Scattering from Metals (MFIE)

$$\vec{E} = i\omega\mu \int_V \left\{ \left(\frac{3}{kR^2} - \frac{3i}{kR} - 1 \right) \hat{R}\hat{R} \cdot \vec{J} + \left[1 + \frac{i}{kR} - \frac{1}{k^2R^2} \right] \overline{\overline{I}} \cdot \vec{J} \right\} \frac{e^{ikR}}{4\pi R} \, d\vec{r}\,'$$

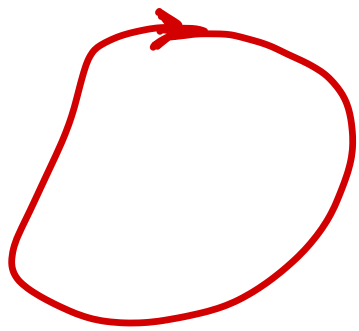
$$\vec{H} = \int_V \left(ik - \frac{1}{R} \right) \frac{e^{ikR}}{4\pi R} \left(\hat{R} \times \vec{J} \right) d\vec{r}\,'$$

$$\hat{n}\times\left[\vec{H}_{tot} = \vec{H}_{inc} + \vec{H}_{sca}\right]$$

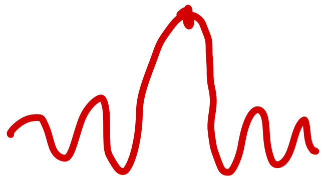
$$\vec{H}_{sca} = \int_S \left(\nabla \times \overline{\overline{G}} \cdot \vec{J} \right) d\vec{r}\,'$$

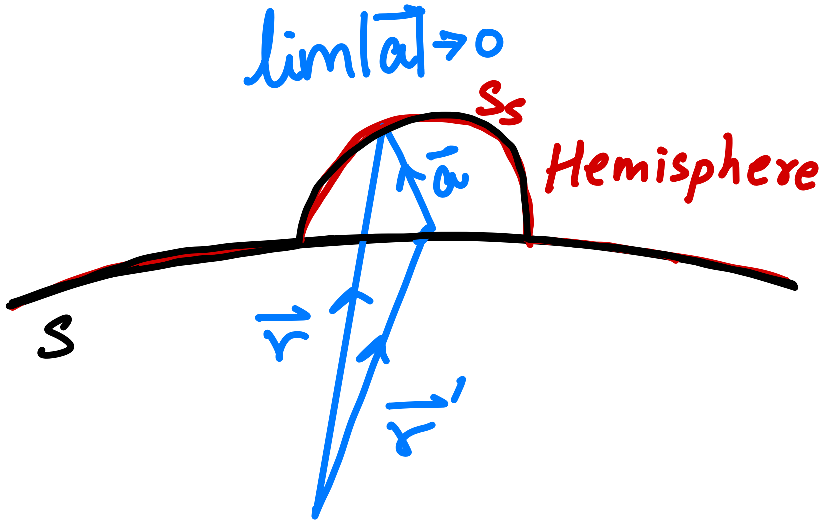
$$\underbrace{\hat{n} \times \vec{H}_{tot}}_{\vec{J}(\vec{r})} = \hat{n} \times \vec{H}_{inc} + \hat{n} \times \int_S \nabla \times \overline{\overline{G}}(\vec{r}, \vec{r}\,') \cdot \vec{J}(\vec{r}\,') \, d\vec{r}\,'$$

$$\mathrm{sinc}(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$



Removing the Singularity





$$\hat{n} \times \int_S \nabla \times \bar{\bar{G}} \cdot \vec{J}\, d\vec{r}'$$

$$= \lim_{a \to 0} \left\{ \hat{n} \times \int_{S-S_s} \nabla \times \bar{\bar{G}} \cdot \vec{J}\, d\vec{r}' + \hat{n} \times \int_{S_s} \nabla \times \bar{\bar{G}} \cdot \vec{J}\, d\vec{r}' \right\}$$

$$= \underbrace{\hat{n} \times \lim_{a \to 0} \int_{S-S_s} \nabla \times \bar{\bar{G}} \cdot \vec{J}\, d\vec{r}'}_{\text{Principal Integral (PI)}} + \underbrace{\hat{n} \times \lim_{a \to 0} \int_{S_s} \nabla \times \bar{\bar{G}} \cdot \vec{J}\, d\vec{r}'}_{\text{Singular Integral (SI)}}$$

$$PI = \hat{n} \times ⨍_S \nabla \times \bar{\bar{G}} \cdot \vec{J}\, d\vec{r}' \triangleq \hat{n} \times \lim_{a \to 0} \int_{S-S_s} \nabla \times \bar{\bar{G}} \cdot \vec{J}\, d\vec{r}'$$

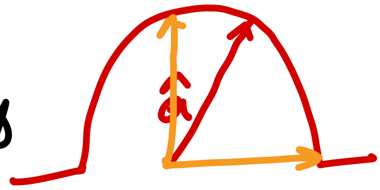
$$S\mathcal{I} = \hat{n}\times\lim_{a\to 0}\int_{S_s}\nabla\times\bar{\bar{G}}\cdot\vec{J}\,d\vec{r}\,'$$

$$\left[\;\nabla\times\bar{\bar{G}} = \left(ik-\frac{1}{a}\right)\frac{e^{ika}}{4\pi a}\,\hat{a}\times\bar{\bar{I}}\;\right] \qquad \vec{a}=\vec{r}-\vec{r}\,'$$

$$\rightarrow S\mathcal{I} = \hat{n}\times\lim_{a\to 0}\int_{S_s}\left(ik-\frac{1}{a}\right)\frac{e^{ika}}{4\pi a}\,\hat{a}\times\vec{J}\,a^2\sin\theta\,d\theta\,d\phi$$

$$= \hat{n}\times\lim_{a\to 0}\int_{S_s}\left(\cancelto{0}{ika}-1\right)\frac{\cancelto{1}{e^{ika}}}{4\pi}\,\cancelto{\hat{n}}{\hat{a}}\times\vec{J}\,\sin\theta\,d\theta\,d\phi$$

$$= -\frac{\hat{n}\times\hat{n}\times\vec{J}}{4\pi}\int_0^{2\pi}\int_0^{\pi/2}\sin\theta\,d\theta\,d\phi$$



$$= -\frac{\hat{n}\times\hat{n}\times\vec{J}}{2} = \frac{1}{2}\left[-\hat{n}(\cancelto{0}{\hat{n}\cdot\vec{J}})+\vec{J}(\cancelto{1}{\hat{n}\cdot\hat{n}})\right]$$

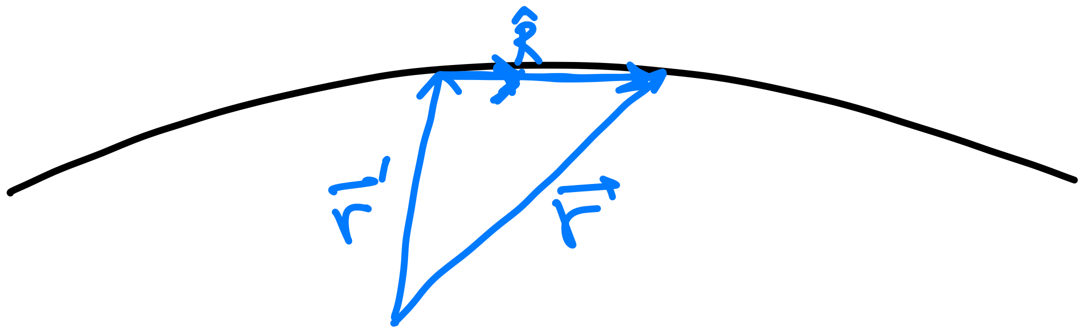
$$= \frac{\vec{J}}{2}$$

MFIE

$$\boxed{\vec{J} = 2(\hat{n}\times\vec{H}_{inc}) + 2\hat{n}\times\int_S\nabla g(\vec{r},\vec{r}\,')\times\vec{J}(\vec{r}\,')\,d\vec{r}\,'}$$

Locally flat approximation

$\nabla g \sim (\ )\nabla R$
$= (\ )\hat{R}$

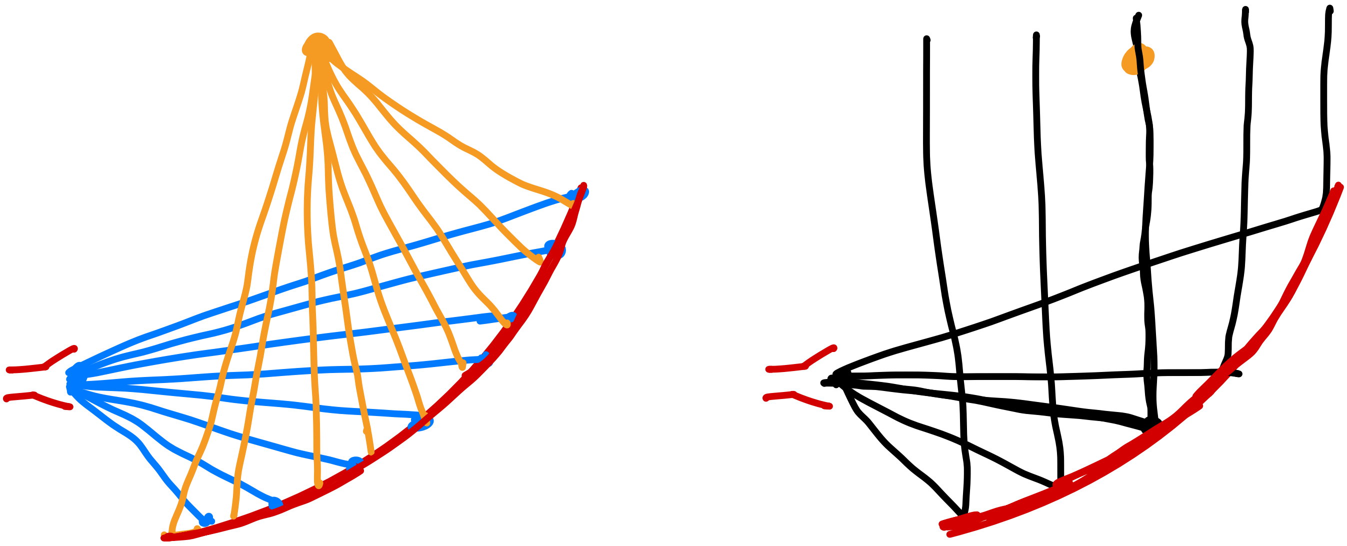


$$\boxed{\vec{J} = 2\,\hat{n}\times\vec{H}_{inc}}$$

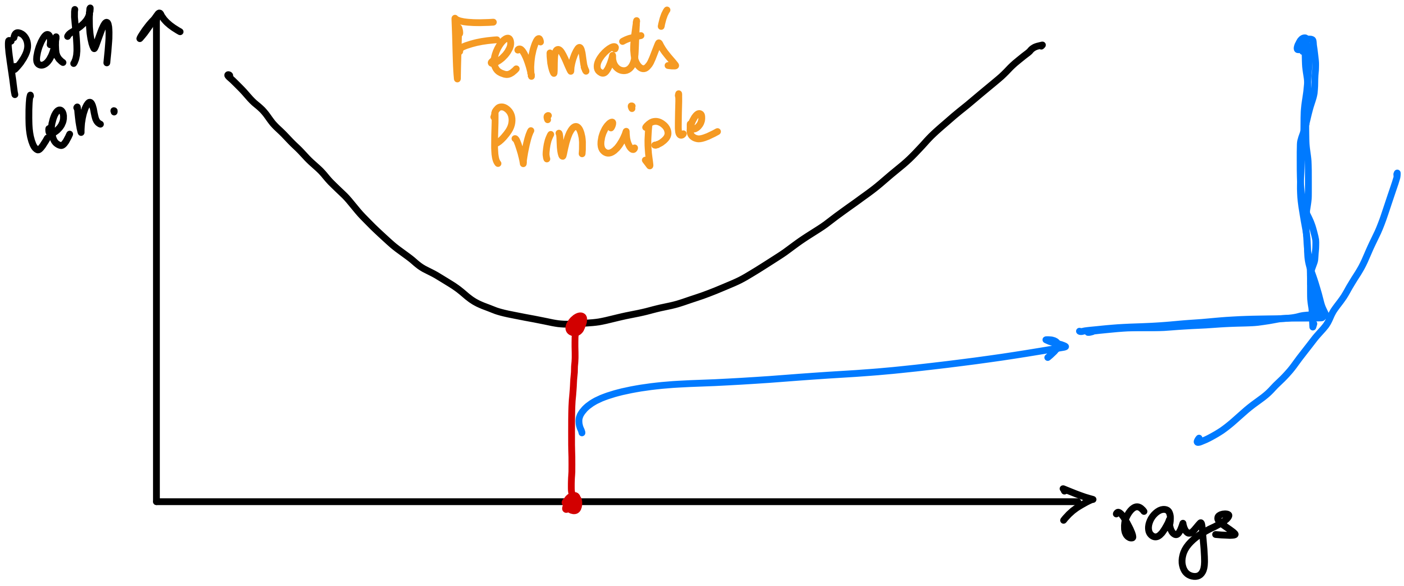
PHYSICAL OPTICS
(PO)

$\vec{J} = \hat{n}\times\vec{H}_{tot}$ BC

$\vec{H}_{tot} = 2\,\vec{H}_{inc} \Rightarrow \vec{H}_{ref}$ or $\vec{H}_{sca} = \vec{H}_{inc}$

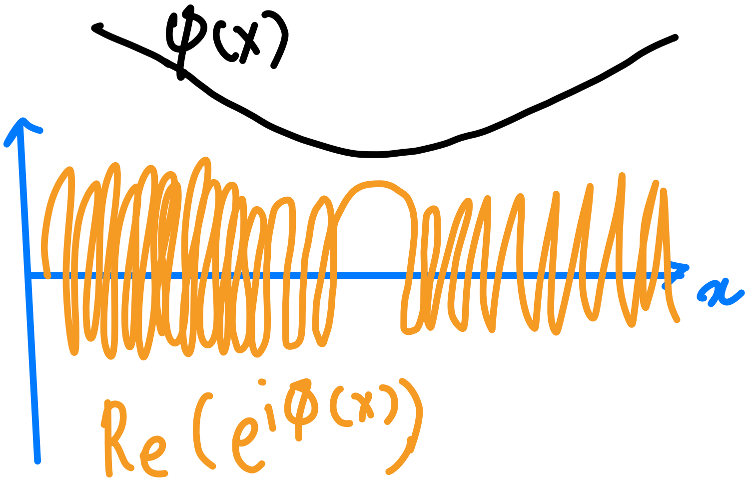


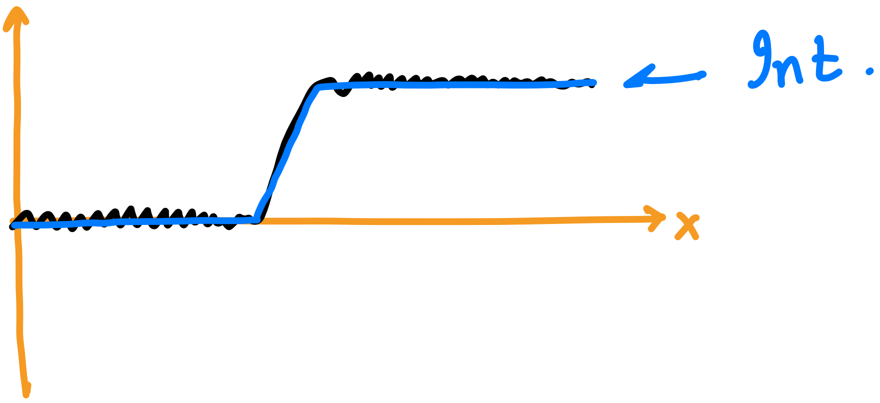
Stationary Phase Approximation.



$$\int_x \underbrace{f(x)}_{\text{slow}}\, e^{i\underbrace{\phi(x)}_{\text{fast}}} \longrightarrow A(x_0)\, f(x_0)\, e^{i\phi(x_0)}$$

where $\phi'(x_0) = 0$





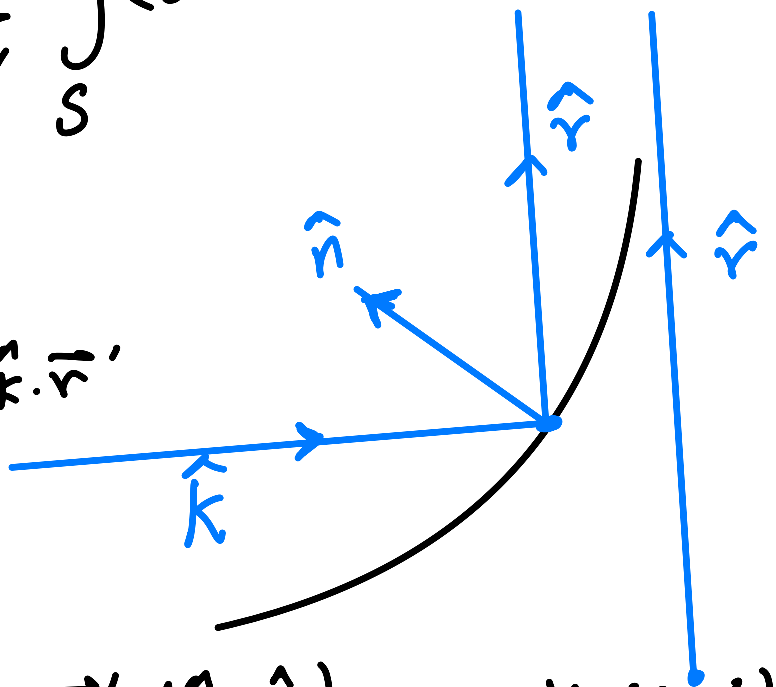
Geometrical Optics

$$\vec{J} = 2\hat{n} \times \vec{H}_{inc}$$

$$\vec{E}_{FF} = -i\omega\mu \frac{e^{ikr}}{4\pi r} \int_S (\vec{J} \times \hat{r} \times \hat{r}) e^{-ik\vec{r}'\cdot\hat{r}} \, d\vec{r}'$$

$$\vec{E}_{inc} = \vec{E}_0 e^{ik\hat{k}\cdot\vec{r}'}$$

$$\vec{H}_{inc} = \frac{1}{\eta} \hat{k} \times \vec{E}_0 e^{ik\hat{k}\cdot\vec{r}'}$$



Phase term is $e^{-ik\vec{r}'\cdot(\hat{r}-\hat{k})} = e^{-ik\phi(\vec{r}')}$

$$\phi(\vec{r}') = (\hat{r} - \hat{k})\cdot\vec{r}' \qquad \nabla(\vec{a}\cdot\vec{r}) = \vec{a}$$

$$\nabla\phi(\vec{r}')\cdot\vec{t}_{an} = 0$$

$\Rightarrow \nabla\phi = \lambda\hat{n}$ for some λ.

$$\nabla((\hat{r}-\hat{k})\cdot\vec{r}') = \hat{r} - \hat{k}$$

$$\hat{r} - \hat{k} = \lambda \hat{n} \Rightarrow |\hat{r}|^2 = |\hat{k}|^2 + \lambda^2 |\hat{n}|^2 + 2\lambda\, \hat{k}\cdot\hat{n}$$

$$\Rightarrow 1 = 1 + \lambda^2 + 2\lambda\, \hat{k}\cdot\hat{n}$$

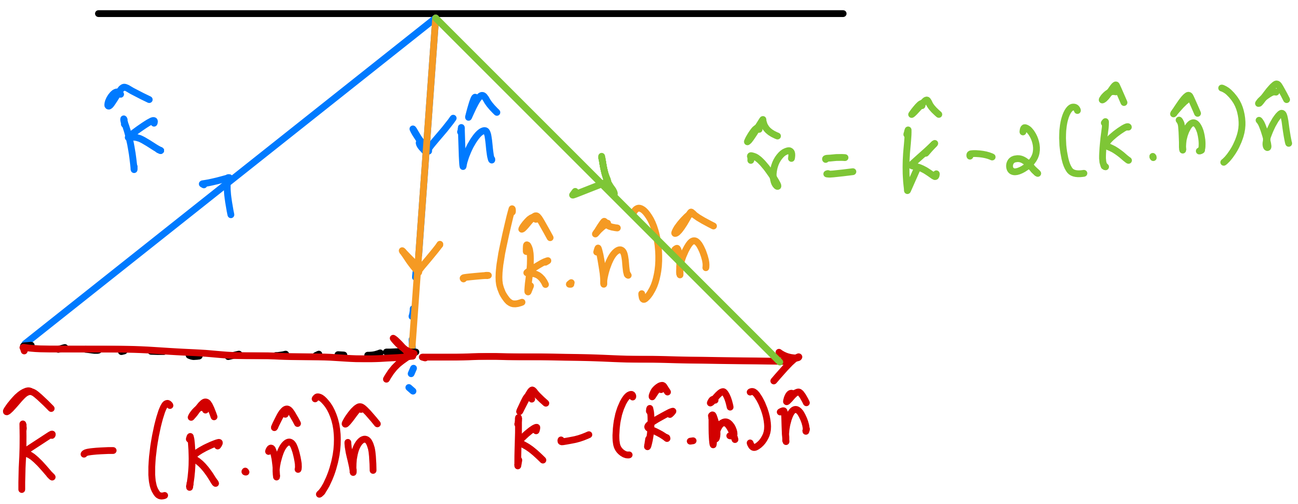
$$\Rightarrow \lambda(\lambda + 2\hat{k}\cdot\hat{n}) = 0$$

$$\Rightarrow \lambda = -2\hat{k}\cdot\hat{n}$$

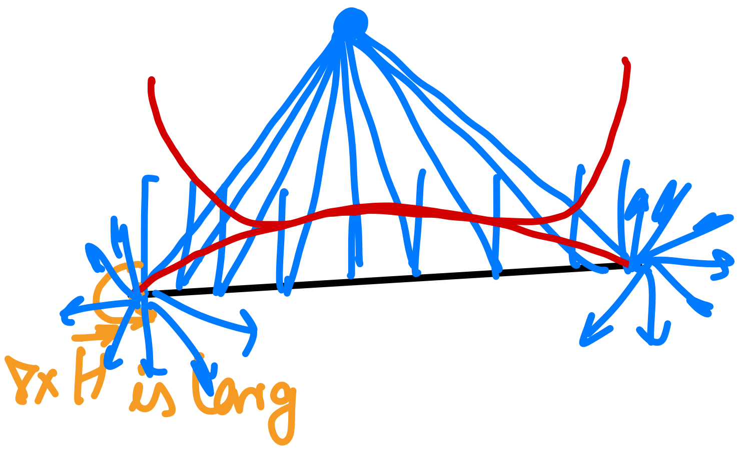
$$\boxed{\hat{r} = \hat{k} - 2(\hat{k}\cdot\hat{n})\hat{n}}$$

Snell's Law of Reflection

$\theta_i = \theta_r$.



> PO, GO do not account for edge diffraction.



PO + PTD — Current Based

GO + GTD (SBR+) — Ray Based.