Lec 13 – Reciprocity

$g(\vec{r},\vec{r}') = g(\vec{r}',\vec{r})$ → Derived in homogeneous.

Assume that ∃ two solutions to ME: a, b.

$$\nabla\times\vec{E_a} = i\omega\bar{\bar{\mu}}\,\vec{H_a} - \vec{J}_{ma}$$
$$\nabla\times\vec{H_a} = -i\omega\bar{\bar{\epsilon}}\,\vec{E_a} + \vec{J_a}$$
$$\nabla\times\vec{E_b} = i\omega\bar{\bar{\mu}}\,\vec{H_b} - \vec{J}_{mb}$$
$$\nabla\times\vec{H_b} = -i\omega\bar{\bar{\epsilon}}\,\vec{E_b} + \vec{J_b}$$

ϵ, μ are functions of space.

$$\nabla\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\nabla\times\vec{E} - \vec{E}\cdot\nabla\times\vec{H}$$

$$\nabla\cdot(\vec{E_a}\times\vec{H_b} - \vec{E_b}\times\vec{H_a})$$
$$= \vec{H_b}\cdot\nabla\times\vec{E_a} - \vec{E_a}\cdot\nabla\times\vec{H_b} - \vec{H_a}\cdot\nabla\times\vec{E_b} + \vec{E_b}\cdot\nabla\times\vec{H_a}$$
$$= \vec{H_b}\cdot(i\omega\bar{\bar{\mu}}\,\vec{H_a} - \vec{J}_{ma}) - \vec{E_a}\cdot(-i\omega\bar{\bar{\epsilon}}\,\vec{E_b} + \vec{J_b})$$
$$- \vec{H_a}\cdot(i\omega\bar{\bar{\mu}}\,\vec{H_b} - \vec{J}_{mb}) + \vec{E_b}\cdot(-i\omega\bar{\bar{\epsilon}}\,\vec{E_a} + \vec{J_a})$$

$$\vec{H}_b \cdot \bar{\bar{\mu}}\, \vec{H}_a = \bar{H}_b^T \bar{\bar{\mu}}\, \bar{H}_a$$
$$= \left(\bar{H}_a^T \bar{\bar{\mu}}^T \bar{H}_b\right)^T$$
$$= \bar{H}_a^T \bar{\bar{\mu}}^T \vec{H}_b$$

If $\bar{\bar{\mu}}^T = \bar{\bar{\mu}}$ & $\bar{\bar{\epsilon}}^T = \bar{\bar{\epsilon}}$,

$$\nabla \cdot (\bar{E}_a \times \vec{H}_b - \vec{E}_b \times \vec{H}_a) = -(\vec{E}_a \cdot \vec{J}_b - \vec{H}_a \cdot \vec{J}_{mb}) + (\bar{E}_b \cdot \bar{J}_a - \bar{H}_b \cdot \vec{J}_{ma})$$

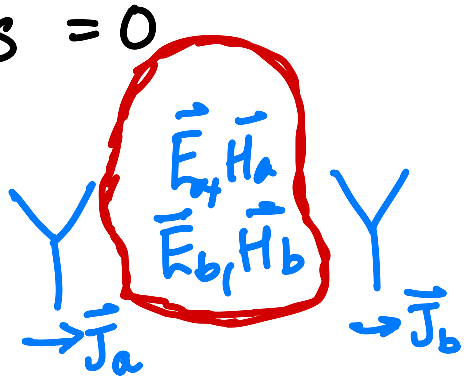
$\int_V$ & Div Thm.

$$\oint\!\!\oint_S (\vec{E}_a \times \vec{H}_b - \vec{E}_b \times \vec{H}_a) \cdot \bar{ds} = -\int_V (\vec{E}_a \cdot \vec{J}_b - \vec{H}_a \cdot \vec{J}_{mb})\, dv + \int_V (\vec{E}_b \cdot \vec{J}_a - \vec{H}_b \cdot \vec{J}_{ma})\, dv$$

<u>Case 1</u> Source free region.

RHS $\to 0$

$$\Rightarrow \oint\!\!\oint_S (\vec{E}_a \times \vec{H}_b - \vec{E}_b \times \vec{H}_a) \cdot \vec{ds} = 0$$



Case 2 $\quad S \longrightarrow \infty$ & it is a sphere.

* Surface fields are TEM to $\hat{r}$.

$$\left(\vec{E}_a \times \vec{H}_b - \vec{E}_b \times \vec{H}_a\right) \cdot \hat{r}\, ds$$

$$= \left[\vec{E}_a \cdot \left(\vec{H}_b \times \hat{r}\right) - \vec{E}_b \cdot \left(\vec{H}_a \times \hat{r}\right)\right] ds$$

$$= \frac{1}{\eta} \left[\vec{E}_a \cdot \vec{E}_b - \vec{E}_b \cdot \vec{E}_a\right] ds = 0$$

$$\Rightarrow \int_V \left(\vec{E}_a \cdot \vec{J}_b - \vec{H}_a \cdot \vec{J}_{mb}\right) dv = \int_V \left(\vec{E}_b \cdot \vec{J}_a - \vec{H}_b \cdot \vec{J}_{ma}\right) dv$$

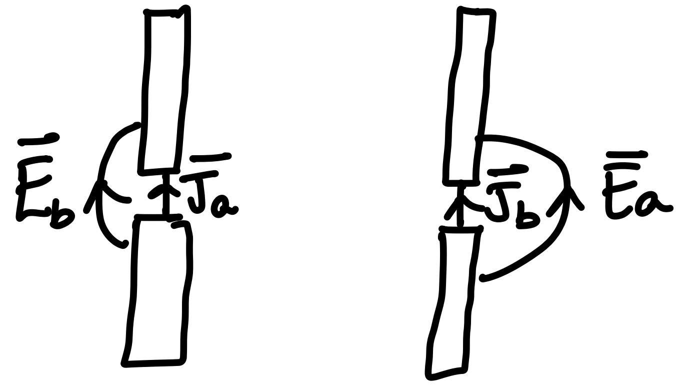
$$\vec{J}_a = I_a \vec{\Delta l}_a \delta\left(\vec{r} - \vec{r}_a\right)$$

$$\vec{J}_b = I_b \vec{\Delta l}_b \delta\left(\vec{r} - \vec{r}_b\right)$$

$$\boxed{\vec{E}_a(\vec{r}_b) \cdot \vec{J}_b(\vec{r}_b) = \vec{E}_b(\vec{r}_a) \cdot \vec{J}_a(\vec{r}_a)}$$

$\Rightarrow \quad Z_{21} = Z_{12}$

$V_a^{oc} = -\int \vec{E}_b \cdot d\vec{l}$



$V_a^{oc} I_a = V_b^{oc} I_b$ from reciprocity.

$V_a^{oc} = Z_{ab} I_b$

$V_a = Z_{aa} I_a + Z_{ab} I_b$

$V_b^{oc} = Z_{ba} I_a$

$\boxed{Z_{ab} = Z_{ba}} \Rightarrow \boxed{S_{21} = S_{12}}$

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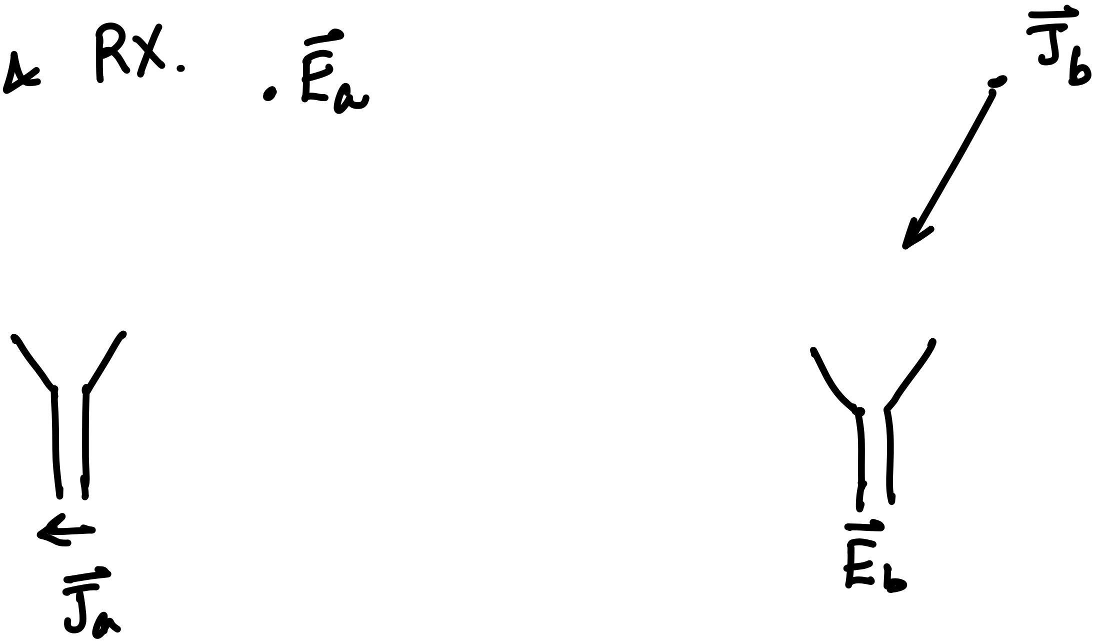
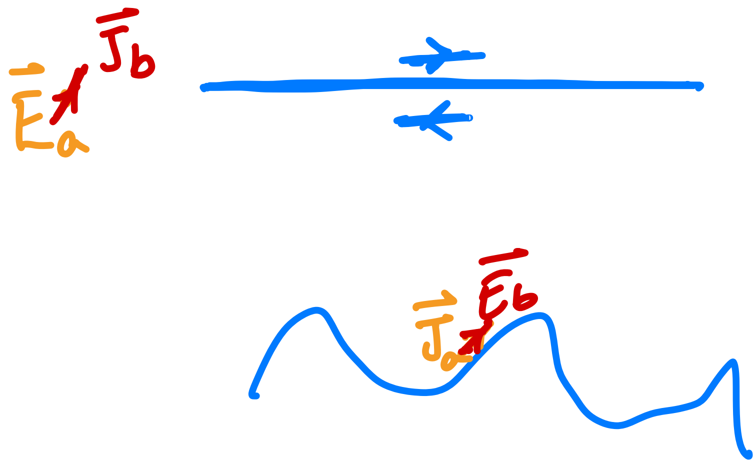
1) Antenna Radiation Pattern is identical in TX & RX.



2) Tangential impressed currents over PEC cannot radiate.

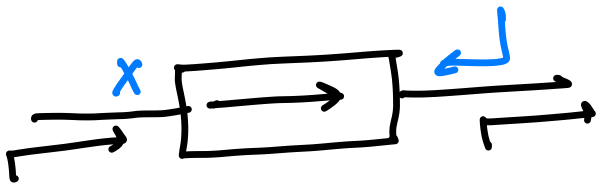
$\vec{E}_a = 0$



3) $\boxed{A_{eff} = \frac{\lambda^2}{4\pi} G}$ (Section 4-7.2)

Breaking Reciprocity

1) Isolator



$S_{21} = 1$ or $0\,dB$

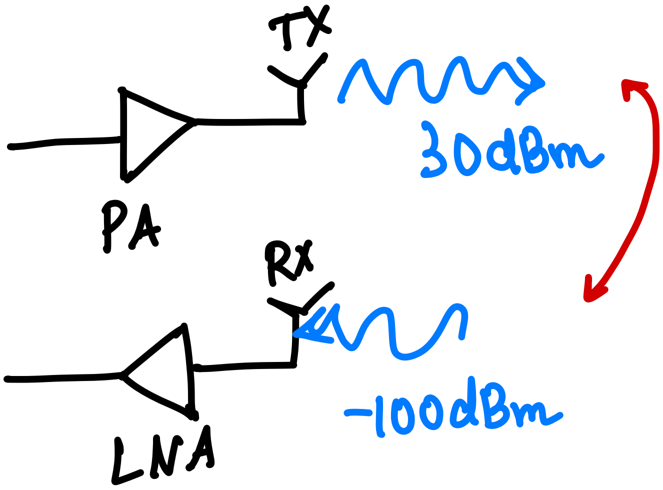
$S_{12} = 0$ or $-\infty$

2) Amplifier

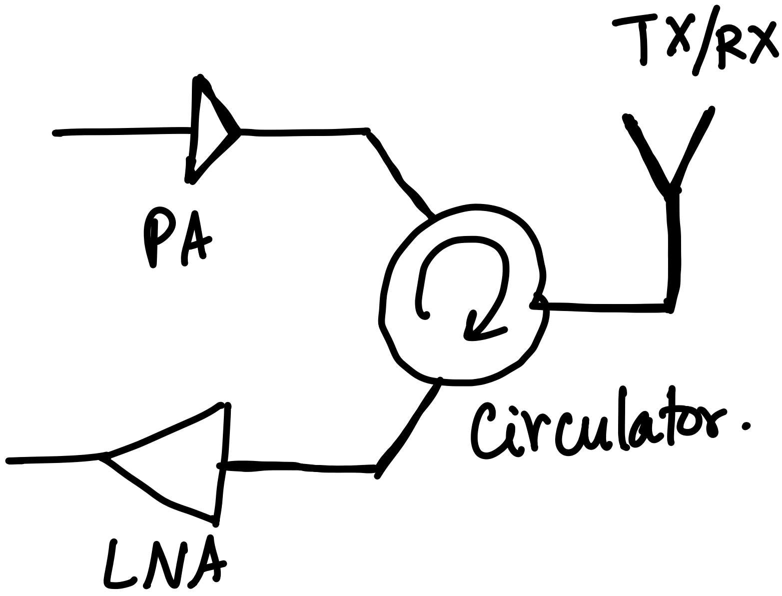
$S_{21} >> 0$, $S_{12} << 0$

3) Circulators

> Full Duplex Comm. or STAR.



130dB!!
(we want high isolation)



3-port device that is lossless & matched at all ports, it must be nonreciprocal

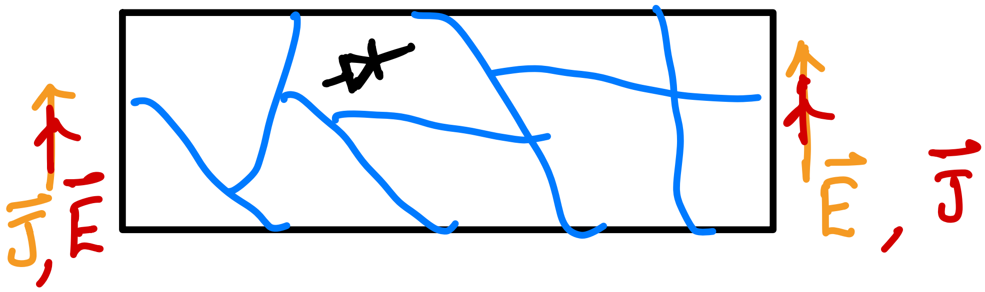
$S_{xy} \neq S_{yx}$ for $x \neq y$.

Breaking Reciprocity

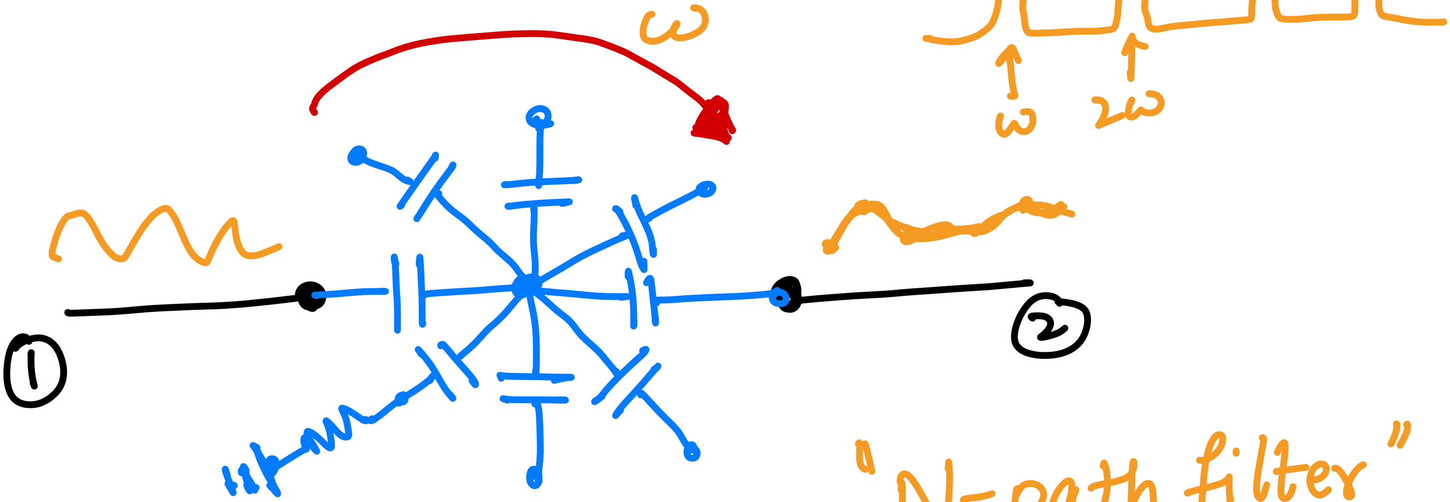
Nonlinearity, Time variation, Nonsymmetric Anisotropy.

> Necessary but not sufficient!

① Nonlinearity

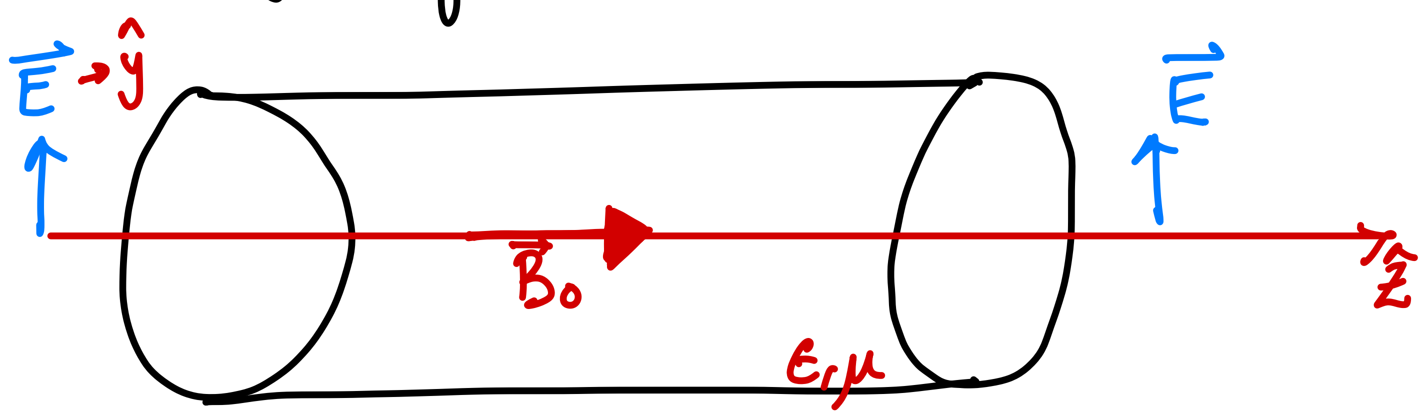


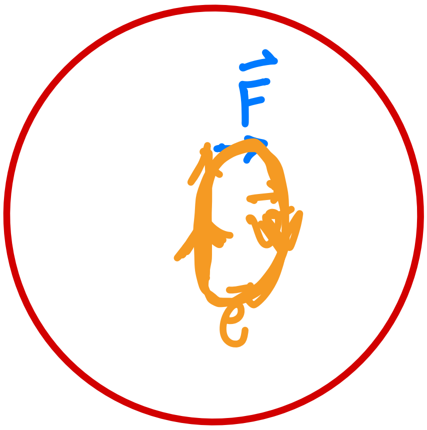
② Time Varying

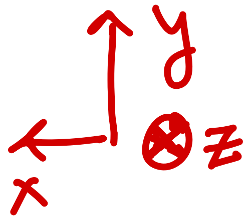


"N-path filter"

③ Non symmetric Anisotropy
(Magnetic Bias)



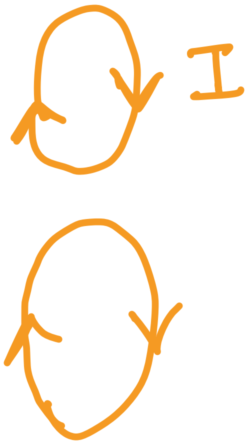




$\hat{z}$ prop

LHEP

$-\hat{z}$ prop



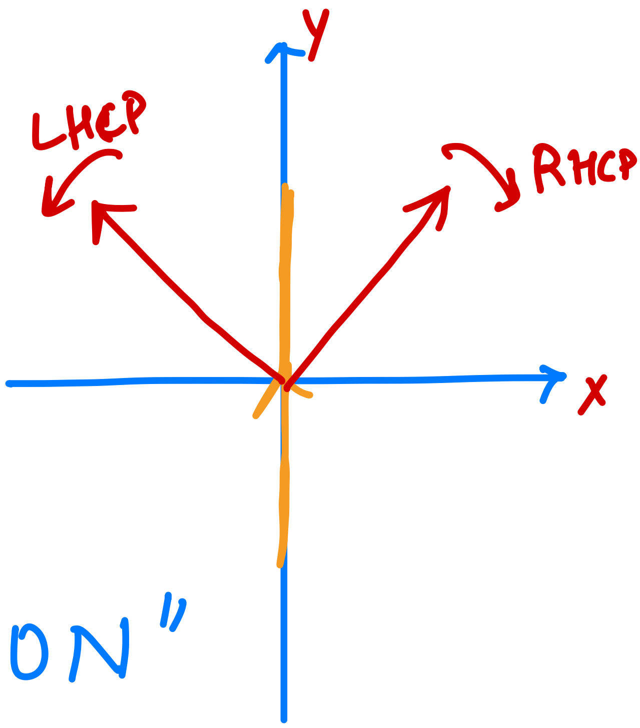
RHEP

$$\underbrace{\bar{\bar{\mu}} = \begin{bmatrix} \mu_x & i\kappa & 0 \\ -i\kappa & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{\text{Polder tensor.}}$$

(Gyrotropic Medium)

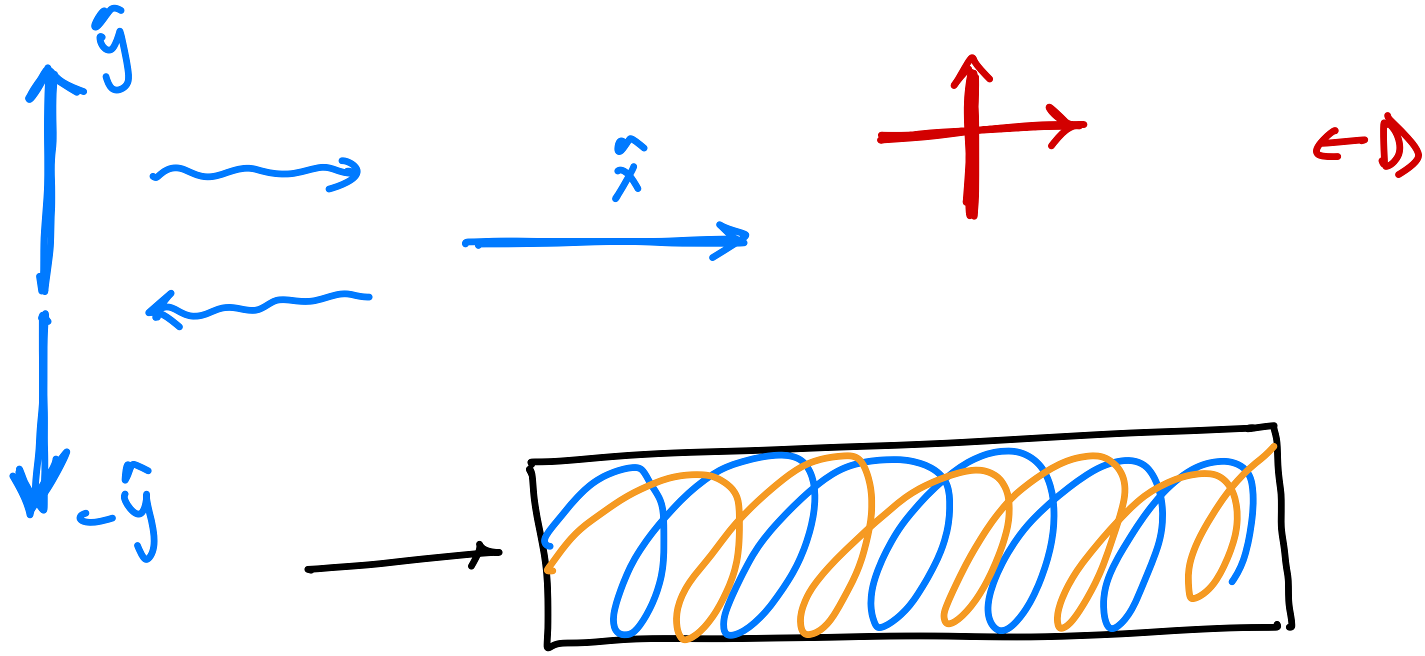
$$K_{LCHP} \longrightarrow K + \delta K$$

$$K_{RHCP} \longrightarrow K - \delta K$$



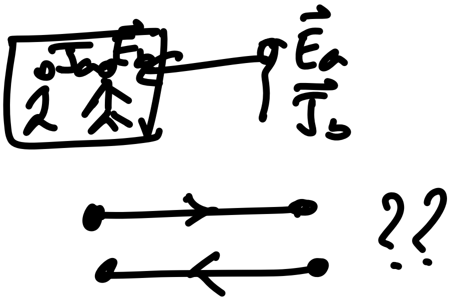
$\uparrow$ = LHCP + RHCP

"FARADAY ROTATION"



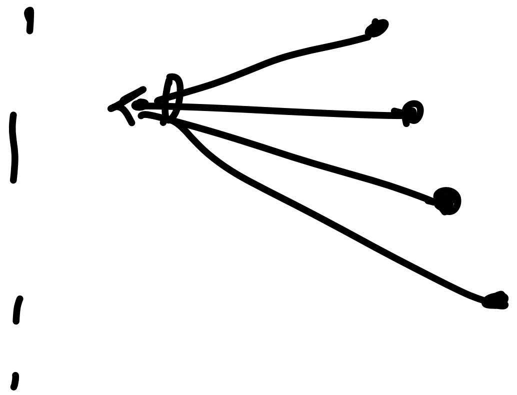
Example

1) One way mirror.



$$\uparrow E_a J_b = E_b J_a \uparrow$$

2) Peep Hole.



Time Reversal Symmetry $\Rightarrow$ Reciprocity.

Not Reciprocity $\Rightarrow$ TRS Breaking.

TRS Breaking $\not\Rightarrow$ Nonreciprocity.

Lossy

All amplifiers are nonreciprocal?

In a common source amp where does the nonreciprocity come from?

Nonlinearity, ? Time variation, X Anisotropy. X