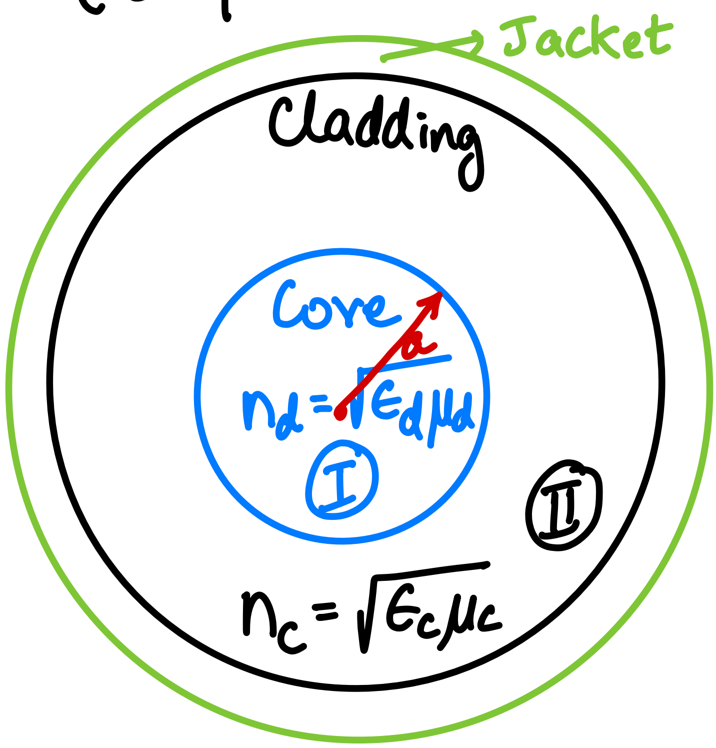
Circular Dielectric Waveguides

(Step Index Fiber Optic Cables)



> 1550nm (loss 0.2dB/km)

> 1310nm (lowest dispersion)

> $n_d \simeq 1.45$ Doped Silica (SiO_2)

> $n_c \simeq 1.44$ Pure Silica

$$\left(k_\rho^{I}\right)^2 = k_d^2 - \beta^2 = \omega^2 \mu_d \epsilon_d - \beta^2$$

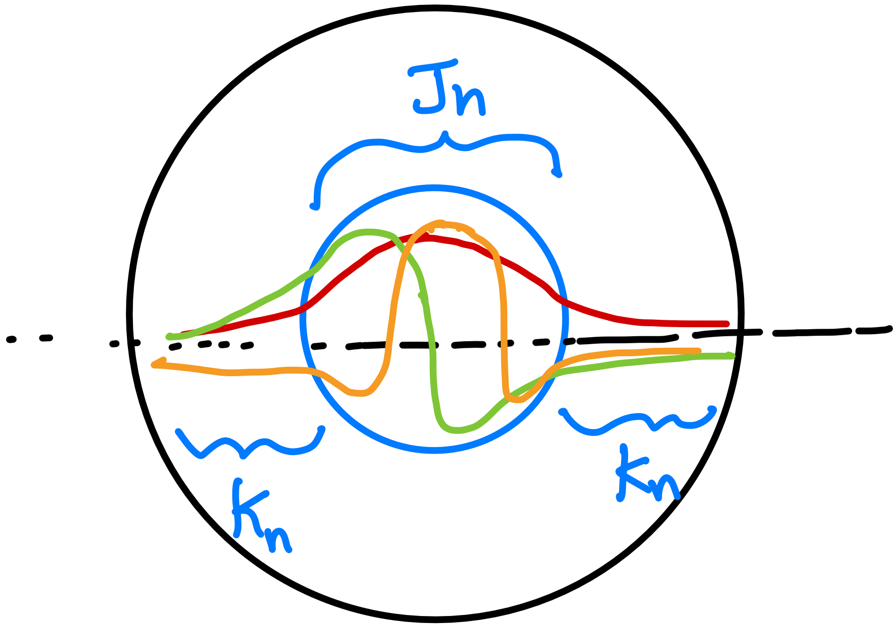
$$\left(k_\rho^{II}\right)^2 = k_c^2 - \beta^2 = \omega^2 \mu_c \epsilon_c - \beta^2 = -\nu^2$$

$$\Rightarrow k_\rho^{II} = i\nu \quad (\nu \text{ is real}).$$

$$\Psi_I = J_n(k_\rho^{I} \rho)\, e^{in\phi} \longrightarrow \text{core}$$

$$\Psi_{II} = H_n^{(1)}(k_\rho^{II} \rho)\, e^{in\phi} \longrightarrow \text{cladding}$$

$$= \frac{2}{\pi}(i)^{n+1} \underbrace{K_n(\nu\rho)}_{\text{modified Bessel fn. of 2nd kind.}} \quad ; \; k_\rho^{II} = i\nu$$



Fields

In general, TE & TM alone cannot satisfy the BCs.

In region I

$$E_z^{(I)} = (k_\rho^I)^2 A_n J_n(k_\rho^I \rho) e^{in\phi} e^{i\beta z}$$

$$H_z^{(I)} = (k_\rho^I)^2 B_n J_n(k_\rho^I \rho) e^{in\phi} e^{i\beta z}$$

In region II

$$E_z^{II} = (k_\rho^{II})^2 C_n H_n^{(1)}(k_\rho^{II}\rho) e^{in\phi} e^{i\beta z}$$

$$H_z^{II} = (k_\rho^{II})^2 D_n H_n^{(1)}(k_\rho^{II}\rho) e^{in\phi} e^{i\beta z}.$$

Similarly we get E_ϕ^{I}, H_ϕ^{I}, E_ϕ^{II} & H_ϕ^{II}.

Applying the boundary conditions:

> $E_z^{I} = E_z^{II}$ & $H_z^{I} = H_z^{II}$ we get

$$(k_\rho^{I})^2 A_n J_n(k_\rho^{I} a) = (k_\rho^{II})^2 C_n H_n^{(1)}(k_\rho^{II} a) \quad \text{— (1)}$$

$$(k_\rho^{I})^2 B_n J_n(k_\rho^{I} a) = (k_\rho^{II})^2 D_n H_n^{(1)}(k_\rho^{II} a) \quad \text{— (2)}$$

> $E_\phi^{I} = E_\phi^{II}$ & $H_\phi^{I} = H_\phi^{II}$ give:

$$\frac{n\beta}{a} J_n(k_\rho^{I} a) A_n + i\omega\mu_d k_\rho^{I} J_n'(k_\rho^{I} a) B_n$$

$$= \frac{n\beta}{a} H_n^{(1)}(k_\rho^{II} a) C_n + i\omega\mu_c k_\rho^{II} H_n^{(1)\prime}(k_\rho^{II} a) D_n . \quad ③$$

$$i\omega\epsilon_d k_\rho^{I} J_n^{\prime}(k_\rho^{I} a) A_n - \frac{n\beta}{a} J_n(k_\rho^{I} a) B_n =$$

$$i\omega\epsilon_c k_\rho^{II} H_n^{(1)\prime}(k_\rho^{II} a) C_n - \frac{n\beta}{a} H_n^{(1)}(k_\rho^{II} a) D_n . \quad ④$$

Sub ①, ② in ③

$$B_n = \frac{in\beta}{\omega}\left[\frac{1}{(k_\rho^{I}a)^2} - \frac{1}{(k_\rho^{II}a)^2}\right]$$

$$\left[\frac{\mu_d}{k_\rho^{I}a}\frac{J_n^{\prime}(k_\rho^{I}a)}{J_n(k_\rho^{I}a)} - \frac{\mu_c}{k_\rho^{II}a}\frac{H_n^{(1)\prime}(k_\rho^{II}a)}{H_n^{(1)}(k_\rho^{II}a)}\right]^{-1} A_n$$

$$u = k_\rho^{I} a \; ; \; iv = k_\rho^{II} a = i\nu a$$

& recall $H_n^{(1)}(i\nu r) = \frac{2}{\pi}(i)^{n+1} K_n(\nu r)$

$$\Rightarrow \frac{H_n^{(1)\prime}(i\nu a)}{H_n^{(1)}(i\nu a)} = -i\frac{K_n^{\prime}(\nu a)}{K_n(\nu a)}$$

$$B_n = \frac{in\beta}{\omega}\left(\frac{1}{u^2}+\frac{1}{v^2}\right)\left[\frac{\mu_d}{u}\frac{J_n'(u)}{J_n(u)} + \frac{\mu_c}{v}\frac{K_n'(v)}{K_n(v)}\right]^{-1} A_n \quad (5)$$

Similarly sub (1), (2) in (4)

$$B_n = \frac{i\omega}{n\beta}\left(\frac{1}{u^2}+\frac{1}{v^2}\right)^{-1}\left[\frac{\epsilon_d}{u}\frac{J_n'(u)}{J_n(u)} + \frac{\epsilon_c}{v}\frac{K_n'(v)}{K_n(v)}\right] A_n \quad (6)$$

Transcendental Equation

$$\omega^2\left[\frac{\mu_d}{u}\frac{J_n'(u)}{J_n(u)} + \frac{\mu_c}{v}\frac{K_n'(v)}{K_n(v)}\right]\left[\frac{\epsilon_d}{u}\frac{J_n'(u)}{J_n(u)} + \frac{\epsilon_c}{v}\frac{K_n'(v)}{K_n(v)}\right]$$

$$= n^2\beta^2\left[\frac{1}{u^2}+\frac{1}{v^2}\right]$$

$$k_p^{I\,2} = k_d^2 - \beta^2 \qquad v^2 = \beta^2 - k_c^2$$

$$\Rightarrow u^2 + v^2 = a^2\left(k_d^2 - k_c^2\right)$$

Solve for $\underbrace{u, v}$

$\downarrow$

$\beta, \frac{B_n}{A_n}, \frac{C_n}{A_n}, \frac{D_n}{A_n}$

TE/TM modes $(n=0) \rightarrow$ Azimuthally invariant!

$$\omega^2 \underbrace{\left[\frac{\mu_d}{u}\frac{J_n'(u)}{J_n(u)} + \frac{\mu_c}{v}\frac{K_n'(v)}{K_n(v)}\right]}_{0}\underbrace{\left[\frac{\epsilon_d}{u}\frac{J_n'(u)}{J_n(u)} + \frac{\epsilon_c}{v}\frac{K_n'(v)}{K_n(v)}\right]}_{0}$$

$$= \cancel{n^2\beta^2\left[\frac{1}{u^2}+\frac{1}{v^2}\right]} \rightarrow 0$$

$$\frac{\mu_d}{u}\frac{J_0'(u)}{J_0(u)} + \frac{\mu_c}{v}\frac{K_0'(v)}{K_0(v)} = 0$$

$\Rightarrow A_0 = 0 \Rightarrow$ TE mode

Similarly

$$\frac{\epsilon_d}{u}\frac{J_0'(u)}{J_0(u)} = -\frac{\epsilon_c}{v}\frac{K_0'(v)}{K_0(v)}$$ $\Rightarrow$ TM modes since $B_0 = 0$.

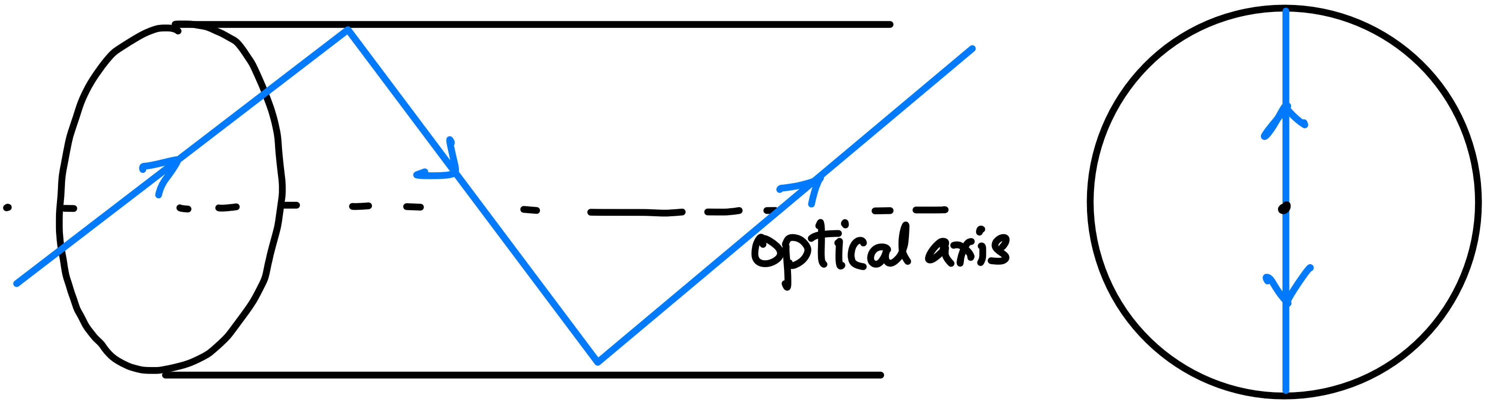
Note that $J_0'(u) = -J_1(u)$ & $K_0'(v) = -K_1(v)$

$\left(J_0' = \frac{1}{2}\left[J_{-1} - J_1\right] \text{ \& } J_{-1} = -J_1\right)$

$$\Rightarrow \frac{\mu_d}{u}\frac{J_1(u)}{J_0(u)} = -\frac{\mu_c}{v}\frac{K_1(v)}{K_0(v)} \longrightarrow TE$$

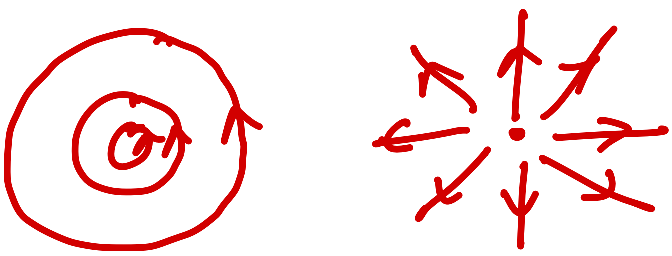
$$\frac{\epsilon_d}{u}\frac{J_1(u)}{J_0(u)} = -\frac{\epsilon_c}{v}\frac{K_1(v)}{K_0(v)} \longrightarrow TM$$

Solve with $u^2+v^2 = a^2(k_d^2-k_c^2)$ for u, v $\Rightarrow \beta$.



"Meridional rays"

→ No azimuthal variation ⇒ null in the center!



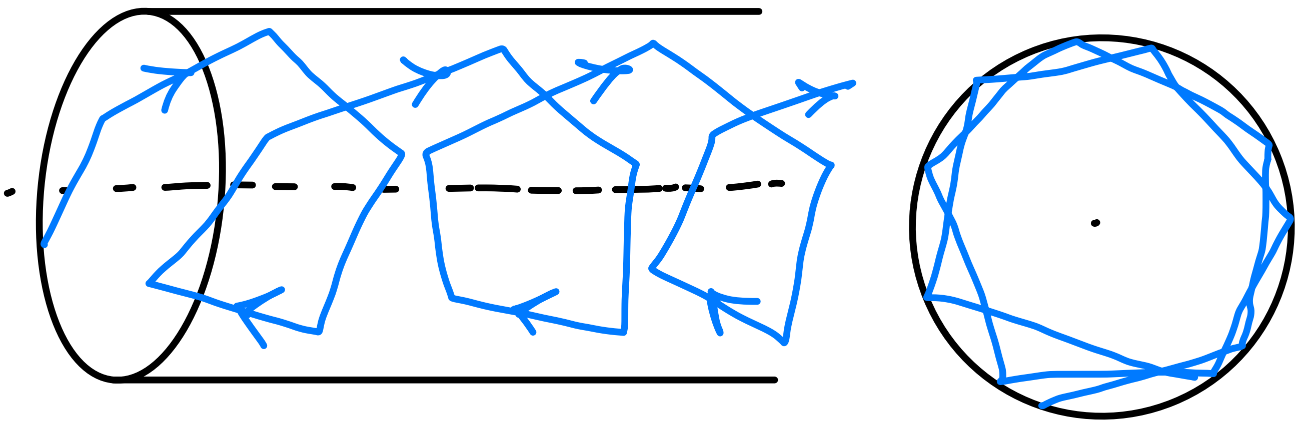
⇒ TE, TM are <u>not</u> the fundamental mode!

⇒ The fundamental mode is TE + TM × is a Hybrid modes.

Hybrid Modes

EH $\Rightarrow$ mostly TM $\quad |H_z| \ll |E_z|$

HE $\Rightarrow$ mostly TE $\quad |E_z| \ll |H_z|$



$$\omega^2 \left[\frac{\mu_d}{u} \frac{J_n'(u)}{J_n(u)} + \frac{\mu_c}{v} \frac{K_n'(v)}{K_n(v)} \right] \left[\frac{\epsilon_d}{u} \frac{J_n'(u)}{J_n(u)} + \frac{\epsilon_c}{v} \frac{K_n'(v)}{K_n(v)} \right]$$

$$= n^2 \beta^2 \left[\frac{1}{u^2} + \frac{1}{v^2} \right]$$

Weakly Guiding Approx.

$n_d \simeq n_c \simeq n \qquad (\epsilon_d \simeq \epsilon_c \simeq \epsilon \ \ \& \ \ \mu_d = \mu_c = \mu)$

Caution: This is only to solve the Trans. Eq.

Under WGA Hybrid modes → Linearly Polarized modes (LP modes).

Transcendental Eq:

$$\left[\frac{J_n'(u)}{u J_n(u)} + \frac{K_n'(v)}{v K_n(v)}\right]^2 = n^2\left(\frac{1}{u^2} + \frac{1}{v^2}\right)$$

$$\left(\beta = \omega\sqrt{\mu\epsilon} \leftarrow \text{WGA}\right)$$

Using Bessel Identities, it reduces to

$$\frac{J_{n\pm1}(u)}{u J_n(u)} = \mp \frac{K_{n\pm1}(v)}{v K_n(v)}$$

$+ \Rightarrow$ EH modes, $- \Rightarrow$ HE modes

Reindexing from n to j

$$u\,\frac{J_{j-1}(u)}{J_j(u)} = -v\,\frac{K_{j-1}(v)}{K_j(v)}$$

$j=1 \Rightarrow$ TE, TM modes $\longrightarrow$ degenerate modes.

$j=n+1 \Rightarrow EH_n$ modes

$j=n-1 \Rightarrow HE_n$ modes

EH_{n-1} is degenerate with HE_{n+1}

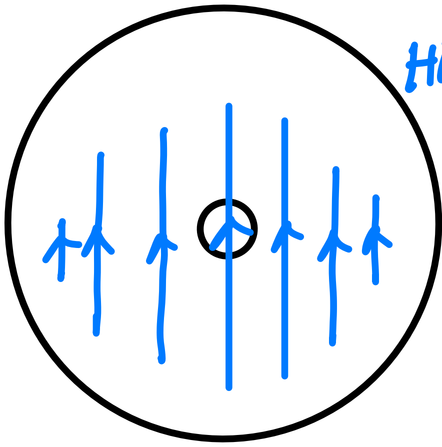
> Combining degeneracies in EH/HE basis to get LP modes.

$LP_{1m} \longrightarrow$ Sum of $(TE_{0m}$ & $TM_{0m})$ & (HE_{2m}, EH_{1m})

$LP_{\nu m} \longrightarrow HE_{n+1,m}$ & $EH_{n-1,m}$.

$LP_{0m} \longrightarrow HE_{1m}$ {special case}

* $LP_{01} \simeq HE_{11}$ mode is the fundamental mode! It has no cut-off.



HE11.

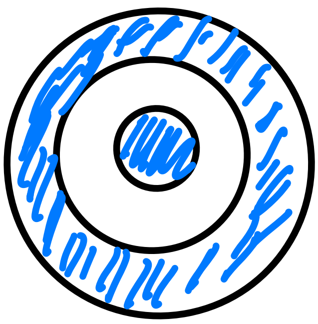
> Modes of the Fiber

① $HE_{11} \Leftrightarrow LP_{01}$

② $\left.\begin{array}{l} TE_{01} \\ TM_{01} \\ HE_{21} \\ EH_{11} \end{array}\right\} \Leftrightarrow LP_{11}$

$V \simeq 2.405$

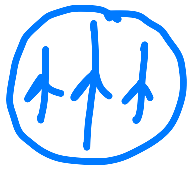
③ $HE_{12} \Leftrightarrow LP_{02}$



$V \simeq 3.83$

V-number: $V = \frac{2\pi}{\lambda} a \sqrt{n_d^2 - n_c^2}$

$V < 2.405 \Rightarrow$ Single mode fiber!



<u>For large V numbers,</u>

$\#\text{ modes} \simeq \dfrac{4V}{\pi^2} \longrightarrow$ multimodal fibers.

▷ Polarization Maintaining (PM) fiber.