Lec 08 - Antennas Concepts

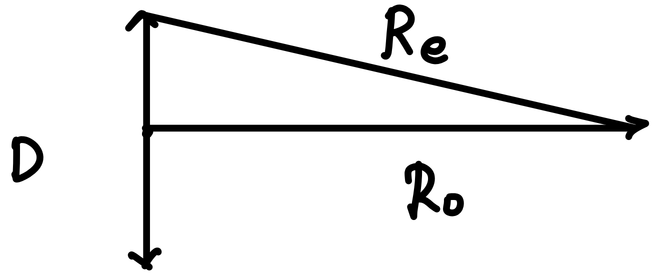
1) Far-field Distance

$Kr \gg 1 \rightarrow$ far-field criteria for small antennas.

$Kr \geq 10$

$\Rightarrow \boxed{r \geq 1.6\lambda} \rightarrow$ independant of size.

"Phase diff. b/w center & edge $\leq \frac{\pi}{8}$" $\rightarrow$ FF criteria for antenna.



$$K(R_e - R_o) \leq \frac{\pi}{8}$$

$$R_e = \sqrt{R_o^2 + \frac{D^2}{4}}$$

$$\simeq R_o + \frac{D^2}{8R_o}$$

$$\frac{KD^2}{8R_o} \leq \frac{\pi}{8}$$

$$\Rightarrow \boxed{R_o \geq \frac{2D^2}{\lambda}} \rightarrow \text{Far-field distance.}$$

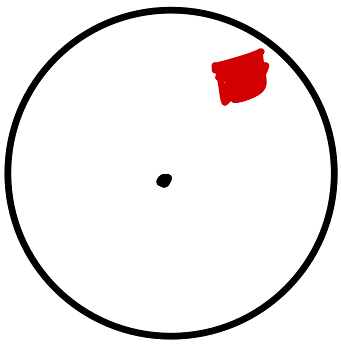
Note: This depends on the coordinate system.

2) Radiation Power Density

(Real part of $\vec{S}$)

$$\vec{S}_r(\vec{r}) = \mathrm{Re}\{\vec{S}\} \overset{F.F}{=} \underbrace{S_r}_{\text{Radiation Power Density.}} \hat{r}$$

Isotropic antenna



$$S_r = \frac{P_{rad}}{4\pi r^2}$$

Q: Is it possible to make an isotropic radiator?

A: (Hairy Ball Thm)

3) Radiation Intensity

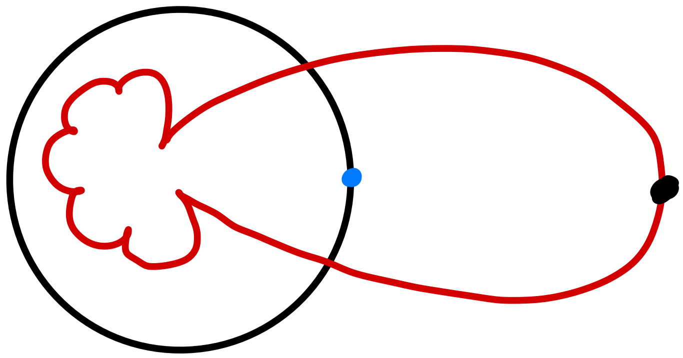
$$U(\theta, \phi) = r^2 S_r$$

→ Power radiated per unit solid angle.

→ In FF, only depends on (θ, ϕ).

4) Directivity

$$D(\theta, \phi) = \frac{S_r(\vec{r})}{P_r/4\pi r^2} = \frac{4\pi U}{P_{rad}}$$



Directivity : 10dBi → max (Directivity).

$$10 \log(D)$$

5) Gain $(G(\theta,\phi))$ dBi

$$G = KD$$

↳ efficiency

6) Efficiency

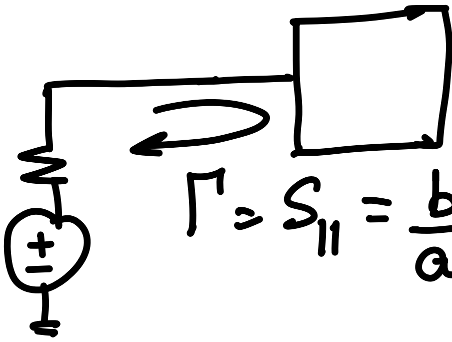
$$\text{Radiation efficiency} = \frac{P_{rad}}{P_{acc}}$$

Antenna (Total) efficiency $= \dfrac{P_{rad}}{P_{sup}}$

$$P_{acc} = P_{sup}\left(1 - |\Gamma|^2\right)$$

$$K_{tot} = \left(1 - |\Gamma|^2\right) K_{rad}$$

P_{sup}



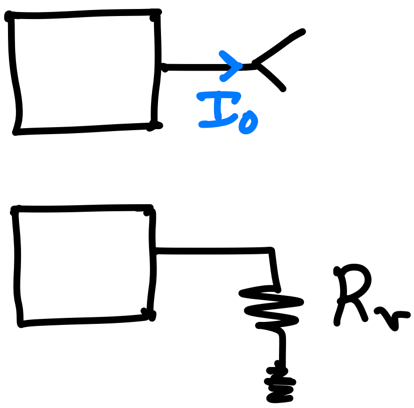
$\Gamma = S_{11} = \dfrac{b}{a}$

7) Radiation resistance

From a circuit POV, antenna looks like a resistor. (even when the antenna is lossless)

$$\boxed{R_r = \frac{2P_{rad}}{I_0^{\,2}}}$$

Current amp. at the antenna terminals.



8) Radiation Pattern

> $G(\theta, \phi)$
> $D(\theta, \phi)$

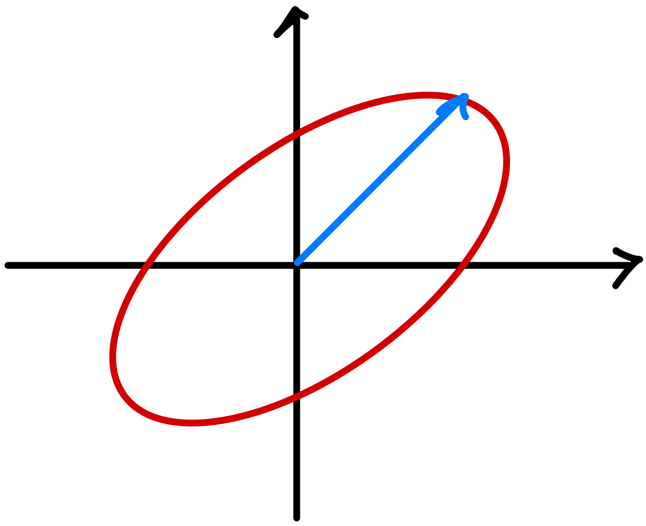
$\left.\right\}$ dBi

> $\dfrac{S(\theta, \phi)}{S_{max}(\theta, \phi)}$ $\left.\right\}$ dB (Normalized)

> $\dfrac{E_\theta(\theta, \phi)}{E_\theta(\theta, \phi)_{max}}$ or $\dfrac{E_\phi(\theta, \phi)}{E_\phi(\theta, \phi)_{max}}$ → normalized Field Pattern (Complex)

9) Polarization

($\vec{E}$-field vector as a fn. of time!)



Linear pol. ← Elliptical pol. → Circular pol.

Axial ratio $= \dfrac{\text{major axis}}{\text{minox axis}}$ of pol. ellipse.

$AR = \infty$ linear pol.

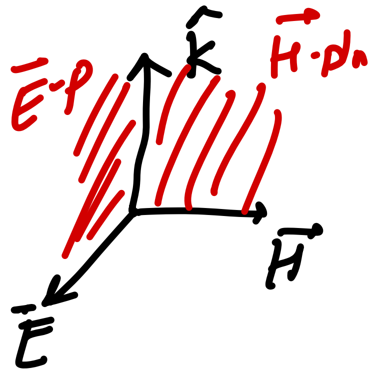
$= 1$ circular pol.

> <u>AR bandwidth</u>, $AR \leq 3\,dB$

$AR \leq 2$

<u>E-plane</u>: Plane $\{\vec{E}, \vec{k}\}$

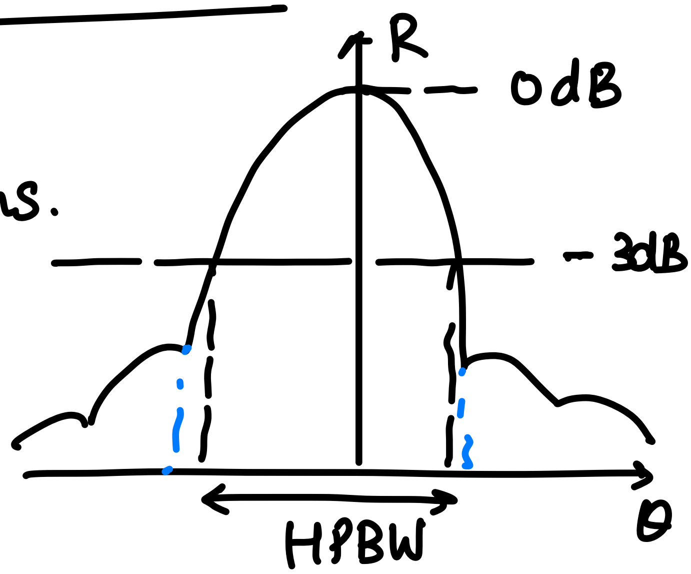
<u>H-plane</u>: Plane $\{\vec{H}, \vec{k}\}$



10) <u>Half Power Beamwidth</u> (HPBW)

$$\theta_{HPBW} \approx \frac{\lambda}{D} \text{ in radians.}$$

D → physical aperture diameter.

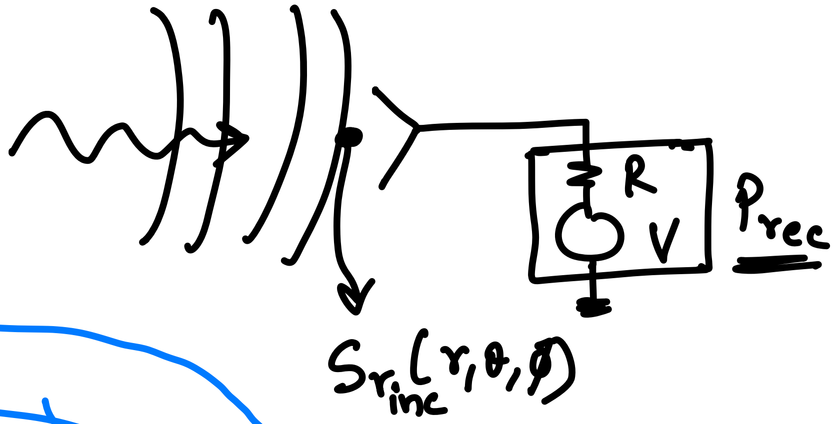


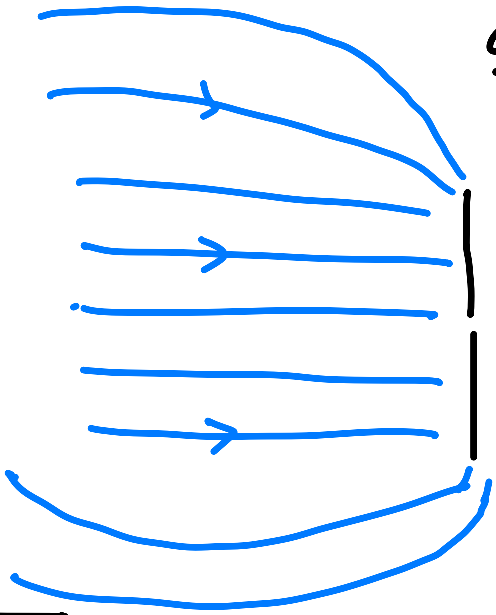
$$\theta_{FNBW} \approx 2\, HPBW.$$

$$\boxed{D \cong \frac{4\pi}{\theta_E \theta_H}} \rightarrow \text{Appr.}$$

11) Effective Aperture (area)

$$A_e = \frac{P_{rec}}{S_{inc}} \quad (m^2)$$

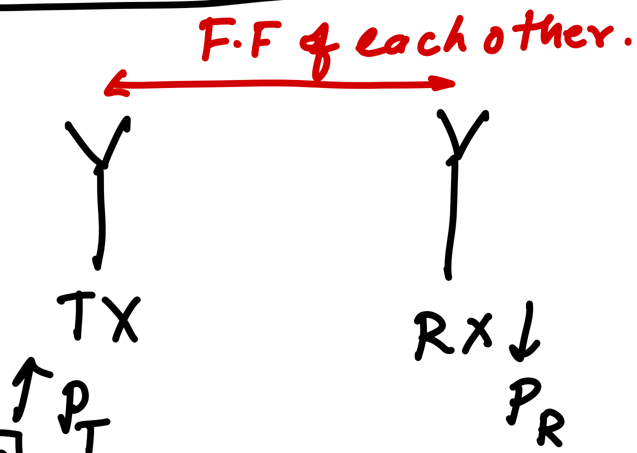




$$\boxed{A_e = \frac{\lambda^2}{4\pi} G}$$

Proved later.

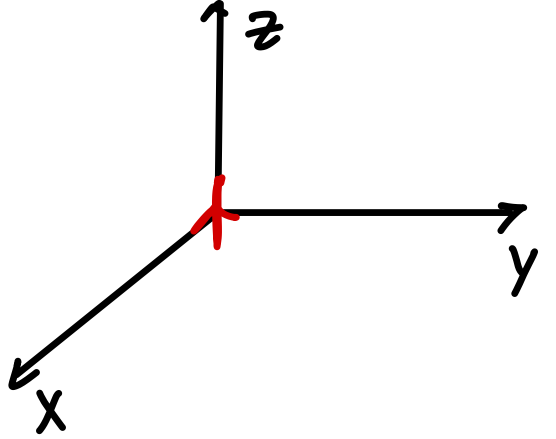
Link Budget (Frii's Transmission)



$$P_R = \left(\frac{\lambda^2}{4\pi} G_R\right) \frac{G_T P_T}{4\pi r^2}$$

$$\boxed{P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi r}\right)^2}$$

Hertzian Dipole



$$\vec{J}(\vec{r},t) = I_0 \Delta l \, \delta(\vec{r}) \hat{z}$$

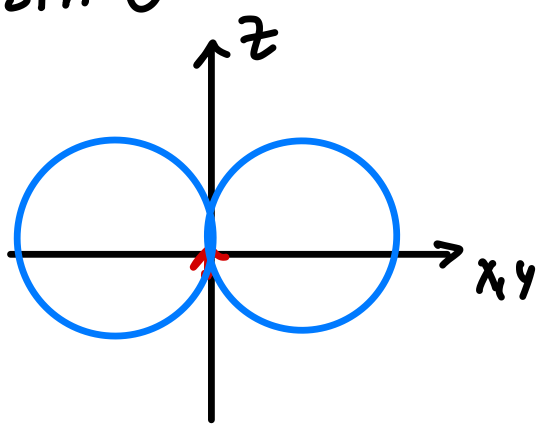
$$\vec{E}(\vec{r}) = \eta I_0 \Delta l \, k \frac{e^{ikr}}{4\pi r} \left[2\left(\frac{1}{kr} + \frac{i}{(kr)^2}\right)\cos\theta \, \hat{r} + \left(-i + \frac{1}{kr} + \frac{i}{(kr)^2}\right) \cdot \sin\theta \, \hat{\theta} \right]$$

$$\vec{H}(\vec{r}) = I_0 \Delta l \, k \frac{e^{ikr}}{4\pi r} \left(-i + \frac{1}{kr}\right) \sin\theta \, \hat{\phi}$$

$$P_r = \int_0^{2\pi} \int_0^{\pi} \mathrm{Re}\left\{ \frac{1}{2} \vec{E}(\vec{r}) \times \vec{H}(\vec{r}) \right\} \cdot \hat{r} \, r^2 \sin\theta \, d\theta \, d\phi$$

$$P_r = \frac{\eta}{3} \frac{|I_0 \Delta l|^2 k^2}{4\pi}$$

$$D(\theta,\phi) = \frac{S_r(\vec{r})}{P_r / 4\pi r^2} = \frac{3}{2} \sin^2\theta$$

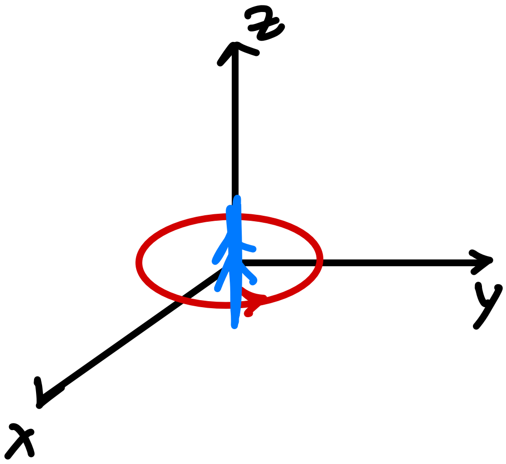


$$R_r = \frac{2P_r}{I_0^2} \simeq 790 \left(\frac{\Delta l}{\lambda}\right)^2$$

$$@\ \Delta l = \frac{\lambda}{10} \Rightarrow R_r \simeq 8\,\Omega$$

$$@\ \Delta l = \frac{\lambda}{100} \Rightarrow \underbrace{R_r \simeq 0.08\,\Omega}_{\text{Revisit Small Antennas.}}$$

Small Current Loop



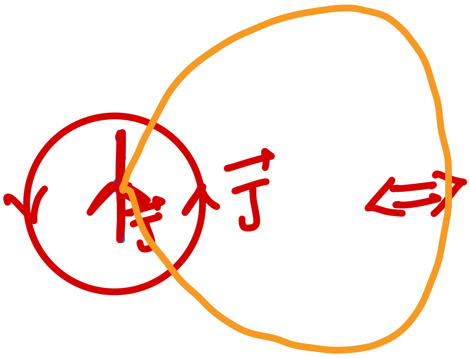
$$\vec{J} = I_0\,\delta(\rho - a)\,\delta(z)\,\hat{\phi}$$

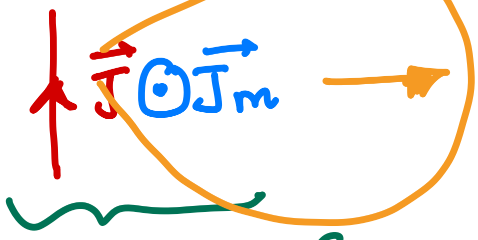
$$\vec{A}(\vec{r}) = \mu_0 I_0 \pi a^2 \frac{e^{ikr}}{4\pi r}\left(-ik + \frac{1}{r}\right)\sin\theta\,\hat{\phi}$$

$$\vec{E}(\vec{r}) = i\omega\mu k I_0 \pi a^2 \frac{e^{ikr}}{4\pi r}\left(-i + \frac{1}{kr}\right)\sin\theta\,\hat{\phi}$$

$$\vec{H}(\vec{r}) = -ik^2 I_0 \pi a^2 \frac{e^{ikr}}{4\pi r}\left[2\left(\frac{1}{kr} + \frac{i}{(kr)^2}\right)\cos\theta\,\hat{r} + \left(-i + \frac{1}{kr} + \frac{i}{(kr)^2}\right)\sin\theta\,\hat{\theta}\right]$$

$$\boxed{I_{om}\,\Delta l = -ik\eta\,\pi a^2 I_0}$$





Huygen's Source!

Huygen's Principle!