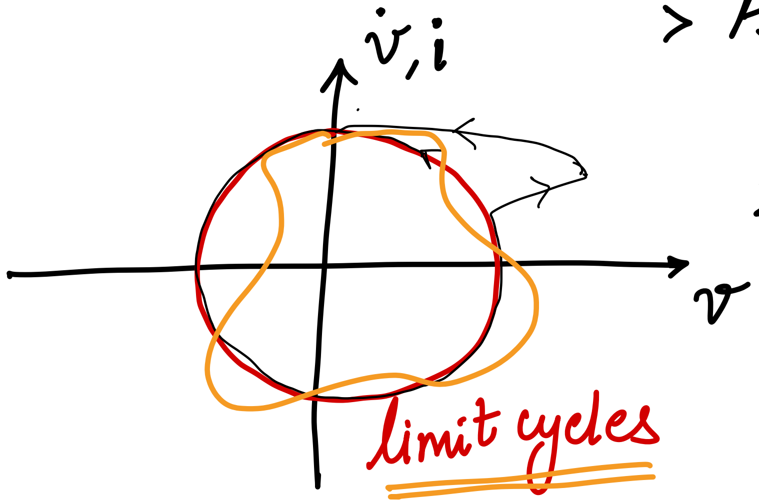
Oscillator Theory - Impulse Sensitivity Function

(ISF model)



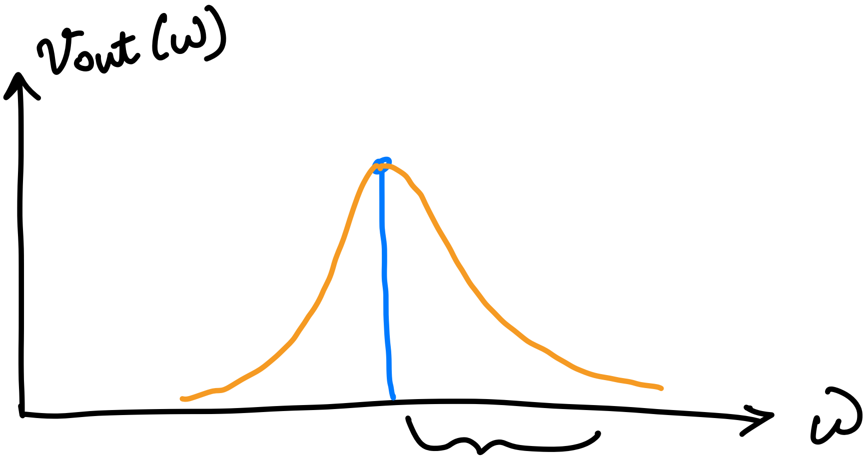
limit cycles

> Amplitude fluctuations are corrected
> Phase fluctuations persist.

$$V_{out}(t) = A\cos(\omega_0 t + \phi) \rightarrow \text{ideal}$$

$$✓\ V_{out}(t) = A(t)\, f[\omega_0 t + \phi(t)] \rightarrow \text{real.}$$

↳ periodic with period 2π.

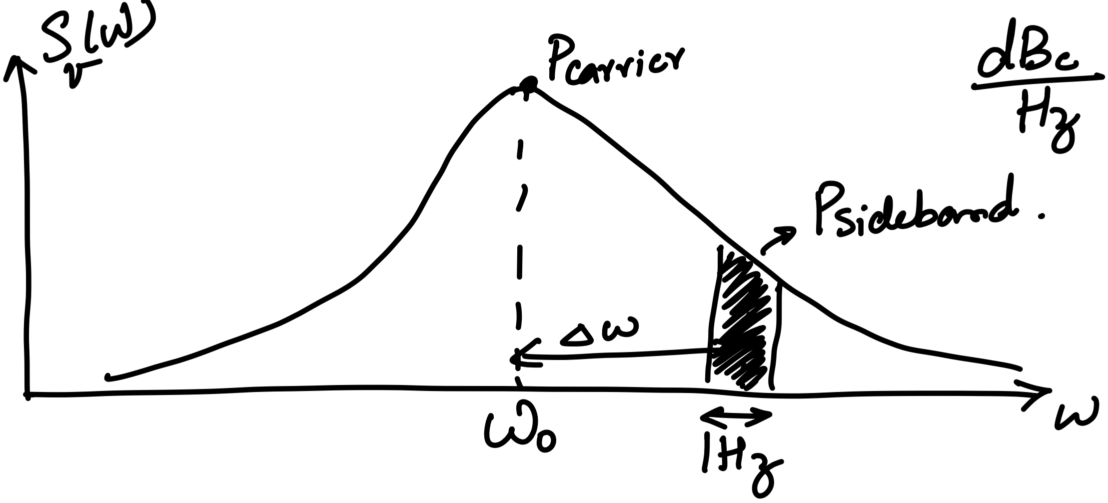


Characterized by single sideband noise spectral density. { Easy to measure }

$$\mathcal{L}_{total}\{\Delta\omega\} = 10\log\left[\frac{P_{sideband}(\omega_0 + \Delta\omega, 1\,Hz)}{P_{carrier}}\right]$$

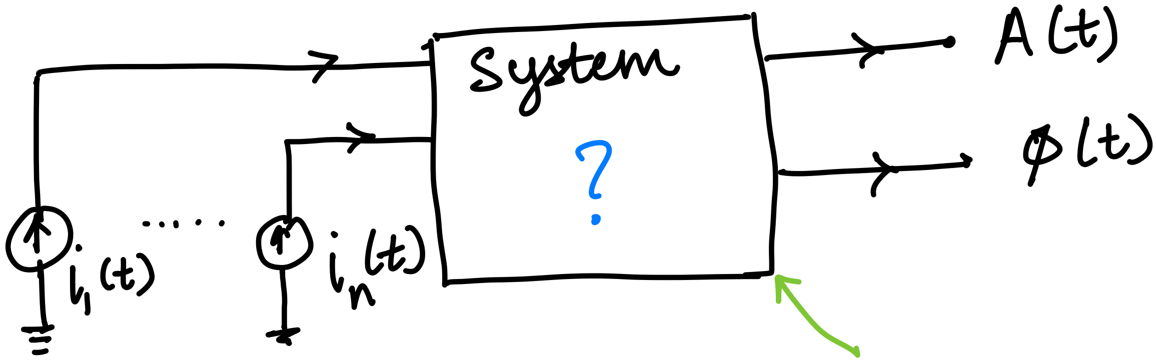
↳ Amp term

↳ phase term. $\rightarrow \mathcal{L}_{phase}\{\Delta\omega\} \rightarrow \mathcal{L}\{\Delta\omega\} \rightarrow$ Phase Noise.



Phase Noise Model (ISF)

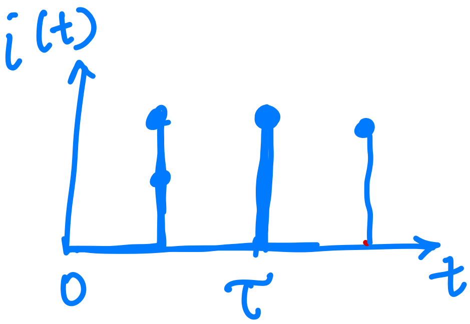
At a given node

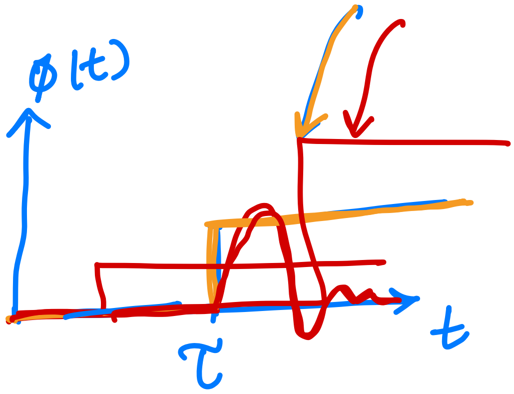


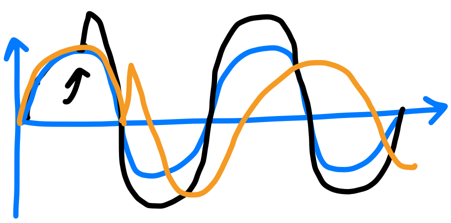
All over the system

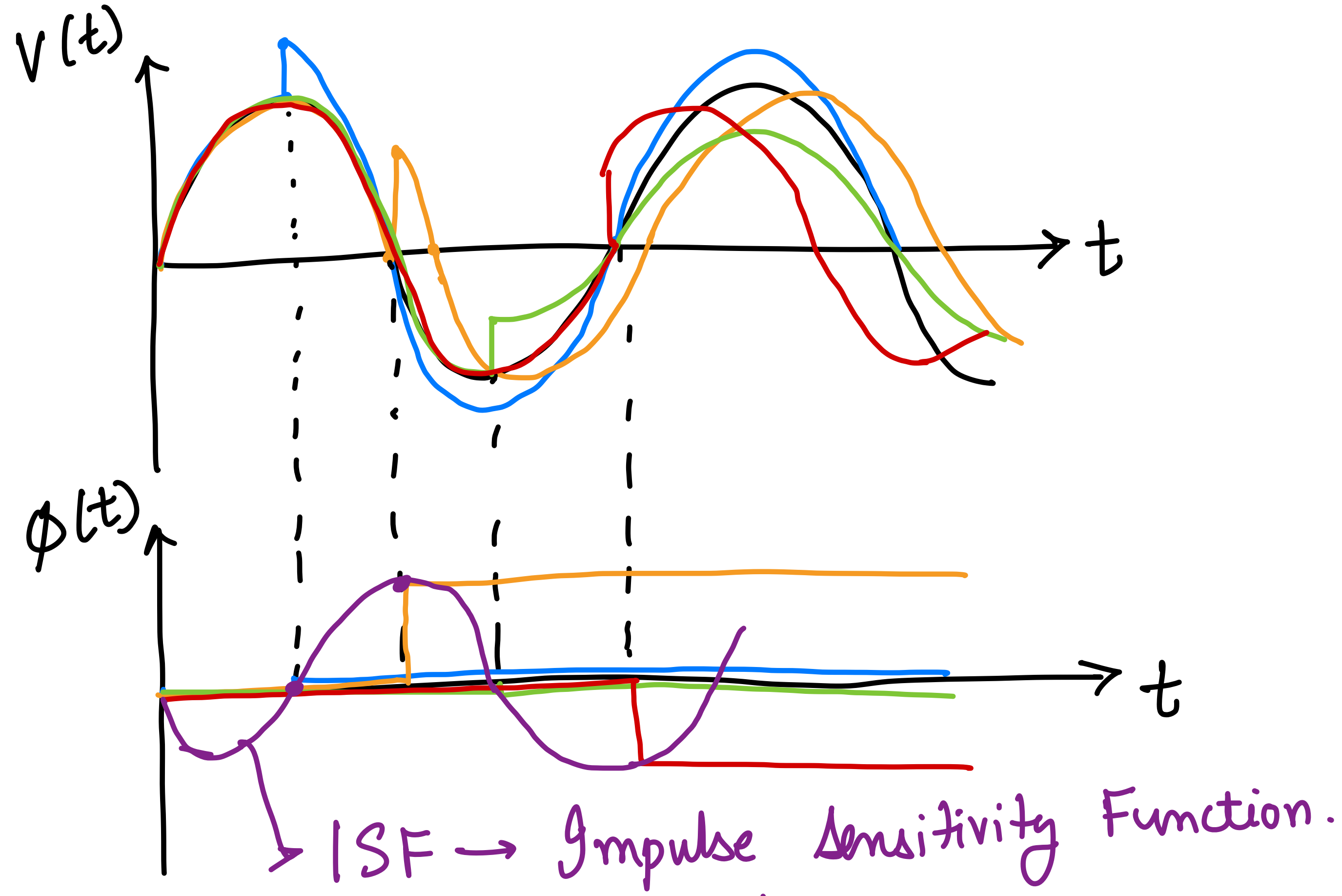
$$i(t) \longrightarrow \boxed{h_\phi(t,\tau)} \longrightarrow \phi(t)$$

$$i(t) \longrightarrow \boxed{h_A(t,\tau)} \longrightarrow A(t)$$









$$h_\phi(t,\tau) = \frac{\overbrace{\Gamma(\omega_0\tau)}^{ISF}}{q_{max}}\, u(t-\tau) \leftarrow$$

$q_{max} \rightarrow$ max charge displaced on the node of interest.

Assumptions

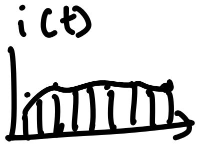
1) $i(t) \rightarrow \boxed{} \rightarrow \phi(t)$ $\quad h_\phi(t,\tau)$ is <u>linear</u>.

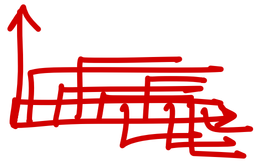
True in practice, especially for phase noise analysis because $i(t)$ is small.

2) $\phi(t)$ is independant of $A(t)$ & takes the form of a step function $u(t-\tau)$.

Excess phase

$$i(t) \longrightarrow \boxed{h_\phi(t,\tau)} \longrightarrow \phi(t)$$





$$\phi(t) = \int_{-\infty}^{\infty} h_\phi(t,\tau)\, i(\tau)\, d\tau$$

$$= \frac{1}{q_{max}} \int_{-\infty}^{t} \underbrace{\Gamma(\omega_0 \tau)}_{\hookrightarrow \text{Periodic}}\, i(\tau)\, d\tau$$

Substitute.

Fourier series expansion of Γ

Limit cycle freq.

$$\Gamma(\omega_0 \tau) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 \tau + \theta_n)$$

$$\Rightarrow \phi(t) = \frac{1}{q_{max}} \left[\frac{C_0}{2} \int_{-\infty}^{t} i(\tau)\, d\tau + \sum_{n=1}^{\infty} C_n \int_{-\infty}^{t} i(\tau) \cos(n\omega_0 \tau)\, d\tau \right]$$