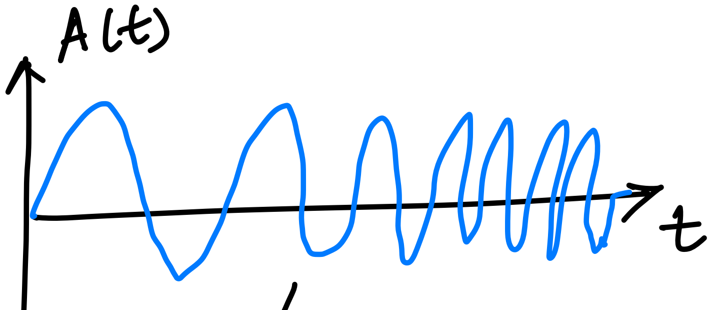
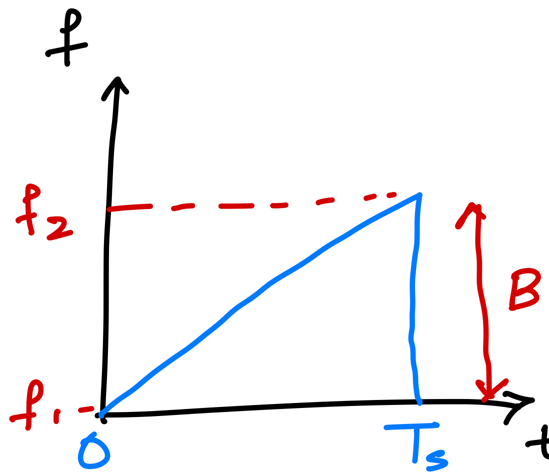
Frequency Modulated Continuous Wave Radar

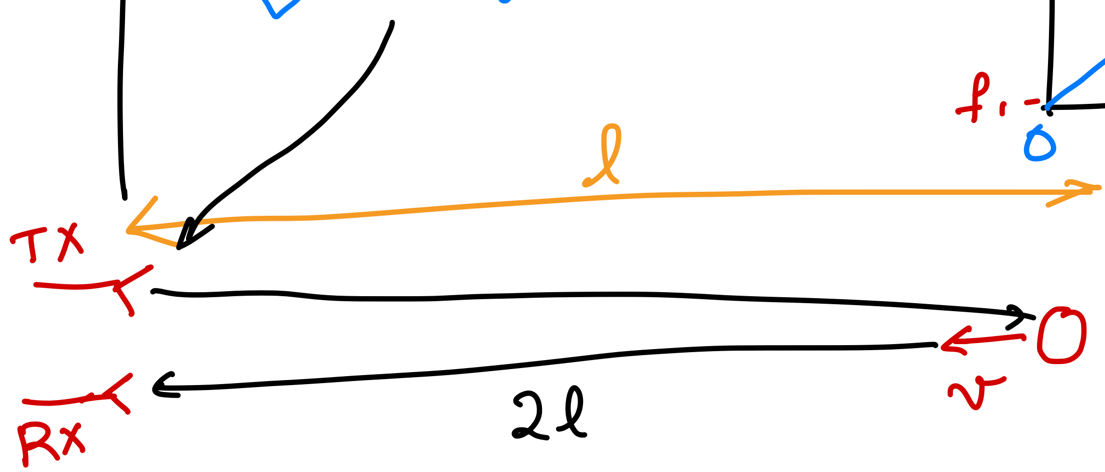
(FMCW radars)

Q: Can we measure both range & velocity with high accuracy simultaneously?..

A: Chirp

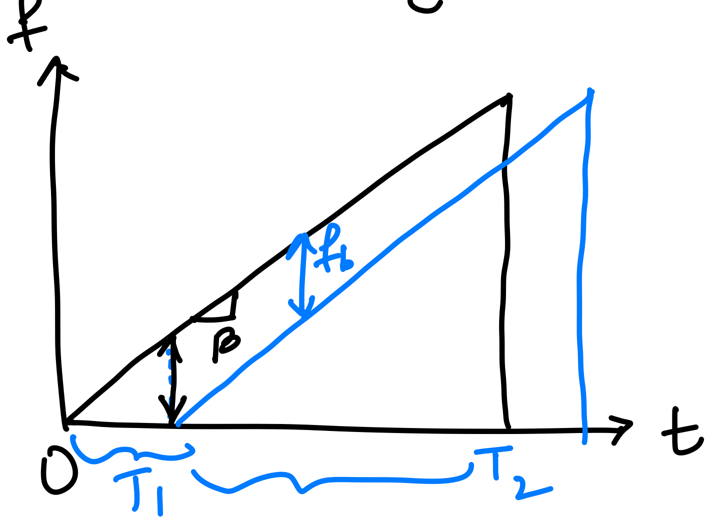




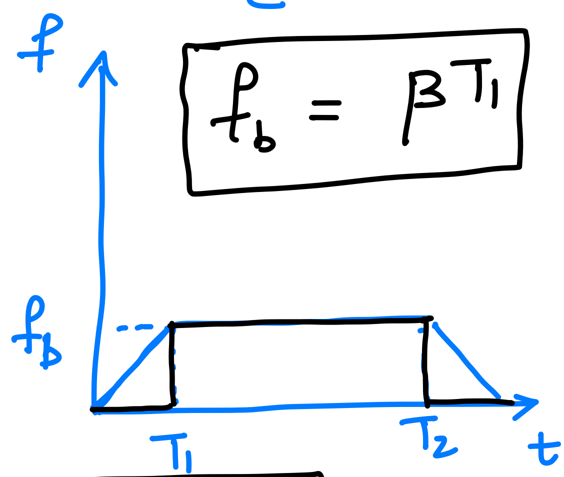


$$t_{rt} = \frac{2l}{c}$$

$$T_1 = t_{rt} = \frac{2l}{c}$$



$$\boxed{f_b = \beta T_1}$$

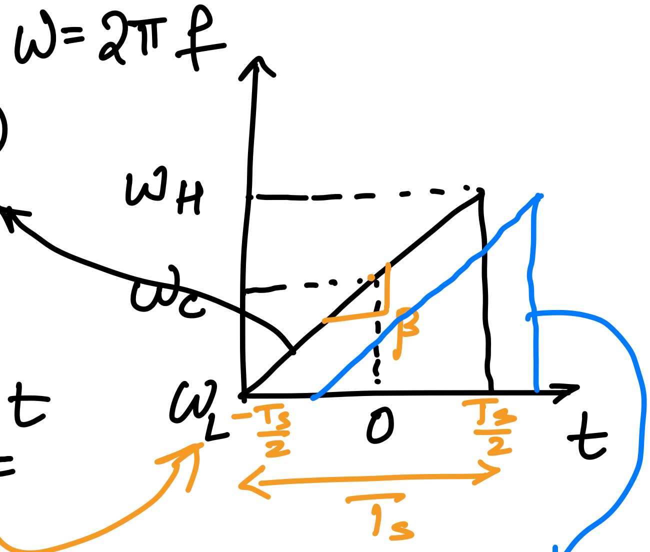


$$\Rightarrow \boxed{f_b = \frac{\beta 2l}{c}}$$

Signal model of FMCW – Single Chirp

Range Estimation

$$A_{TX}(t) = A_0 \cos\Big(\underbrace{\omega_c t + \frac{\beta}{2} t^2}_{\phi_{TX}(t)}\Big)$$



$$\omega_{TX}(t) = \frac{\partial \phi_{TX}(t)}{\partial t} = \underline{\underline{\omega_c + \beta t}}$$

$$A_{RX}(t) = A_0' \cos\left(\omega_c\left(t - \frac{2l}{c}\right) + \frac{\beta}{2}\left(t - \frac{2l}{c}\right)^2\right)$$

$$A_{IF}(t) = A_{TX}(t) \times A_{RX}(t)$$

$$= \frac{A_0 A_0'}{2} \cos\Big(\underbrace{\frac{\omega_c 2l}{c} + \beta\left(\frac{2lt}{c}\right) - \beta\left(\frac{2l^2}{c^2}\right)}_{\phi_{IF}(t)}\Big)$$

$$+ \text{ HF term} \longrightarrow \text{filtered by LPF.}$$

$$\boxed{\phi_{IF}(t) = \frac{2\omega_c l}{c} + \frac{2\beta l}{c} t - \frac{2\beta l^2}{c^2}}$$

RVP (ignore)

$$\omega_{IF}(t) = \frac{\partial \phi_{IF}(t)}{\partial t}$$

"Assume l is independant of t" — ?

$$\boxed{\omega_{IF}(t) = \frac{2\beta l}{c}} \rightarrow \text{Beat Frequency!}$$

What if $l = -vt + c \Rightarrow \frac{dl}{dt} = -v$

Then,

$$\omega_{IF}(t) = \frac{\partial \phi_{IF}(t)}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\omega_c 2l}{c} + \frac{2\beta l t}{c} - \frac{2\beta l^2}{c^2}\right)$$

$$= -\frac{2\omega_c v}{c} + \frac{2\beta l}{c} - \frac{2\beta t v}{c} + \frac{4\beta l v}{c^2}$$

Which terms actually matter?

$$\omega_c v \longleftrightarrow \beta l \longleftrightarrow \beta t v \longleftrightarrow \cancel{\frac{2\beta l v}{c}}$$

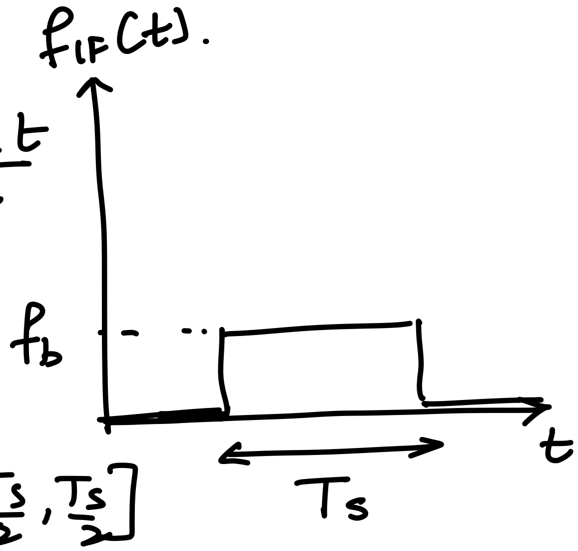
$10^9 \times 10$ | $\{10^{12} - 10^{15}\} \times 10 - 10^2$ | $\{10^{12} - 10^{15}\} \times 10^{-4} \times 10$ | $\{10^{12} - 10^{15}\} \times \{10 - 10^2\} \times \{10^{-7}\}$

$\sim 10^{10}$ | $10^{13} - 10^{17}$ | $\underbrace{10^9 - 10^{12}}$ | 10^5

$$\boxed{\omega_b \approx \frac{2\beta l}{c}}$$

$$\Rightarrow \boxed{l = \frac{\omega_b c}{2\beta}}$$

Range Characteristics.



$$A_{IF}(t) = \frac{A_0 A_0'}{2}\cos\left(\frac{2\omega_c l}{c} + \frac{2\beta l t}{c} - \cancel{\frac{2\beta l^2}{c^2}}\right)$$

$$\times \ \text{rect}\{T_s\}$$

$$\text{rect}(T_s) = \begin{cases} 1 & t \in \left[-\frac{T_s}{2}, \frac{T_s}{2}\right] \\ 0 & \text{otherwise} \end{cases}$$

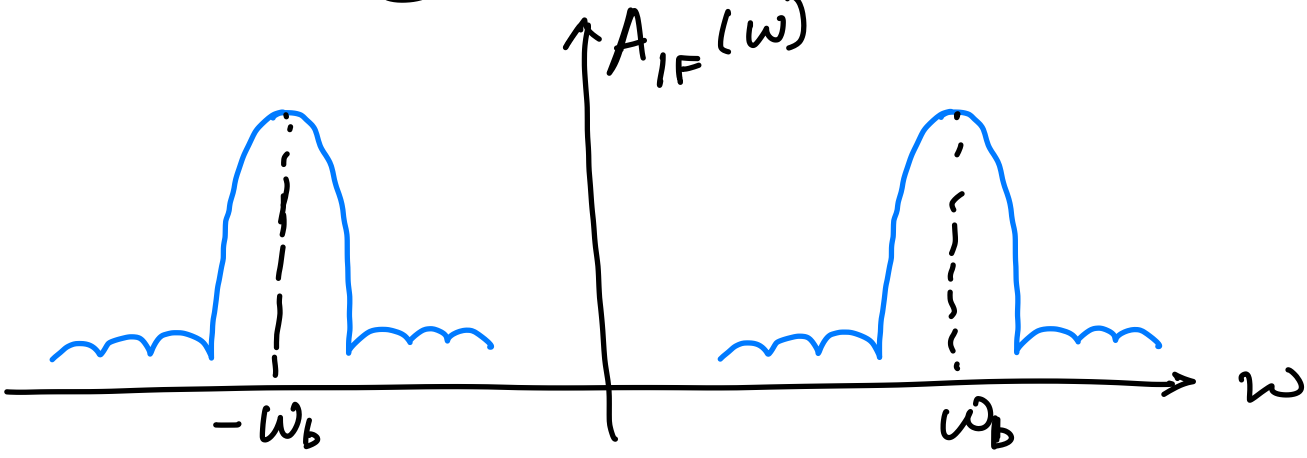
$$A_{IF}(t) = \frac{A_0 A_0'}{2}\cos\left(\frac{2\omega_c l}{c} + \frac{2\beta l t}{c}\right)\text{rect}\{T_s\}$$

$$\mathcal{F}(\cos(\omega_0 t + \theta)) = \pi\left\{e^{j\theta}\delta(\omega - \omega_0) + e^{-j\theta}\delta(\omega + \omega_0)\right\}$$

$$\mathcal{F}(\text{rect}(T_s)) = T_s \ \text{sinc}\left(\frac{\omega T_s}{2}\right)$$

$$A_{IF}(\omega) = \frac{A_0 A_0' \pi T_s}{2}\left\{e^{j\theta}\,\text{sinc}\left[\frac{(\omega - \omega_0)T_s}{2}\right] + e^{-j\theta}\,\text{sinc}\left[\frac{(\omega + \omega_0)T_s}{2}\right]\right\}$$

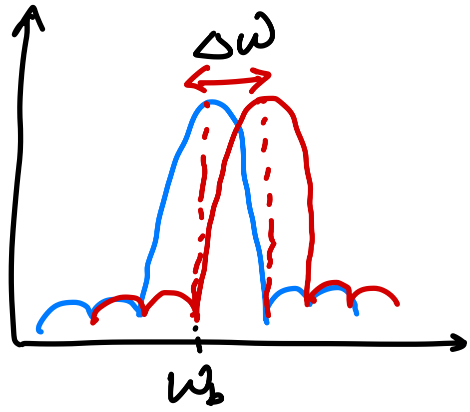
where, $\theta = \dfrac{2\omega_c l}{c}$ & $\omega_0 = \dfrac{2\beta l}{c} = \omega_b$



Range resolution

$\Delta l \rightarrow$ range resolution.

$$\text{sinc}\left(\frac{(\omega-\omega_b)T_s}{2}\right) = 0?$$



$$\frac{(\omega-\omega_b)T_s}{2} = \pi$$

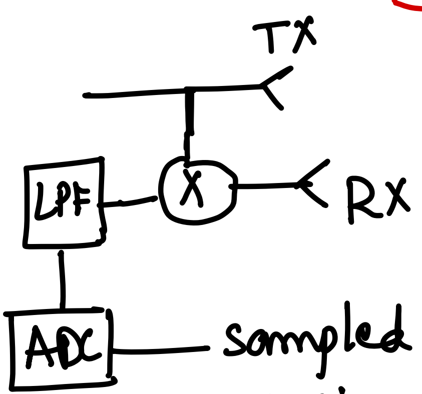
$$\Rightarrow \Delta\omega = \frac{2\pi}{T_s}$$

$$\omega = \frac{2\beta l}{c} \Rightarrow \Delta\omega = \frac{2\beta\Delta l}{c} \Rightarrow \Delta l = \frac{c}{2\beta}\cdot\Delta\omega$$

$$\Rightarrow \Delta l = \frac{c}{2\beta}\cdot\frac{2\pi}{T_s} = \frac{c}{2B}$$

$B \hookrightarrow$ Bandwidth in Hz.

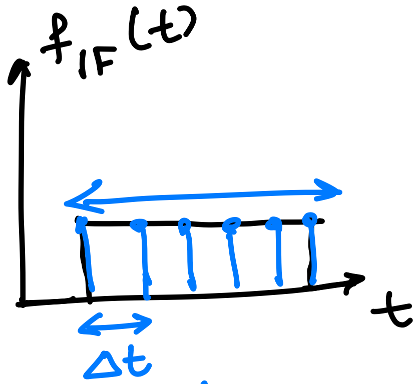
$$\boxed{\Delta l = \frac{c}{2B}}$$



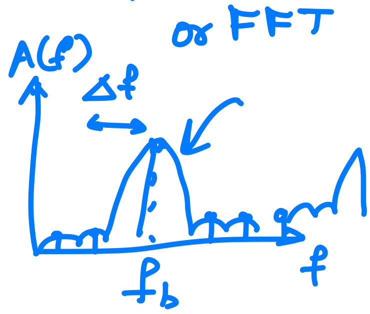
sampled version of $A_{IF}(t)$.

$$N = \frac{T_s}{\Delta t}$$

$$\Delta f = \frac{1}{T_s}$$

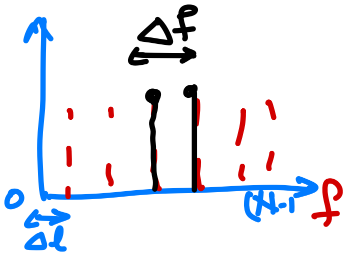


$\downarrow$ DFT or FFT



$$\Rightarrow \Delta\omega = \frac{2\pi}{T_s}$$

$$\Rightarrow \boxed{\Delta l = \frac{c}{2\beta}\Delta\omega = \frac{c}{2B}}$$



$$\text{Max range} = \Delta l \times (N-1) \simeq N\Delta l = \frac{Nc}{2B}$$

Doppler Processing

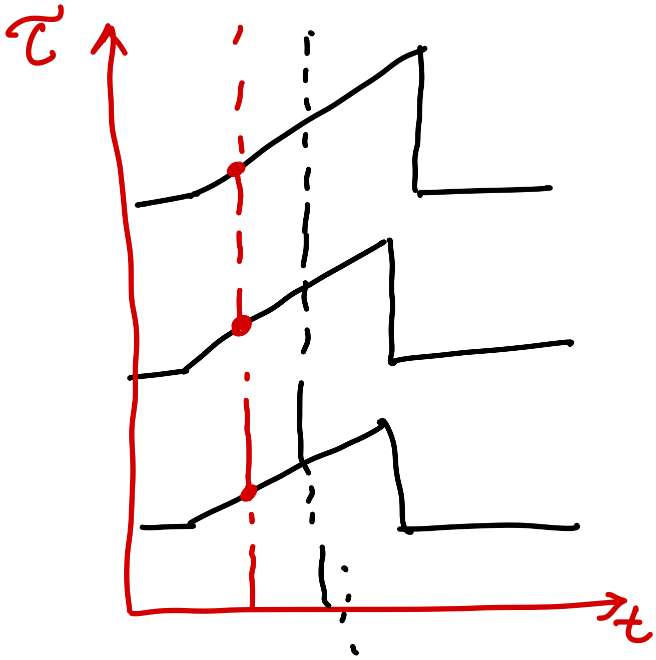
(l, v)

t, τ → independant variable (Slowtime)

t → (Fast time)

$$t \in \left[-\frac{T_s}{2}, \frac{T_s}{2}\right]$$

τ → Time period between multiple chirps.



$$\phi_{IF}(t) = \frac{2\omega_c l}{c} + \frac{2\beta l t}{c} - \frac{2\beta l^2}{c^2}$$

$$\phi_{IF}(t,\tau) = \frac{2\omega_c(l - v\tau)}{c} + \frac{2\beta t}{c}(l - v\tau) - \frac{2\beta}{c^2}(l - v\tau)^2$$

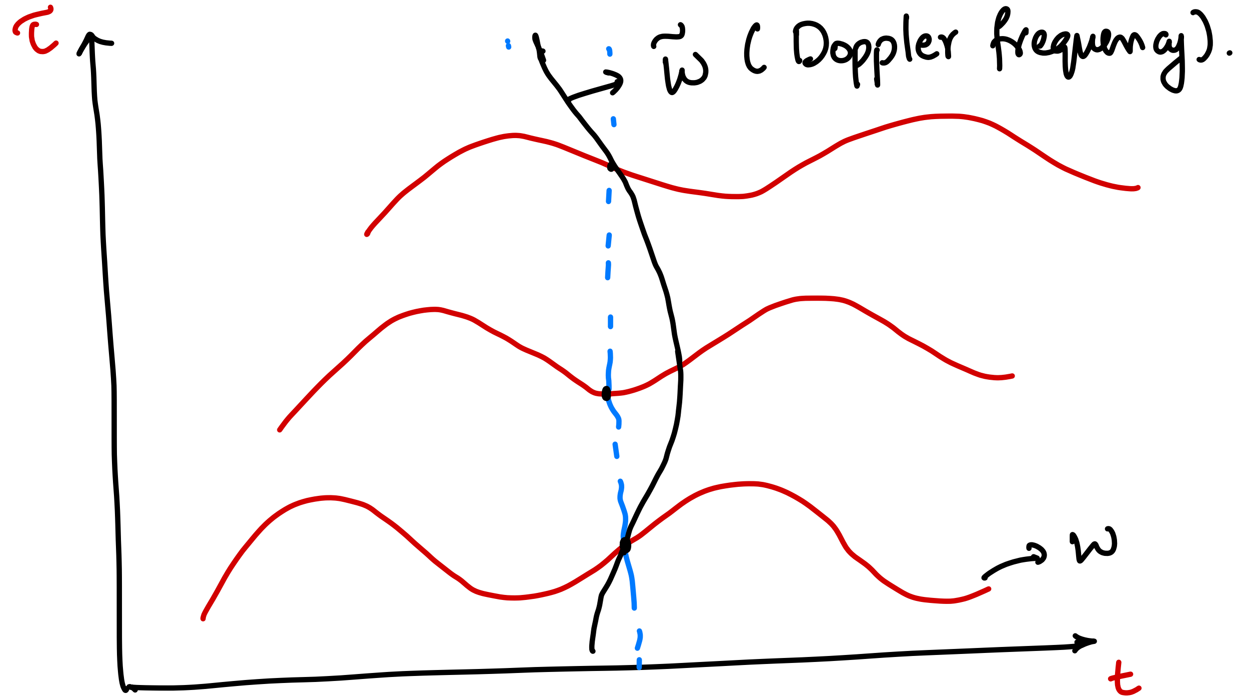
$\omega_c\,(10^9)$

$$\tilde{\omega} = \frac{\partial \phi_{IF}(t,\tau)}{\partial \tau} = -\frac{2v\omega_c}{c} - \cancel{\frac{2\beta t v}{c}} + \cancel{\frac{4\beta v l}{c^2}} - \cancel{\frac{4\beta v^2 \tau}{c^2}}$$

$\frac{\beta l}{c}$ (10^4) $\quad$ $\frac{\beta v \tau}{c}$ (10^1)

$\underline{\underline{t = 0}}$

$$\boxed{\tilde{\omega} = -\frac{2\omega_c v}{c}}$$



Velocity characteristics.

$$A_{IF}(\omega) = \tilde{A}_0 \, e^{j\theta} \, \mathrm{sinc}\left(\frac{(\omega-\omega_0)T_s}{2}\right)$$

$$\theta = \frac{2\omega_c l}{c} \; ; \quad \omega_0 = \omega_b = \frac{2\beta l}{c} \; ; \quad \tilde{A}_0 = \frac{A_0 A_1' \pi T_s}{2}$$

$$l \longrightarrow l - v\tau$$

$$A_{IF}(\omega) = \tilde{A}_0 \, e^{j\underbrace{\left(\frac{2\omega_c(l - v\tau)}{c}\right)}_{\theta}} \, \mathrm{sinc}\left(\frac{(\omega-\omega_0)T_s}{2}\right)$$

$$\underline{\underline{\tilde{\omega}(\tau)}} = \frac{\partial\theta}{\partial\tau} = \frac{-2\omega_c v}{\underbrace{c}} \Rightarrow \boxed{v = -\frac{\tilde{f}\lambda_c}{2}}$$

> FFT across chirps. Say we have M chirps.

$$\Delta\tilde{\omega} = \frac{2\pi}{M T_s} = \frac{2\pi}{T_F} \rightarrow \text{Frame time.}$$

$$v = \frac{-\tilde{\omega} c}{2\omega_c} \Rightarrow \Delta v = \frac{2\pi c}{2\omega_c T_F} \Rightarrow \boxed{\Delta v = \frac{\lambda_c}{2 T_F}}$$

$$v_{max} - v_{min} = 2v_{max} = \Delta v \cdot M$$

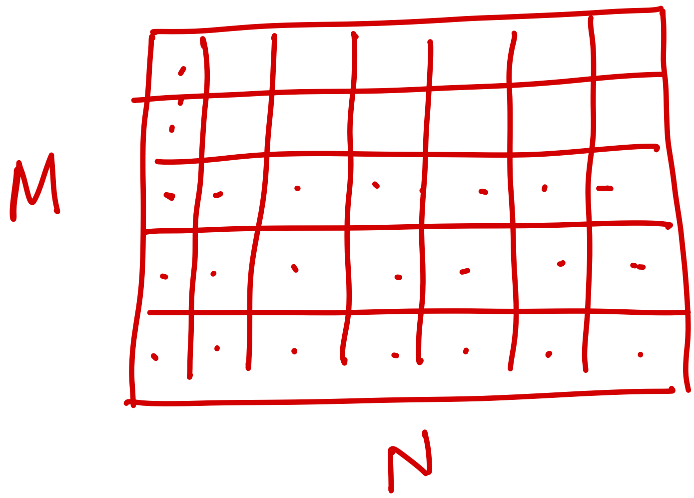
$$\Rightarrow \boxed{v_{max} = \frac{\lambda_c}{4T_s}}$$

Summary

$$l = \frac{\omega_b c}{2B} \; ; \quad \Delta l = \frac{c}{2B} \; ; \quad l_{max} = \frac{Nc}{2B}$$

$$v = \frac{\tilde{f} \lambda_c}{2} \; ; \quad \Delta v = \frac{\lambda_c}{2T_F} \; , \quad v_{max} = \frac{\lambda_c}{4T_s}$$

Computation?



$\longrightarrow$ 2D FFT

$$l : \quad 0 \longrightarrow \Delta l (N-1)$$

$$v : \quad -\Delta v \left(\frac{M}{2}\right) \longrightarrow \Delta v \left(\frac{M}{2} - 1\right)$$

Assumptions?

> Range is small (ignored RVP)
> velocity is "low"
> Chirp is linear.
> Phase stable
> Phase noise?
> Target is point scatterer
> velocity is constant over frame.
> Ignored dispersion.
> ...