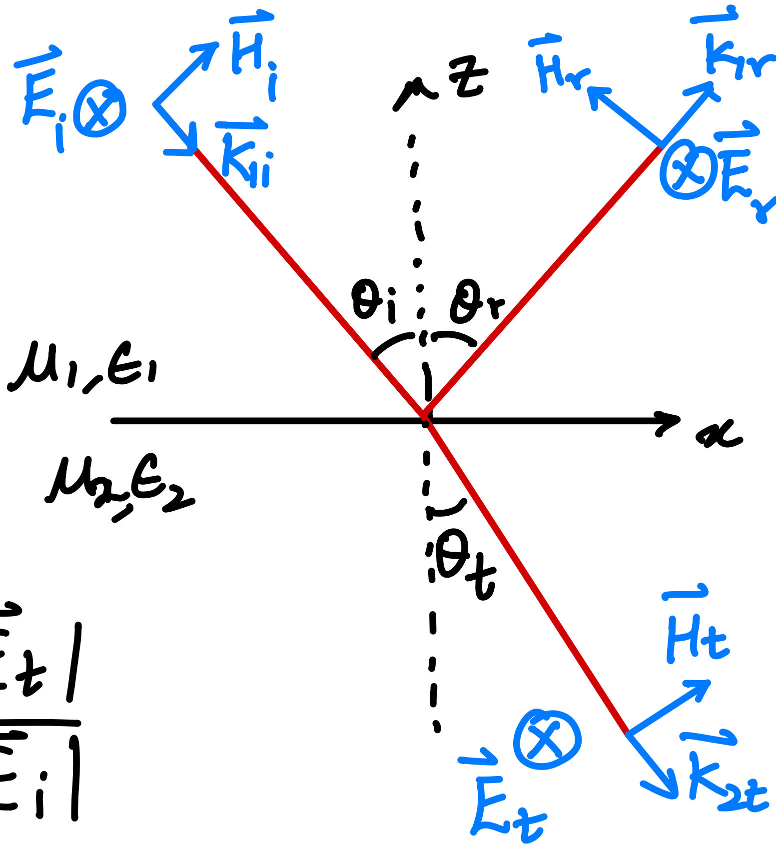
Fresnel Reflection & Transmission

TE_z Transverse

Electric to z

S-polarization or

Perpendicular pol.



$$\Gamma_{TE} = \frac{|\vec{E}_r|}{|\vec{E}_i|}, \quad \tau_{TE} = \frac{|\vec{E}_t|}{|\vec{E}_i|}$$

$$\vec{E}_i = E_0 \, e^{i\vec{k}_{1i}\cdot\vec{r}} \, \hat{y}$$

$$\vec{E}_r = \Gamma_{TE} E_0 \, e^{i\vec{k}_{1r}\cdot\vec{r}} \, \hat{y}$$

$$\vec{E}_t = \tau_{TE} E_0 \, e^{i\vec{k}_{2t}\cdot\vec{r}} \, \hat{y}$$

$$\vec{k}_{1i}\cdot\vec{r} = k^i_{1x} x - k^i_{1z} z = k_1 x \sin\theta_i - k_1 z \cos\theta_i$$

$$\vec{k}_{1r}\cdot\vec{r} = k^r_{1x} x + k^r_{1z} z = k_1 x \sin\theta_r + k_1 z \cos\theta_r$$

$$\vec{k}_{1t}\cdot\vec{r} = k^t_{2x} x - k^t_{2z} z = k_2 x \sin\theta_t - k_2 z \cos\theta_t$$

$$\vec{H}_i = \frac{E_0}{\eta_1} e^{i\vec{k}_i \cdot \vec{r}} (\cos\theta_i \hat{x} + \sin\theta_i \hat{z})$$

$$\vec{H}_r = \frac{\Gamma_{TE} E_0}{\eta_1} e^{i\vec{k}_{1r} \cdot \vec{r}} (-\cos\theta_r \hat{x} + \sin\theta_r \hat{z})$$

$$\vec{H}_t = \frac{\tau_{TE} E_0}{\eta_2} e^{i\vec{k}_{2t} \cdot \vec{r}} (\cos\theta_t \hat{x} + \sin\theta_t \hat{z})$$

Boundary Conditions

$$E_0 e^{ik_1(x\sin\theta_i)} + \Gamma_{TE} E_0 e^{ik_1(x\sin\theta_r)} = \tau_{TE} E_0 e^{ik_2(x\sin\theta_t)}$$

$$x = 0 \Rightarrow \boxed{1 + \Gamma_{TE} = \tau_{TE}}$$

$$\boxed{k_1 \sin\theta_i = k_1 \sin\theta_r = k_2 \sin\theta_t}$$

PHASE MATCHING!

$$k = \frac{\omega}{v} = \frac{\omega}{c} n$$

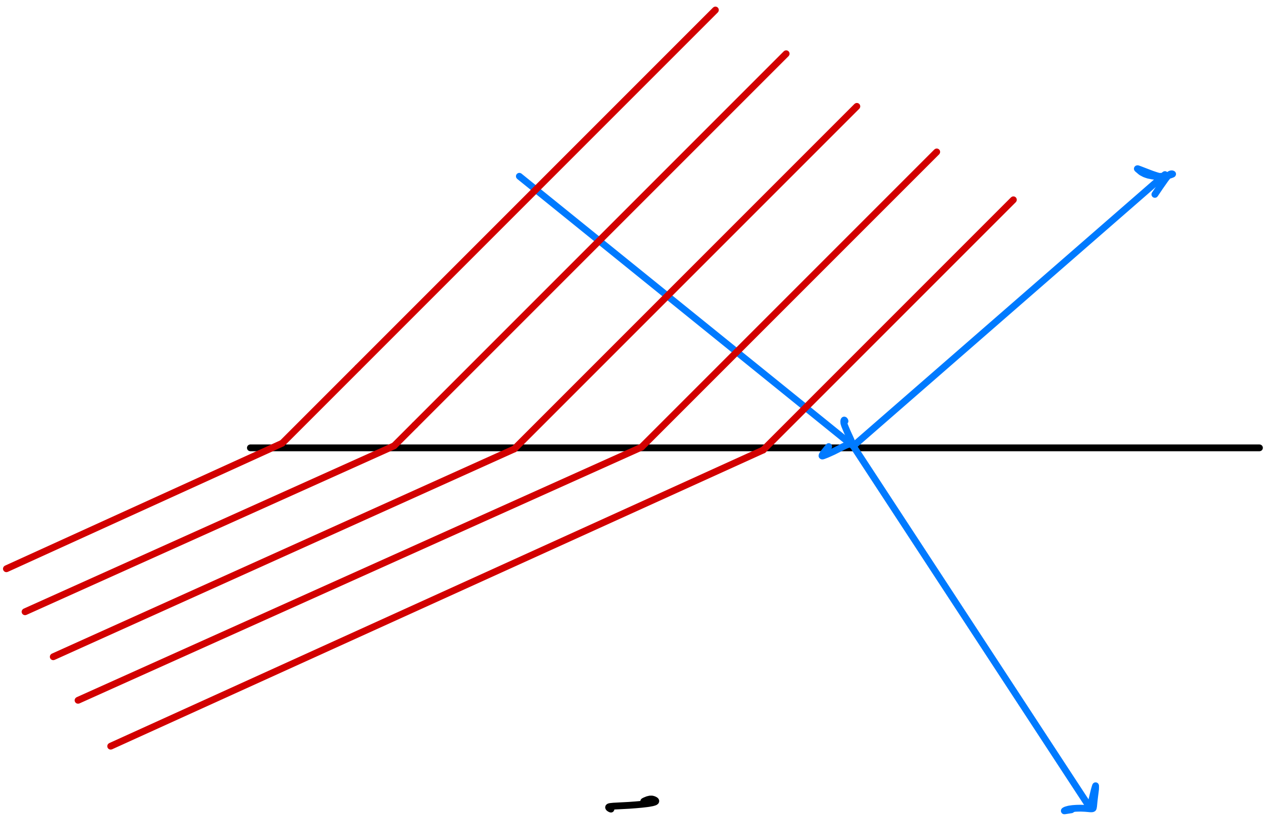
$n \rightarrow$ refractive index

$$k = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r} = \frac{\omega}{c}n.$$

$$n = \frac{c}{v} = \frac{\sqrt{\mu_0\epsilon_0}^{\,-1}}{\sqrt{\mu_0\epsilon_0}^{\,-1}\sqrt{\epsilon_r}^{\,-1}} = \sqrt{\epsilon_r}$$

$$\boxed{\theta_i = \theta_r} \qquad \boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

SNELL'S LAWS OF REFLECTION & TRANSMISSION.



Applying BC for $\bar{H}$ we get

$$\frac{\cos\theta_i}{\eta_1} - \frac{\Gamma_{TE}\cos\theta_r}{\eta_1} = \frac{\tau_{TE}\cos\theta_t}{\eta_2}$$

$$\frac{\cos\theta_i}{\eta_1}\left(1-\Gamma_{TE}\right) = \frac{\cos\theta_t}{\eta_2}\left(1+\Gamma_{TE}\right)$$

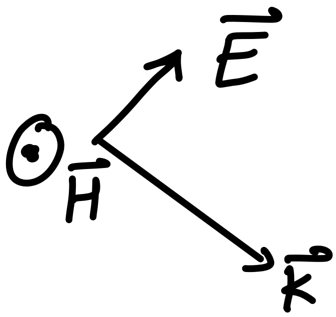
$$\boxed{\Gamma_{TE} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

$$\boxed{\tau_{TE} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

Fresnel Reflection & Transmission Coefficients.

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\,\frac{1}{\sqrt{\epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\cdot\frac{1}{n}$$

<u>TM_z pol.</u>



$$\Gamma_{TM} = \frac{-\eta_1\cos\theta_i + \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}$$

$$\tau_{TM} = 2\eta_2\cos\theta_i \,/\left(\eta_1\cos\theta_i + \eta_2\cos\theta_t\right)$$

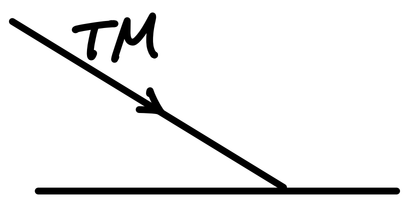
Brewster Angle

$$\Gamma_{TM} = 0 \Rightarrow \cos\theta_i = \frac{\eta_2}{\eta_1}\cos\theta_t$$

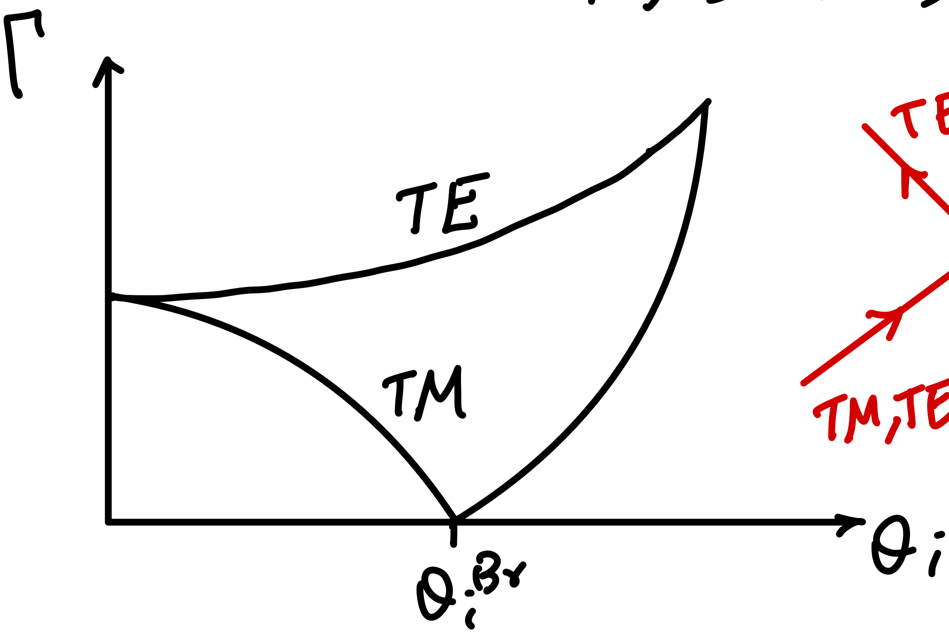
$$\sqrt{1-\sin^2\theta_t}$$

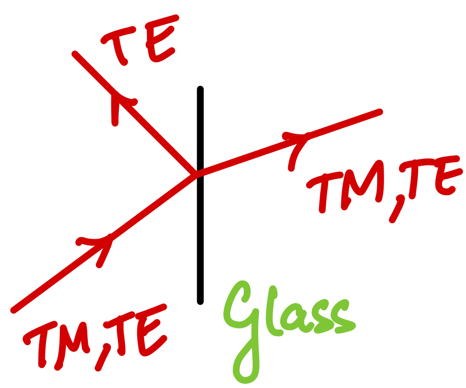
$$= \frac{\eta_2}{\eta_1}\sqrt{1-\frac{k_1^2}{k_2}\sin^2\theta_i}$$

$$\theta_i^{Br} = \sin^{-1}\sqrt{\frac{\epsilon_2(\mu_1\epsilon_2-\epsilon_1\mu_2)}{\mu_1(\epsilon_2^2-\epsilon_1^2)}} \underset{\mu_1=\mu_2}{=} \sqrt{\frac{\epsilon_2}{\epsilon_1+\epsilon_2}}$$

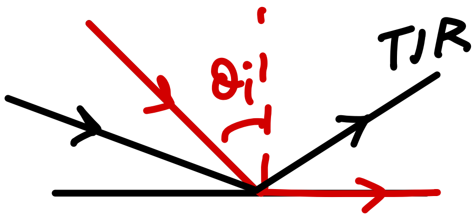


$$\Gamma_{TE} = 0 \Rightarrow \theta_i^{Br} = \sin^{-1}\sqrt{\frac{\mu_2(\mu_2\epsilon_1-\mu_1\epsilon_2)}{\epsilon_1(\mu_2^2-\mu_1^2)}} = \infty \quad (\mu_1=\mu_2)$$





Total Internal Reflection



$\mu_1 = \mu_2 \rightarrow$ assume.

$n_1 > n_2 \rightarrow$ WLOG

$$\theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_i^c\right) = 90^\circ$$

$$\theta_i^c = \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \rightarrow \text{Critical Angle.}$$

What happens to E_t beyond θ_c?

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\frac{\sin^2\theta_i}{\sin^2\theta_c}}$$

Since $\theta_i > \theta_c \Rightarrow \cos\theta_t$ is now imaginary!

$$\Rightarrow \cos\theta_t = i\alpha = i\sqrt{\frac{\sin^2\theta_i'}{\sin^2\theta_c}-1}$$

$$\alpha = \sqrt{\frac{\sin^2\theta_i}{\sin^2\theta_c}-1} = \frac{n_1}{n_2}\sqrt{\sin^2\theta_i - \frac{n_2^2}{n_1^2}}$$

$$= \frac{\eta_2}{\eta_1}\sqrt{\sin^2\theta_i - \frac{\eta_1^2}{\eta_2^2}}$$

$$E_t = \tau_{TM} E_0 e^{i\vec{k}_t \cdot \vec{r}}$$

$$= \tau_{TM} E_0 e^{i k_t (x \sin\theta_t + z\cos\theta_t)}$$

$$= \tau_{TM} E_0 e^{i k_t x \sin\theta_t} \underbrace{e^{-\alpha k_t z}}_{\text{evanescent along } z.}$$

$$\Gamma_{TM} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{\overbrace{i\alpha\eta_2}^{A} - \overbrace{\eta_1\cos\theta_i}^{B}}{i\alpha\eta_2 + \eta_1\cos\theta_i}$$

$$= \frac{iA - B}{iA + B} = -\frac{B - iA}{B + iA} = -e^{-i2\tan^{-1}\left(\frac{A}{B}\right)}$$

$$= e^{i\left(\pi - 2\tan^{-1}\left(\frac{A}{B}\right)\right)}$$

$$\Rightarrow \left|\Gamma_{TM}\right| = 1 \Rightarrow \text{TIR.}$$

$$\phi_{TM}(\theta_i) = \pi - 2\tan^{-1}\left(\frac{A}{B}\right)$$

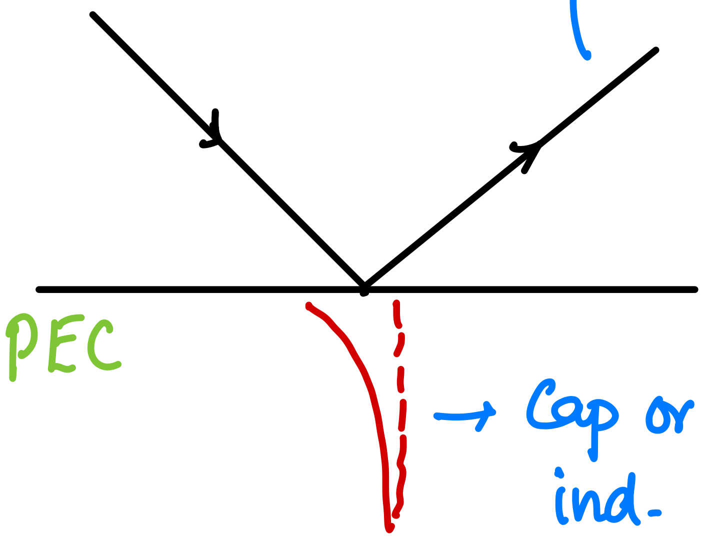
$$= 2\tan^{-1}\left(\frac{B}{A}\right) \quad \text{when } \frac{B}{A} > 0$$

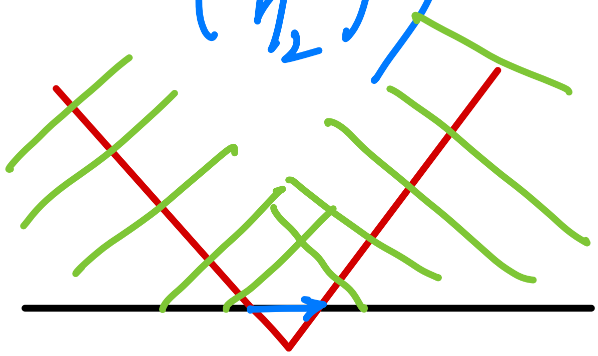
$$\phi_{TM}(\theta_i) = 2\tan^{-1}\left(\frac{\eta_1\cos\theta_i}{\alpha\,\eta_2}\right)$$

$$\Rightarrow \boxed{\phi_{TM}(\theta_i) = 2\tan^{-1}\left(\frac{\eta_1^2\cos\theta_i}{\eta_2^2\sqrt{\sin^2\theta_i - \frac{\eta_1^2}{\eta_2^2}}}\right)}$$

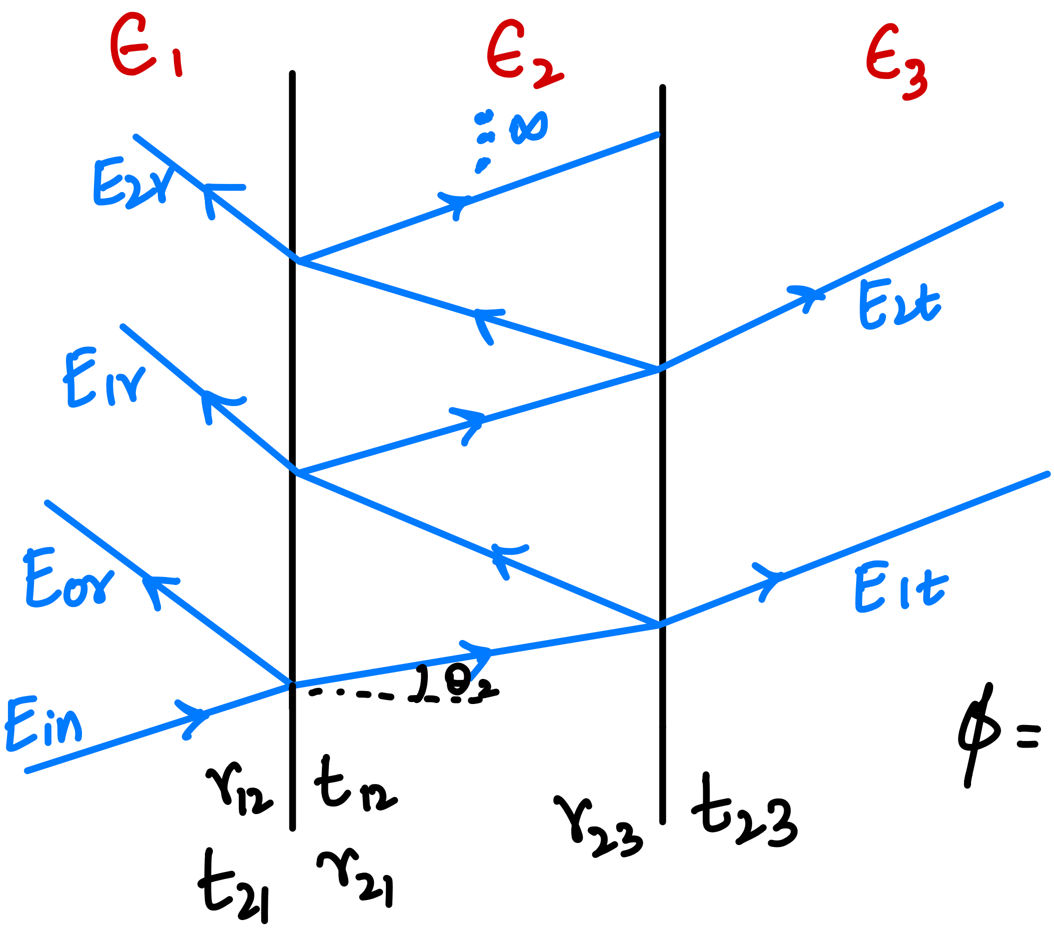
Goos Hanchen Phase Shift.

$$\phi_{TE} = 2\tan^{-1}\left(\frac{\eta_2\cos\theta_i}{\eta_1\sqrt{\sin^2\theta_i-\left(\frac{\eta_1}{\eta_2}\right)^2}}\right)$$





Three Dielectric Interface



$$\phi = k_2 \cos\theta_2 \, d$$

$$E_{1t} = E_{in} t_{12} e^{i\phi} t_{23}$$

$$E_{2t} = E_{in} t_{12} e^{i\phi} r_{23} e^{i\phi} r_{21} e^{i\phi} t_{23}$$

$$E_{(m+1)t} = E_{in} t_{12} t_{23} e^{i\phi} \left(r_{23} r_{21} e^{2i\phi} \right)^m$$

$$E_t = \sum_{m=0}^{\infty} E_{mt} = E_{in} t_{12} t_{23} e^{i\phi} \sum_{m=0}^{\infty} \left(r_{12} r_{23} e^{2i\phi} \right)^m$$

$$t = \frac{E_t}{E_{in}} = \frac{t_{12} t_{23} e^{i\phi}}{1 - r_{21} r_{23} e^{2i\phi}}$$

$$E_{0r} = r_{12} E_{in}$$

$$E_{1r} = t_{12} r_{23} t_{21} e^{2i\phi} E_{in}$$

$$E_{mr} = t_{12} t_{21} r_{23} e^{2i\phi} E_{in} \left(r_{21} r_{23} e^{2i\phi} \right)^m$$

$$\Rightarrow r = r_{12} + \frac{t_{12} t_{21} r_{23} e^{2i\phi}}{1 - r_{21} r_{23} e^{2i\phi}}$$

Let $3 = 1$, note $t_{12} t_{21} = 1 - r_{12}^2$ & $r_{21} = -r_{12}$

$$\boxed{r = \frac{r_{12}\left(1 - e^{2i\phi}\right)}{1 - r_{12}^2 e^{2i\phi}}}$$

When $2i\phi = 2m\pi \Rightarrow \boxed{k_2 d \cos\theta_t = m\pi}$

Fabry Perot Resonator!

Free spectral range.

$$FSR = \frac{c}{2 n_2 d \cos\theta}$$