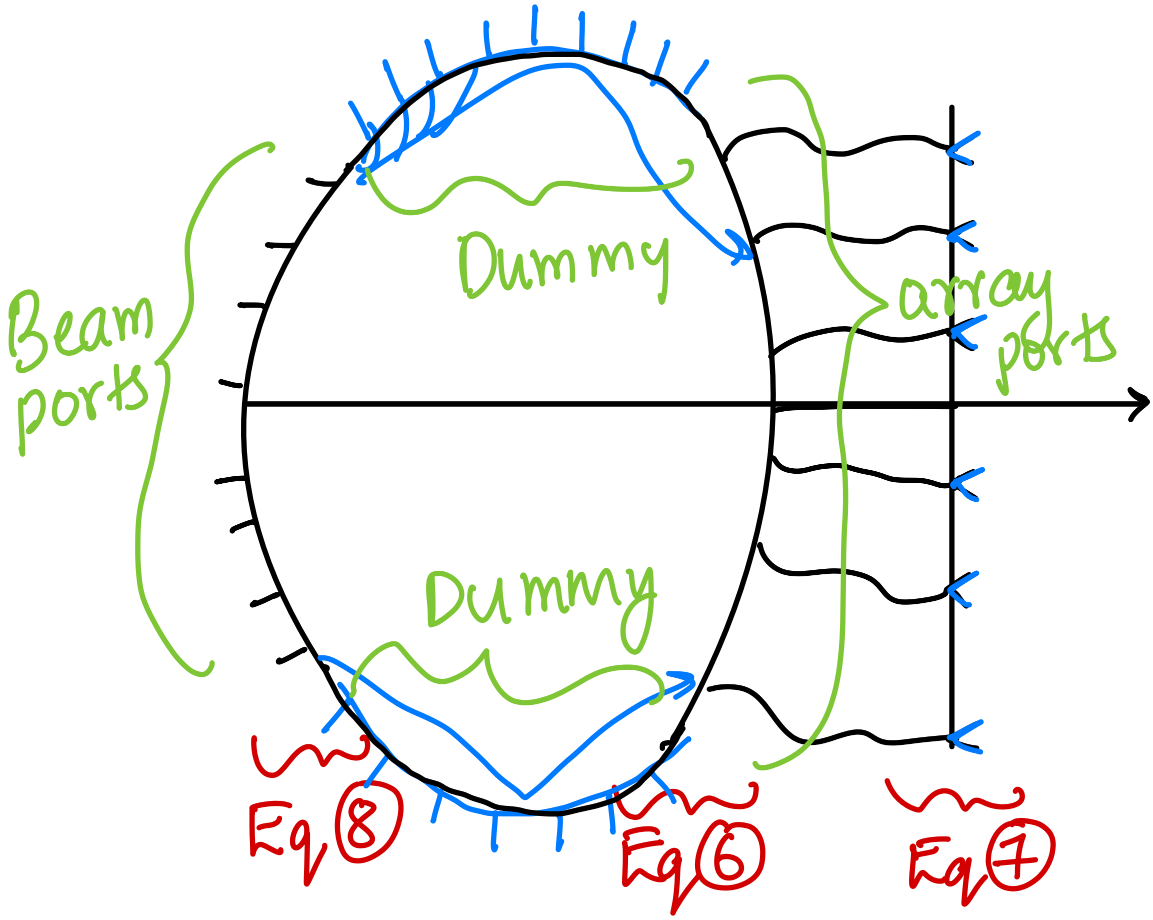
$$\frac{\sin\phi}{1-\rho_0} = \frac{\sin\alpha'}{\rho_0} \Rightarrow \boxed{\phi = \sin^{-1}\left(\frac{1-\rho_0}{\rho_0}\sin\alpha'\right)}$$

$\vec{q} = \vec{u} + \vec{v}$

$= (\rho_0, 0) + \left(\rho_0 \cos(180-\phi-\alpha'), \ \rho_0 \sin(180-\phi-\alpha')\right)$



$$\boxed{\begin{aligned} x_0 &= \rho_0\left[1-\cos(\alpha'+\phi)\right] \\ y_0 &= \rho_0 \sin(\alpha'+\phi) \end{aligned}} \quad (8)$$



Given,
$\alpha, \gamma, f_1,$
$\beta \equiv \frac{2}{2+\alpha^2}$