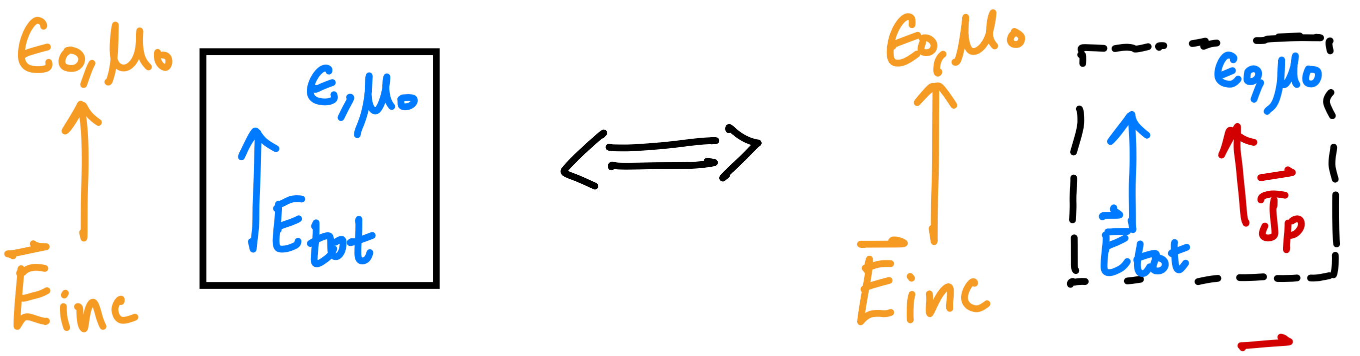
Ewald Oseen Extinction Theorem.



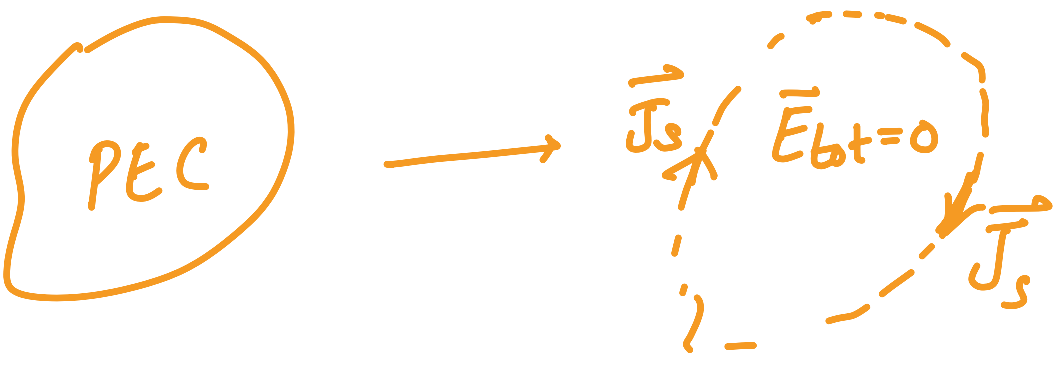
$$\vec{J}_p = -i\omega\epsilon_0\chi_e \vec{E}_{tot}$$

$$\vec{E}_{sca} \triangleq i\omega\mu \int_V \bar{\bar{G}} \cdot \vec{J}_p \, dv'$$

$$= \vec{E}_{tot} - \vec{E}_{inc}$$

$$\nabla \times \nabla \times \vec{E}_{tot} - k^2 \vec{E}_{tot} = 0$$

$$k \rightarrow \omega\sqrt{\mu_0 \epsilon}$$



Proof: $\vec{E}_{sca} = \vec{E}_{tot} - \vec{E}_{inc} = i\omega\mu \int_V \bar{\bar{G}} \cdot \vec{J}_p \, dv'$

$$\vec{E}_{tot} = \vec{E}_{inc} + i\omega\mu_0 \int_V \bar{\bar{G}} \cdot \left(-i\omega\epsilon_0 \chi_e \vec{E}_{tot}\right) \underset{d\vec{r}\,'}{\underset{\downarrow}{dv'}}$$

$$\Rightarrow \boxed{\vec{E}_{tot} = \vec{E}_{inc} + k_0^2 \chi_e \int_V \bar{\bar{G}} \cdot \vec{E}_{tot} \, d\vec{r}\,'} \quad \text{EFIE}$$

$\mathcal{L} : (\nabla \times \nabla \times \; - k_0^2)$ $\qquad$ $\mathcal{L}(\vec{E}) = 0$

$$\mathcal{L}\vec{E}_{tot} = \underbrace{\mathcal{L}\vec{E}_{inc}}_{0} + \underbrace{k_0^2 \chi_e \int_V (\underbrace{\mathcal{L}\,\bar{\bar{G}}}_{\bar{\bar{I}}\,\delta(\bar{r}-\bar{r}')}) \cdot \vec{E}_{tot}(\vec{r}\,') \, d\vec{r}\,'}_{k_0^2 \chi_e \underbrace{\int_V \vec{E}_{tot}(\vec{r}\,')\,\delta(\vec{r}-\vec{r}\,')\,d\vec{r}\,'}_{\vec{E}_{tot}(\vec{r})}}$$

$$\mathcal{L}\vec{E}_{tot} = k_0^2 \chi_e \vec{E}_{tot}$$

$$\Rightarrow \nabla \times \nabla \times \vec{E}_{tot} - \underbrace{k_0^2 (1 + \chi_e)}_{k^2} \vec{E}_{tot} = 0$$

Integral Equations (EFIE).

$$\vec{E}_{tot} = \frac{i}{\omega \epsilon_0 \chi_e} \vec{J}$$ (Dielectrics)

$$\vec{E}_{tot} = 0$$ (PEC)

In general $\vec{E}_{tot} = \bar{\bar{Z}}_s \vec{J}_s$

$$\bar{\bar{Z}}_s \vec{J}_s = \vec{E}_{inc} + i\omega\mu \int_V \bar{\bar{G}}(\bar{r}, \bar{r}') \cdot \vec{J}(\bar{r}') d\bar{r}'$$

$$\boxed{\vec{J}_s \approx \sum_{n=1}^{N} I_n \underbrace{\vec{f}_n(\vec{r}')}_{\text{Basis functions.}}}$$

Basis expansion

$$\bar{\bar{Z}}_s \sum_{n=1}^{N} I_n \vec{f}_n(\vec{r}) = \vec{E}^{inc}(\vec{r})$$

$$+ i\omega\mu \sum_{n=1}^{N} I_n \int_V \bar{\bar{G}}(\vec{r}, \vec{r}') \vec{f}_n(\vec{r}') d\vec{r}'$$

$$\vec r = \vec r_m , \quad m = 1, 2, \dots, N.$$

$$\overbrace{\underset{3\times3}{\overline{\overline{Z}}_s} \sum_{n=1}^{N} I_n \,\bar f_n(\vec r_m)}^{W_m\ (m\times1)} = \overbrace{\vec E_{inc}(\vec r_m)}^{\vec V_m \quad m\times1}$$

$$+ \sum_{n=1}^{N} I_n \, i\omega\mu \int_V \overline{\overline{G}}(\vec r_m, \vec r\,') \,\bar f_n(\vec r\,') \, d\vec r\,'$$

Z_{mn}

$$\vec W_m = \underset{m\times n}{\tilde Z} \, \vec I$$

$$\tilde Z \vec I = \vec V + \tilde Z' \vec I$$

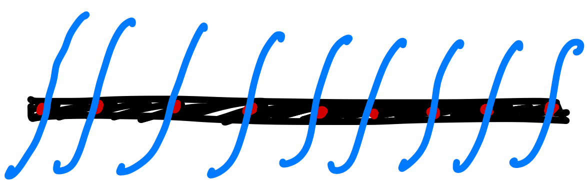
$$\vec V = \underbrace{(\tilde Z - \tilde Z')}_{\overline{\overline{Z}}} \vec I$$

$$\Rightarrow \boxed{\vec V = \overline{\overline{Z}} \, \vec I}$$

$$\boxed{\vec I = \overline{\overline{Z}}^{-1} \vec V}$$

$$V_m = \sum_{n=1}^{N} Z_{mn} I_n$$

$$\begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ \vdots \end{matrix} = \begin{matrix} Z_{11} I_1 + Z_{12} I_2 + \cdots \\ Z_{21} I_1 + Z_{22} I_2 + \cdots \\ \\ \\ \end{matrix}$$



Method of Moments

$$\overbrace{\bar{\bar{Z}}_s \sum_{n=1}^{N} I_n \vec{f}_n(\vec{r})}^{E_{tot}} = \vec{E}^{inc}(\vec{r})$$

$$+ i\omega\mu \sum_{n=1}^{N} I_n \int_V \bar{\bar{G}}(\vec{r},\vec{r}')\,\vec{f}_n(\vec{r}')\,d\vec{r}'$$

$$\int_{V_m} (\qquad)\,\underbrace{\vec{f}_m(\vec{r})}\,d\vec{r}$$ on bothsides.

testing functions or weighting functions.

$$\overbrace{\int_{V_m} \vec{E}_{tot}(\vec{r}).\vec{f}_m(\vec{r})\,d\vec{r}}^{W_m} = \overbrace{\int_{V_m} \vec{E}_{inc}(\vec{r}).\vec{f}_m(\vec{r})\,d\vec{r}}^{V_n}$$

$$+ \sum_{n=1}^{N} I_n \underbrace{\left[i\omega\mu \int_{V_m}\int_{V} \vec{f}_m(\vec{r}).\bar{\bar{G}}(\vec{r},\vec{r}')\,\vec{f}_n(\vec{r}')\,d\vec{r}'\,d\vec{r} \right]}_{Z_{mn}}$$

$$\vec{W} = \vec{V} + \tilde{\bar{Z}}' I_n \Rightarrow \boxed{\vec{V} = \bar{\bar{Z}}\,\vec{I}}$$ // MoM

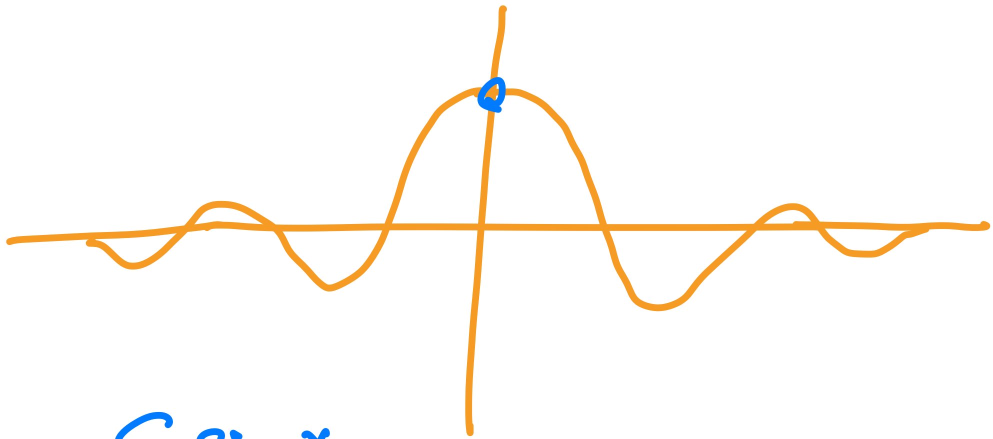
Practical Considerations

1) $$Z_{mn} = i\omega\mu \int_{V_m}\int_{V} \vec{f}_m(\vec{r}) \cdot \bar{\bar{G}}(\vec{r},\vec{r}\,') \cdot \vec{f}_n(\vec{r}\,')\, d\vec{r}\,' \, d\vec{r}$$

$\vec{r} = \vec{r}\,'$ has a singularity in $\bar{\bar{G}}$!

However it is removable.

$\frac{\sin x}{x}$

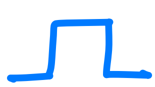


$$\text{Sinc}(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

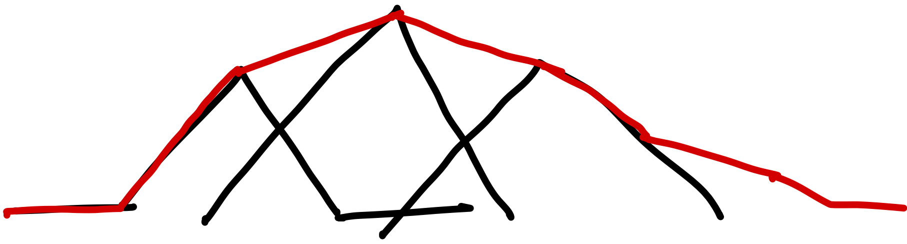
Basis functions & Testing functions

1) Galerkin's Method: $\vec{f}_m = \vec{f}_n$.

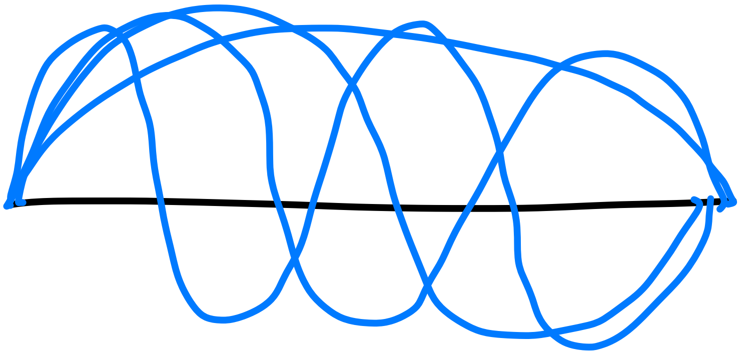
2) Subdomain

- Dirac delta (Point Matching)
- Pulse basis 

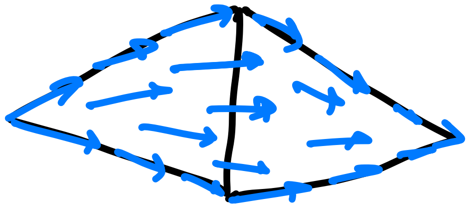
• Triangular basis



Entire domain



▷ Rao Wilton Glisson (RWG)



> Vector valued
> Triangular basis-like
> Enforces current cont. automatically
> Subdomain

> Jordan Budhu (videos on MoM).
