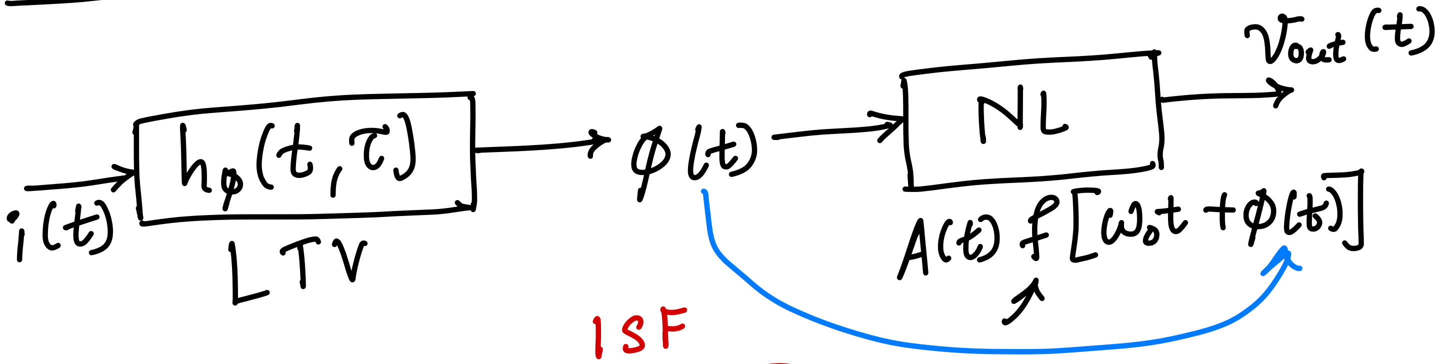
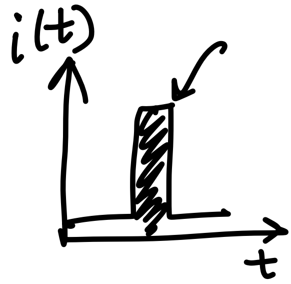
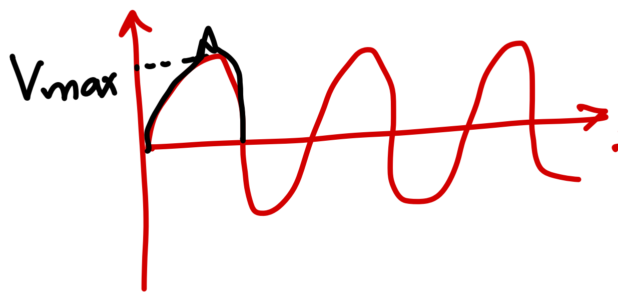
Oscillator Phase Noise - ISF model



$$h_\phi(t,\tau) = \frac{\overbrace{\Gamma(\omega_0\tau)}^{\text{ISF}}}{q_{max}}\, u(t-\tau)$$

$q_{max} \rightarrow C_{eq} V_{max}$





Assumptions

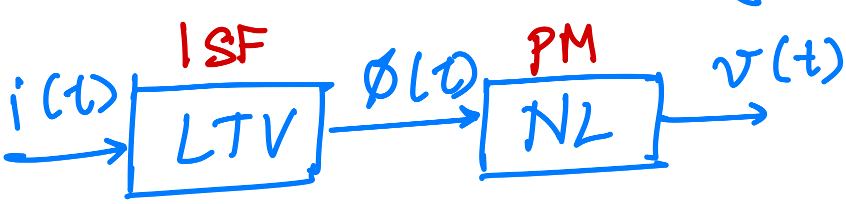
1) $h_\phi(t,\tau)$ is linear.

   $i(t) \longrightarrow \phi(t)$.

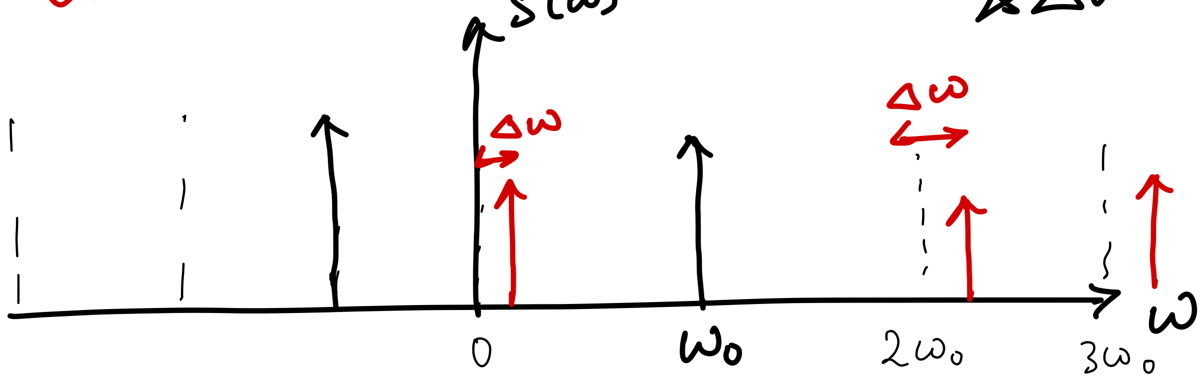
2) $\phi(t)$ is independant of $A(t)$. $\phi(t)$ takes the form of a step function.

$$\Gamma(\omega_0\tau) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n \cos(n\omega_0\tau + \underline{\theta_n})$$

$$\phi(t) = \frac{1}{q_{max}}\left[\frac{C_0}{2}\int_{-\infty}^{t} i(\tau)\,d\tau + \sum_{n=1}^{\infty} C_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau)\,d\tau\right]$$



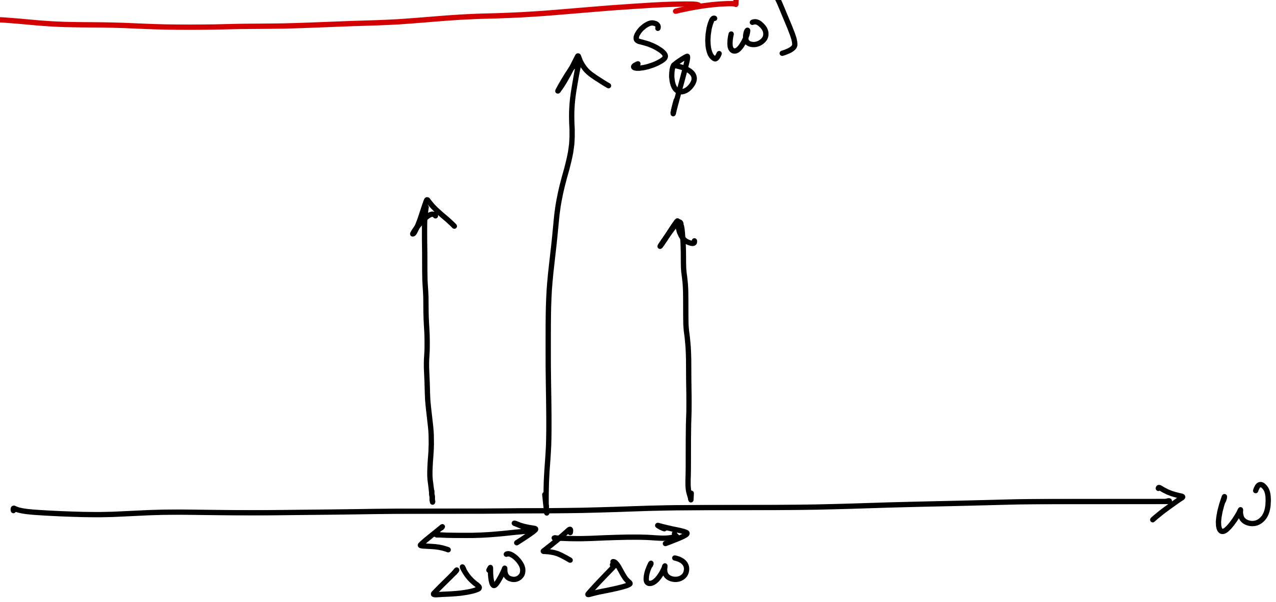
$$i(t) = \underbrace{I_n}\cos\left(\left(\underbrace{m}\omega_0 + \underbrace{\Delta\omega}\right)t\right) \quad \text{for } m \in \mathbb{Z} \;\&\; \Delta\omega \ll \omega_0$$



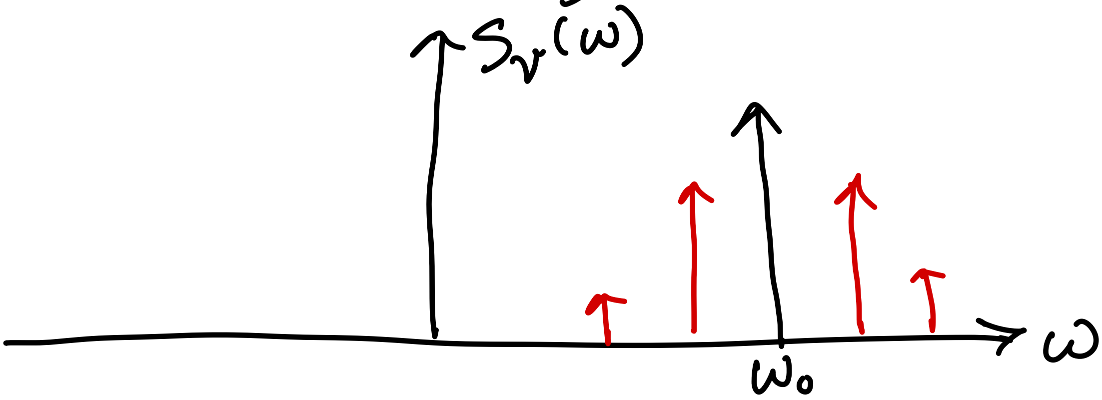
$$\phi(t) = \frac{1}{q_{max}}\left[\frac{C_0}{2}\int_{-\infty}^{t} I_n \cos\left(\underbrace{(m\omega_0+\Delta\omega)}_{\substack{\text{fast when } m\neq 0\\ \text{slow when } m=0}}\tau\right)d\tau + \sum_{n=1}^{\infty} C_n \int_{-\infty}^{t} I_n \underbrace{\cos((m\omega_0+\Delta\omega)\tau)\cdot\cos(n\omega_0\tau)}_{\substack{m\neq n \text{ very fast}\\ m=n \text{ very slow}}}\,d\tau\right]$$

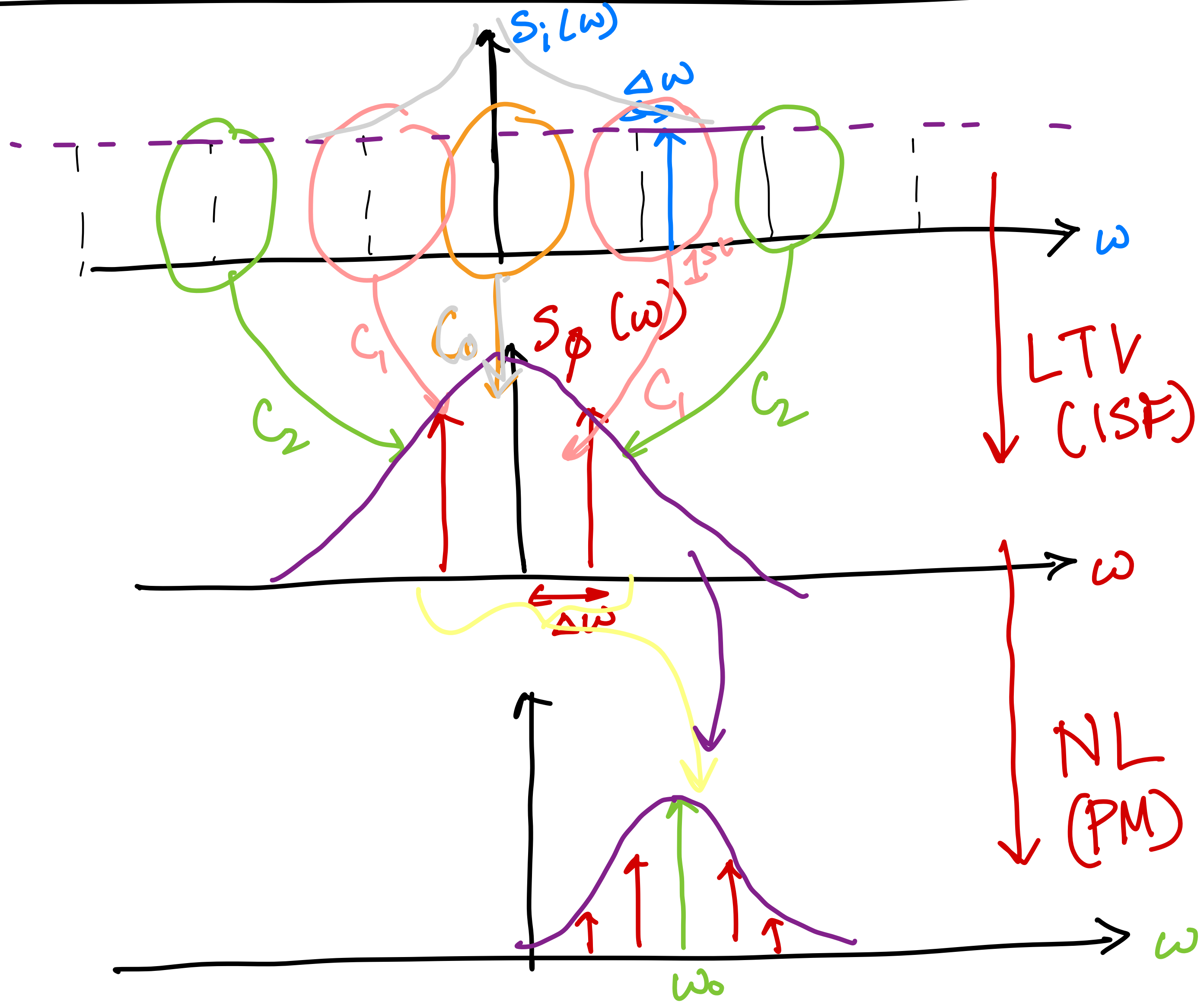
$$\int \cos(\Delta\omega\tau) \to \frac{\sin(\Delta\omega\tau)}{\Delta\omega}$$

$$\boxed{\phi(t) = \frac{I_n C_n}{2 q_{max}\Delta\omega}\sin(\Delta\omega t)}$$

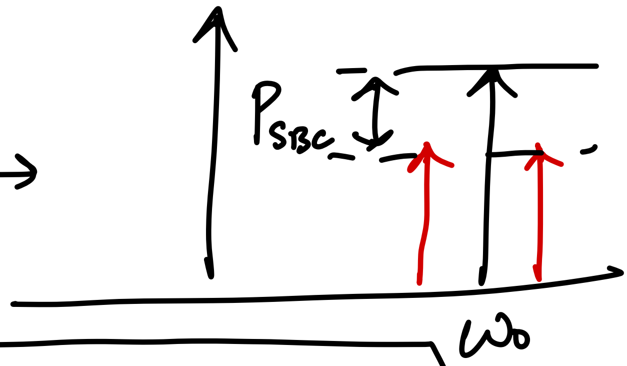


$$\phi(t) \longrightarrow A_0 \cos(\omega_0 t + \underbrace{\phi(t)}_{\sin(\Delta\omega t)}) \longrightarrow v(t)$$





$$\phi(t) \cong \frac{I_n c_n \sin(\Delta\omega t)}{2 q_{max} \Delta\omega} \longrightarrow \text{PM}$$



$$\boxed{P_{SBC}(\Delta\omega) \simeq 10\log\left(\frac{I_n C_n}{4 q_{max} \Delta\omega}\right)^2}$$

$1/f^2$ term comes from Thermal noise

Phase Noise Power

Given a noise source (current) i_n, the PSD is $\frac{\overline{i_n^2}}{\Delta f}$. For $\Delta f = 1Hz$,

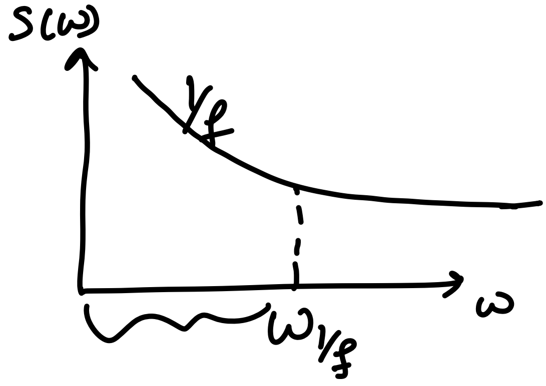
$$\frac{I_n^2}{2} = \overline{i_n^2}.$$

$$\mathcal{L}\{\Delta\omega\} = 10\log\left(\frac{\frac{\overline{i_n^2}}{\Delta f}\sum_{n=0}^{\infty} C_n^2}{8 q_{max}^2 \Delta\omega^2}\right)$$

Parseval's Thm:

$$\sum_{n=0}^{\infty} c_n^2 = \frac{1}{\pi}\int_0^{2\pi} |\Gamma(x)|^2 dx = 2\Gamma_{rms}^2$$

$$\underbrace{\mathcal{L}\{\Delta\omega\}}_{\left(\frac{1}{f^2}\text{ part}\right)} = 10\log\left(\frac{\Gamma_{rms}^2}{q_{max}^2}\;\frac{\overline{i_n^2}/\Delta f}{4\Delta\omega^2}\right)$$



Flicker Noise

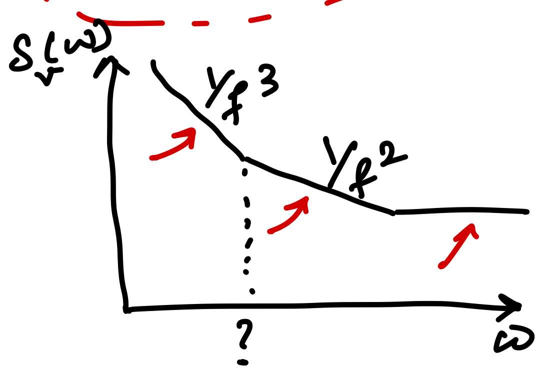
$$\overline{i_{n,1/f}^2} = \overline{i_n^2}\,\frac{\omega_{1/f}}{\Delta\omega} \quad \text{when } \Delta\omega < \omega_{1/f}$$

↳ 1/f corner freq.

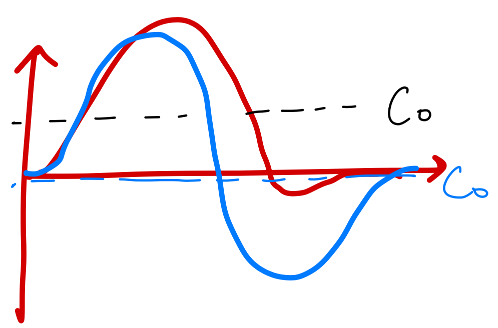
$$\mathcal{L}\{\Delta\omega\} = 10\log\left(\frac{c_0^2}{q_{max}^2}\cdot\frac{\overline{i_n^2}/\Delta f}{8\Delta\omega^2}\cdot\frac{\omega_{1/f}}{\Delta\omega}\right)$$

$$\underbrace{8\Delta\omega^2 \cdot \Delta\omega}_{\Delta\omega^3}$$

$$\boxed{\Delta\omega = \omega_{1/f^3} = \omega_{1/f}\cdot\frac{c_0^2}{2\Gamma_{rms}^2}}$$



If $\underbrace{c_0}_{\text{DC component of ISF}} \to 0 \Rightarrow \omega_{1/f^3} \to 0$



Cyclostationary Noise

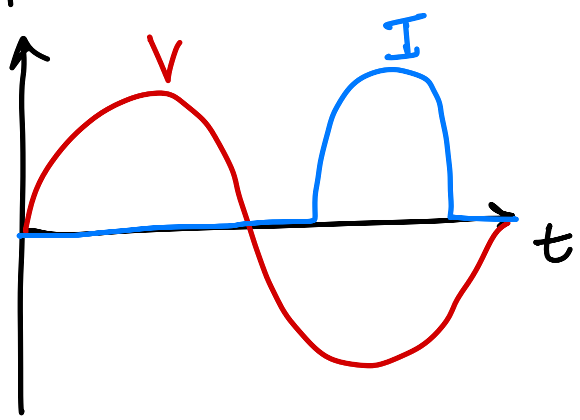
Noise properties (statistics, mean & var) are periodic. Eg: Shot noise

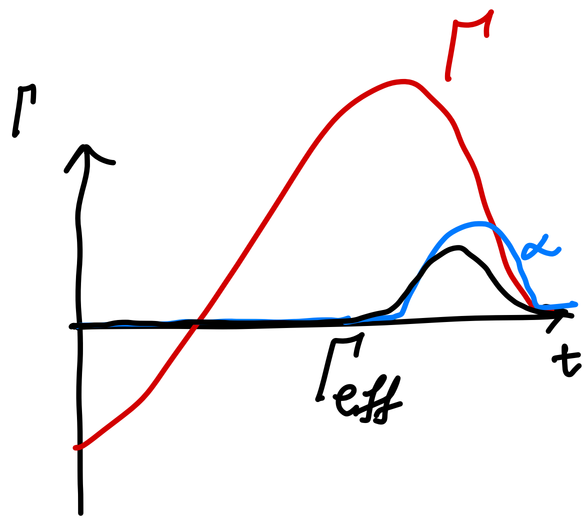
$$\underbrace{i_n(t)}_{C\text{-}S} = \underbrace{i_{n_0}(t)}_{S}\;\underbrace{\alpha(\omega_0 t)}_{\text{periodic}} \rightarrow \max(\alpha) = 1$$

$$\phi(t) = \int_{-\infty}^{t} \frac{\Gamma(\omega_0\tau)}{q_{max}}\, i_n(\tau)\, d\tau$$

$$= \int_{-\infty}^{t} \frac{\overbrace{\alpha(\omega_0\tau)\Gamma(\omega_0\tau)}^{\Gamma_{eff}(\omega_0\tau)}}{q_{max}}\, i_{n_0}(\tau)\, d\tau$$

Colpitts





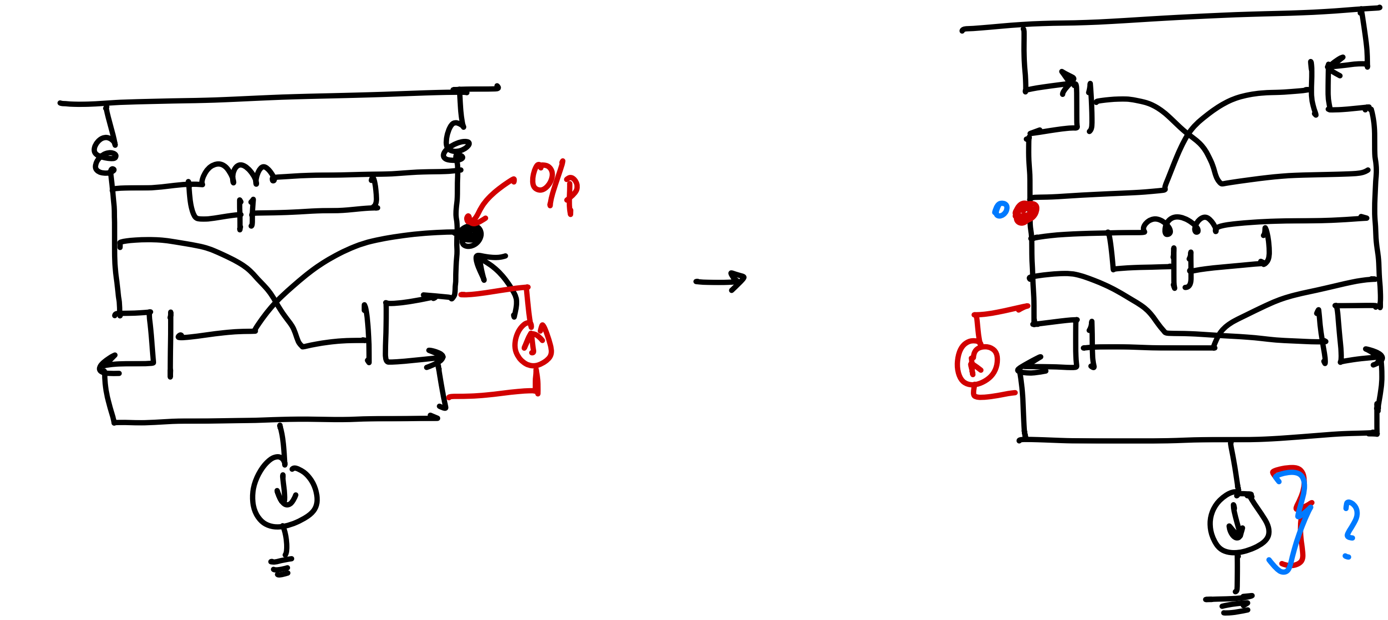
Design implications

> Larger q_{max} ⇒ smaller PN

> Reduce interference around $n\omega_0$.

> Reduce C_0 term to reduce $1/f^3$ part of PN.

$\Rightarrow$ Make ISF symmetric.



Design Procedure

1) Identify noise sources (cyclostationary, correlated).
2) Simulate Γ at o/p w.r.t all the noise source.
3) Find $\mathcal{L}\{\Delta\omega\}$ for each source & identify the "bad" sources.
4) Modify the circuit topology to improve the PN.