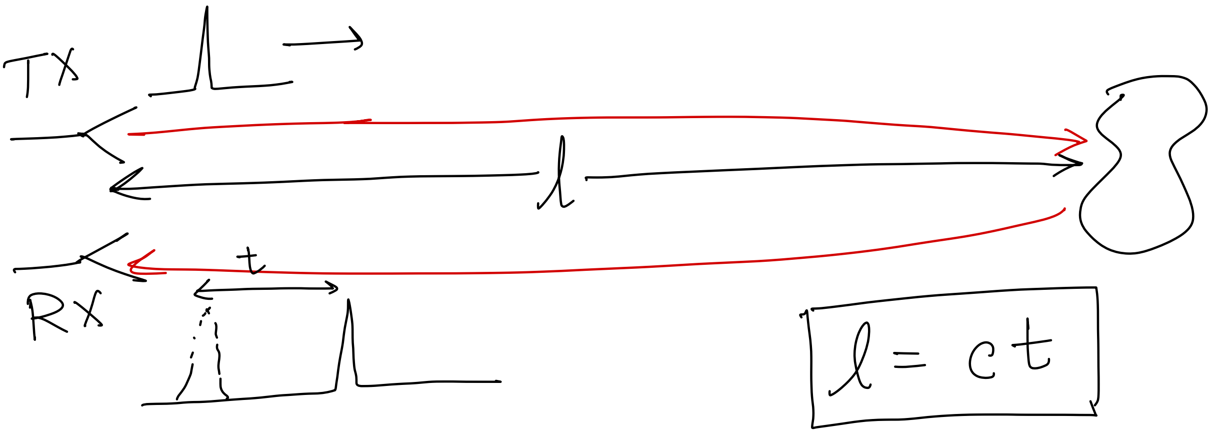
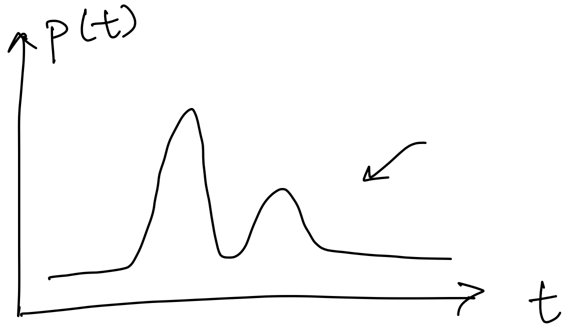
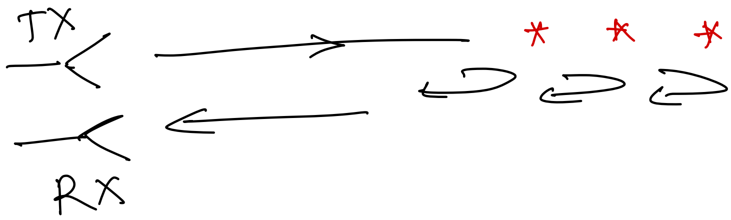
II Pulsed radars & Matched Filtering

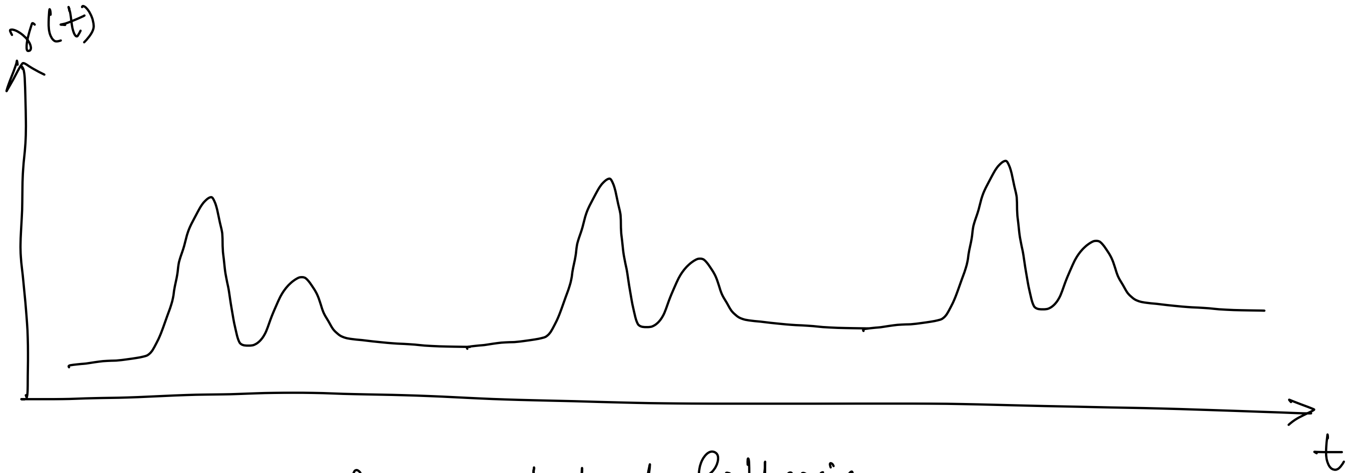
2.1 Intuition



$$\boxed{l = ct}$$







2.2 Signal model for matched filtering.

$$\int_{-\infty}^{\infty} r(t)\, p(t-\tau)\, dt = s(\tau)$$

Convolution

$$s(t) = \int_{-\infty}^{\infty} r(\tau)\, p(\tau - t)\, dt = \underbrace{r(t) * p(-t)}_{\text{Correlation.}}$$

$$s(t) = \mathcal{F}^{-1}\left[\mathcal{F}(r(t))\,\mathcal{F}(p(-t))\right]$$

matched filter.

$$\boxed{s(t) = \mathcal{F}^{-1}\left[R(\omega)\; P^{*}(\omega)\right]}$$

Matched Filtering

For a single target at 'l',

$$r(t) = p\left(t - \frac{2l}{c}\right)$$

Ignoring $\frac{1}{l^2}$ amplitude dependance.

$$s(t) = \mathcal{F}^{-1}\left[R(\omega)\,P^{*}(\omega)\right]$$

$$\mathcal{F}\left[r(t)\right] = \mathcal{F}\left[p\left(t - \frac{2l}{c}\right)\right]$$

$$= \mathcal{F}^{-1}\left[P(\omega)\,P^{*}(\omega)\,e^{-\frac{2jl\omega}{c}}\right]$$

$$k = \frac{\omega}{c}$$

$$= \mathcal{F}^{-1}\left[|P(\omega)|^{2} e^{-j2kl}\right]$$

$$\boxed{S(\omega) = |P(\omega)|^{2} e^{-j2kl}}$$

linear in l

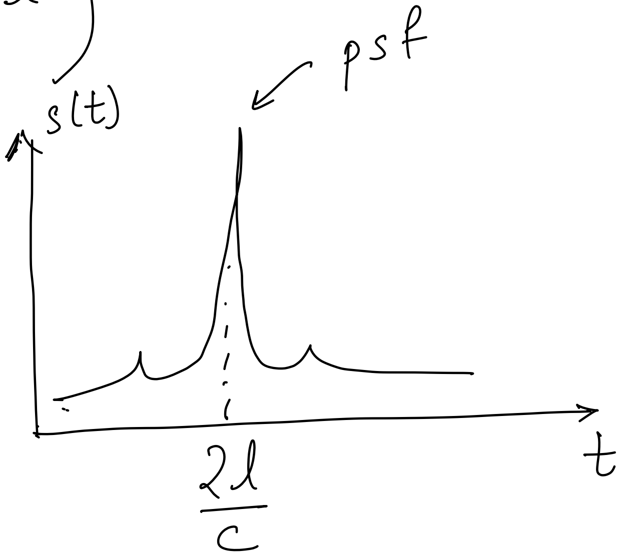
$2kl$ phase

Define $\mathcal{F}^{-1}\left[|P(\omega)|^{2}\right] = psf(t)$

Point Spread Function!

$$s(t) = \mathcal{F}^{-1}\left(|P(\omega)|^2 e^{-j2kl}\right)$$

$$\boxed{s(t) = psf\left(t - \frac{2l}{c}\right)}$$



PSF $\Longleftrightarrow$ Green's function

$\Longleftrightarrow$ Impulse response

For a distributed target, assume it is a collection of point targets. (assume N such point targets).

$$r(t) = \sum_{n=1}^{N} \sigma_n \, p\left(t - \frac{2l_n}{c}\right)$$

$l_n \rightarrow$ dist to n^{th} target

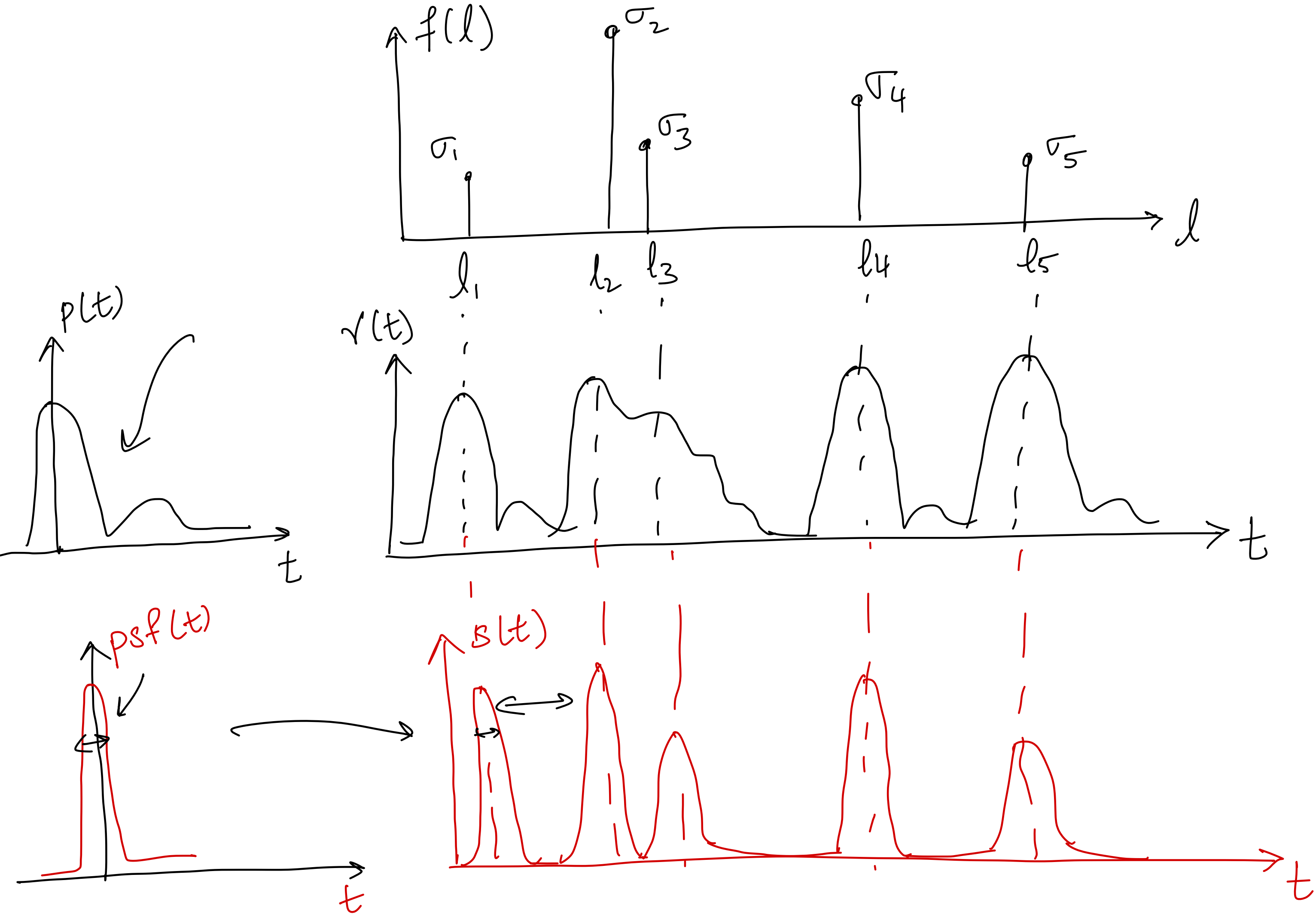
$\sigma_n \rightarrow$ response to n^{th} target.

Assume σ_n is independant of frequency.

$$R(\omega) = \sum_{n=1}^{N} \sigma_n \, P(\omega) \, e^{-2j\frac{\omega l_n}{c}}$$

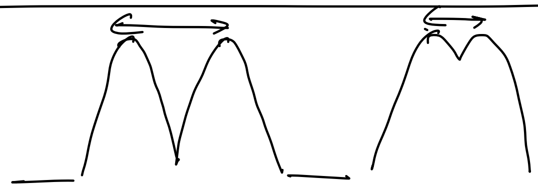
$$S(\omega) = \sum_{n=1}^{N} \sigma_n \, |P(\omega)|^2 e^{-2j\frac{\omega l_n}{c}}$$

$$\boxed{s(t) = \sum_{n=1}^{N} \sigma_n \, psf\left(t - \frac{2l_n}{c}\right)}$$

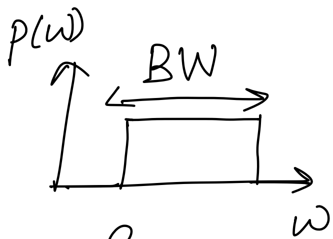


"PULSE COMPRESSION"

2.3 Range resolution

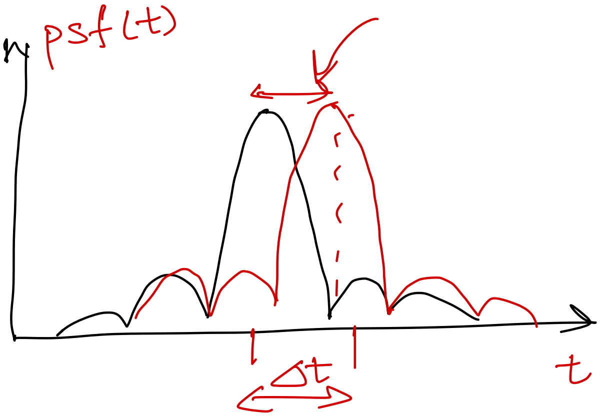


$$psf = \mathcal{F}^{-1}\left(|P(\omega)|^2 \right)$$



Assume $|P(\omega)|^2$ is "rectangular" with a bandwidth of $(\Delta\omega)$. Then the psf is a sinc $\rightarrow$ $\text{sinc}\left(\frac{\Delta\omega t}{2}\right)$.

$$\Rightarrow \frac{\Delta\omega(\Delta t)}{2} = \pi$$



$$t = \frac{2l}{c} \Rightarrow \Delta t = \frac{2\Delta l}{c}$$

$$\Rightarrow \frac{\Delta\omega(\Delta t)}{2} = \pi \Rightarrow \frac{\cancel{\Delta\omega}^{\,2\Delta f}}{\cancel{2}}\left(\frac{2\Delta l}{c}\right) = \cancel{\pi}$$

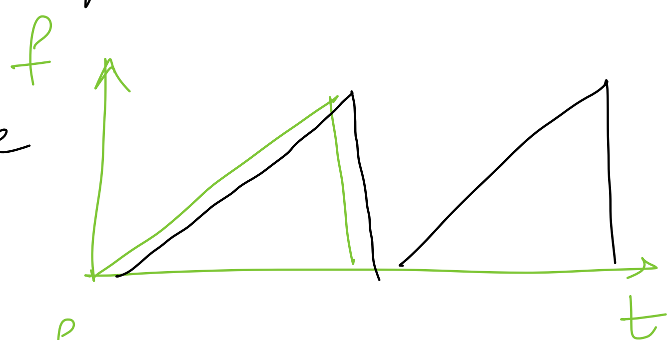
$$\Rightarrow \boxed{\Delta l = \frac{c}{2\Delta f} = \frac{c}{2BW}} \rightarrow \text{Same as FMCW!}$$

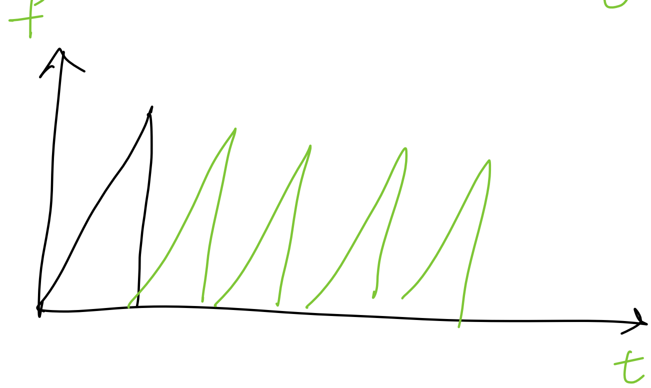
2.4 Advantages & Limitations of Pulsed radars (with matched filtering)

* ① Max SNR in the presence of additive white noise.

② Short to long range operation.

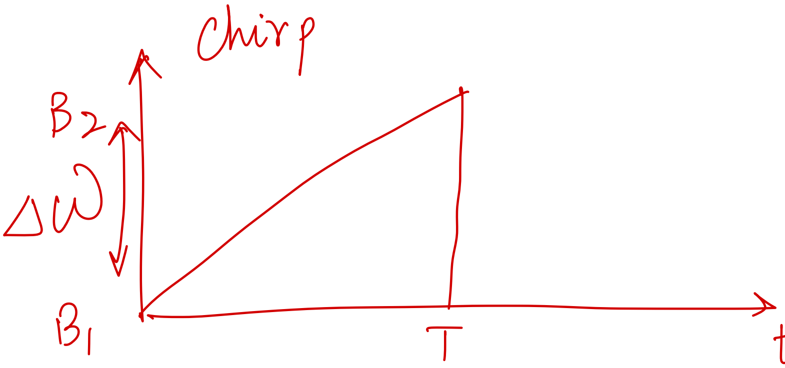
③ TX-RX leakage can be basically 0.

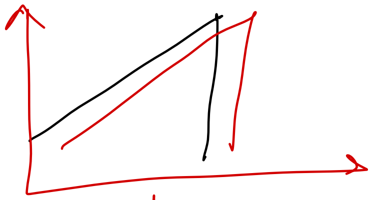




4) <u>Versatile signals</u> → complex coding but requires accurate knowledge of TX signal.

5) Bandwidth (range resolution) is limited.





↳ IF → upto few MHz.

$\boxed{2(\Delta\omega)}$

15 GHz → $\dfrac{3\times10^8}{30\times10^9} = \underline{1cm}$

30 GHz → 60 GHz

5) Doppler processing is complicated.

Ambiguity function.

> <u>Future videos</u>

i) Stepped frequency radars.

ii) Imaging algorithms for FMCW × Pulsed/stepped

iii) Back-projection, TDC, ω-k, RMA, P-D, ...