Oscillator Theory - Limit Cycles

> "Nonlinear Dynamics"

> A qualitative description of limit cycles.

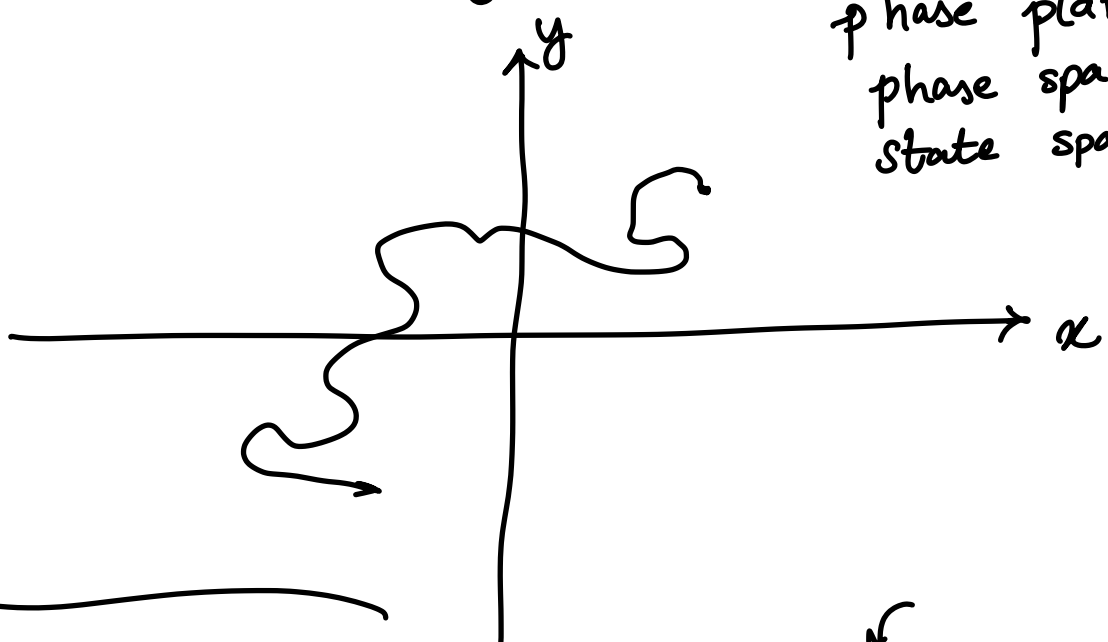
1-D

$$\dot{x}(t) = \frac{dx(t)}{dt} = f(x) = ax + b. \checkmark$$

$$f(ax + by) = af(x) + bf(y) \Rightarrow \text{linear.}$$

2-D

$x(t), y(t)$; $x \in \mathbb{R}$; $y \in \mathbb{R}$.



phase plane
phase space
state space.

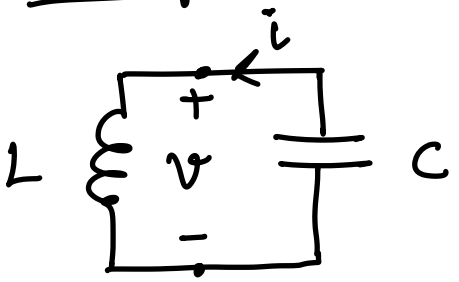
$$\frac{dx}{dt} = \dot{x}(t) = ax + by$$

$$\frac{dy}{dt} = \dot{y}(t) = cx + dy$$

$$\dot{\vec{x}}(t) = \underline{\underline{A}}\,\vec{x}$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

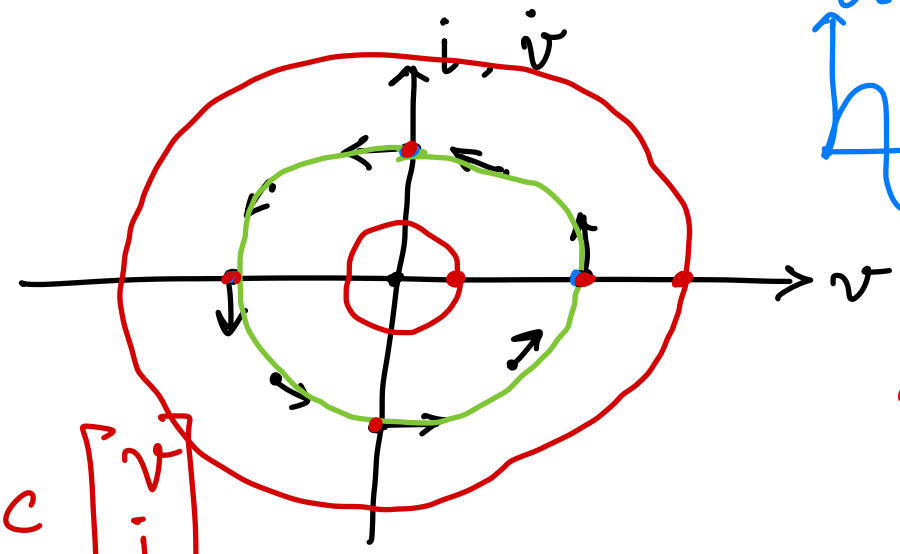
Examples

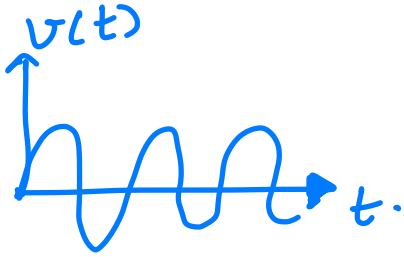


$$i = -C\frac{dv}{dt}$$

$$v = L\frac{di}{dt}$$

$$\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix}$$

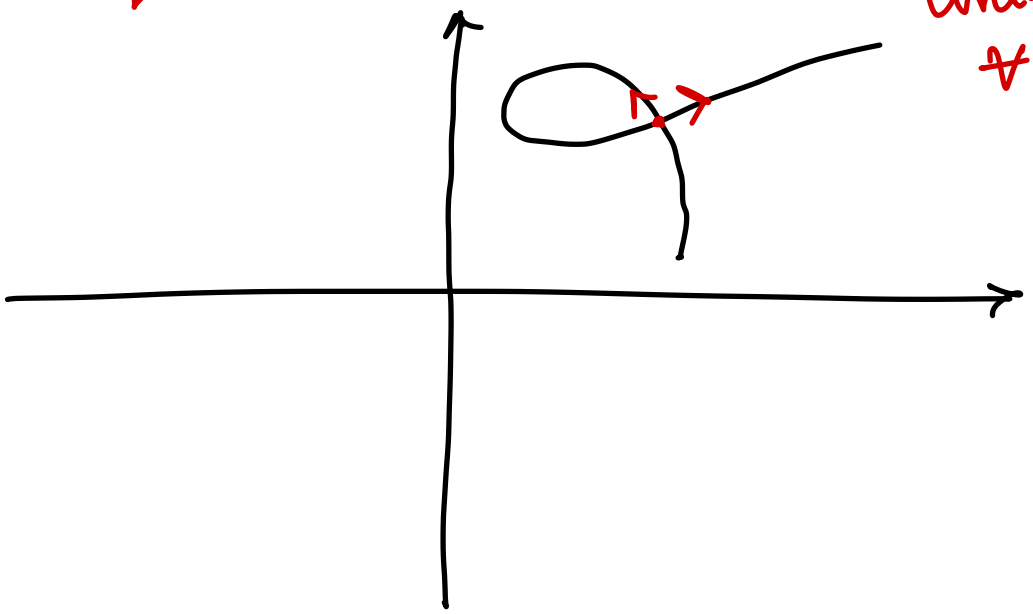




$q = cv$

$$\begin{bmatrix} v \\ i \end{bmatrix} \rightarrow c\begin{bmatrix} v \\ i \end{bmatrix}$$

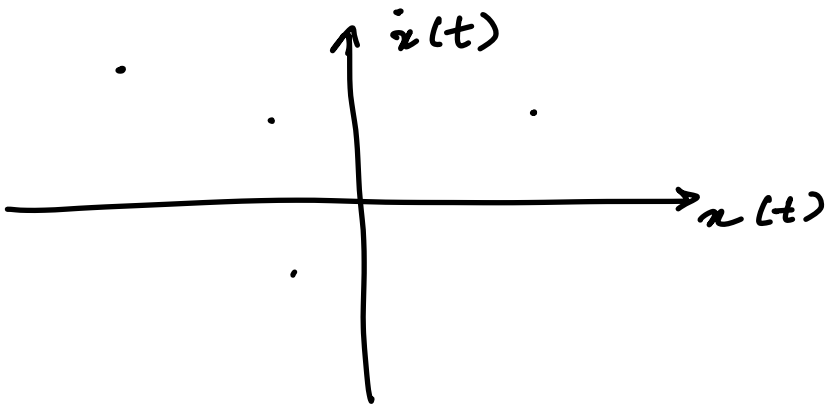
v v

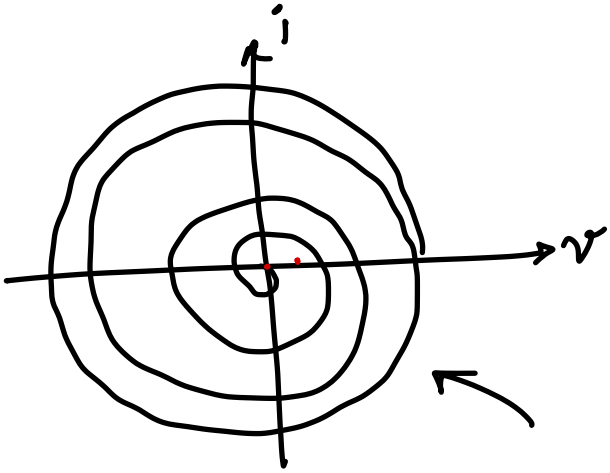


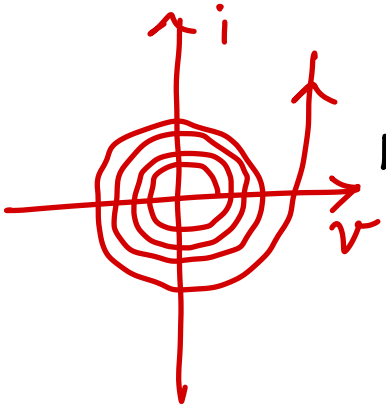
"unique" $\forall t$

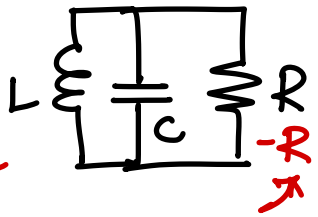
Dynamics of Phase space.

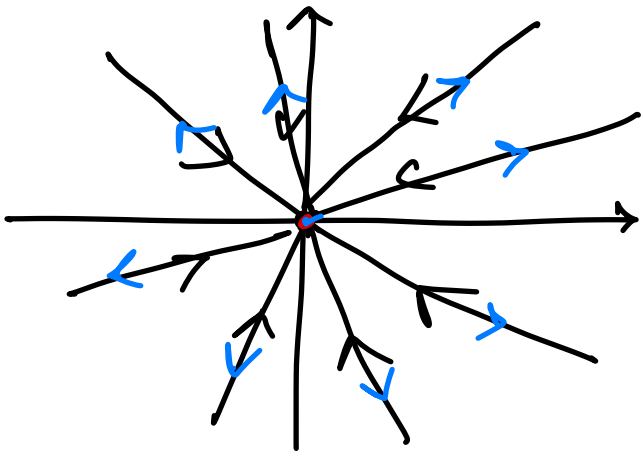
$\dot{\vec{x}} = 0 \quad \forall t,$





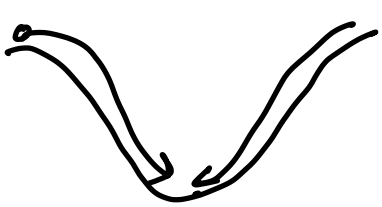


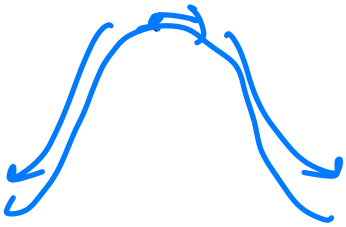


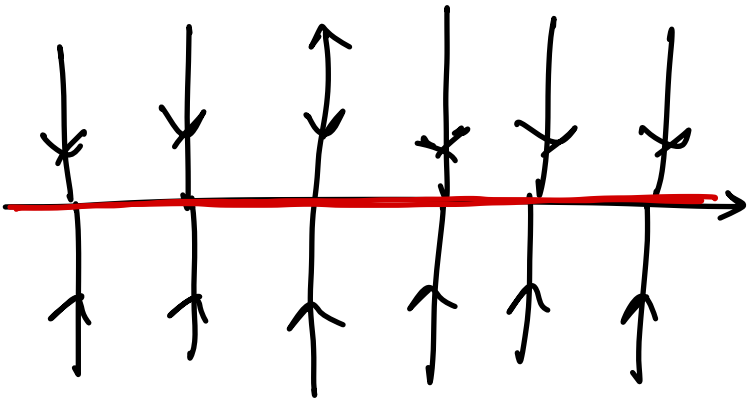


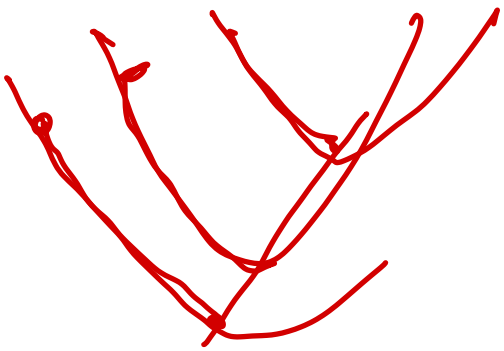
Stationary pt.
fixed pt.

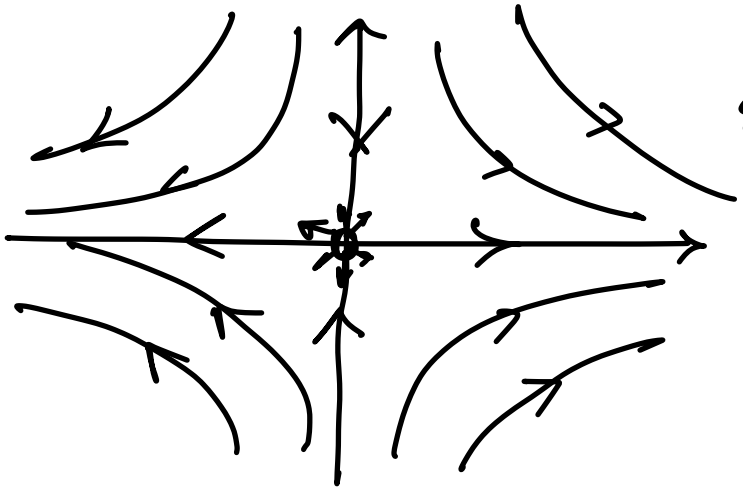
$\dot{\vec{x}}(0) = 0$



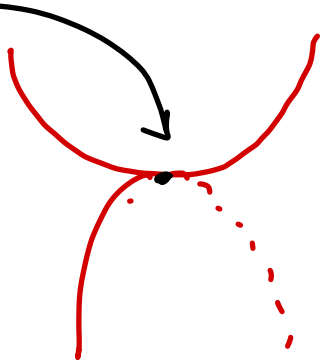




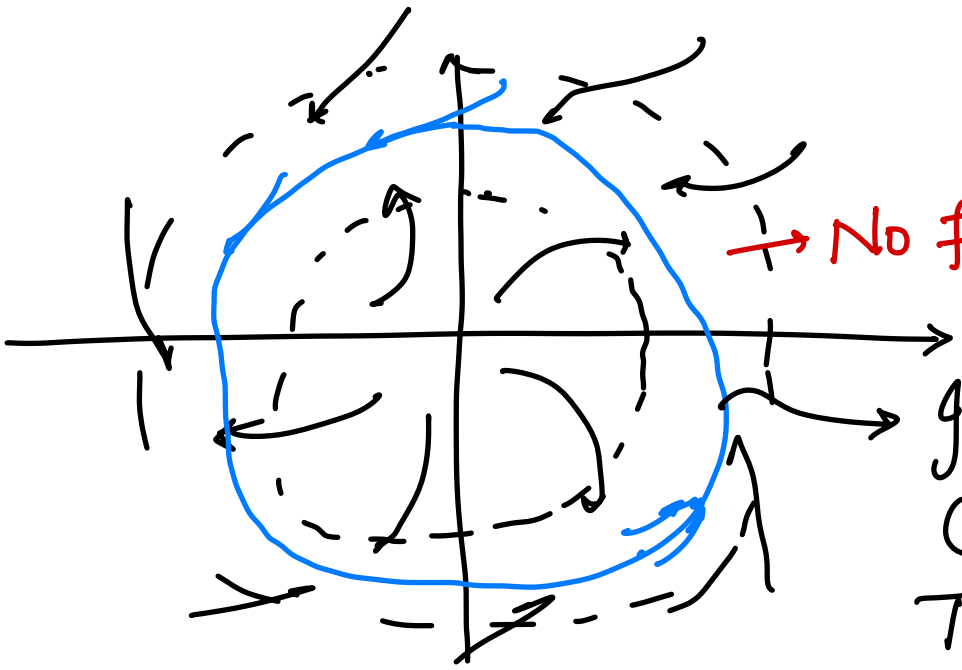




Saddle pt.

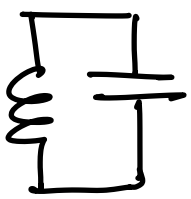


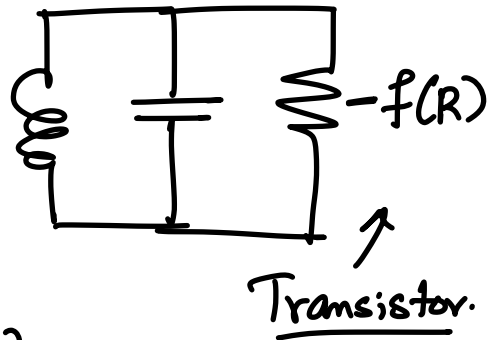
(Nonlinear) phase space.



Limit cycles: "Nonlinear" & "2-D or more"

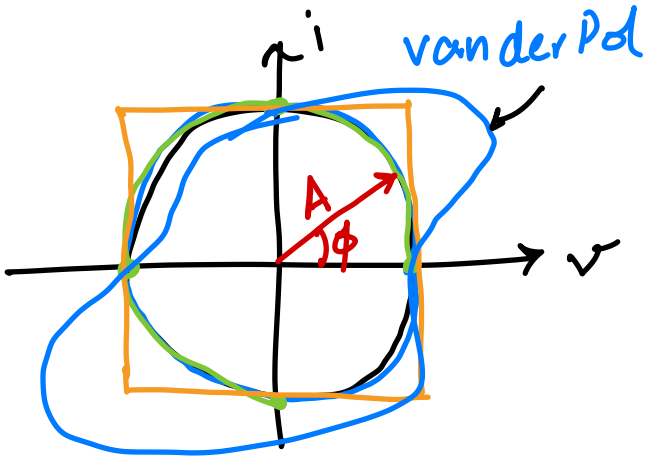
(Necessary)

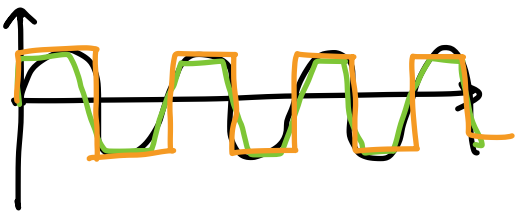
>  Resonator

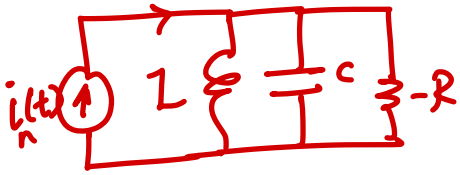


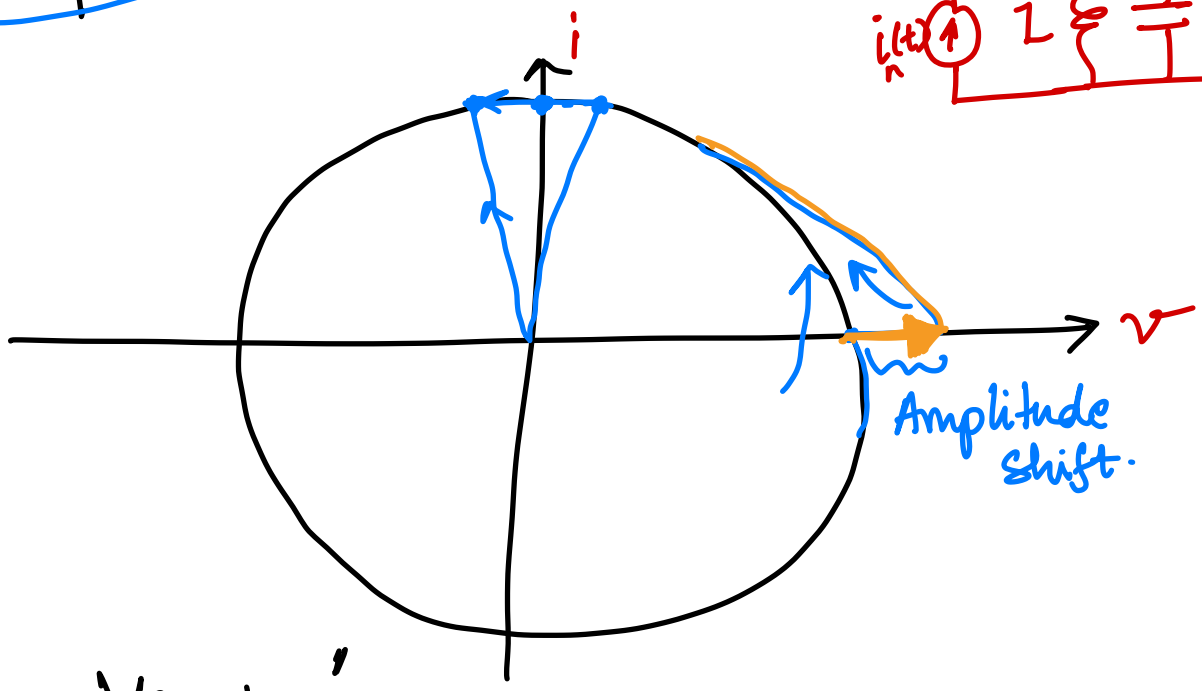
> An electrical oscillator (has) is a stable limit cycle in phase space.

> To understand the dynamics (phase noise, injection locking/pulling, etc.) we need to study the behaviour of these limit cycles.









> "Time Varying"

> Amplitude Fluctuations: Quickly corrected

> Phase Fluctuations: Persist!

> $f(q)$ → Amp. / phase change } ISF (LTV)

> ISF → Phase Noise
Injection locking/Pulling } Hajimiri

> PPV → Alper Demir. } Stochastic D.E (Perturbation An.) Non-lin.

> Attractors (Fixed pts., Fixed lines, planes, limit cycles)

> 3-D : "Stronge attractor", "Fractal" in phase space → Chaos.