EM18 - Plane Waves 2

Plane Waves in a Lossy Medium.

Loss $\Rightarrow$ complex ϵ $\Rightarrow$ $\epsilon = \epsilon_r + i\epsilon_i$

$K = \omega\sqrt{\mu\epsilon}$ $\Rightarrow$ $\boxed{k \text{ is also complex.}}$

$K = \beta + i\alpha$ $\rightarrow$ waves in 1-D problems.

$$\boxed{\vec{k} = \vec{k}' + i\vec{k}''} \quad ; \quad K = |\vec{k}|$$

$$\beta \neq |\vec{k}'| \; ; \; \alpha \neq |\vec{k}''|$$

$$\beta = \text{real}\left(|\vec{k}' + i\vec{k}''|\right)$$

$> \phi = \vec{k}\cdot\vec{r} \Rightarrow \nabla\phi = \vec{k}$ $\rightarrow$ what does a complex direction mean?

$$\vec{E} = \vec{E}_0 e^{i\vec{k}\cdot\vec{r}} = \vec{E}_0 e^{i(\vec{k}' + i\vec{k}'')\cdot\vec{r}}$$

$$= \vec{E}_0 e^{i\vec{k}'\cdot\vec{r}} e^{-\vec{k}''\cdot\vec{r}}$$

$$\vec{E} = \underbrace{E_0 e^{-\vec{k}''\cdot\vec{r}}}_{\text{Amplitude}} \; \underbrace{e^{i\vec{k}'\cdot\vec{r}}}_{\text{Phase}} \; \hat{E}_0$$

$\hat{E}_0 \rightarrow$ Polarization

$\vec{k}'$ is normal to equiphase surfaces.

$\vec{k}''$ is normal to equiamplitude surfaces.

$\vec{k}' \parallel \vec{k}'' \rightarrow$ Uniform Plane Waves.

$\vec{k}' \nparallel \vec{k}'' \rightarrow$ Nonuniform Plane Waves.

Examples

i) $\vec{k}'' = \vec{0} \Rightarrow$ classical plane waves.

ii) $\vec{k}'' \perp \vec{k}'$ Say $\vec{k}'' = k_z \hat{z}$

$\vec{k}' = k_x \hat{x} + k_y \hat{y}$

$$\vec{E} = \vec{E}_0 \, e^{-k_z z} e^{i(k_x x + k_y y)}$$

$\rightarrow$ evanescent wave.

$\rightarrow \hat{z}$

> Evanscent waves at the interface b/w two media

$\Rightarrow$ <u>Surface Waves</u>

ϵ_2

ϵ_1

$\Rightarrow$ <u>Surface Plasmons</u>

ϵ

metal

$\epsilon < 0$ (plasma)

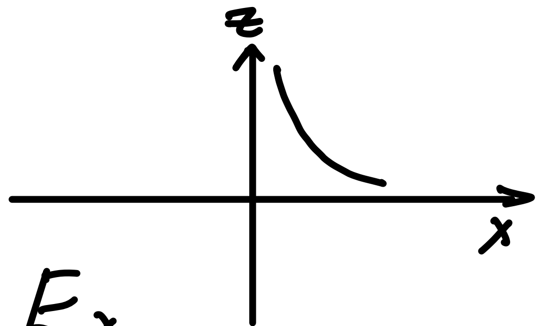
(Surface Plasmon Polaritons)

<u>Poynting Vector of Evanescent</u>

$$\vec{E} = \vec{E}_0 e^{ik_x x - k_z z}$$

, Let $\vec{E}_0 \xleftarrow{} $ TM: $\vec{E}_0 = E_x \hat{x} + E_z \hat{z}$

$$\vec{H} = H_y \hat{y}$$



$$\nabla \cdot \vec{E} = 0 \Rightarrow E_z = \frac{ik_x}{k_z} E_x$$

$$\vec{H} = \frac{i}{\omega\mu} \nabla \times \vec{E} \Rightarrow H_y = \frac{ik^2}{\omega\mu k_z} E_x.$$

$$\vec{E} = E_x\left(\hat{x} + i\frac{k_x}{k_z}\hat{z}\right)e^{ik_x x - k_z z}$$

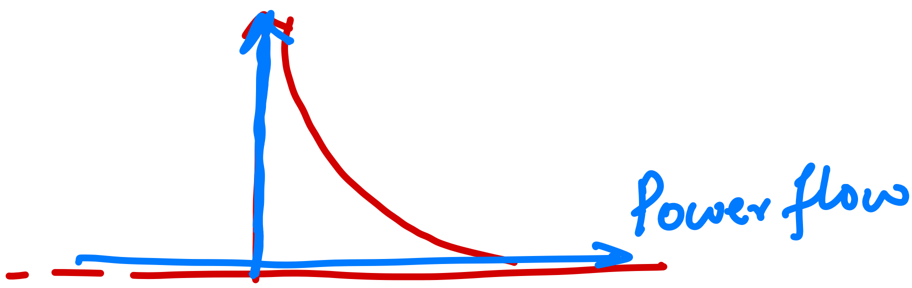
$$\vec{H} = \frac{ik^2}{\omega\mu k_z} E_x \hat{y}\, e^{ik_x x - k_z z}$$

$$\vec{S} = \vec{E} \times \vec{H}^* = S_x\hat{x} + S_z\hat{z}$$

$$S_x = \frac{k_x k^2}{\omega\mu k_z^2}|E_x|^2 e^{-2k_z z} \Big\} \text{ Real.}$$

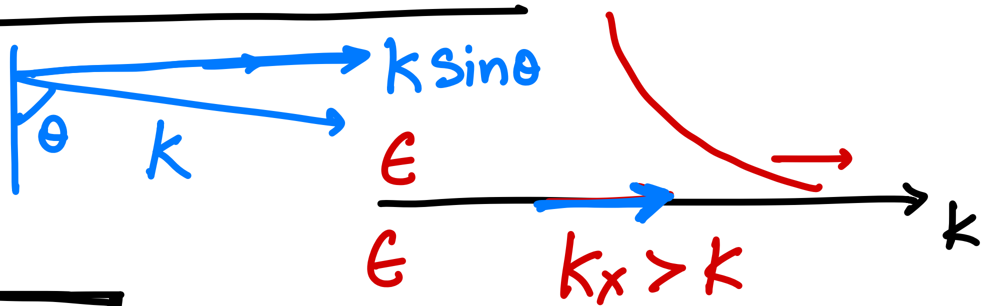
$$S_z = \frac{-ik^2}{\omega\mu k_z}|E_x|^2 e^{-2k_z z} \Big\} \text{ Imaginary.}$$

$\langle S \rangle = \frac{1}{2}\mathrm{Re}\{S\} = S_x$ component.



Exciting Evanescent Waves

$k_x > k$



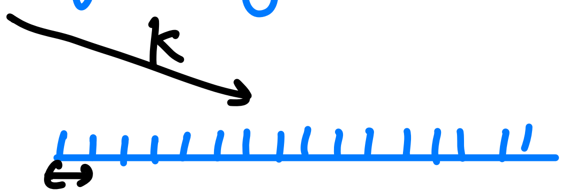
Phase matching

$$\boxed{\underbrace{k_x}_{>k} = \underbrace{k \sin\theta}_{\leq k}}$$

1) Use a dielectric

$k > k_0 \quad ; \; k = \sqrt{\epsilon_r}\, k_0$

2) Grating coupler



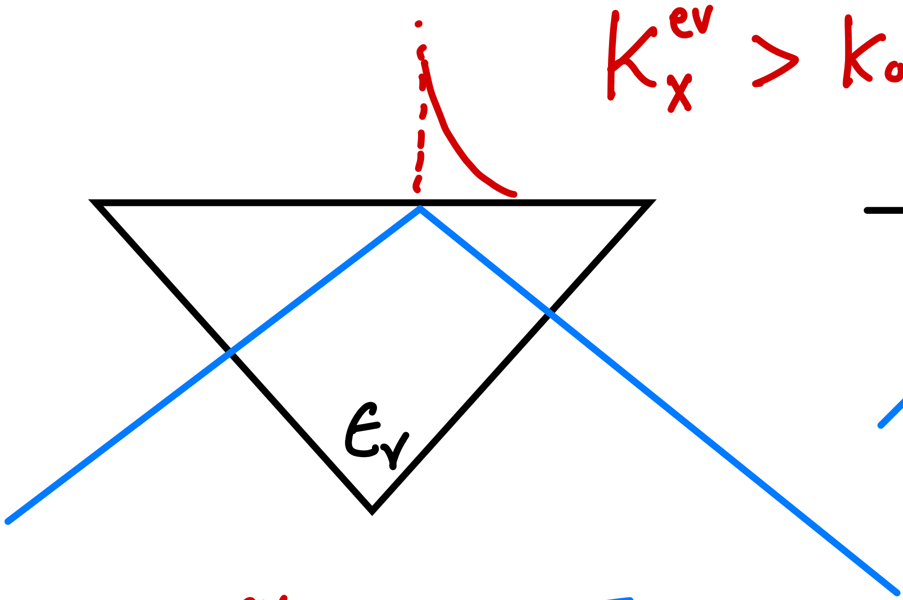
$\frac{2\pi}{\Lambda} = k_\Lambda$

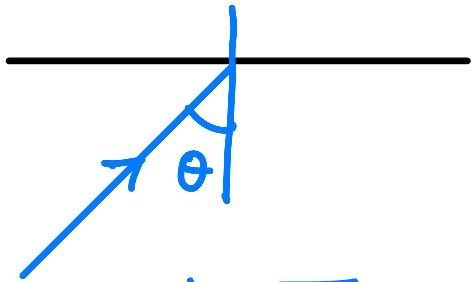
3) Slow wave structure.

$$k = \frac{\omega}{v}$$

i) Using a Dielectric

$K_x^{ev} > k_0$



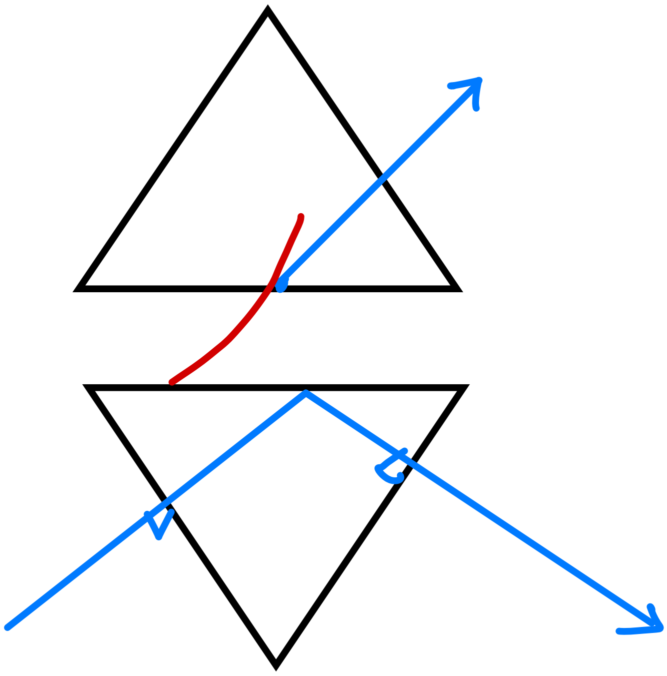


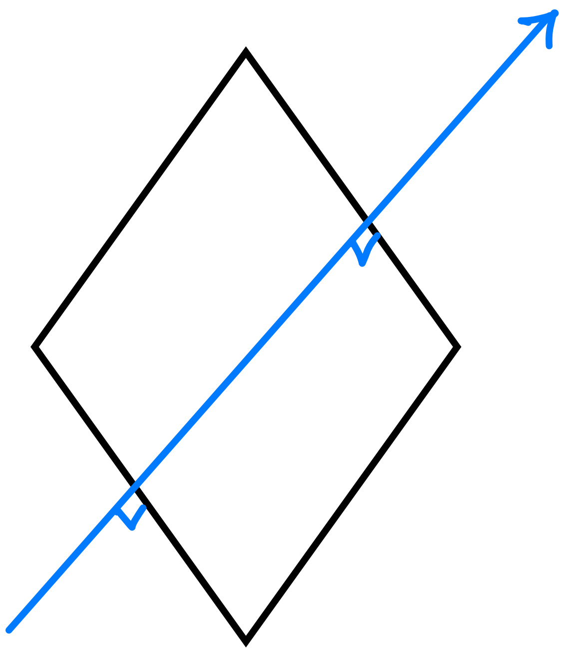
$k_0\sqrt{\epsilon_r}\sin\theta$

$> k_0$

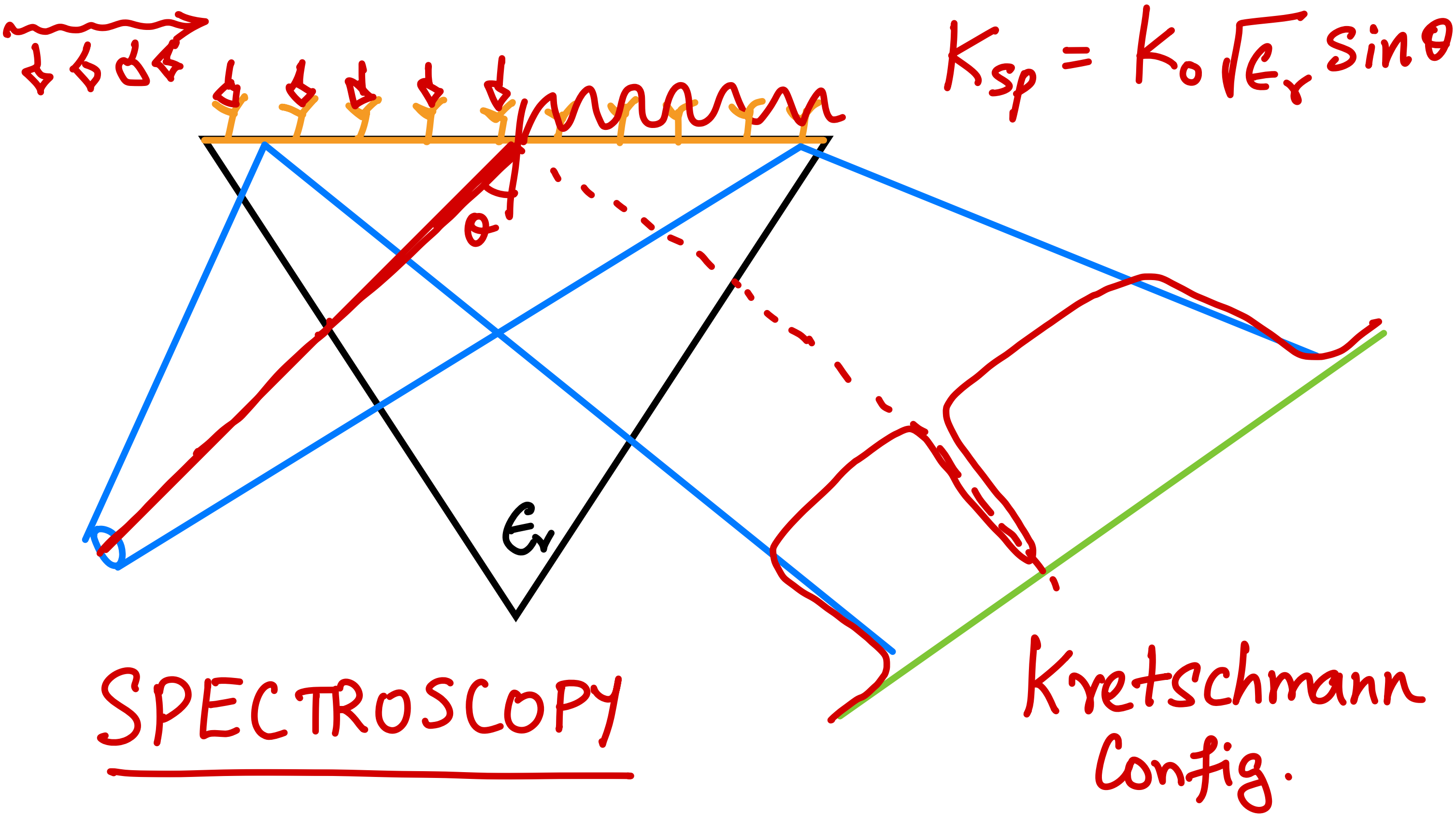
$$K_x^{ev} = k_0\sqrt{\epsilon_r}\sin\theta$$

Thought Exp





Surface Plasmon Resonance (SPR)



SPECTROSCOPY

Plane Waves in Anisotropic Media

$$\vec{D} = \bar{\bar{\epsilon}}\,\vec{E}, \quad \vec{B} = \bar{\bar{\mu}}\,\vec{H} \quad ; \quad \bar{\bar{\mu}} \rightarrow \mu$$

$$\nabla \times \nabla \times \vec{E} - k^2 \bar{\bar{\epsilon}}_r \vec{E} = 0$$

We want solutions of the form $\vec{E}_0 e^{i\vec{k}\cdot\vec{r}}$.

$$\nabla \times \quad \begin{pmatrix} 0 & -\partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & -\partial/\partial x \\ -\partial/\partial y & \partial/\partial x & 0 \end{pmatrix}$$

$$\begin{pmatrix} k_y^2+k_z^2-k^2\epsilon_{xx} & -(k_yk_x+k^2\epsilon_{xy}) & -(k_xk_z+k^2\epsilon_{xz}) \\ -(k_yk_x+k^2\epsilon_{yx}) & k_x^2+k_y^2-k^2\epsilon_{yy} & -(k_yk_z+k^2\epsilon_{yz}) \\ -(k_xk_z+k^2\epsilon_{zx}) & -(k_yk_z+k^2\epsilon_{zy}) & k_x^2+k_y^2-k^2\epsilon_{zz} \end{pmatrix} \cdot \vec{E}_0 = 0$$

$$\Delta = | \quad | = 0$$ gives a dispersion relation.

Special case (Uniaxial medium)

$$\bar{\bar{\epsilon}}_r = \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_x & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$$

$$\Delta = -k^2(\epsilon_x k^2 - k_x^2 - k_y^2 - k_z^2)[\epsilon_x(\epsilon_z k^2 - k_x^2 - k_y^2) - \epsilon_z k_z^2] = 0$$

$$\Rightarrow \begin{cases} k_x^2 + k_y^2 + k_z^2 = k^2 \epsilon_x & \longrightarrow \text{Ordinary wave.} \\ k_x^2 + k_y^2 + \dfrac{\epsilon_z}{\epsilon_x} k_z^2 = k^2 \epsilon_z & \longrightarrow \text{Extra ordinary wave.} \end{cases}$$

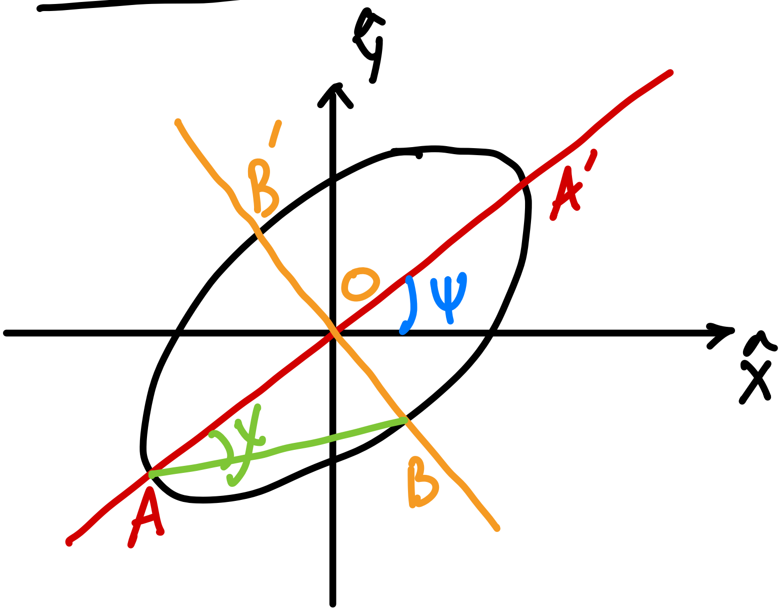
BIREFRINGENCE.

OW $\rightarrow$ k-space is a sphere.

EOW $\rightarrow$ k-space is Spheroid

$\boxed{\text{Wave Plate}}$

Poincaré Sphere



$\psi \rightarrow$ Tilt angle.

$\chi \rightarrow$ Ellipticity angle.

$$AR = \frac{OA}{OB}$$

$$\Rightarrow \chi = \tan^{-1}\left(\frac{1}{AR}\right)$$

$\chi = 0 \Rightarrow$ linear.

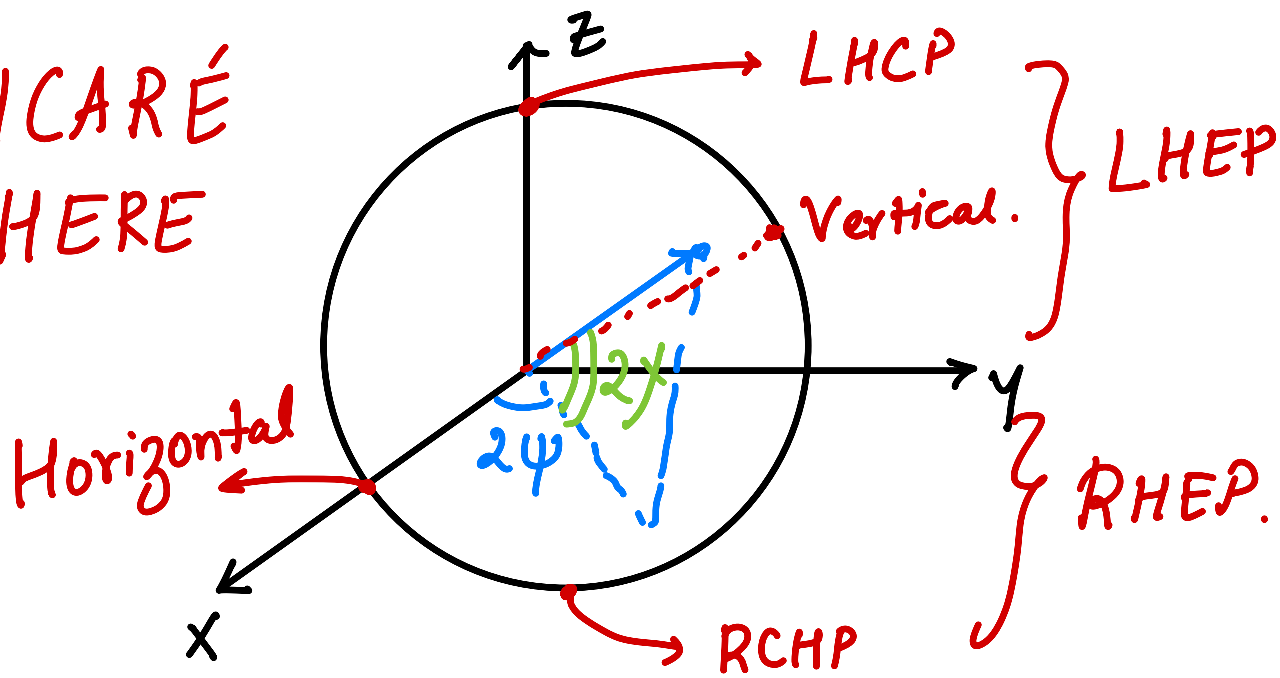
$\chi = 45° \Rightarrow$ Circular

$\underset{\text{RHCP}}{-45} \le \chi \le \underset{\text{LHCP}}{45°}$ } IEEE def.

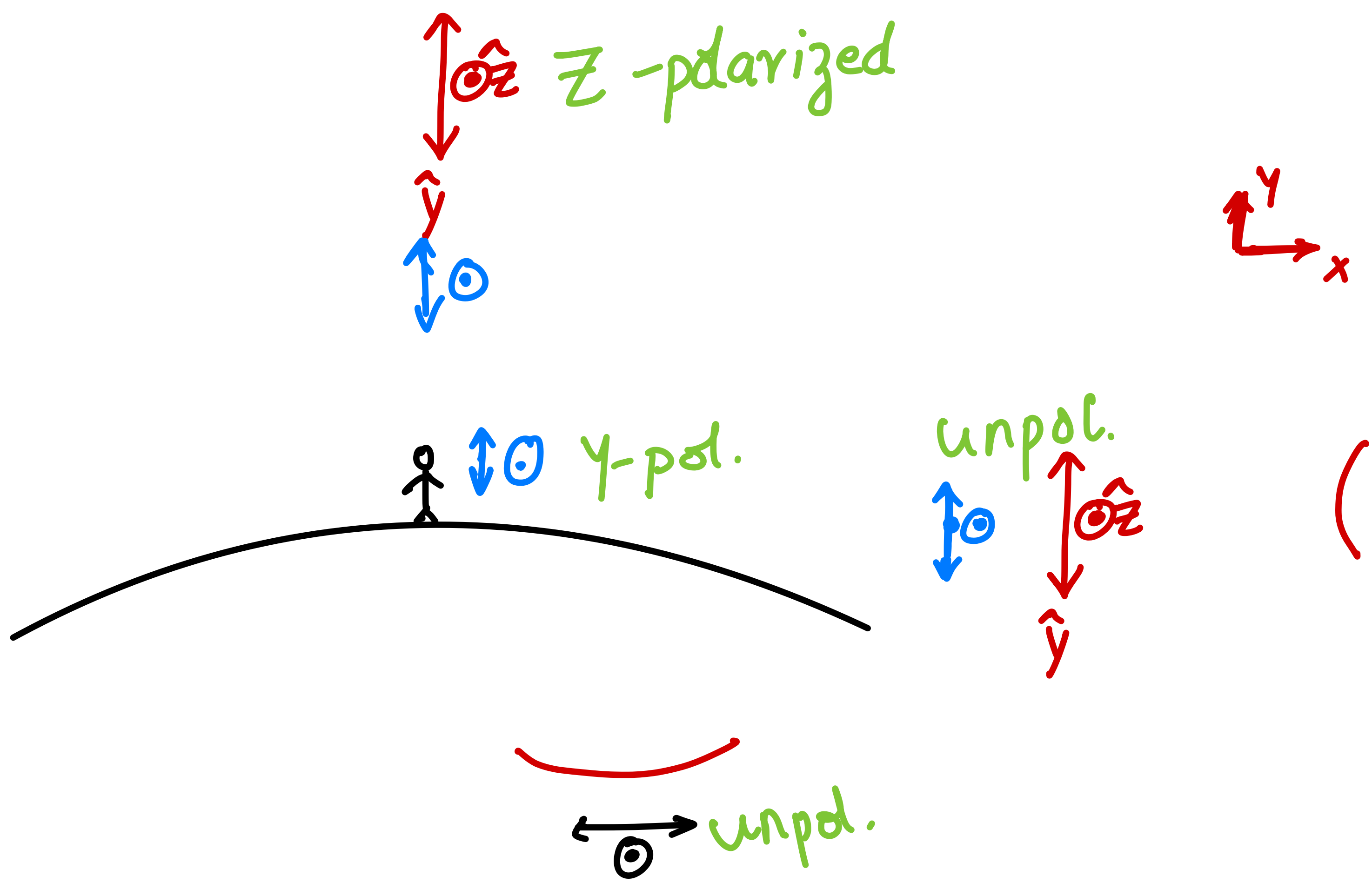
$\psi \Rightarrow \quad 0° \le \psi \le 180°$

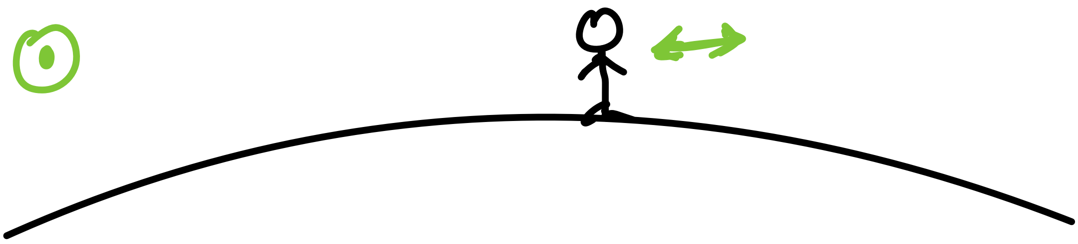
Note: $\quad 0 \le 2\psi \le 360°$; $-90° \le 2\chi \le 90°$

POINCARÉ SPHERE



Polarization of the sky





Bees use this pol. map to navigate

> Haidinger's Brush