EM04 - EM Tools

Equivalent Magnetic Currents and Charges

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0(1+\chi_m)\frac{\partial \vec{H}}{\partial t}$$

$$= -\mu_0 \frac{\partial \vec{H}}{\partial t} \underbrace{-\mu_0 \frac{\partial \vec{M}}{\partial t}}_{\vec{J}_m}$$

$$\vec{J}_m = \mu_0 \frac{\partial \vec{M}}{\partial t}$$

$$\boxed{\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} - \vec{J}_m}$$

$$\nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \mu_0(\vec{H} + \vec{M}) = 0$$

$$\Rightarrow \mu_0 \nabla \cdot \vec{H} = \underbrace{-\mu_0 \nabla \cdot \vec{M}}_{\rho_m}$$

$$\Rightarrow \nabla \cdot \vec{H} = \frac{\rho_m}{\mu_0}$$

$$\boxed{\begin{aligned} \vec{J}_m &= \mu_0 \frac{\partial \vec{M}}{\partial t} \\ \rho_m &= -\mu_0 \nabla \cdot \vec{M} \end{aligned}}$$

These are not actual mag. Sources.

$$\mu \longrightarrow \mu_0 + \vec{J}_m + \rho_m$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \underbrace{-\frac{\partial \mu \vec{H}}{\partial t} + \frac{\partial \tilde{\mu} \vec{H}}{\partial t}}_{\vec{J}_m} - \frac{\partial \tilde{\mu} \vec{H}}{\partial t}$$

$$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \tilde{\mu} \vec{H}}{\partial t} - \vec{J}_m$$

$$\hookrightarrow (\mu - \tilde{\mu}) \frac{\partial \vec{H}}{\partial t}$$

$$\boxed{\begin{aligned} \vec{J}_m &= (\mu - \tilde{\mu}) \frac{\partial \vec{H}}{\partial t} \\ \rho_m &= -(\mu - \tilde{\mu}) \nabla \cdot \vec{H} \end{aligned}}$$

$$\mu \rightarrow \tilde{\mu} + \vec{J}_m + \rho_m$$

$$\boxed{\begin{aligned} \vec{J}_p &= (\epsilon - \tilde{\epsilon}) \frac{\partial \vec{E}}{\partial t} \\ \rho_p &= -(\epsilon - \tilde{\epsilon}) \nabla \cdot \vec{E} \end{aligned}}$$

$$\epsilon \rightarrow \tilde{\epsilon} + \vec{J}_p + \rho_p$$

Note: $\vec{J}_p, \vec{J}_m, \rho_p, \rho_m$ are dependant on the fields.

$\Rightarrow$ $\vec{J}_p$ & $\vec{J}_m$ are not independant (Book Eqns. 2.36 & 2.37)

Duality Relations

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} - \vec{J}_m$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{H} = \frac{\rho_m}{\mu}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

Duality relations

$\vec{E} \longrightarrow \vec{H}$	$\vec{J} \longrightarrow \vec{J}_m$
$\vec{H} \longrightarrow \vec{E}$	$\vec{J}_m \longrightarrow -\vec{J}$
$\mu \longrightarrow \epsilon$	$\rho \longrightarrow \rho_m$
$\epsilon \longrightarrow \mu$	$\rho_m \longrightarrow -\rho$

Normalized Duality Relations

$\vec{E} \to \sqrt{\frac{\mu}{\epsilon}}\,\vec{H}$ $\qquad$ $\vec{J}_m \to -\sqrt{\frac{\mu}{\epsilon}}\,\vec{J}$ $\qquad$ $\epsilon \to \epsilon$

$\vec{H} \to -\sqrt{\frac{\epsilon}{\mu}}\,\vec{E}$ $\qquad$ $\rho \to \sqrt{\frac{\epsilon}{\mu}}\,\rho_m$ $\qquad$ $\mu \to \mu$

$\vec{J} \to \sqrt{\frac{\epsilon}{\mu}}\,\vec{J}_m$ $\qquad$ $\rho_m \to -\sqrt{\frac{\mu}{\epsilon}}\,\rho$

Boundary Conditions revisited

$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$ $\to$ derived from MAL.

$\downarrow$ duality.

$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = -\vec{J}_{sm}$

Similarly,

$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$

$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = \rho_{sm}$

Perfect Magnetic Conductor (PMC)

$\vec{J}_m = \sigma_m \vec{H} \Rightarrow \sigma_m \to \infty \Rightarrow$ PMC.

$$\left(\hat{n}\times\vec{H}=0\;;\quad \hat{n}\times\vec{E}=-\vec{J}_{sm}\;;\quad \hat{n}\cdot\vec{B}=\rho_{sm}\;;\quad \hat{n}\cdot\vec{D}=0\right)$$

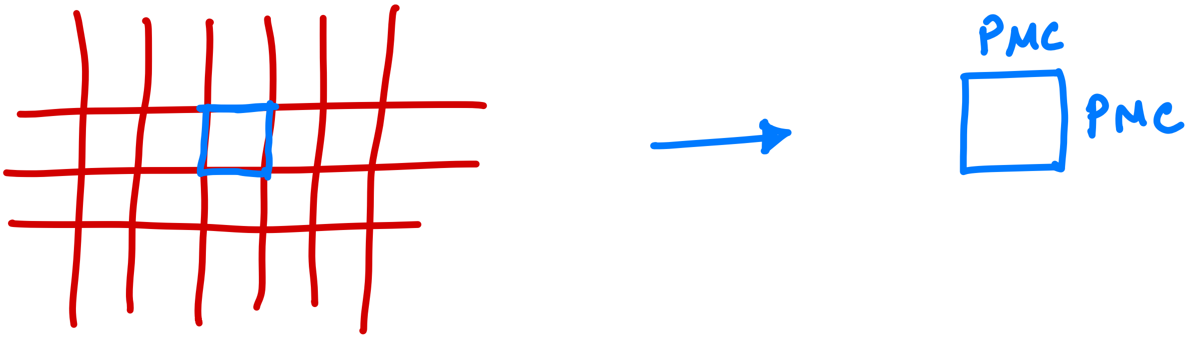


Image Theory

Uniqueness Theorem

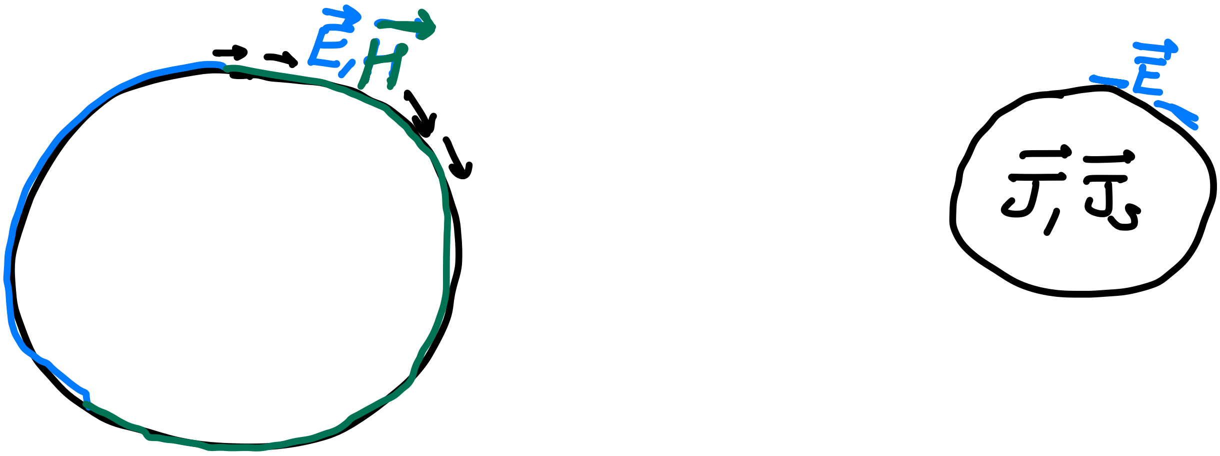
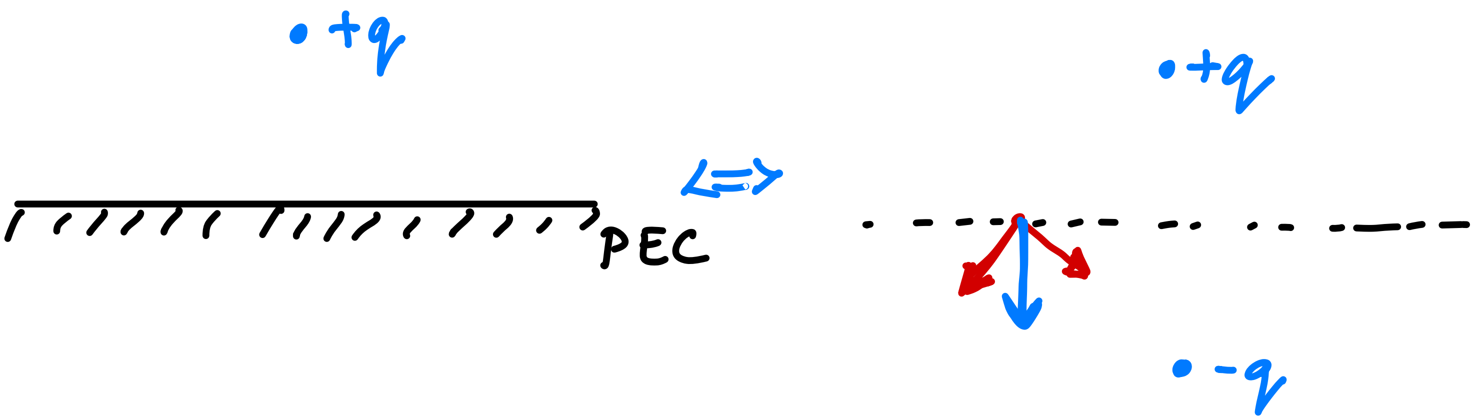
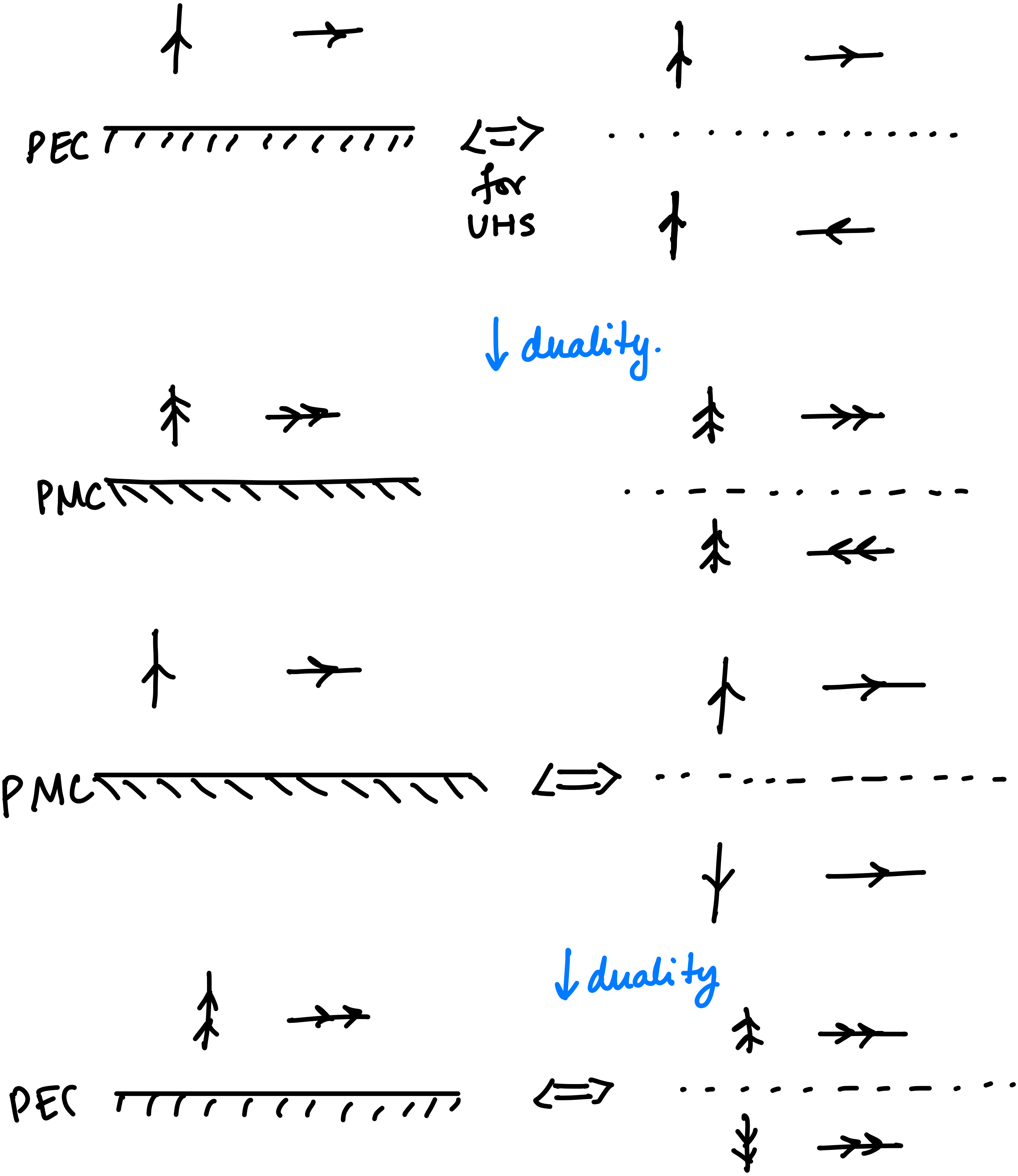
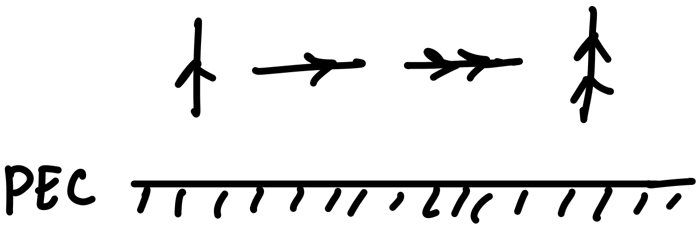


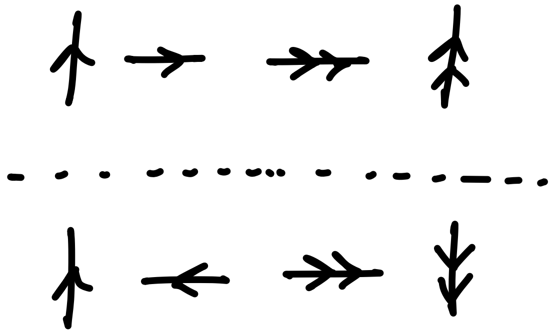
Image Theory

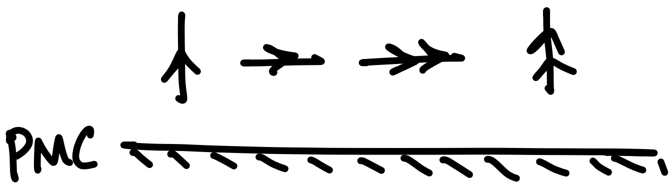


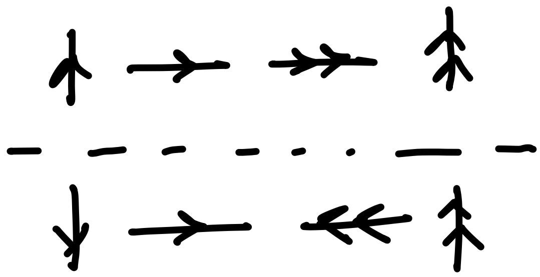

PEC
⟺
for
UHS
duality.
PMC
PMC
⟺
duality
PEC
⟺

In summary





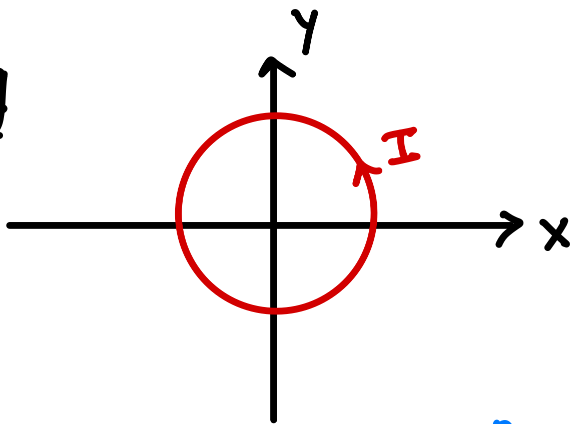




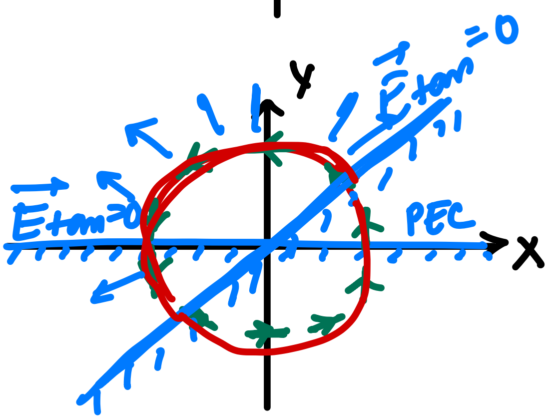
> Valid planar ∞ boundaries.

> Valid for impressed currents.

Eg



Prove that $E_z = 0$.



$E_z = 0$, $E_\rho = 0$

$\Rightarrow \vec{E} = \vec{E}_\phi \hat{\phi}$

Induced currents vs. Impressed currents.

Currents

Impressed currents

Source currents used in MEq. to solve for the fields in a given problem.

Eg: Lumped ports, Wave ports etc.

Induced currents

Currents generated by the fields that are solved for in a given problem.

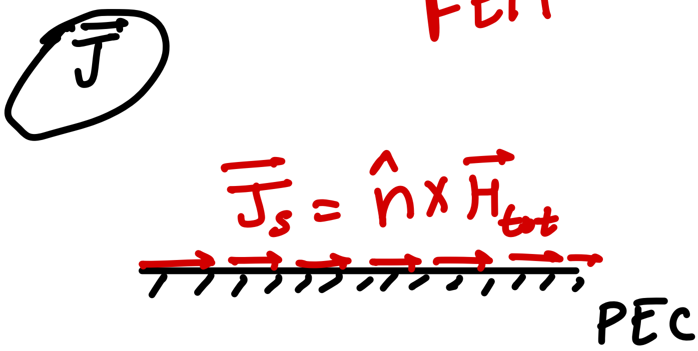
Eg: 1) Conduction current
$\vec{J} = \sigma \vec{E}$

2) Polarization currents
$\epsilon, \mu \rightarrow \vec{J}_p \;\&\; \vec{J}_m$

3) Surface currents.
$\vec{J} = \hat{n} \times (\vec{H}_1 - \vec{H}_2)$
$\vec{J}_m = -\hat{n} \times (\vec{E}_1 - \vec{E}_2)$

Example

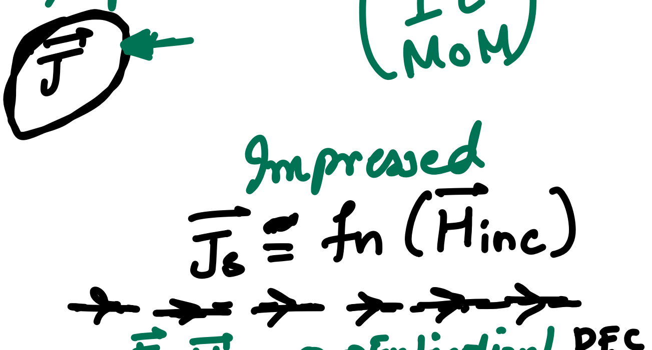
FEM



$\vec{J}_s = \hat{n} \times \vec{H}_{tot}$

(IE / MoM)

Impressed



Impressed

$\vec{J}_s \doteq fn(\vec{H}_{inc})$

$\vec{E}, \vec{H} = 0 \Rightarrow$ Extinction!