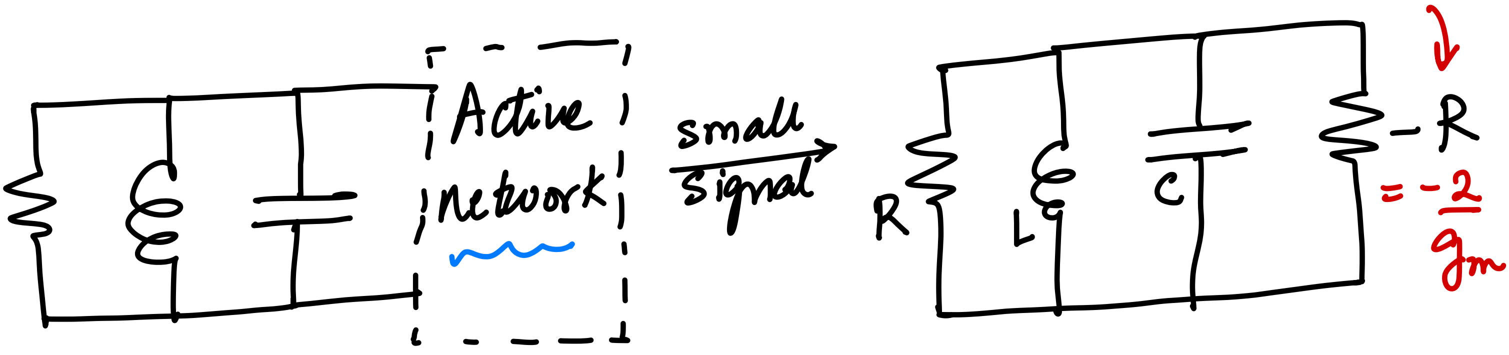
LC oscillators - Nonlinear model

Pre-req: MMIC 04, 09

I) Linear Negative Resistance Model



Problems: Linear model ⇒ Cannot generate a "limit cycle!"

Limit cycles require 2^{nd} order nonlinear system.

But it is OK to study the startup condition.

II) Nonlinear "Negative Resistance" Model

$$V = f_1(I) = IR$$

$$I = f_2(V) = -\frac{V}{R} \Rightarrow f_2(x) = -\frac{x}{R}$$

VCCS

$I = f_3(V)$ where $f_3(x)$ is nonlinear.

$I = V + 3$

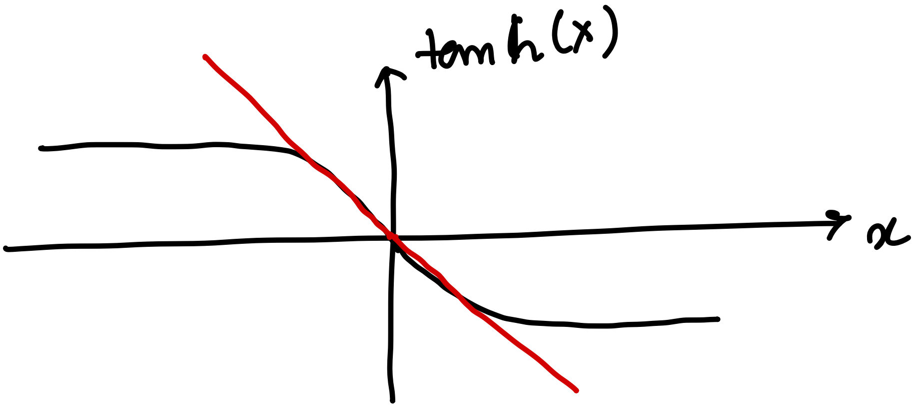
- $f_3 \rightarrow$ odd function $f_3(-V) = -f_3(V) = -I$
- $f_3 \rightarrow$ limiting behavior.
- $f_3 \rightarrow$ approximately linear for small V.

$$f_3(x) \longrightarrow -\frac{x}{R} \quad \text{when } x \rightarrow 0.$$

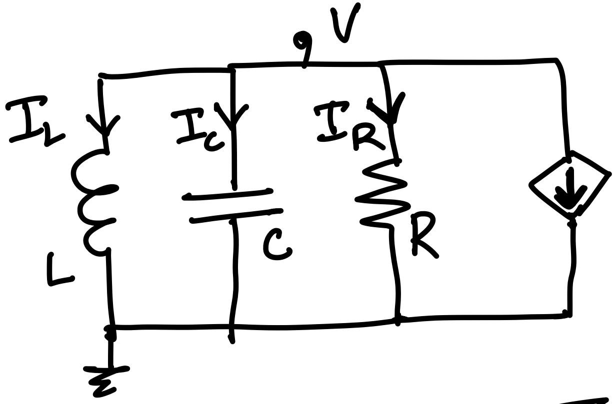
$$f_3(x) = \tanh\left(-\frac{x}{R}\right)$$

$$\tanh(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} + O(7)$$

$\rightarrow$ odd fn.
$\rightarrow$ limiting
$\rightarrow \approx x$ for $x \rightarrow 0$



<u>Note</u>: In reality f_3 must be derived from the large signal I-V characteristics of the active network.



$I_s = \tanh(-V)$

$V = I_R R$

$$I_c = C\frac{dV}{dt}$$

$$V = L\frac{dI_L}{dt}$$

$I_s = \tanh(-V)$

$$V = -L\frac{d}{dt}\left[I_c + I_s + I_R\right]$$

$$= -L\frac{d}{dt}\left[\tanh(-V) + \frac{V}{R} + \frac{C\,dV}{dt}\right]$$

$$= L\left[\left(1 - \tanh^2(V)\right)\frac{dV}{dt} - \frac{1}{R}\frac{dV}{dt} - \frac{C\,d^2V}{dt^2}\right]$$

$$\Rightarrow \boxed{\ddot{V}(LC) + \dot{V}\left[\tanh^2(V) + \frac{1}{R} - 1\right]L + V = 0}$$

2^{nd} order nonlinear diff. eqn. $\Rightarrow$ limit cycle can exist.

Let $x = V$ & $\dot{x} = \dot{V} = y$

$$\boxed{\begin{aligned} \dot{x} &= y \\ \dot{y} &= -\frac{\dot{x}\left[\tanh^2(x) + \frac{1}{R} - 1\right]}{C} - \frac{x}{LC} \end{aligned}}$$