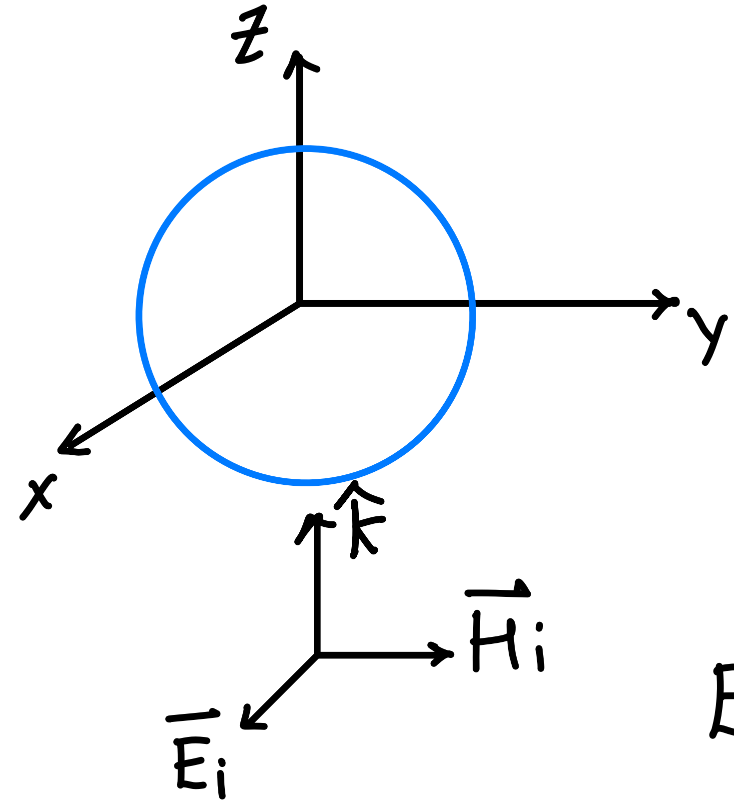
Plane Wave Scattering from Spheres.



$$\vec{E}^i = E_0 \hat{x} e^{ikz}$$

$$\vec{H}^i = \frac{E_0}{\eta} \hat{y} e^{ikz}$$

$$\vec{E}^i = E_r \hat{r} + E_\theta \hat{\theta} + E_\phi \hat{\phi}$$

$$E_r^i = \vec{E}^i \cdot \hat{r}$$

$$= E_0 \cos\phi \sin\theta \, e^{ikr\cos\theta}$$

$$= \frac{E_0 \cos\phi}{-ikr} \frac{d}{d\theta} e^{ikr\cos\theta}$$

Expanding plane wave in sph. modes.

$$e^{ikr\cos\theta} = \sum_{n>1}^{\infty} (i)^n (2n+1) j_n(kr) P_n(\cos\theta).$$

(Sec 8-2.1)

$$E_r^i = \frac{E_0 \cos\phi}{-i(kr)^2} \sum_{n=1}^{\infty} (i)^n (2n+1) \underbrace{\hat{j}_n(kr)}_{kr j_n(kr)} \underbrace{P_n'(\cos\theta)}_{\frac{d}{d\theta} P_n^0 = P_n^1}$$

$$E_r = -\frac{1}{i\omega\mu\epsilon}\left(\frac{\partial^2}{\partial r^2} + k^2\right)A_r$$ → Spherical Gauge.

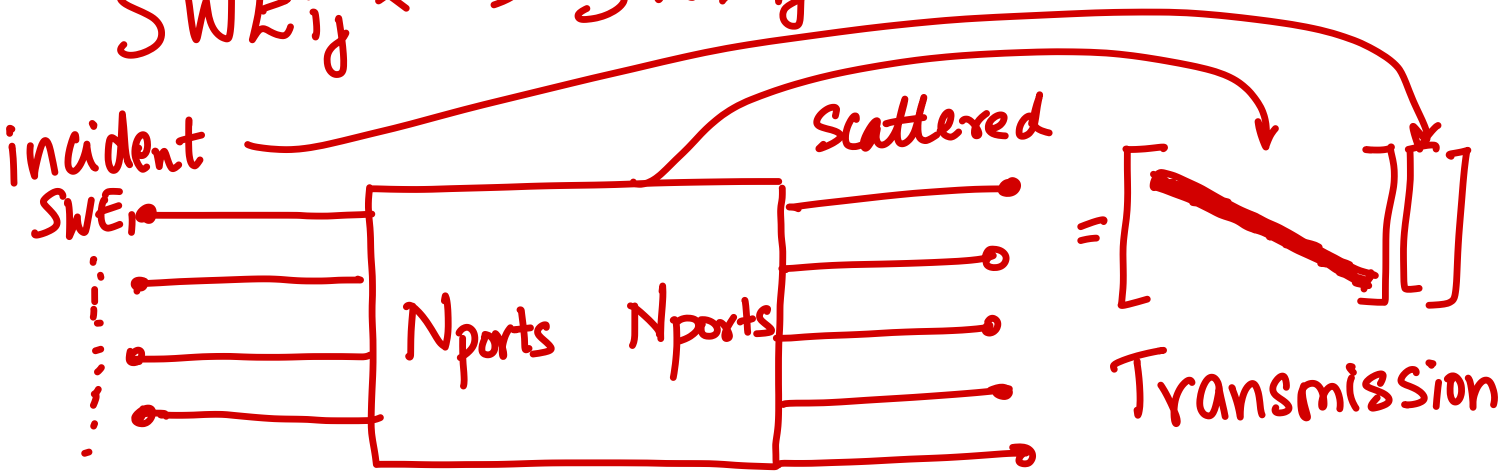
$$A_r^i = \frac{E_0}{\omega}\cos\phi \sum_{n=1}^{\infty} \frac{(i)^n(2n+1)}{n(n+1)} \hat{j}_n(kr)\, P_n'(\cos\theta).$$

$$\frac{d^2}{dr^2}\hat{j}_n(kr) + k^2 \hat{j}_n(kr) = \frac{n(n+1)}{r^2}\hat{j}_n(kr)$$

$$A_{mr}^i = \frac{E_0}{\omega\eta}\sin\phi \sum_{n=1}^{\infty} \frac{(i)^n(2n+1)}{n(n+1)} \underbrace{\hat{j}_n(kr)\, P_n'(\cos\theta)}_{SWE_{ij}}.$$

> SWF are orthogonal!

$SWE_{ij} \longleftrightarrow SWE_{ij}$



$$A_r^s = \frac{E_0}{\omega} \cos\phi \sum_{n=1}^{\infty} a_n \, \hat{h}_n^{(1)}(kr) P_n'(\cos\theta)$$

$$A_{mr}^s = \frac{E_0}{\omega\eta} \sin\phi \sum_{n=1}^{\infty} b_n \, \hat{h}_n^{(1)}(kr) P_n'(\cos\theta)$$

We apply BC to find a_n, b_n.

$$E_\theta^i + E_\theta^s \Big|_{r=a} = 0 \; ; \; E_\phi^i + E_\phi^s \Big|_{r=a} = 0 .$$

$$a_n = -\frac{(i)^n (2n+1)}{n(n+1)} \frac{\hat{j}_n'(ka)}{\hat{h}_n^{(1)\prime}(ka)}$$

$$b_n = -\frac{(i)^n (2n+1)}{n(n+1)} \frac{\hat{j}_n(ka)}{\hat{h}_n^{(1)}(ka)}$$

$\vec{E}^s, \vec{H}^s$ can be derived from A_r^s, A_{mr}^s

(Eq: 8.139 Pg 549)

Far-Field

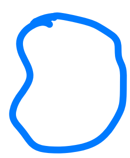
$$\lim_{kr\to\infty} \hat{h}_n^{(1)\prime}(kr) \simeq (-i)^n e^{ikr}$$

$$E_\theta^s = -iE_0 \cos\phi \frac{e^{ikr}}{kr} \sum_{n=1}^{\infty} (-i)^n \left(a_n \sin\theta\, P_n''(\cos\theta) - b_n \frac{P_n'(\cos\theta)}{\sin\theta} \right)$$

$$E_\phi^s = -iE_0 \sin\phi \frac{e^{ikr}}{kr} \sum_{n=1}^{\infty} (-i)^n \left(a_n \frac{P_n'(\cos\theta)}{\sin\theta} - b_n \sin\theta\, P_n''(\cos\theta) \right)$$

Radar cross section (RCS)

$$\sigma_c(\theta,\phi) = \lim_{r\to\infty} 4\pi r^2 \frac{|E_\theta^s|^2}{|E_0|^2}$$

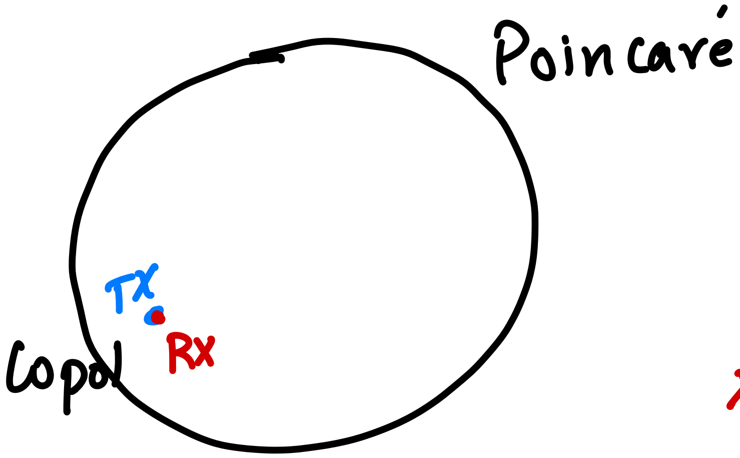


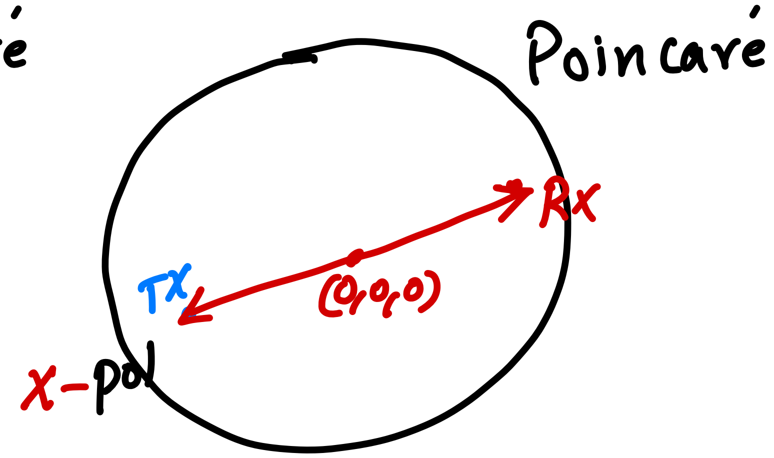
monostatic RCS : TX< RX<

bistatic RCS : TX<

RX

$\sigma(\theta_T, \phi_T; \theta_R, \phi_R)$



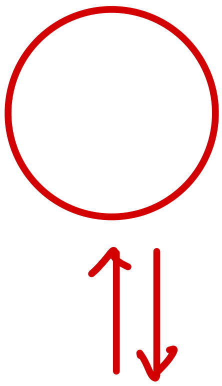


$$\sigma_c(\theta,\phi) = \frac{\lambda^2\cos^2\phi}{\pi}\left|\sum_{n=1}^{\infty}(-i)^n\left(a_n \sin\theta\, P_n''(\cos\theta) - b_n \frac{P_n'(\cos\theta)}{\sin\theta}\right)\right|^2$$

$$\sigma_x(\theta,\phi) = \cdots$$

$\theta = \pi$, co-pol : $\phi = 0$

$\phi = \pi/2$



$$\sigma_c = \frac{\lambda^2}{4\pi}\left|\sum_{n=1}^{\infty}(i)^n n(n+1)(a_n - b_n)\right|^2$$

$$\sigma_x = 0$$

Low-freq approx.

$a \ll \lambda$; $a < 0.1\lambda$

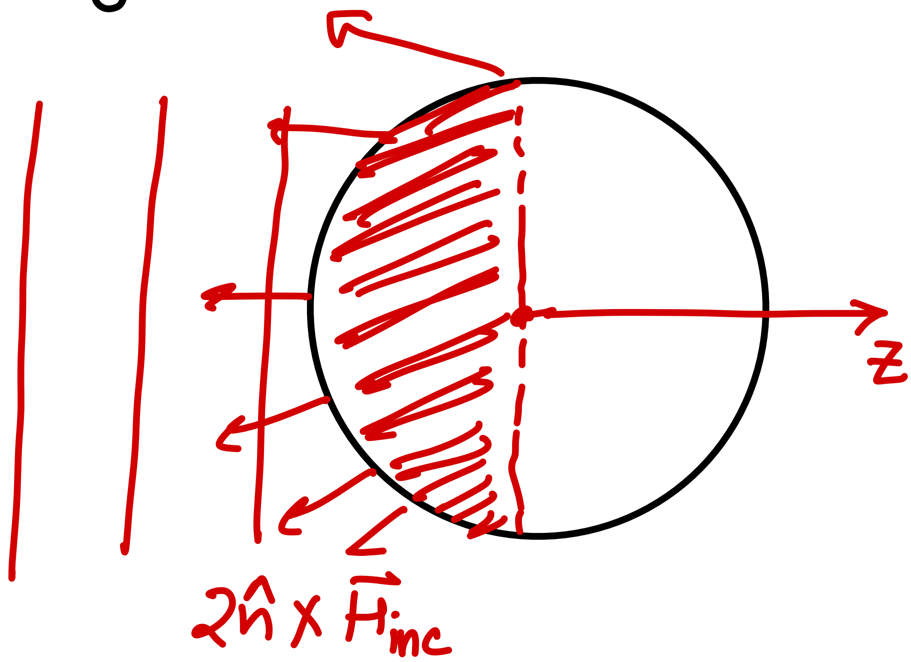
$$\sigma_c^{LF} = \frac{9\lambda^2}{4\pi}(ka)^6$$

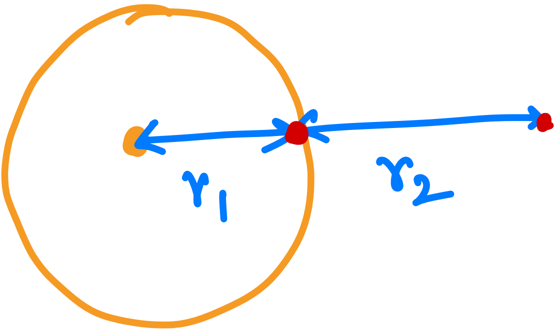
$$\boxed{\sigma_c^{LF} \propto \frac{1}{\lambda^4}}$$

High freq approx

$a \gg \lambda \Rightarrow$ Physical Optics (PO)

$\sigma_c^{PO} = \pi a^2$





$$\frac{1}{r_1^2} \cdot \frac{1}{r_2^2}$$

$$\frac{1}{r^2} \qquad \frac{1}{(r_1+r_2)^2}$$