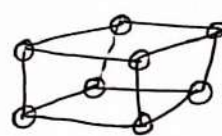


Lec 2 Energy bands in Solids.

> (Crystalline) semiconductors.

- Cubic lattice
 - Simple cubic (sc)
 - BCC
 - FCC



No. of atoms per unit cell.

1

2

4

> What is the no. of valence electrons per unit volume?

a → inter-atomic spacing = 3-7 Å . Say 5 Å.

⇒ volume of unit cell = $125 \text{ Å}^3 = 125 \times 10^{-24} \text{ cm}^3$.

> ⇒ No. of atoms per unit cell = $0.8 \times 10^{22} / \text{cc}$

$\approx 10^{22} / \text{cc}.$

> Since semiconductors have 4 electrons (valence), the no. of valence electrons per unit volume is

$$\boxed{4 \times 10^{22} / \text{cc}} \rightarrow \text{Si \& Ge.}$$

Gallium - 31 & Arsenic - 33.

⇒ Answer: $10^{22} - 10^{23} / \text{cc}.$

Solid $\rightarrow$ state of matter

$\hookrightarrow$ collection of atoms or molecules.

$\downarrow$ given by Schrödinger Eq.

$$H\psi = E\psi$$

Hamiltonian.

Total energy operator.

$$KE + PE$$

$$\frac{p^2}{2m} + V \rightarrow \left(-\frac{\hbar^2 \nabla^2}{2m} + V \right) \psi = E\psi.$$

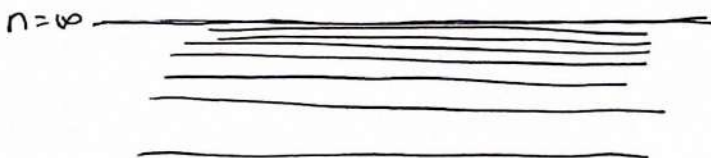
$$\Rightarrow \boxed{\nabla^2 \psi - \frac{2m}{\hbar^2} (E - V) \psi = 0.}$$

$\hookrightarrow$ For a given potential we can find an Energy (E) eigenvalue.

Eg: H-atom : $V = k \frac{q_1 q_2}{r} = -\frac{e^2}{r} k.$

Solving SE using $V \Rightarrow E_n = - \left(\frac{m e^4}{8 \epsilon_0^2 h^2} \right) \frac{1}{n^2}$

$\hookrightarrow$ discrete E vals (energy levels).



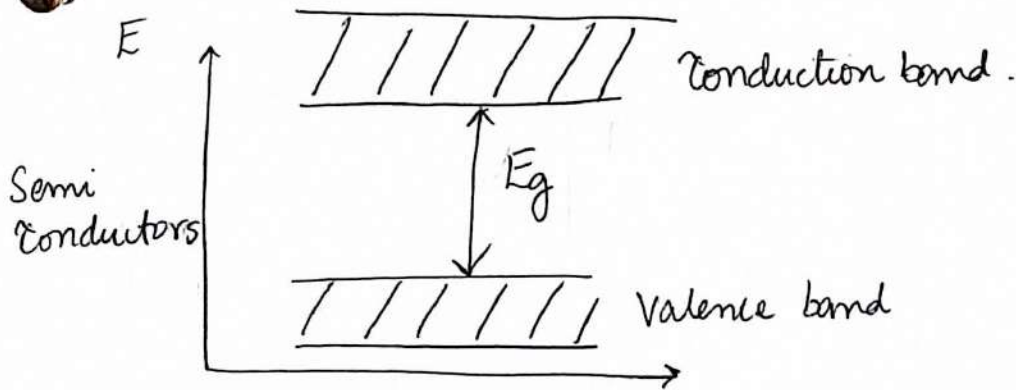
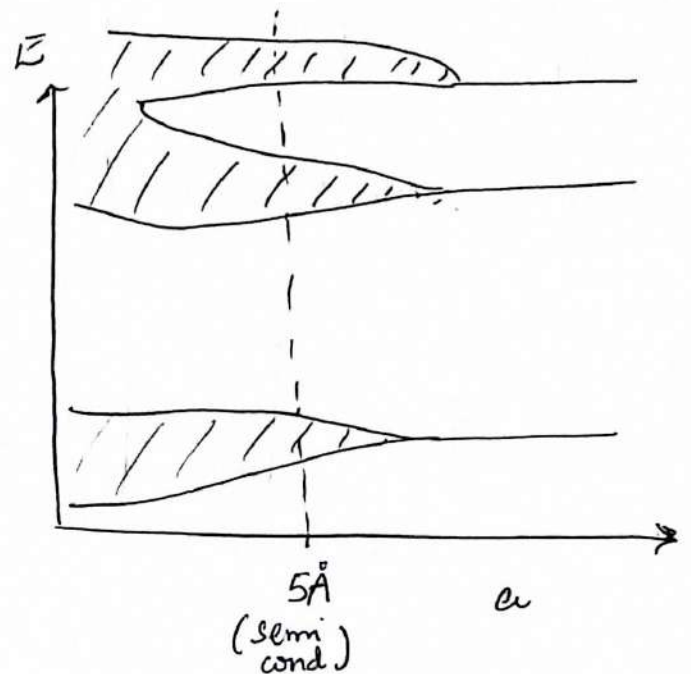
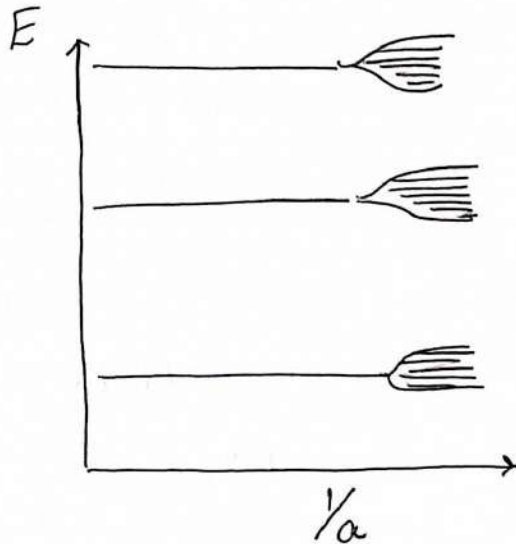
$$= -13.6 \left(\frac{1}{n^2} \right) \text{eV}$$

$n=2$

-13.6eV $n=1 \rightarrow$ Ground state for isolated H-atom.

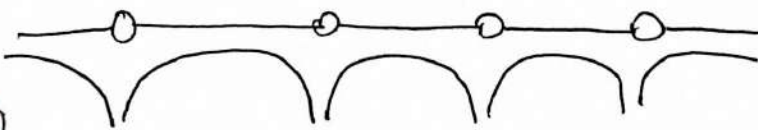
In a gas : $a \sim 100 \text{ \AA} - 1 \mu\text{m}$.

> Band splitting when two atoms come closer (Pauli's)



Kronig Penney Model.

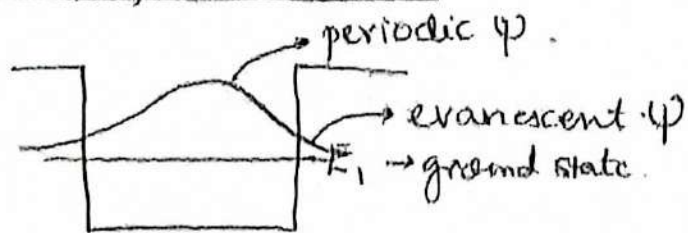
A 1-D lattice has a potential distribution like.



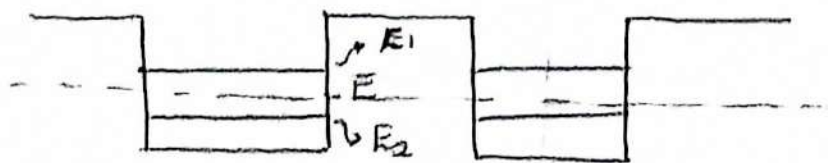
↓ Approx. as periodic potential well.



1-D potential well.



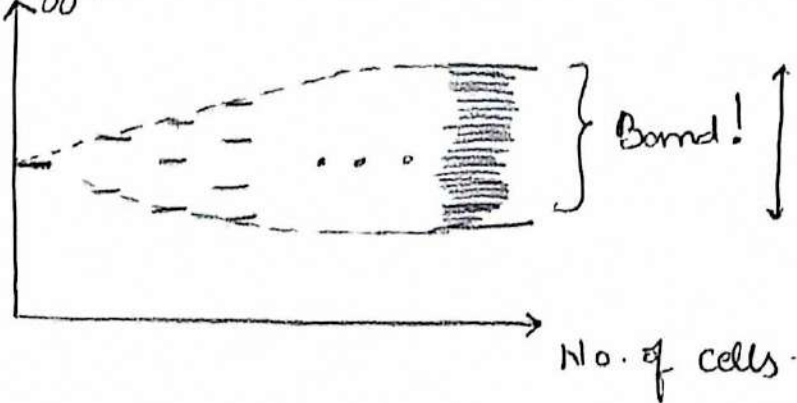
> When 2 wells are brought in close proximity.



→ Solving the problem leads to mode splitting + each well has 2 solutions.

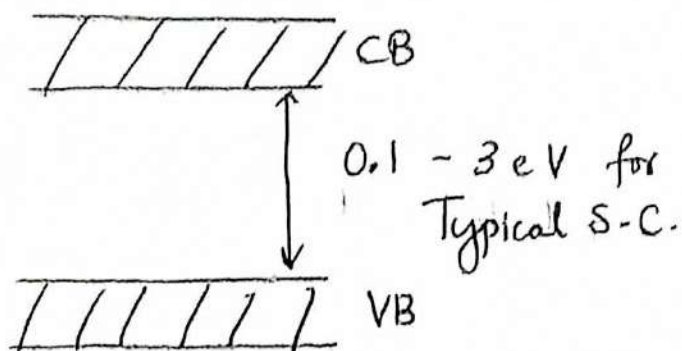
> N interacting wells gives N states.

Energy states

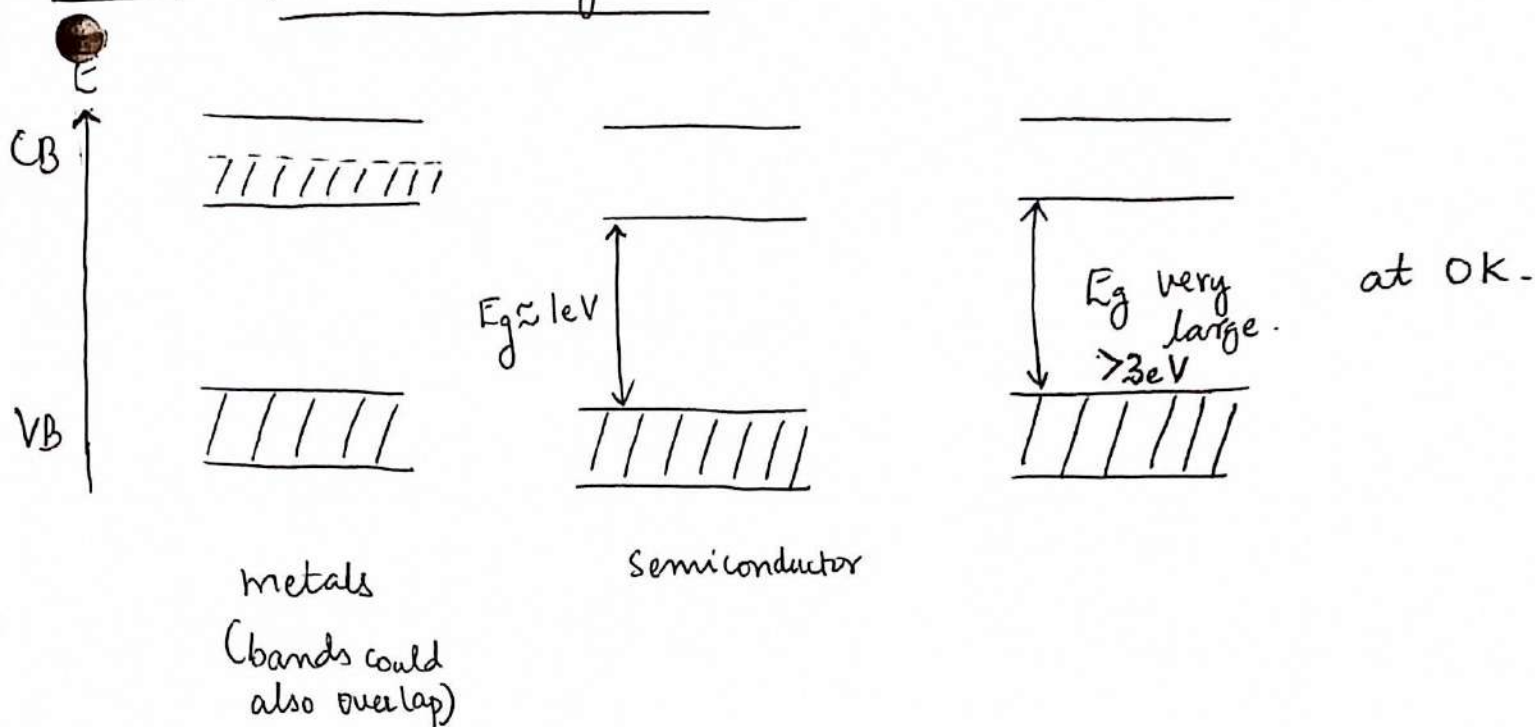


$\Delta E \rightarrow$ same as that from Kronig Penny Model.

Summary



Lec 3 E-k Diagram



At 300K

$$\text{Ge} - 0.67\text{eV} - 10^{23}/\text{cc}$$

$$\text{Si} - 1.1\text{eV} - 10^{10}/\text{cc}$$

$$\text{GaAs} - 1.42\text{eV} - 10^6/\text{cc}$$

$$E_e \approx 0.025\text{eV} \quad (\text{avg. kinetic energy of an electron})$$

How are so many electrons reaching the conduction band?

> Since a 1-D lattice is periodic, F. Bloch postulated

that ψ must also be periodic.

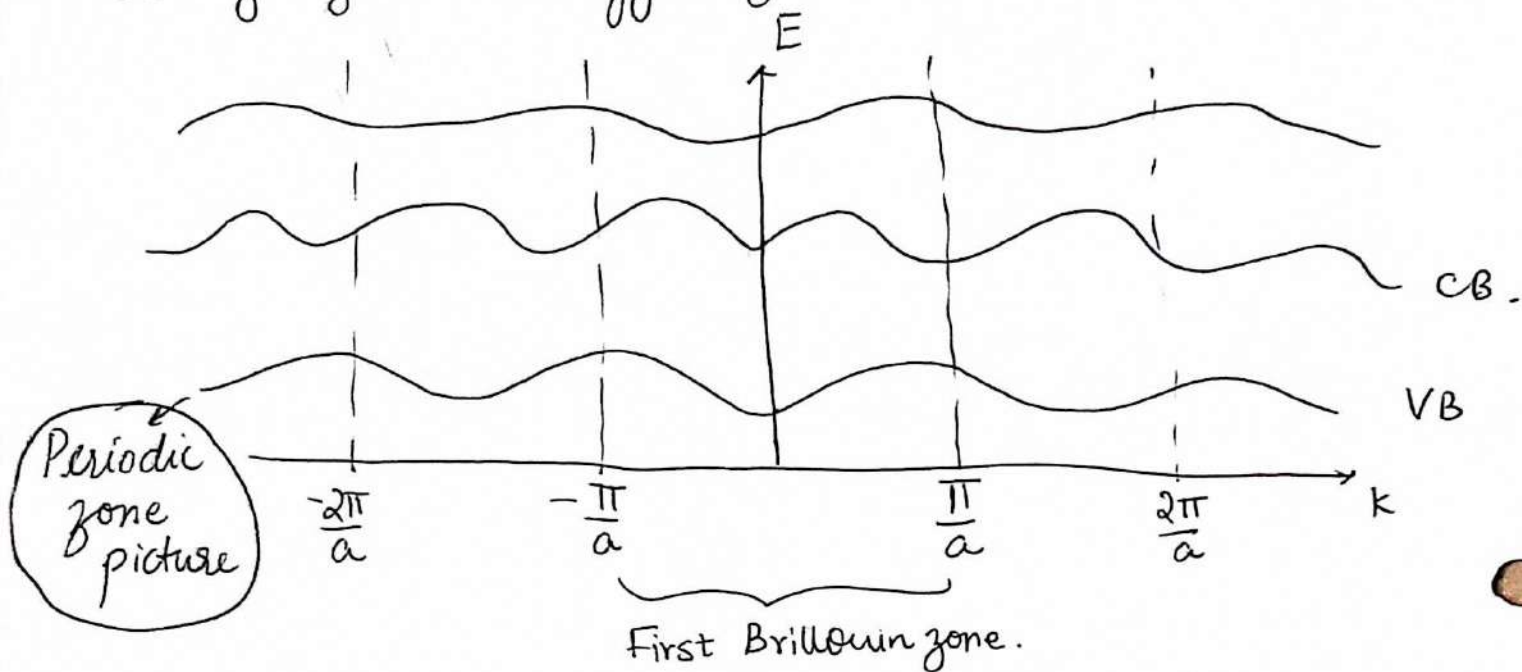
$$\psi_k(r) = u_k(r) e^{i\vec{k} \cdot \vec{r}} \quad \text{where } u_k(r) = u_k(r+a).$$

↳ Periodic cell function

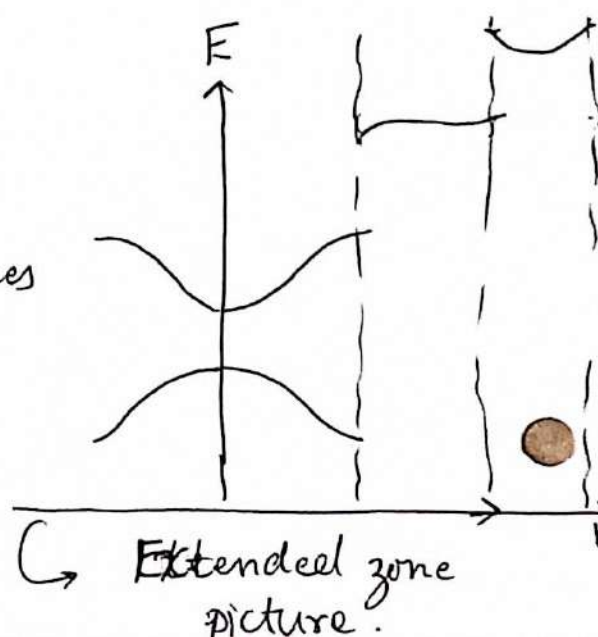
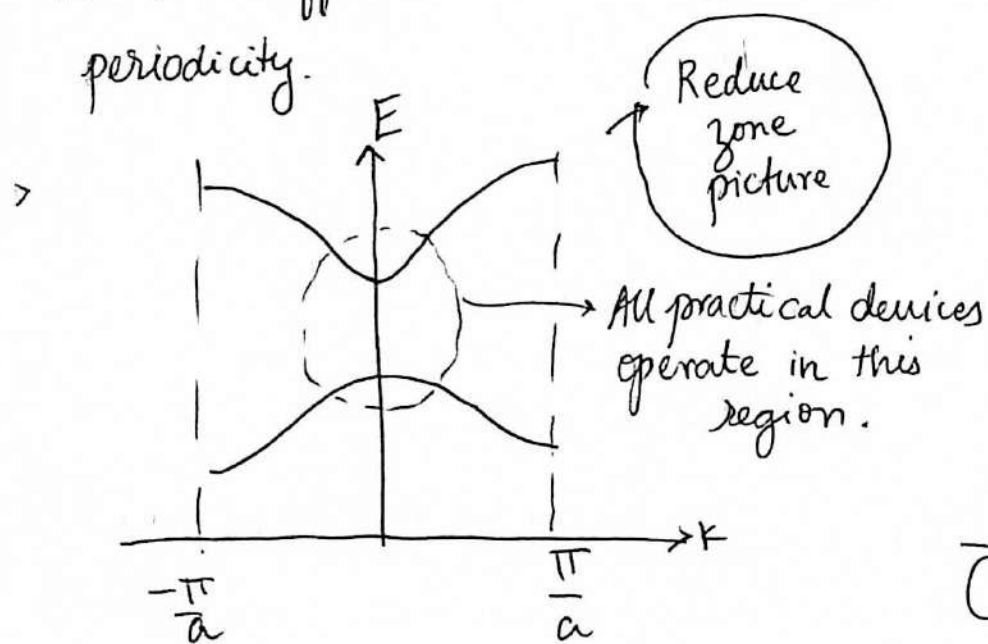
Substituting into Schrödinger,

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

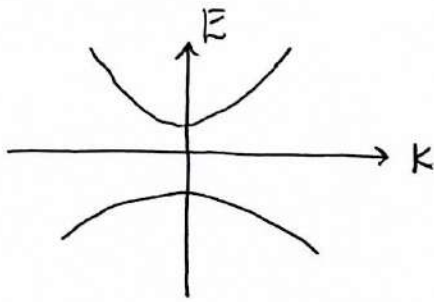
Solving gives energy eigenvalues that are also periodic!



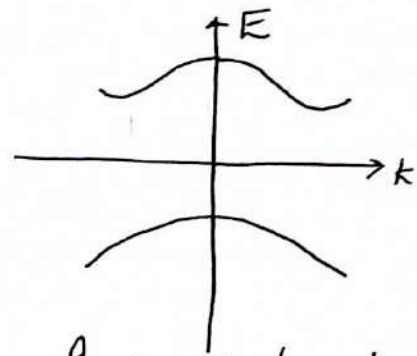
- > In the First Brillouin zone, $k=0$ is the Γ -point.
- > It is sufficient to study the first Brillouin zone due to periodicity.



(9)

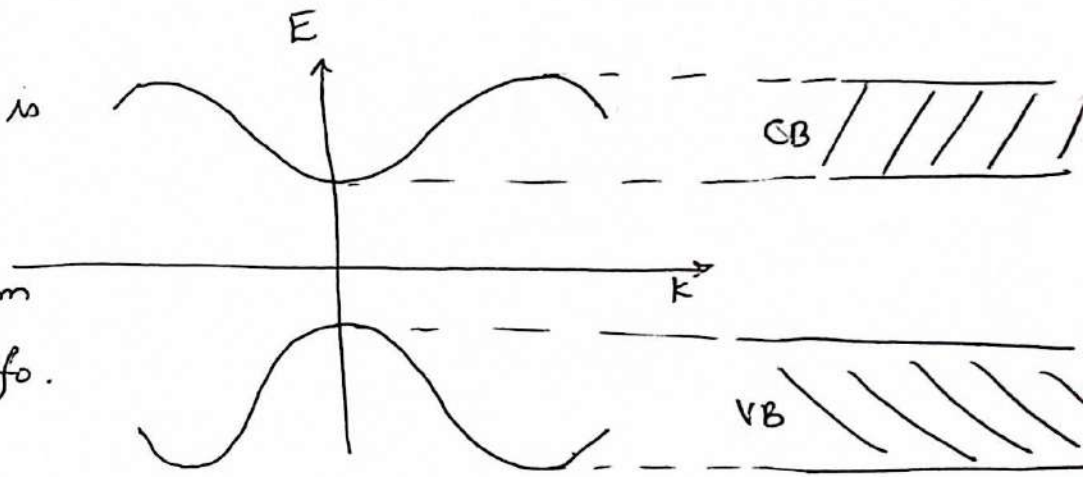


Direct band-gap
semiconductors



Indirect band-gap
semi-conductors.

> E-k diagram is related to the Band-gap diagram & gives more info.



> $|\vec{k}| = \frac{2\pi}{\lambda} \rightarrow$ de Broglie wavelength.

$\vec{p} = \hbar \vec{k} \rightarrow$ "crystal momentum".

But what is $\vec{k}$?

> $\vec{k}$ is the "crystal momentum". If an external E-field is applied to the crystal, the electron moves under the influence of the external field & the internal field. The internal field is from the other electrons & ions, but $\vec{k}$ is defined by the momentum due to E_{ext} alone.

i.e. $\frac{dp}{dt} = F_{ext} = -e E_{ext}.$

> The electron is therefore assumed to move as a free particle with an effective mass " m^* ".

$$m^* = \left(\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} \right)^{-1}$$

This comes from the $E-k$ diagram.
 $\hookrightarrow$ Super imp!

> Using this m^* mass, we can assume e^- is free.

Why does this matter?

(What is the current under an applied field?)

$$I = J A^{\rightarrow \text{area.}}$$

$$J = n q \mu_e E$$

$n \rightarrow$ carrier concentration.

$q \rightarrow$ charge.

$\mu_e \rightarrow$ mobility

$E \rightarrow$ Applied field.

$$n = \int_{E_c}^{\infty} n(E) dE$$

where $n(E) \rightarrow$ carrier density

$$n(E) = \rho(E) f(E) \rightarrow \text{probability of occupation.}$$

$\hookrightarrow$ density of states.

$\hookrightarrow$ (next lecture)

$\tau_c \rightarrow$ carrier recombination time

$$\mu_e = \frac{e \tau_c}{m^*}$$

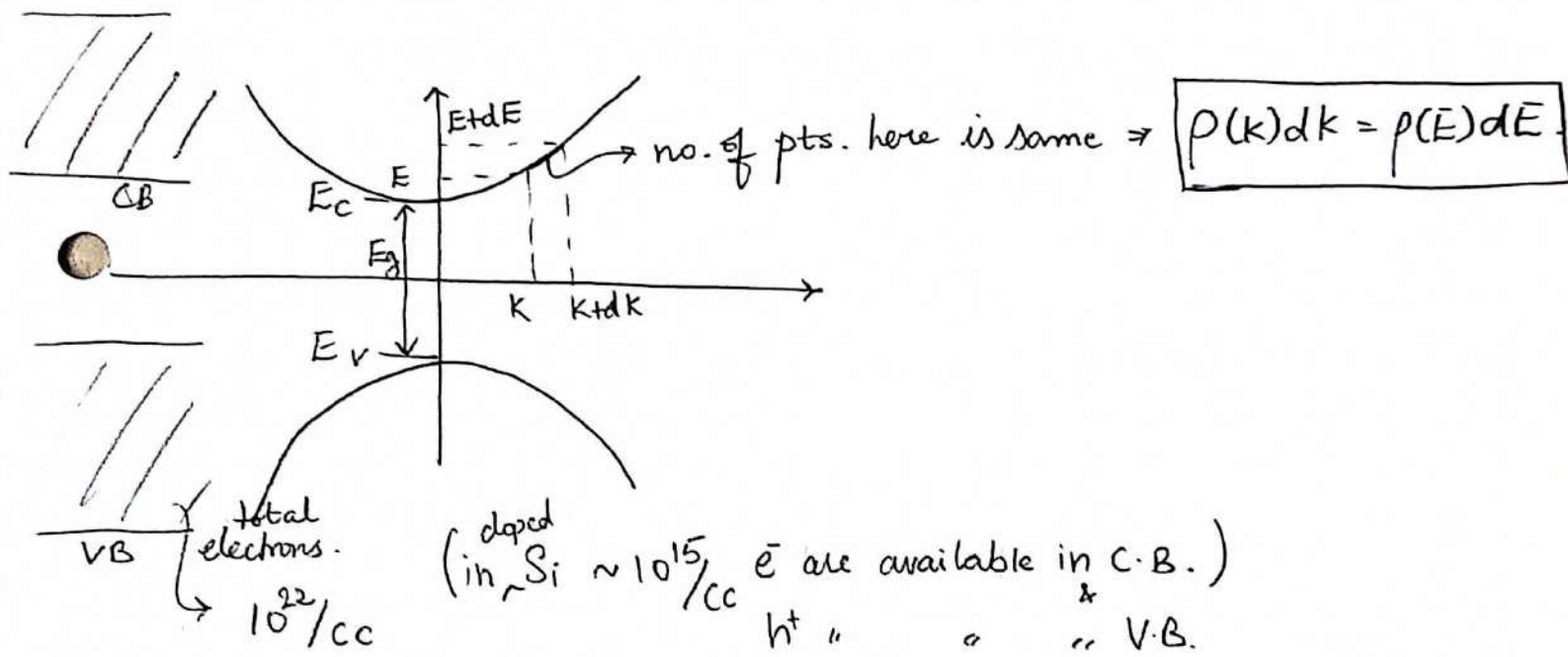
$m^* \rightarrow$ effective mass (from $E-k$ diagram).

Lecture 4 Density of States

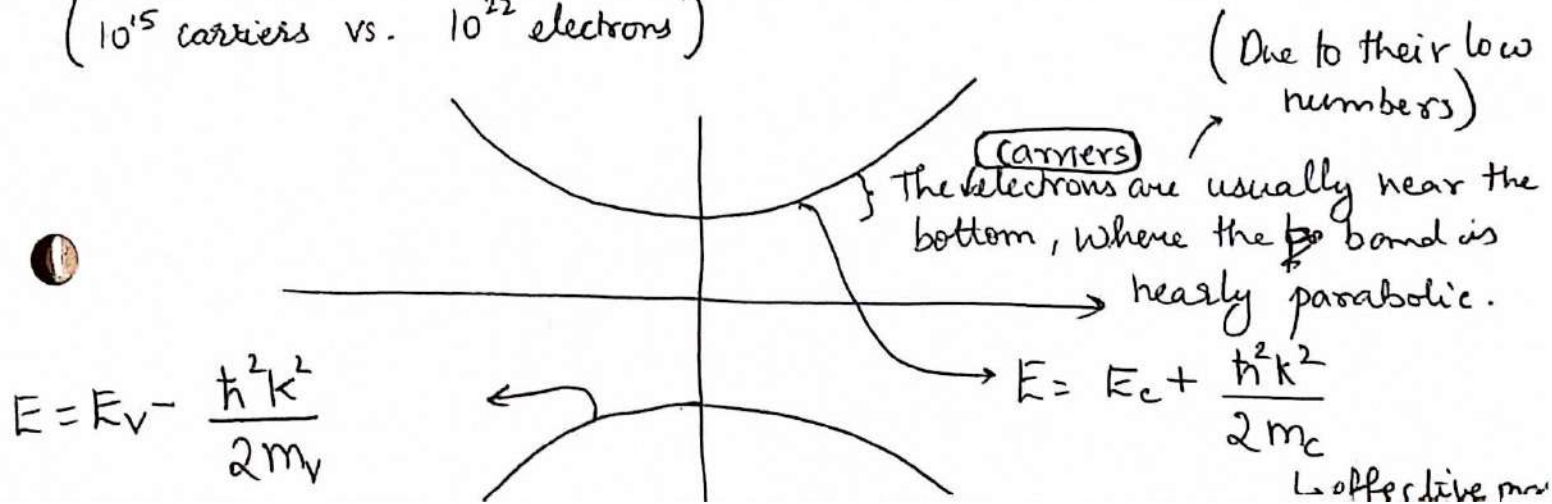
Density of States: $P(k)$ & $P(E)$

$P(k)dk \rightarrow$ no. of states between k & $k+dk$ per unit volume of the material.

$P(E)dE \rightarrow$ "



$\Rightarrow$ Carriers are much smaller than total no. of electrons in that band (10^{15} carriers vs. 10^{22} electrons)



Effective mass

$$m_{e,v} = \left(\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} \right)^{-1}$$

	m_c	m_v	μ_c	μ_v
Si	$0.98 m_0$	$0.49 m_0$	1450	450
GaAs	$0.07 m_0$	$0.45 m_0$	8500	400
Ge	$0.08 m_0$	$0.28 m_0$	3900	1900

$$m_0 = 9.1 \times 10^{-31} \text{ kg.}$$

($\mu \uparrow$ since $m \downarrow$).

What values can k have?

Bloch's Theorem $\psi_k = \underbrace{u_k(r)}_{\text{periodic Bloch cell function}} \underbrace{e^{i\vec{k} \cdot \vec{r}}}_{\text{plane wave}}$

probability = $|\psi|^2$.

periodic Bloch cell function.

For standing waves inside a cubic crystal we need

$$\Rightarrow 2 \vec{k} \cdot \vec{r} = N \cdot 2\pi \quad \text{Let } \vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z} = (k_x, k_y, k_z)$$

$$\Rightarrow 2 k_x L_x = m \cdot 2\pi$$

$$2 k_y L_y = p \cdot 2\pi$$

$$2 k_z L_z = q \cdot 2\pi$$

Where $L_x, L_y, \text{ and } L_z$ are the side lengths of the crystal. Each component must form a standing wave.

$$\Rightarrow k_x = \left(\frac{\pi}{L_x}\right)m; \quad k_y = \left(\frac{\pi}{L_y}\right)p; \quad k_z = \left(\frac{\pi}{L_z}\right)q.$$

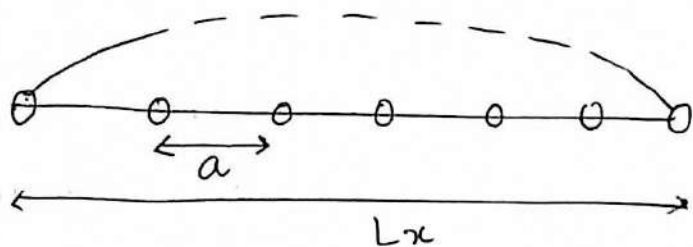
$\Rightarrow \vec{k}$ is discrete!

Eg:

Let $L_x = L_y = L_z = 1\text{mm}$.

$$\Rightarrow k_x = \frac{\pi}{1\text{mm}} m = \underline{10^3 \pi \text{ m}^{-1} \text{ (smallest)}}.$$

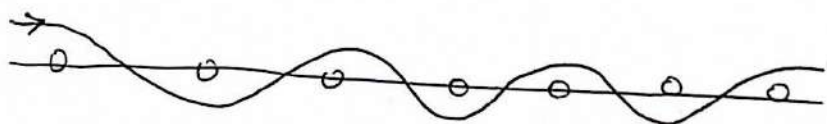
What is its largest value?



→ lowest mode.

$\Delta \rightarrow$ deBroglie wavelength

$$\Delta = 2L_x \quad \& \quad k = \frac{2\pi}{\Delta} = \frac{\pi}{L_x}$$



→ Highest mode.

Electron entering from left can "at most" be scattered such that it bends around each atom. In this case,

deBroglie $\Delta = 2a \Rightarrow k = \frac{\pi}{a} \rightarrow$ which is also the edge of the first Brillouin zone! $\underline{k_{x\text{max}} = 2 \times 10^9 \pi \text{ m}^{-1}}$

Therefore k_x varies from $10^3\pi$ to $\sim 10^9\pi$ in steps of $10^3\pi$

→ There are 10^6 states in each of k_x, k_y, k_z .

→ There are 10^{18} states for k !

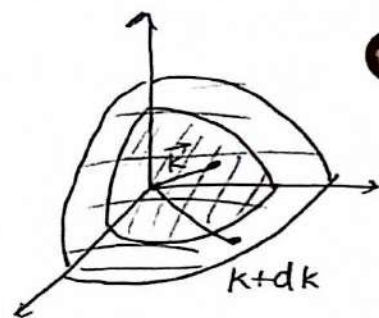
Lec 5 Density of States (contd.)

> Since spacing of states is $\frac{\pi}{L_x}, \frac{\pi}{L_y}$ & $\frac{\pi}{L_z}$, the volume occupied by one state is $\left(\frac{\pi^3}{L_x L_y L_z}\right)$.

> Volume between k & $k+dk$

$$= \frac{1}{8} \cdot 4\pi k^2 \cdot dk$$

↳ only need consider one octant in k -space since $-ve$ k_x, k_y, k_z are already present in a standing wave.



> No. of states b/w k & $k+dk = \frac{\frac{1}{8} \cdot 4\pi k^2 dk}{\left(\frac{\pi}{L_x}\right)\left(\frac{\pi}{L_y}\right)\left(\frac{\pi}{L_z}\right)} \times 2$

$$\Rightarrow \rho(k) dk = \frac{\pi k^2 dk}{\pi^3} \frac{(L_x L_y L_z)}{(L_x L_y L_z)} = \boxed{\frac{k^2 dk}{\pi^2}}$$

due to 2 spin states.

$$\Rightarrow P(k) = \frac{k^2}{\pi^2}$$

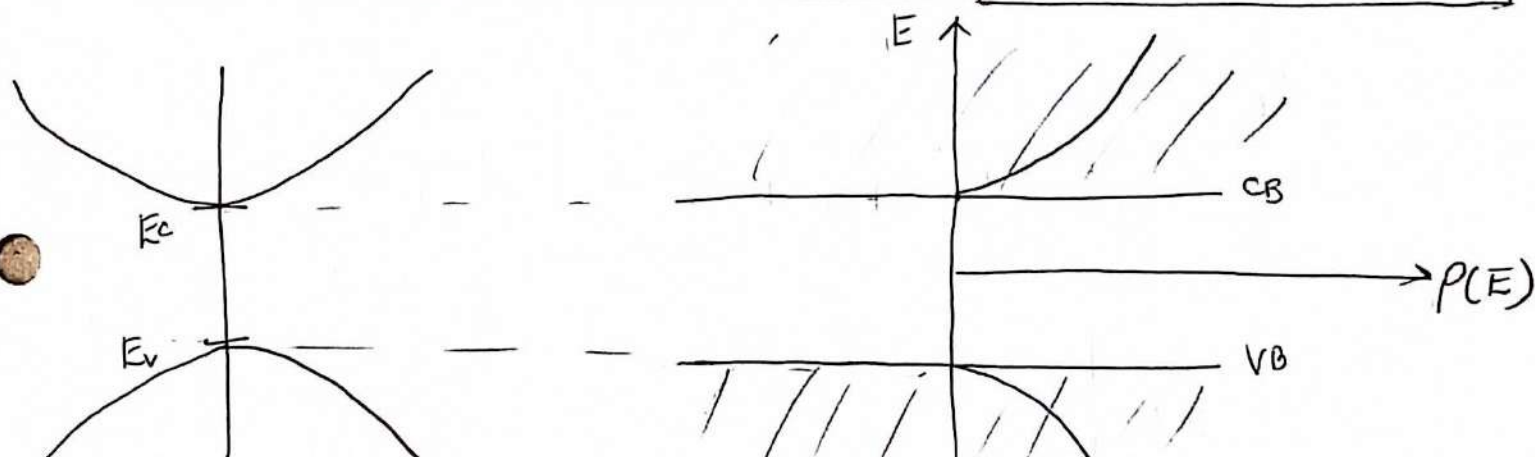
$$\Rightarrow P(E) = \frac{P(k)}{(dE/dk)} = \frac{k^2}{\pi^2} \cdot \frac{1}{(dE/dk)}$$

Parabolic approx. $\Rightarrow E = E_c + \frac{\hbar^2 k^2}{2m_c} \rightarrow$ conduction band.

$$\Rightarrow \frac{dE}{dk} = \frac{\hbar^2}{2m_c} 2k = \frac{\hbar^2}{2m_c} \cdot 2 \left[(E - E_c)^{1/2} \left(\frac{2m_c}{\hbar^2} \right)^{1/2} \right]$$

$$\Rightarrow P(E) = \frac{m_c}{\pi^2 \hbar^2} (E - E_c)^{1/2} \left(\frac{2m_c}{\hbar^2} \right)^{1/2}$$

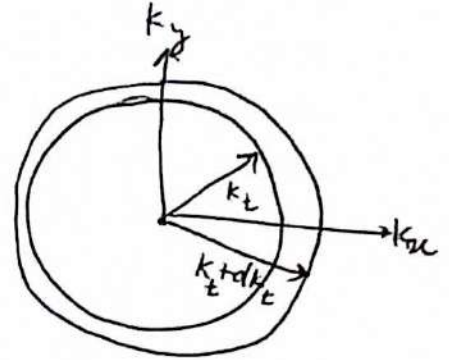
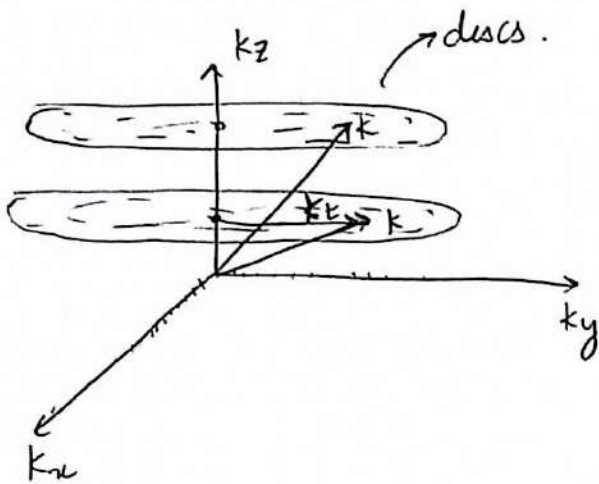
$$\Rightarrow P(E) = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2} \quad \text{for CB} \quad \rightarrow \quad \frac{1}{2\pi^2} \left(\frac{2m_v}{\hbar^2} \right)^{3/2} (E_v - E)^{1/2} \quad \text{for VB}$$



Lec 6 Density of States in a Quantum Well Structure.

> When one dimension of the semiconductor becomes very small, then the discreteness in that dimension's k becomes important.

k-spaces



$$\vec{k} = \vec{k}_t + k_z \hat{z}$$

$$E = E_c + E(q=1) + \frac{\hbar^2 k_z^2}{2m_c}$$

$\Rightarrow P(k)dk = P(k_t)dk_t$ & we only consider one quadrant!
 $\rightarrow$ quadrant.

$\Rightarrow$ Area of quadrant annulus is $\frac{1}{4} \cdot 2\pi k_t dk_t$.

$$\Rightarrow P(k_t)dk_t = \frac{\frac{1}{4} \cdot 2\pi k_t dk_t \times 2}{\left(\frac{\pi}{L_x}\right)\left(\frac{\pi}{L_y}\right)} \quad \begin{matrix} \rightarrow 2 \text{ spins.} \\ \swarrow \\ L_x L_y L_z \\ \rightarrow \text{total volume.} \end{matrix}$$

$$\Rightarrow \boxed{P(k_t) = \frac{k_t}{\pi L_z}}$$

$$\frac{dE}{dk_t} = \frac{\hbar^2 k_t}{m_c}$$

$$\Rightarrow P(E) = \frac{\left(\frac{k_t}{\pi L_z} \right)}{\frac{\hbar^2 k_t}{m_c}}$$

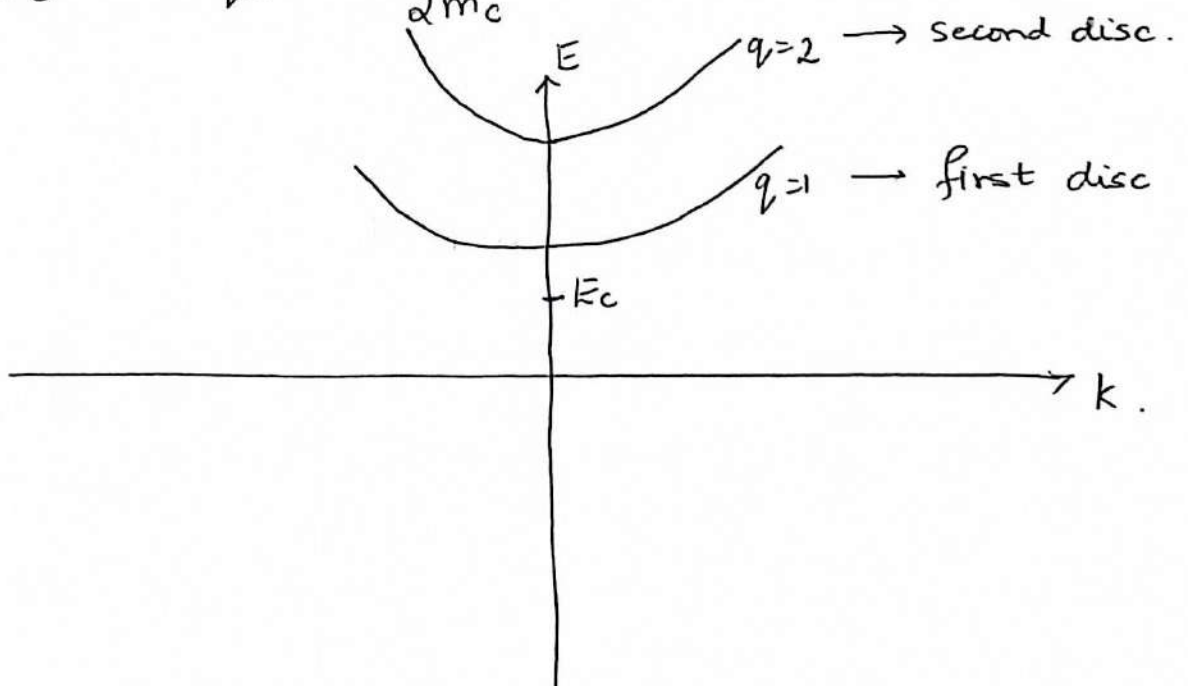
$$\Rightarrow P(E) = \frac{m_c}{\hbar^2 \pi L_z}$$

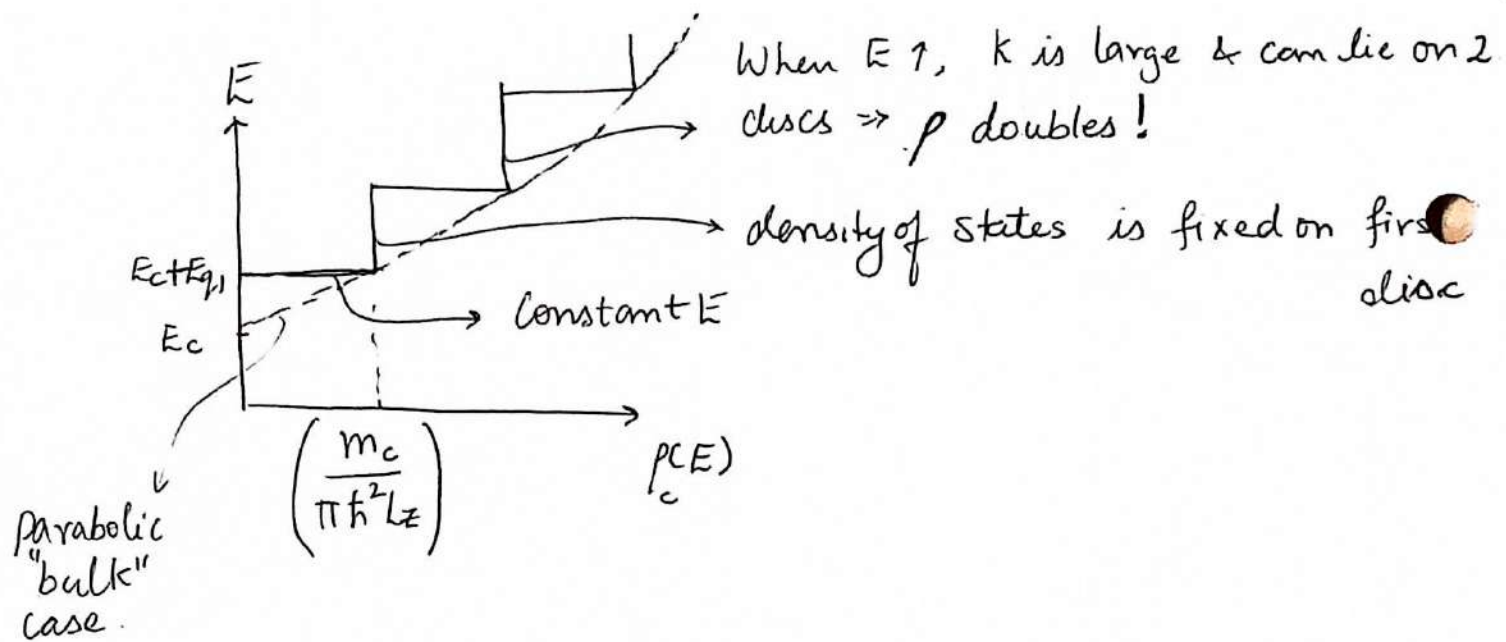
Such thin semiconductors are also called quantum well $\rightarrow$ (will make sense later).

no energy dependence!

$$E = E_c + E_{q1} + \frac{\hbar^2 k_t^2}{2m_c} \quad \text{when } q=1$$

$$E = E_c + E_{q2} + \frac{\hbar^2 k_t^2}{2m_c}$$





> Note that bandgap increases, it is more than $E_c - E_v$.

> Number of states: $N_c = \int_{E_c}^{E_2 = E_c + 0.1 \text{ eV}} \rho_c(E) dE = \frac{1}{3\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E - E_c)^{3/2} \Big|_{E_c}^{E_2}$

$\Rightarrow N_c = \frac{1}{3\pi^2} \cdot \left(2 \cdot \frac{m_c}{\hbar^2} \right)^{3/2} (0.1 \text{ eV})^{3/2}$ let $m_c = 0.067 m_0$

$\approx 10^{18} / \text{cc}$

> $n = \text{total no. of carriers} = \int_{E_c}^{E_2} n(E) dE$

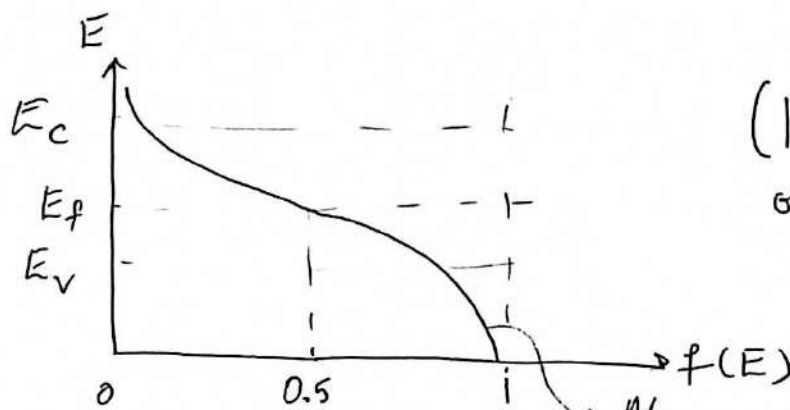
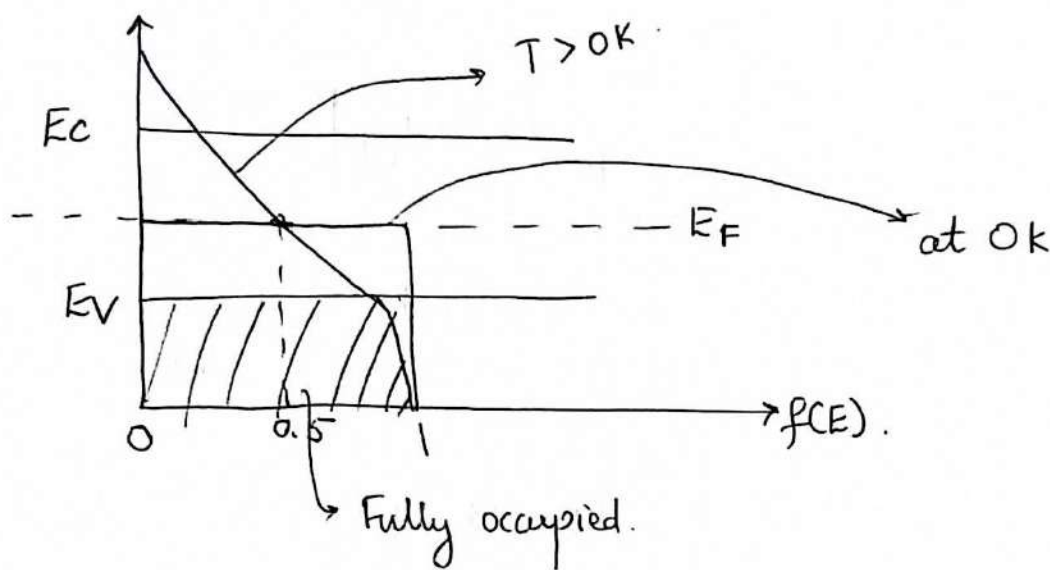
density of carriers $\rightarrow$
 $\hookrightarrow \rho(E) f(E)$
 Probability of occupation.
 (Fermi Function)

> Basically each electron occupies a single "resonant mode" of the crystal & 2 electrons must occupy different modes/states.

Lec 7 Occupation probability and Carrier Concentration (19)

Fermi function $\rightarrow f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$

$E_F \rightarrow$ Fermi energy.



$(1 - f(E))$ is the probability of occupation of a hole.

Always crosses E_F at 0.5.

$\infty \rightarrow$ (since $f(E) \rightarrow 0$ quickly)

Total number of carriers: $n = \int_{E_C}^{\infty} P(E) f(E) dE$

$$\Rightarrow n = \int_{E_C}^{\infty} \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E - E_C)^{1/2} \cdot \frac{1}{e^{(E-E_F)/kT} + 1} dE.$$

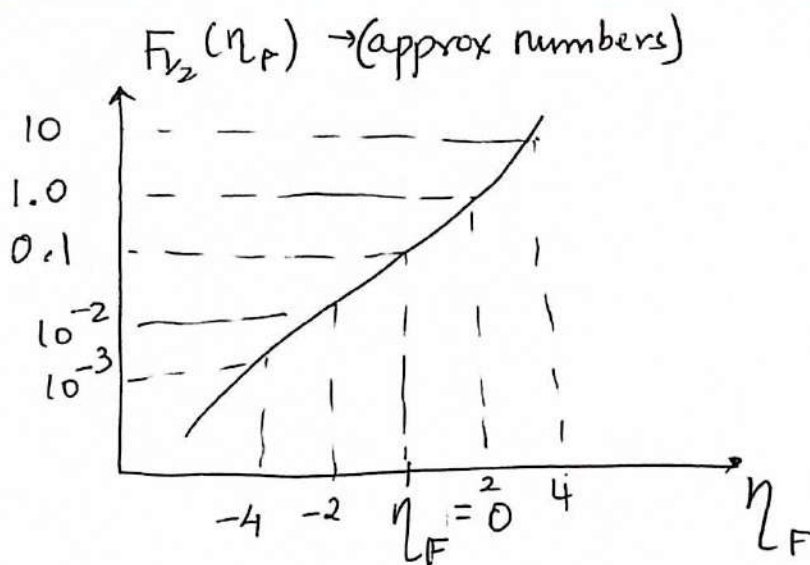
$$\Rightarrow n = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} \cdot \frac{1}{e^{(E - E_F)/kT} + 1} dE.$$

Let $\eta = (E - E_c)/kT$ & $\eta_F = \frac{E_c - E_F}{kT}$

$$\Rightarrow n = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (kT)^{3/2} \underbrace{\int_0^{\infty} \frac{\eta^{1/2} d\eta}{[e^{(\eta - \eta_F)} + 1]}}_{F_{1/2}(\eta_F)}$$

$F_{1/2}(\eta_F) \rightarrow$ Fermi Half Integral.

$$\Rightarrow n = \left(\frac{1}{2\pi^2} \right) \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (kT)^{3/2} F_{1/2}(\eta_F)$$



Boltzmann approx.

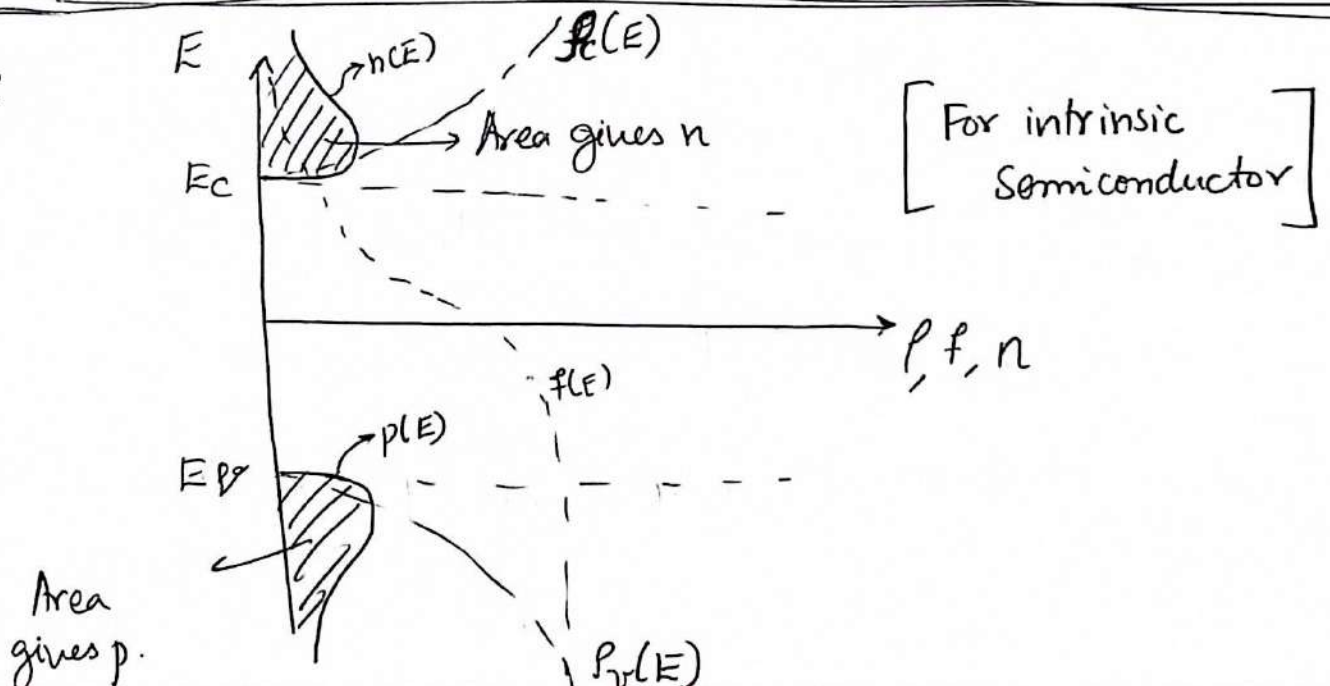
$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \approx e^{-(E-E_F)/kT} \quad \text{when } E-E_F \gg kT$$

$$\Rightarrow n = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \int_{E_c}^{\infty} (E-E_c)^{1/2} e^{-(E-E_F)/kT} dE.$$

$$\Rightarrow n = 2 \left(\frac{m_c kT}{2\pi \hbar^2} \right)^{3/2} e^{(E_F-E_c)/kT} \rightarrow \text{Conduction band.}$$

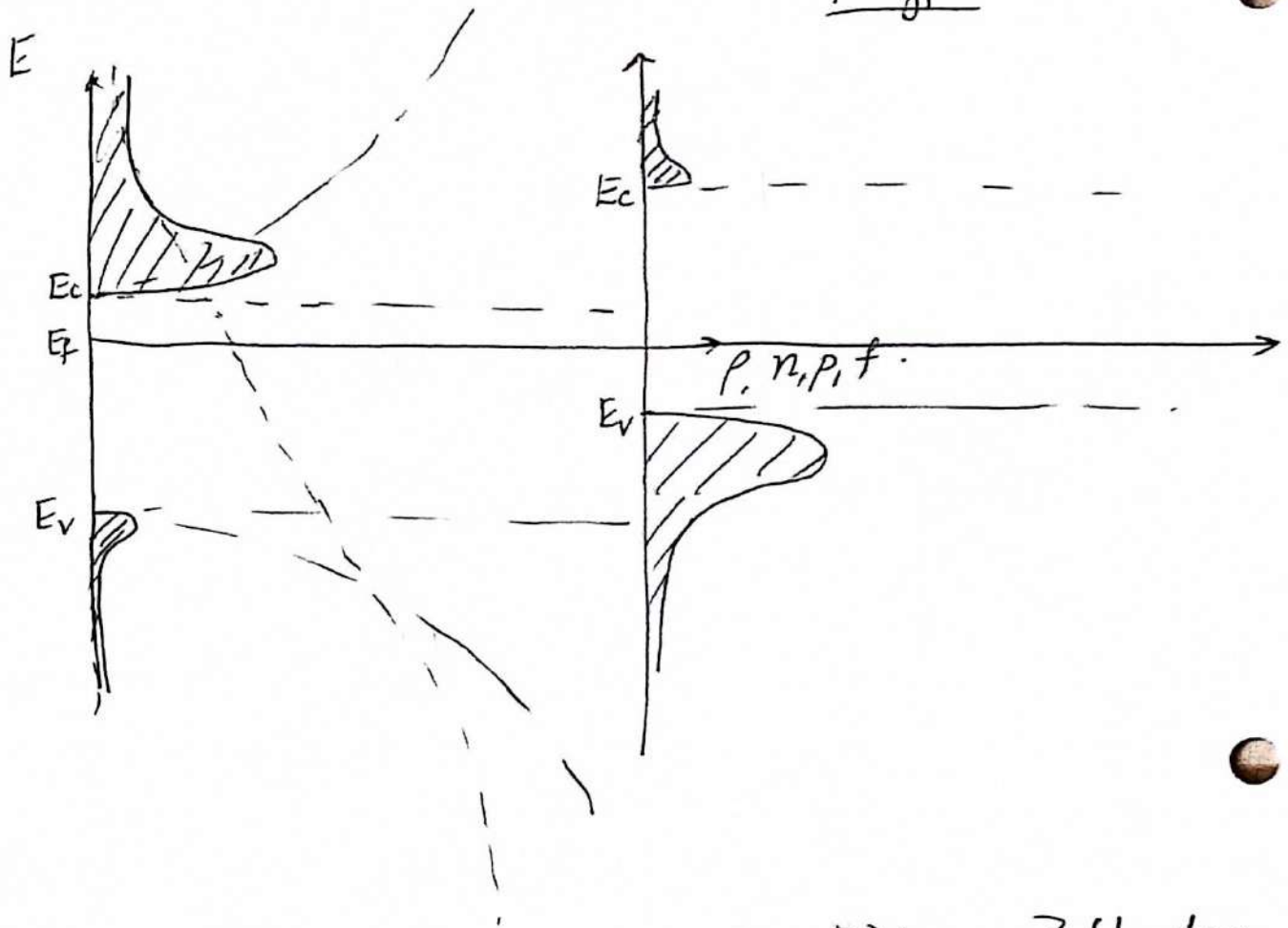
$$p = 2 \left(\frac{m_v kT}{2\pi \hbar^2} \right)^{3/2} e^{(E_v-E_F)/kT} \rightarrow \text{Valence band.}$$

Lec 8:



N type S.C

P type



$$p = N_v e^{(E_v - E_f)/kT} \quad n = N_c e^{(E_f - E_c)/kT} \quad \left. \vphantom{p = N_v e^{(E_v - E_f)/kT}} \right\} \text{Under Boltzmann}$$

$$np = N_c N_v e^{-E_g/kT}$$

For intrinsic semiconductor $n = p = n_i$

$$\Rightarrow n_i^2 = N_c N_v e^{-E_g/kT}$$

$$\Rightarrow \boxed{np = n_i^2} \rightarrow \text{Under Boltzmann approx. (\forall \text{ semiconductor})}$$

↳ Law of mass action.

$$\Rightarrow n_i = (N_c N_v)^{1/2} e^{-E_g/2kT} \Rightarrow \underline{n_i \text{ depends on } E_g.}$$

Typical Values

	Si	9n P	GaAs.
N_c	2.9×10^9	5.4×10^{17}	4.4×10^{17}
N_v	1.1×10^{19}	1.2×10^{19}	8.2×10^{18}
n_i	1.0×10^{10}	1.2×10^7	2.5×10^6
E_g	1.1 eV	1.35 eV	1.42 eV

$$E_{fi} = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln \left(\frac{m_v}{m_c} \right)$$

(intrinsic)

(After some algebra...)

→ usually very small.

For n type,

$$E_f = E_c + kT \ln \left(\frac{n}{N_c} \right)$$

• P type $E_f = E_v - kT \ln \left(\frac{p}{N_v} \right)$

These approx. are under Boltzmann, i.e. when lightly doped.

In general, (w/o Boltzmann).

$$n = M \cdot F_{1/2}(\eta_F).$$

$\downarrow$ constant

$$\& \eta_F = \left(\frac{E_F - E_c}{kT} \right)$$

$$\Rightarrow E_F = E_c + kT \eta_F = E_c + kT F_{1/2}^{-1}\left(\frac{n}{M}\right) //$$

Joyce-Dixon approximation of E_F (Empirical)

$$E_F = E_c + kT \left[\ln\left(\frac{n}{N_c}\right) + \frac{1}{\sqrt{8}} \frac{n}{N_c} \right]$$

$$E_F = E_v - kT \left[\ln\left(\frac{p}{N_v}\right) + \frac{1}{\sqrt{8}} \frac{p}{N_v} \right]$$

Quasi Fermi Levels

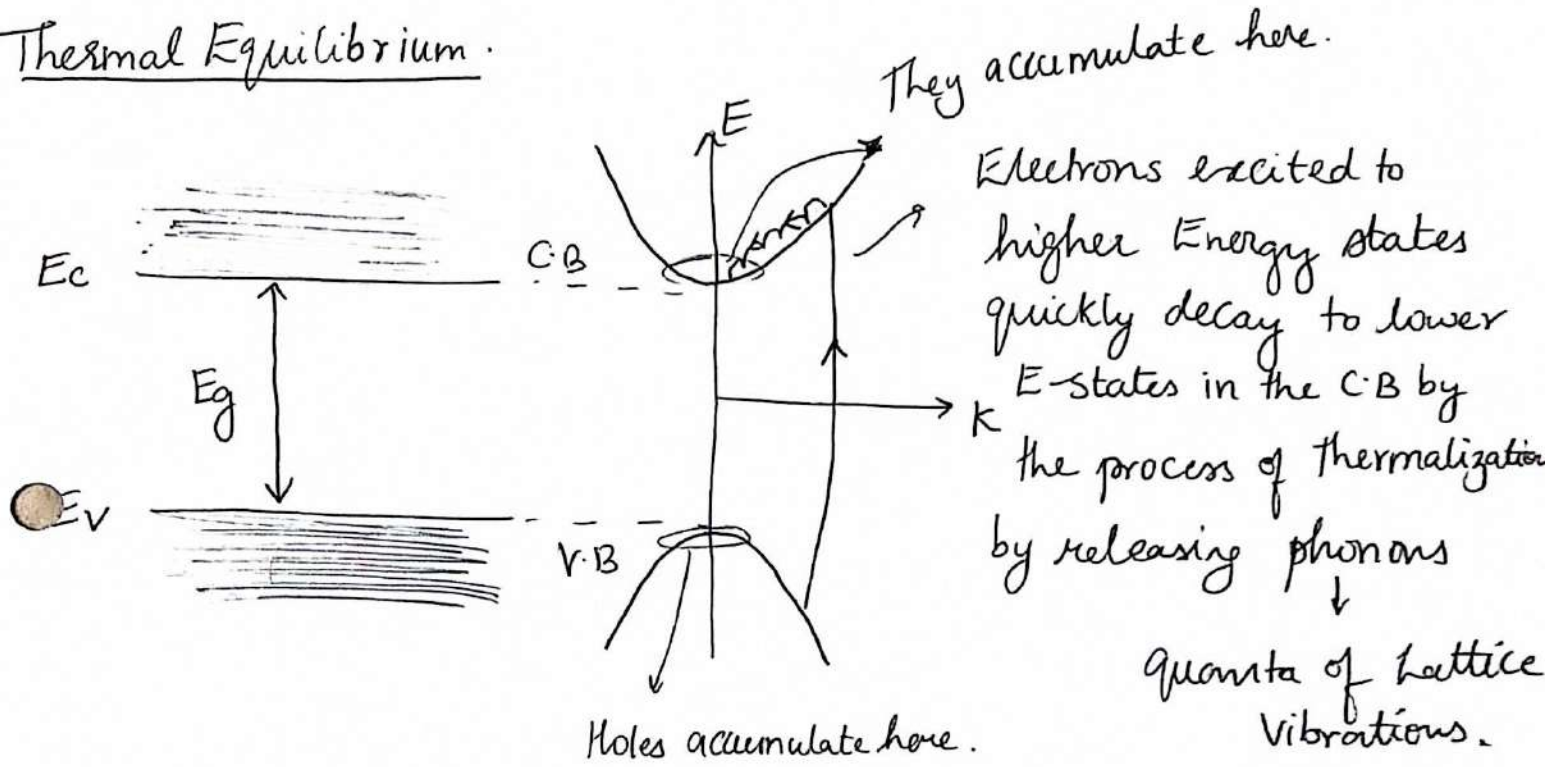
> When both CB & VB have a large number of free carriers,
we need QF Levels ≡

Lec 9 Quasi Fermi Levels

(25)

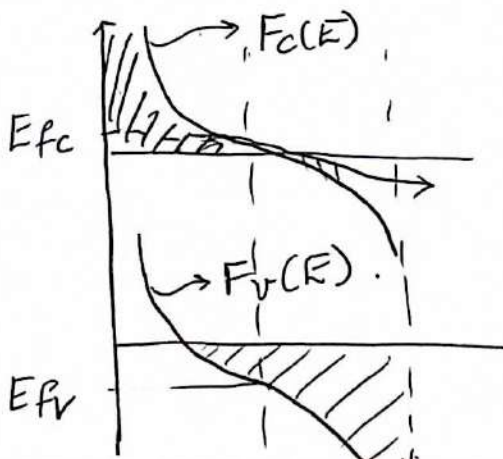
It refers to the 2 fermi levels which describe the probability of occupation in the conduction band and the valence band in "Quasi Equilibrium".

Thermal Equilibrium.



> Phonon transitions are very fast $\tau \approx 10^{-12} - 10^{-13}$ s.
(Intraband lifetime)

> Interband transitions $\tau \approx 10^{-8} - 10^{-9}$ s.



Two different Fermi functions $F_c(E)$ & $F_v(E)$ are used when an additional "pump" to move electrons to CB is present.

$$f_c(E) = \frac{1}{[e^{(E-E_{fc})/kT} + 1]}$$

$$f_v(E) = \frac{1}{[e^{(E-E_{fv})/kT} + 1]}$$

At Thermal Equilibrium, Boltzmann approx was

$$n = N_c e^{(E_{fc}-E_c)/kT}$$

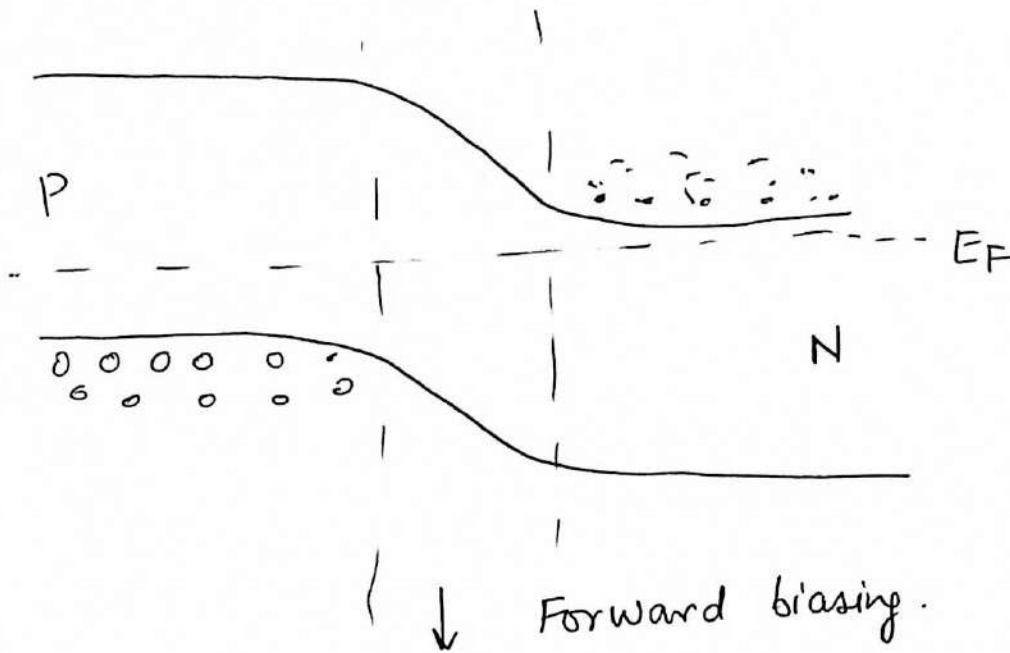
At Quasi Equilibrium,

$$n = N_c e^{(E_{fc}-E_c)/kT} \quad \& \quad p = N_v e^{(E_v-E_{fv})/kT}$$

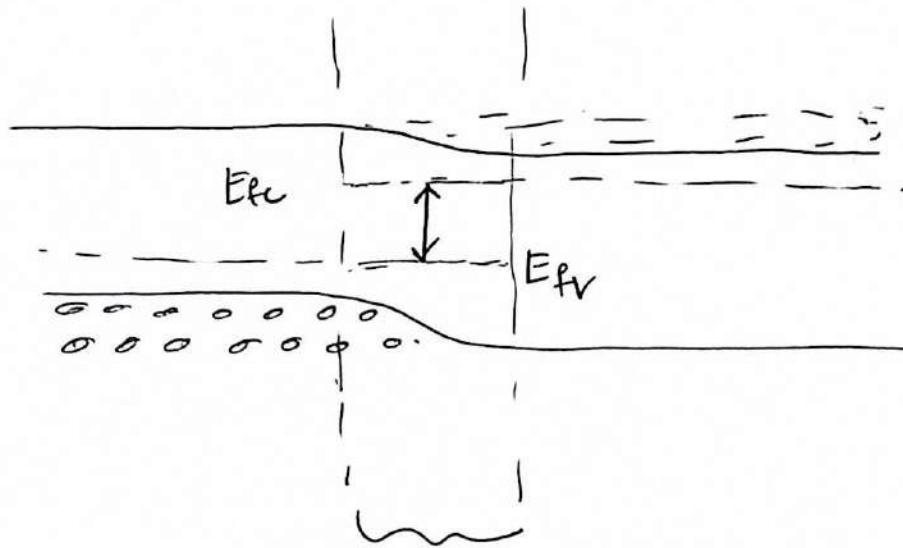
$$\Rightarrow np = N_c N_v e^{-E_g/kT} \cdot e^{(E_{fc}-E_{fv})/kT}$$

$$\boxed{np = n_i^2 e^{(E_{fc}-E_{fv})/kT}} \rightarrow \text{(Under Boltzmann approx} \\ \Rightarrow \text{lightly doped)}$$

PN junction (preview)



↓ Forward biasing.



In this region now the carrier concentrations have gone up! Both electrons & holes. There are now 2 Fermi functions E_{FV} & E_{FC} . Here $E_{FC} - E_{FV} > 0$ & $np > n_i^2$. If reverse biased we get $E_{FV} - E_{FC} > 0$ & $np < n_i^2$.

Lec 10 Semiconductor Materials.

- ① Elemental Semiconductors: Si, Ge, Sn $\rightarrow$ Group IV
- ② Binary Semiconductors (III-V semiconductors).
or (II-VI semiconductors).

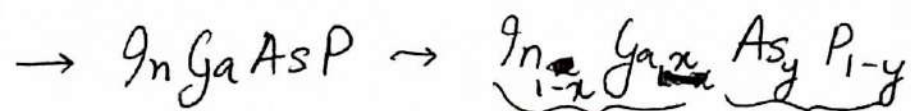
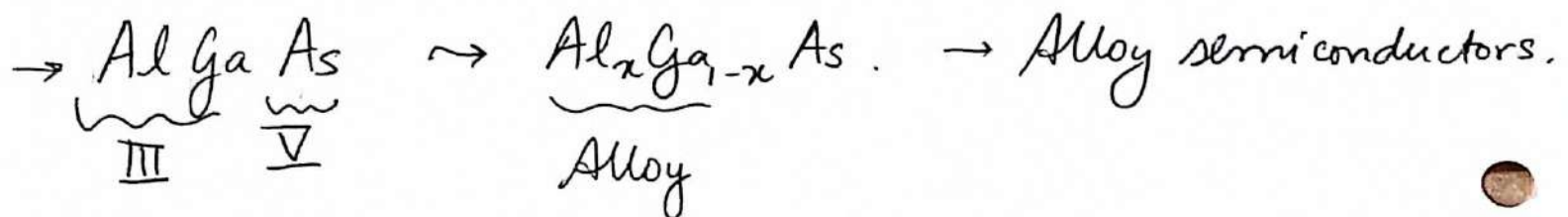
$$\lambda_g = \frac{hc}{E_g} \rightarrow \text{Bandgap wavelength.}$$

$\hookrightarrow$ Largest wavelength that can cause band transition.

$$\boxed{\lambda_g \text{ (in } \mu\text{m})} \approx \frac{1.24}{E_g \text{ (eV)}}$$

$$\lambda_g (\text{GaN}) \approx \frac{1.24}{3.39} \approx 0.35 \mu\text{m} \Rightarrow \text{Important for blue light.}$$

- ③ Ternary Compounds & Quaternary Compounds
(Allows continuous variation in bandgap E_g).



(29)

$\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y} \Rightarrow \text{InP}$ is the starting material since x, y are usually less than 0.5.

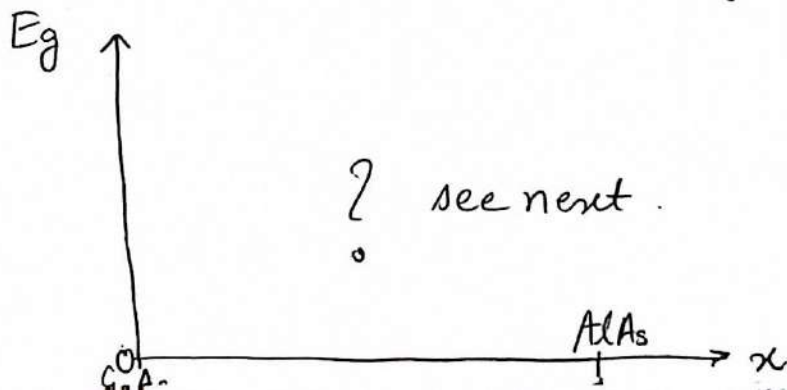
Why Ternary & Quaternary compounds?

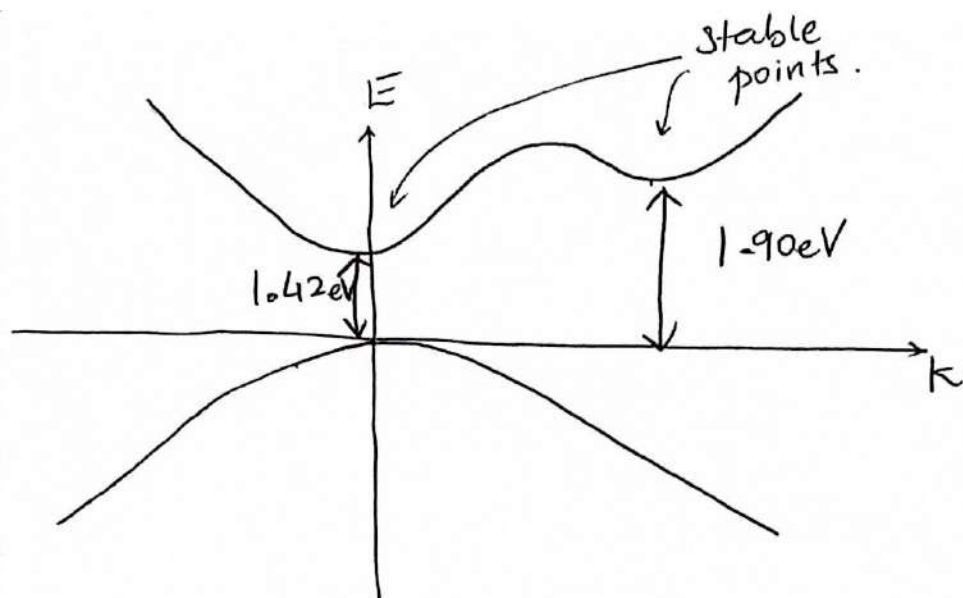
- ① For bandgap modification.
- ② Enables formation of lattice matched heterostructures.

Bandgap modification.

$\text{Al}_{1-x}\text{Ga}_x\text{As} / \text{GaAs} \rightarrow$ means GaAs is the starting material & Al is added to it.

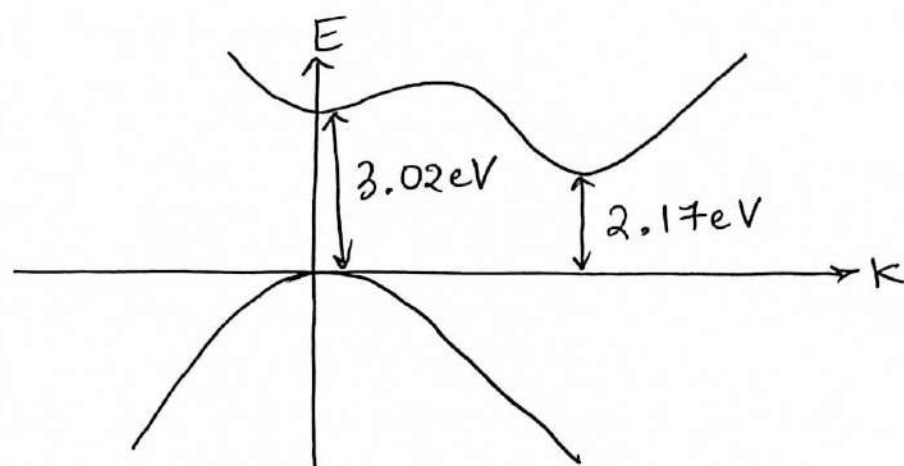
- > $\lambda_g \approx 780\text{nm}$ & is the most widely used photonic semiconductor (laser diodes).
- > For optical comm. λ_g is 1550nm & InGaAsP is used. Because this is the low loss window of optical fibers.





GaAs

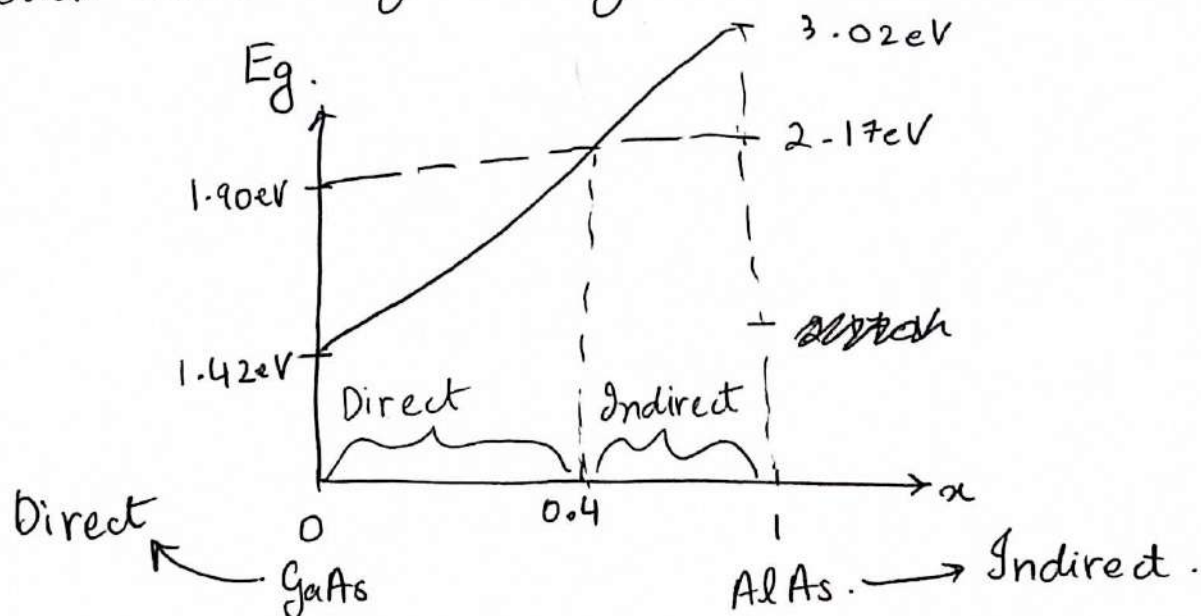
This is a Direct Bandgap material since the global min is at $k=0$.



AlAs

This is an indirect bandgap semiconductor.

At $k=0$ there is always a maxima in valence band but conduction band may or may not have a local minima.



Lec 11 Semiconductor Heterostructures - & -

(31)

Lattice Matched Layers.

Doping vs. Alloy

Doping : > Substitutional element is from the adjacent group whereas alloys are from the same group.
vs
Alloy

> Doping is to change the conductivity & alloying is for bandgap modification.

> Alloys are much higher in concentration in terms of the substitutional elements concentration.

Doping $10^{12} - 10^{19} / \text{cc}$ & Alloys $10^{22} / \text{cc}$.

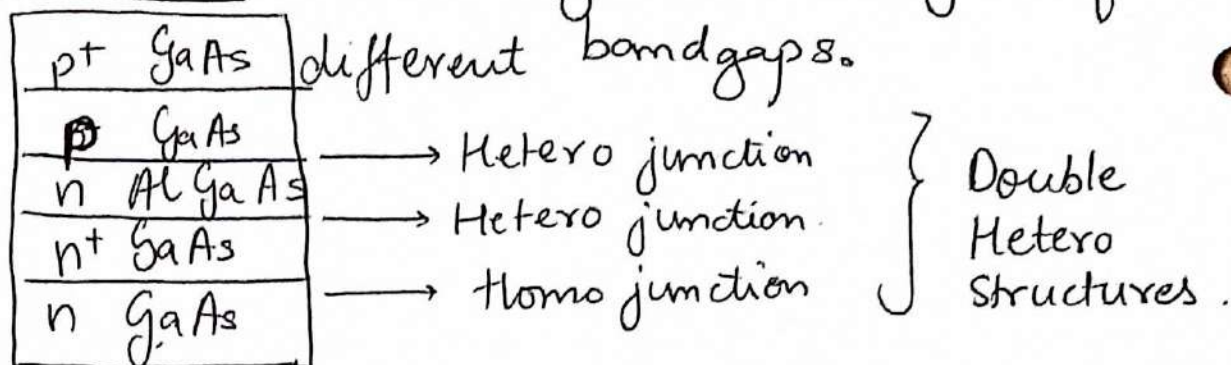
> Fabrication of Devices $\begin{cases} \rightarrow \text{Optoelectronics} \rightarrow \text{much larger.} \\ \rightarrow \text{Electronics} \end{cases}$

> Electronics uses diffusion (P, n, p⁺, n⁺...) & epitaxy
 $\hookrightarrow$ deposition of SiO₂ & other layers

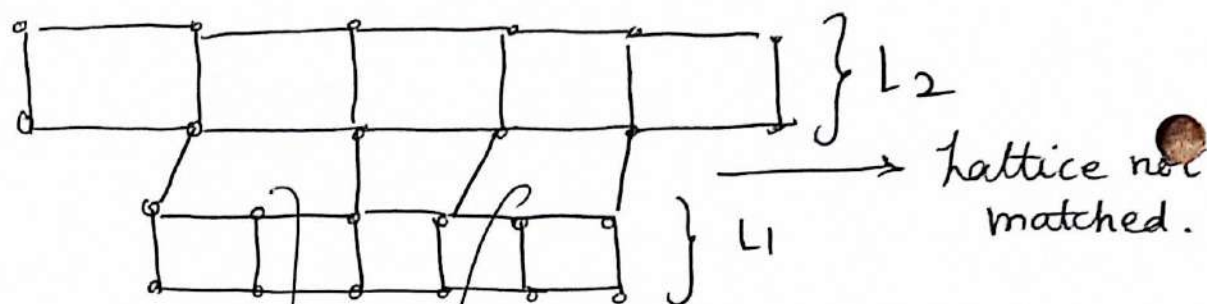
> Photonics mostly uses epitaxy

$\begin{cases} \rightarrow \text{LPE} & \text{Liquid phase epitaxy} \\ \rightarrow \text{VPE} & \text{Vapour phase epitaxy or CVD - chemical vapor dep.} \\ \rightarrow \text{MBE} & \text{Molecular beam epitaxy.} \end{cases}$

> Heterostructure: Device using material layers of different bandgaps.



> Lattice matching is required for the Hetero junction to be free of defects. (i.e. $a_L = a_L$).
 ↳ (very bad!! for optoelectronics)

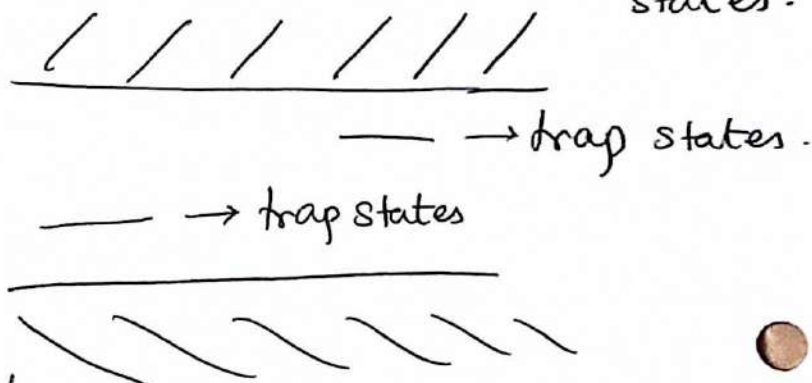


Therefore, trap states can be formed even with 2 pure materials.

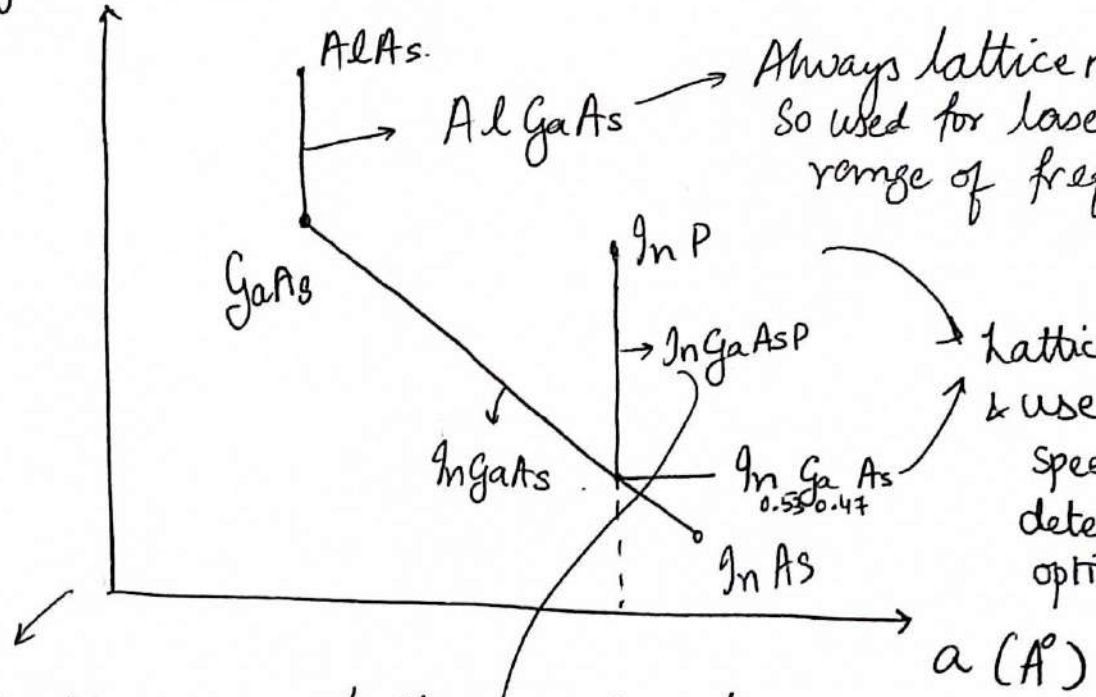
They increase nonradiative transitions.

These transitions happen using photon emission/absorption.

Dangling bonds → electrons here have different energy → form trap states.



E_g (eV)



Always lattice matched
So used for lasers over
range of frequencies.

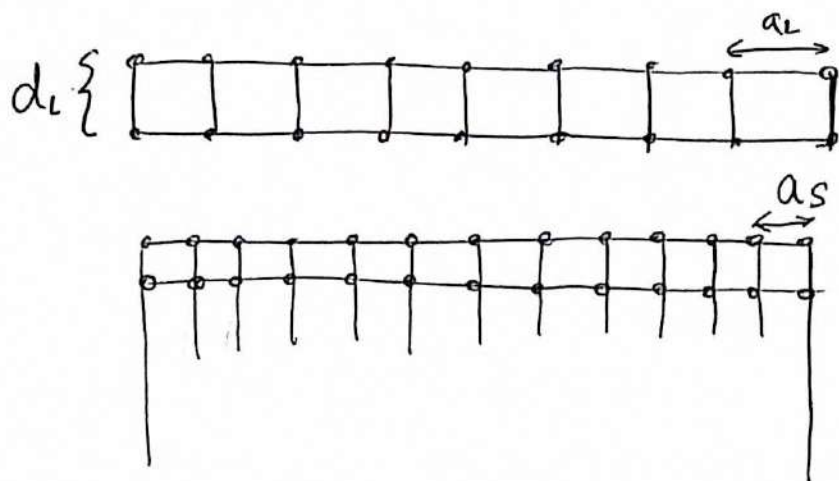
Lattice matched
& used as high
speed photo
detectors for
optical comm.

Vertical lines are lattice matched.

This vertical line defines all optical
Comm. photo detectors from 1.3-1.55 μ m

Lec 12 Strained Layer Epitaxy and Quantum Well Structure

> Strained Layer epitaxy.



→ These atoms are "squeezed" or strained in $\leftrightarrow$ axis to match with a_L . They also experience a tensile strain in $\uparrow$ axis to compensate. This is only possible for thin layers.

If $d_L < d_c \rightarrow$ critical thickness.

$$d_c \approx \frac{a_s}{2|\epsilon|} \quad \text{where } \epsilon = \frac{a_L - a_s}{a_L}$$

↘ mismatch parameter.

High mismatch $\Rightarrow d_c$ is low.

> $d_L \sim 100 \text{ \AA} \Rightarrow$ Quantum well structures

↪ If $d_L \leq \Lambda_{\text{deBroglie}} \rightarrow$ (de Broglie wavelength)

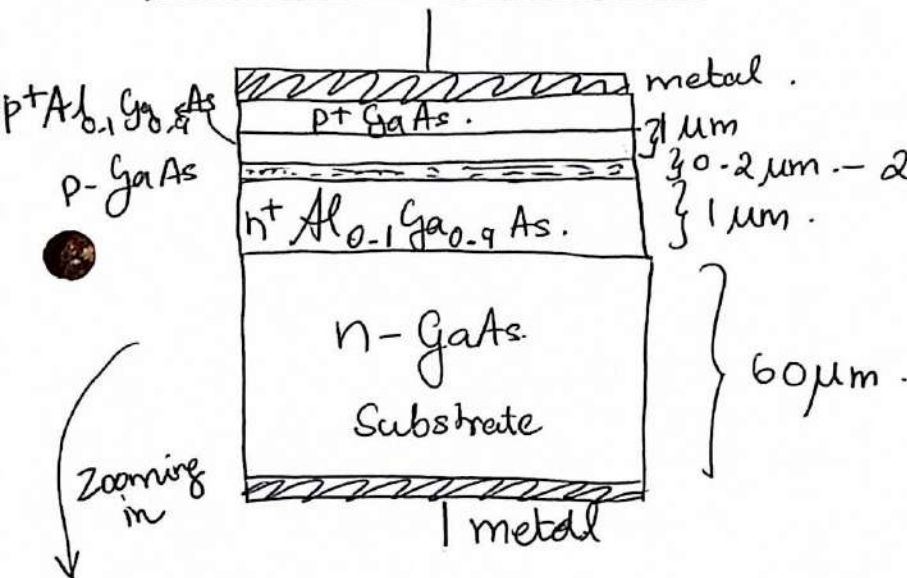
Thermal energy of electrons $E \approx kT \approx 0.025 \text{ eV}$

$$E = \frac{p^2}{2m^*} \Rightarrow p = \sqrt{2m^*E} \propto \Lambda = \frac{h}{p} = \frac{h}{\sqrt{2m^*E}}$$

> For GaAs, $m^* = 0.067 m_0$.

$$\Rightarrow \Delta_{dB_r} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 0.067 \times 9.1 \times 10^{-31} \times 0.025 \times 1.602 \times 10^{-19}}} \\ \approx 300 \text{ \AA}.$$

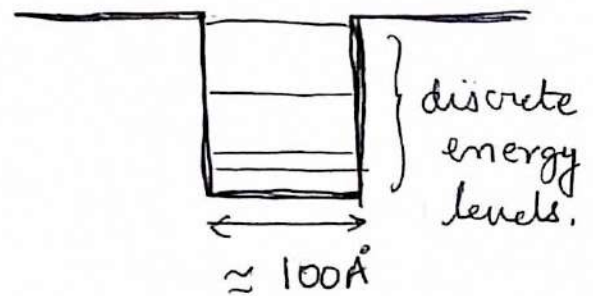
> Double Hetero structure



if this $< d_c$ then this becomes a quantum well structure.

when this becomes small, we see discrete modes as in a quantum well.

1.55 eV	1.42 eV	1.55 eV
← 1 μ →	← 0.2 μ →	← 1 μ →
Al Ga As	Ga As	Al Ga As 0.1 0.9

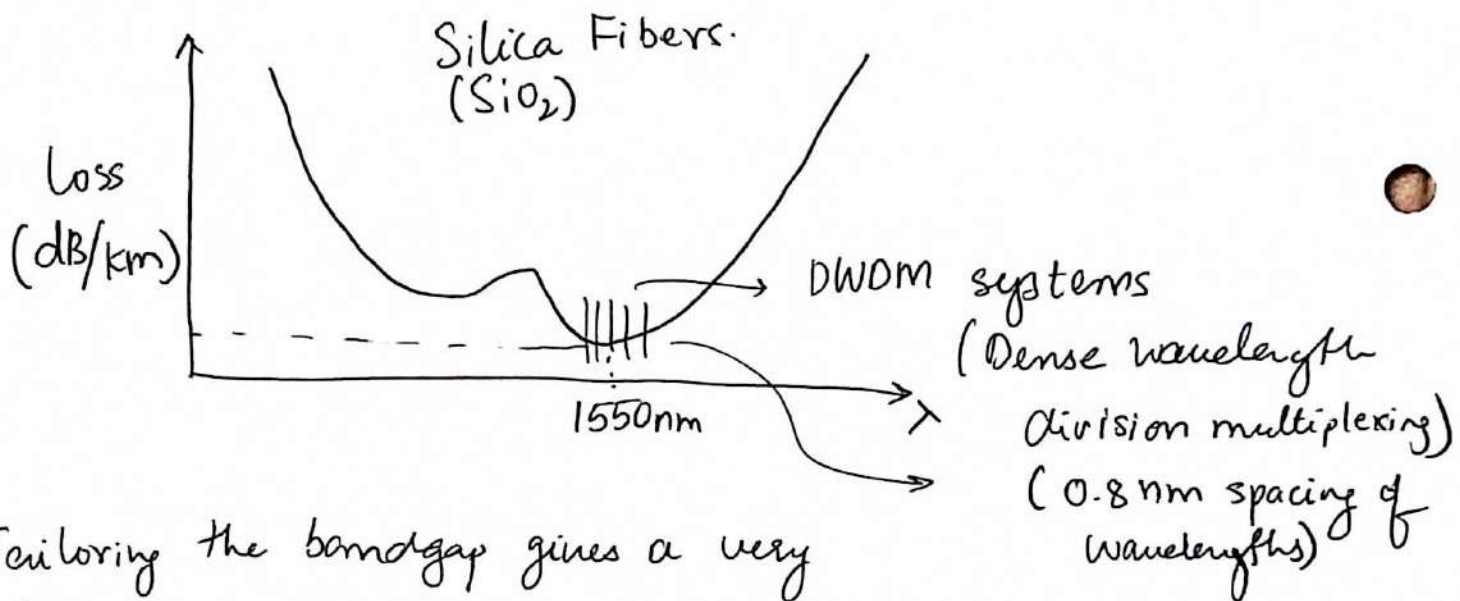


Lec 13: Bandgap Engineering.

- ① Alloy semiconductors
- ② Use of Quantum well structures
- ③ Use of Strained Layer materials (Pseudomorphic materials).

Why do we need Bandgap Engineering?

Optical Fibers.



> Tailoring the bandgap gives a very fine control on the λ to realize DWDM.

Going back to Quantum Wells

V

E_2

E_1

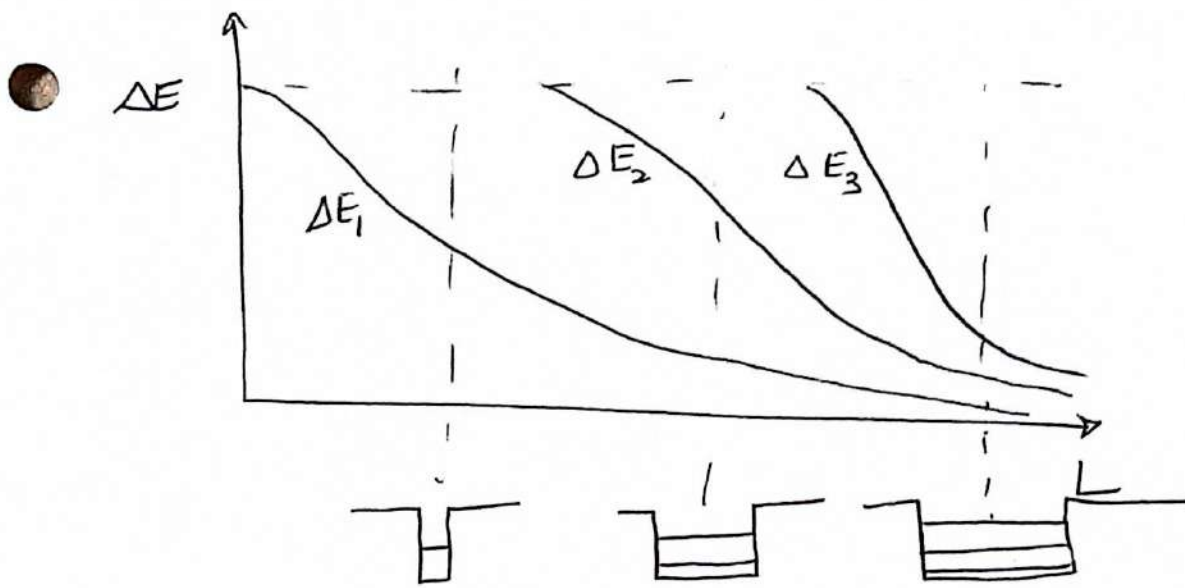
ΔE_1

ΔE_2

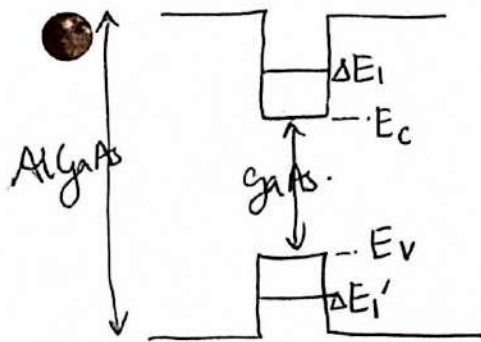
L

$\tan kL = \frac{\gamma}{k} \Rightarrow \boxed{kL = \tan^{-1}\left(\frac{\gamma}{k}\right) + n\pi}$

where $k^2 = \frac{2m}{\hbar^2} E$ & $\gamma^2 = \frac{2m}{\hbar^2} (V - E)$



What is the effective bandgap in a quantum well?



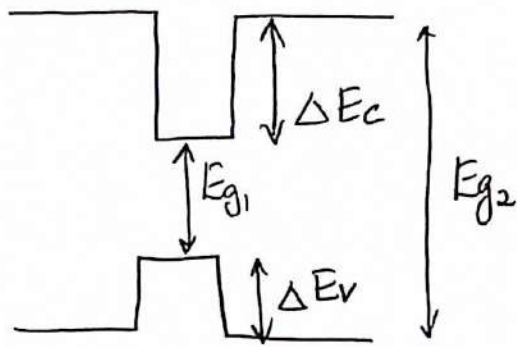
Since ΔE_1 & $\Delta E_1'$ are the first possible occupiable states, the effective bandgap is now

$$E_g^{\text{eff}} = (E_c - E_v) + \Delta E_1 + \Delta E_1'.$$

> MBE ~~also~~ allows monolayer deposition so thickness control is very practical.

Example $\text{Al}_{0.35}\text{Ga}_{0.65}\text{As}$ & $\text{GaAs} \rightarrow 1.42\text{eV}$.
 $\hookrightarrow 1.860\text{eV}$.

$\Rightarrow \Delta E_g = 0.436\text{eV}$. How much λ tuning this gives.



$$\Delta E_c \approx 0.65 \Delta E_g$$

$$\Delta E_v \approx 0.35 \Delta E_g$$

These numbers are typical for opto-electronic materials.

$$\Delta E_c \approx 0.29 \text{ eV}$$

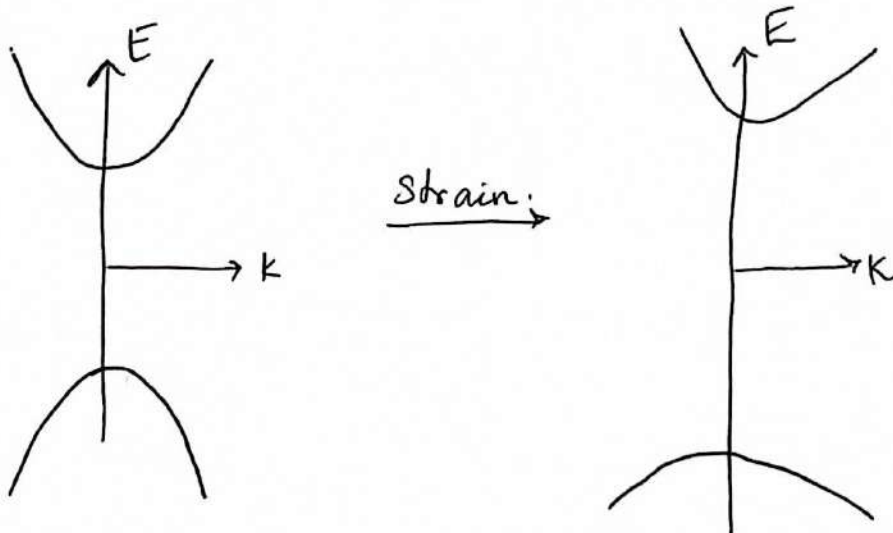
$$\Delta E_v \approx 0.15 \text{ eV}$$

$$\Rightarrow \lambda_g \rightarrow 1.424 \rightarrow 0.88 \mu\text{m}$$

$$\lambda_g \rightarrow 1.86 \rightarrow 0.65 \mu\text{m}$$

$\Rightarrow \Delta \lambda_g = 230 \text{ nm}$. & if $\delta \lambda_g$ is 0.8 nm in DWDM we can have over 250 channels!

③ Use of Strained layers.

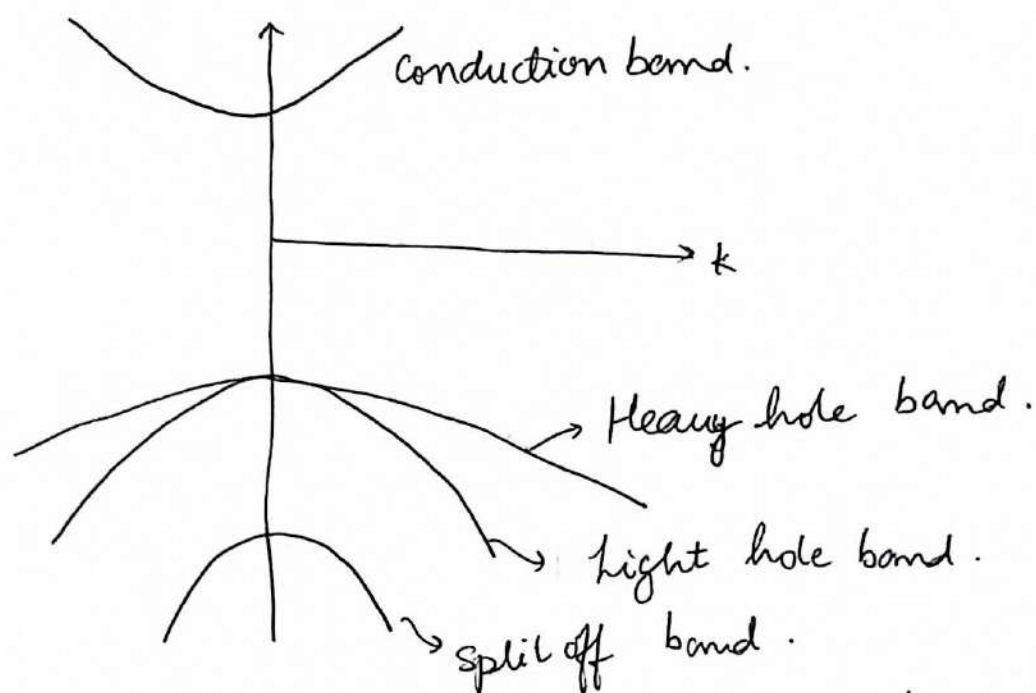


> Strain affects bandgap.

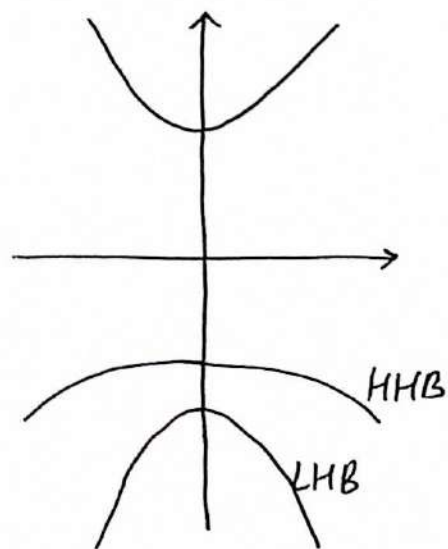
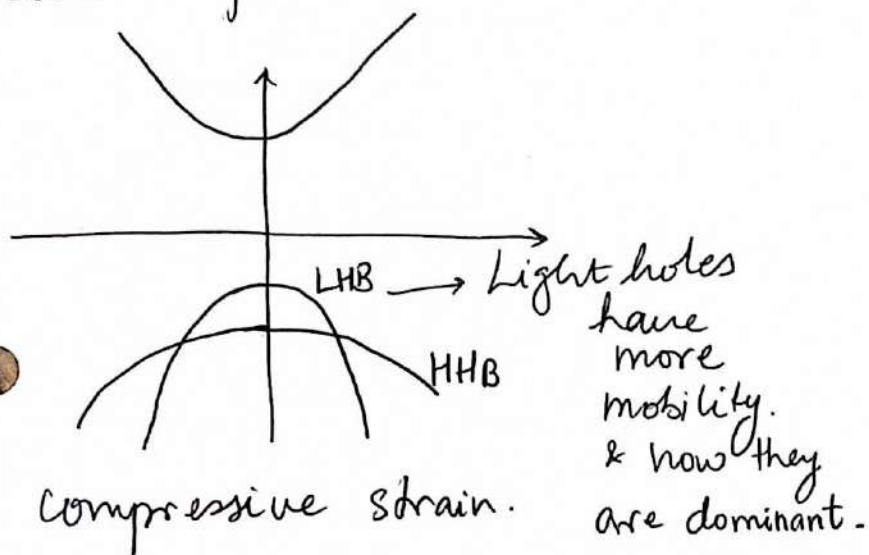
$\Rightarrow$ Bandgap can be controlled by changing the lattice mismatch.

- > Why use composition changes to induce strain for bandgap engineering when composition changes can directly change the bandgap? Reality is more complex.

Valence band is more complex.



When strain is applied, Heavy hole & light hole bands split.



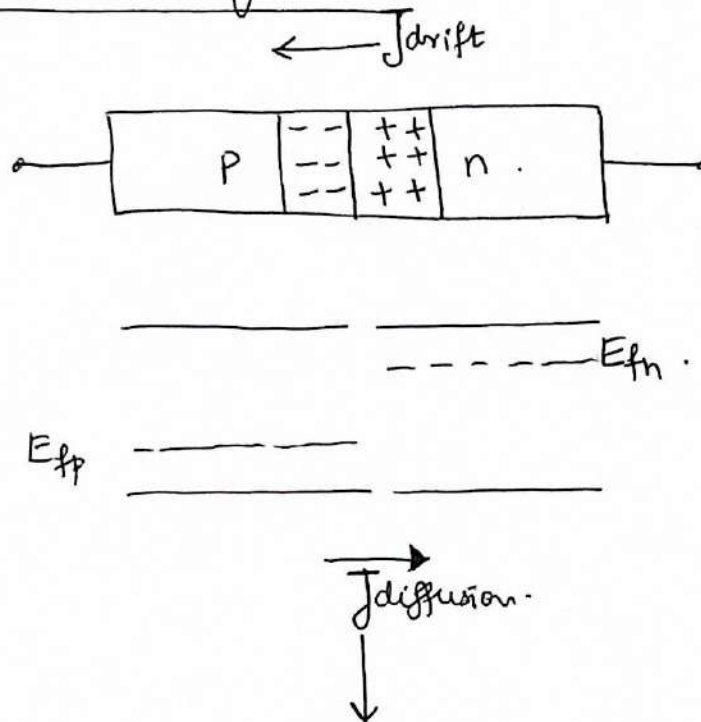
Lec 14 - Heterojunction PN junctions.

- p-n junction devices for active control.

- Fabrication $\left\{ \begin{array}{l} \text{Diffusion.} \\ \text{Ion implantation.} \\ \text{Epitaxy} \rightarrow \text{abrupt junctions.} \end{array} \right\}$ of dopants. $\left. \vphantom{\begin{array}{l} \text{Diffusion.} \\ \text{Ion implantation.} \\ \text{Epitaxy} \end{array}} \right\}$ lead to graded junctions.

- Classification $\left\{ \begin{array}{l} \text{Homojunction} \\ \text{Heterojunction.} \\ \text{Schottky junction.} \end{array} \right.$

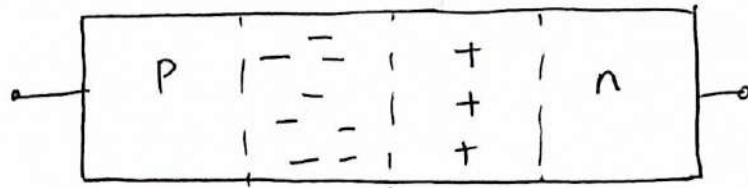
- Energy Band diagrams.



At equilibrium,
 $J_{diff} + J_{drift} = 0$
 $\Rightarrow$ can be shown that

$$\frac{dE_F}{dx} = 0$$

$\Rightarrow E_F$ is constant throughout.
 $\Rightarrow$ Band must bend.



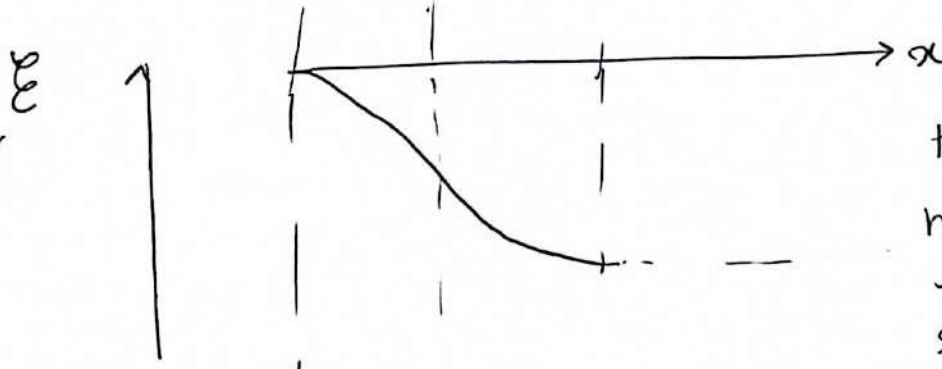
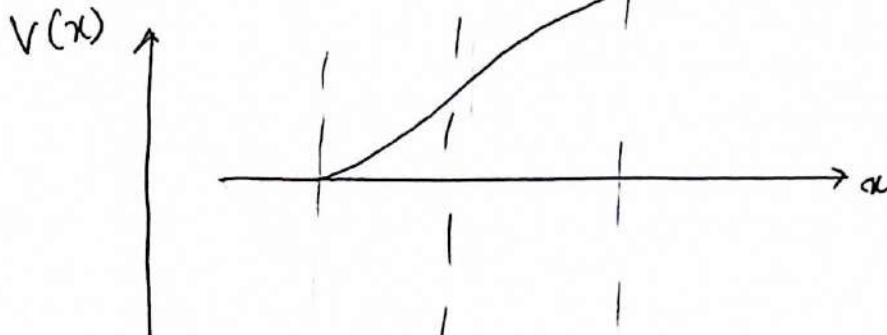
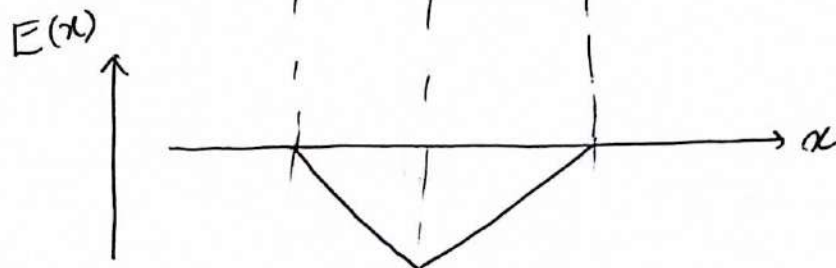
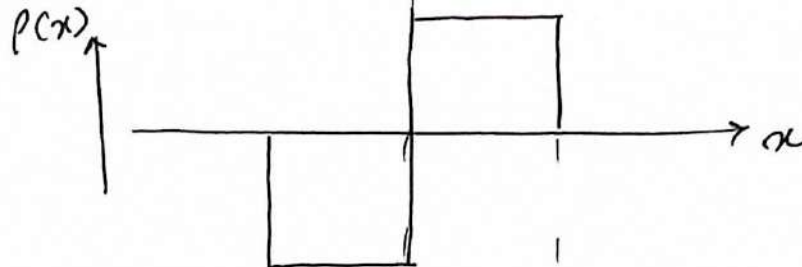
$$\text{Since } \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon}$$

$$\Rightarrow E = \int \frac{\rho(x) dx}{\epsilon}$$

$$V = - \int E(x) dx$$

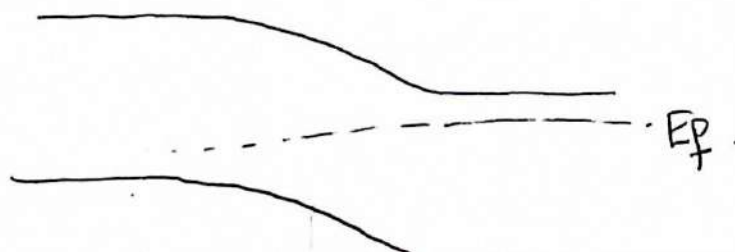
$$\mathcal{E} = -eV$$

pot energy



add this
to band
diagram.

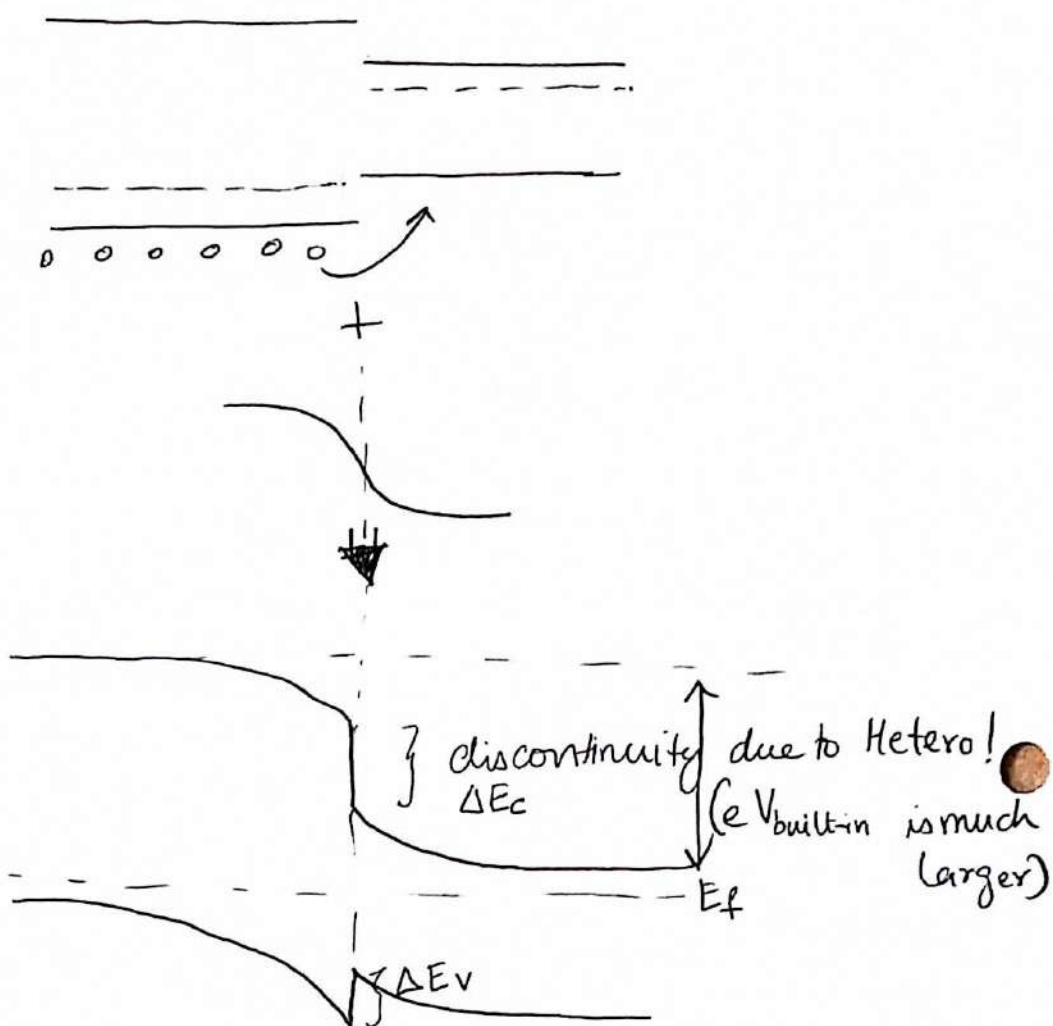
$\mathcal{E}_{\text{total}}$



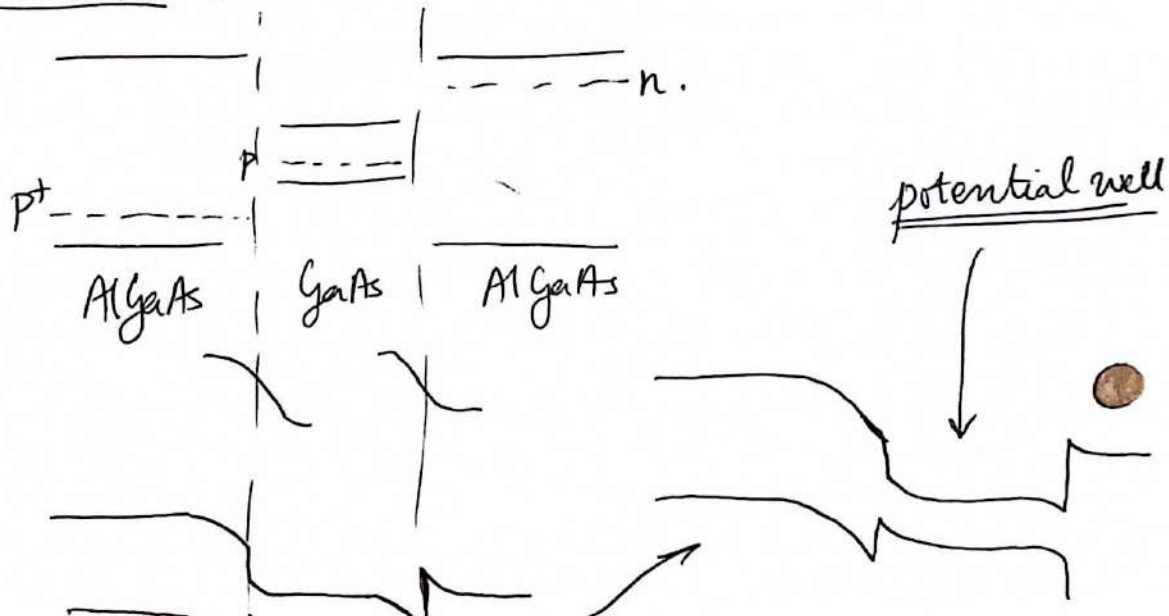
Note that
the n-type
material's
energy drops
since hole enters
 $\Rightarrow V \uparrow \Rightarrow$
 $\mathcal{E} \downarrow$.

Heterojunction

p- $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ | n- GaAs



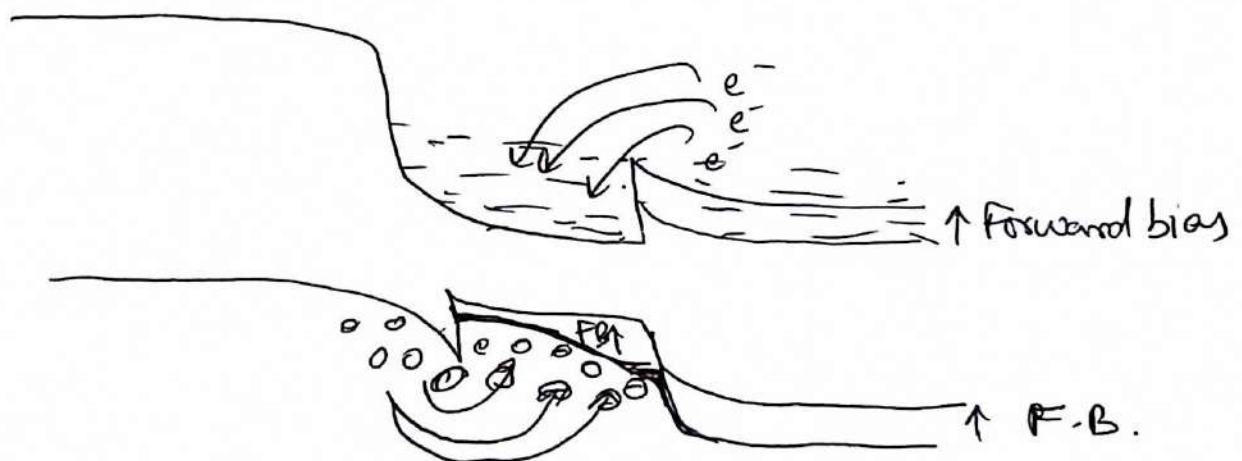
Double Hetero structure.



Why double heterostructures are important?

- > When a forward bias is added, a large number of carriers are poured into the central quantum well. These carriers don't constitute a current since there is still a large potential barrier. We therefore end up with a large carrier concentration in a very narrow region. In PN jns. the width of the depletion region is not designed explicitly, but the q -well width can be controlled.

- > Carrier confinement is the big advantage.

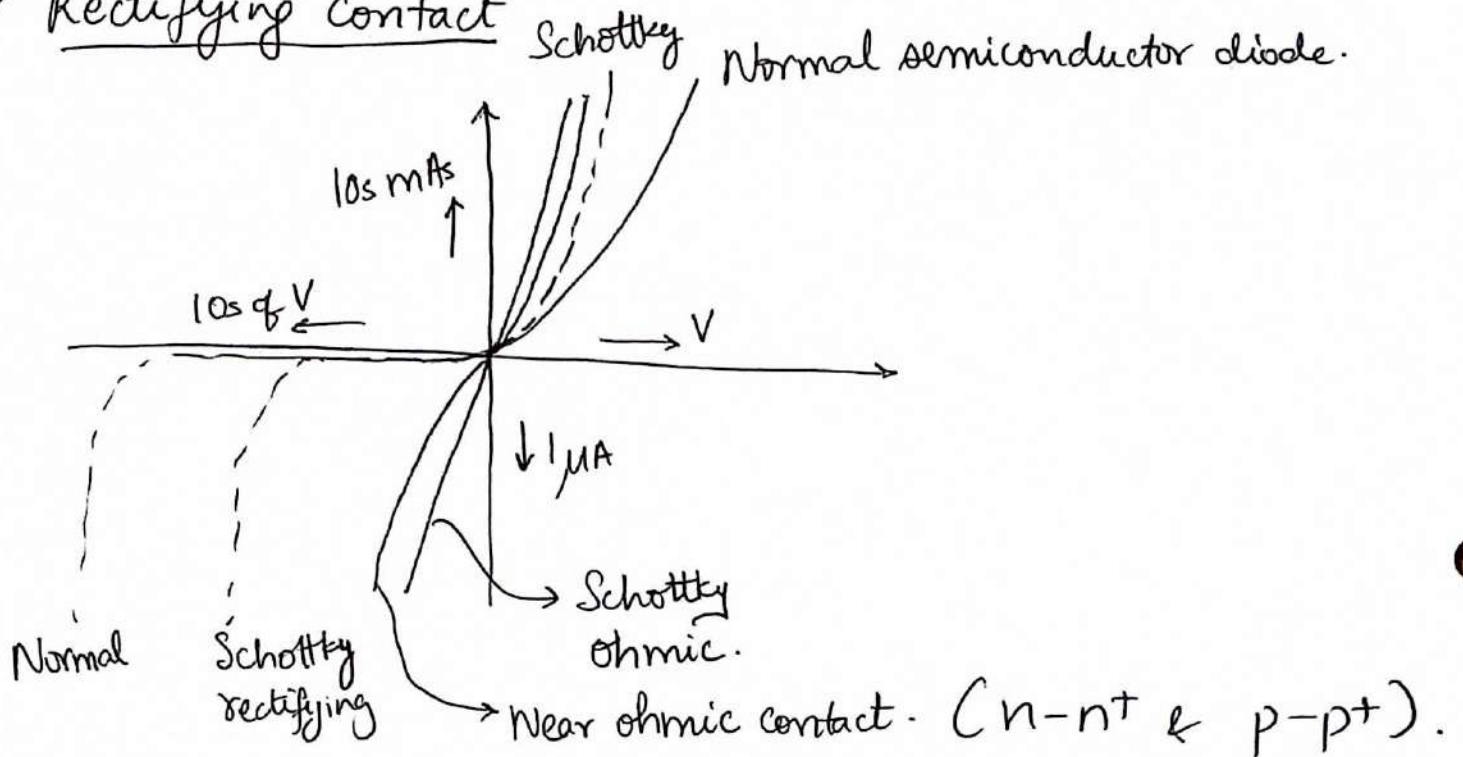


Lec 15 : Schottky Junction & Ohmic Contacts.

↳ jn. b/w metal & semiconductor.

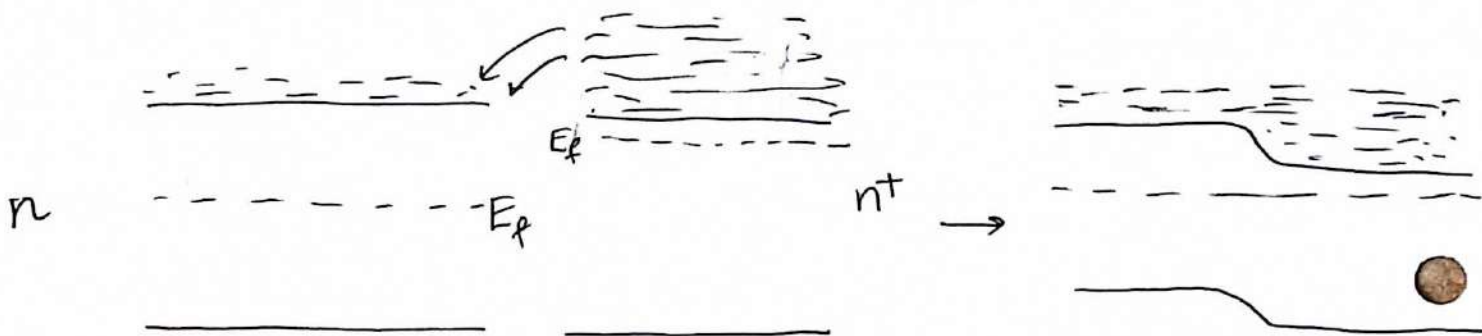
→ Could be rectifying or Ohmic.

> Rectifying contact



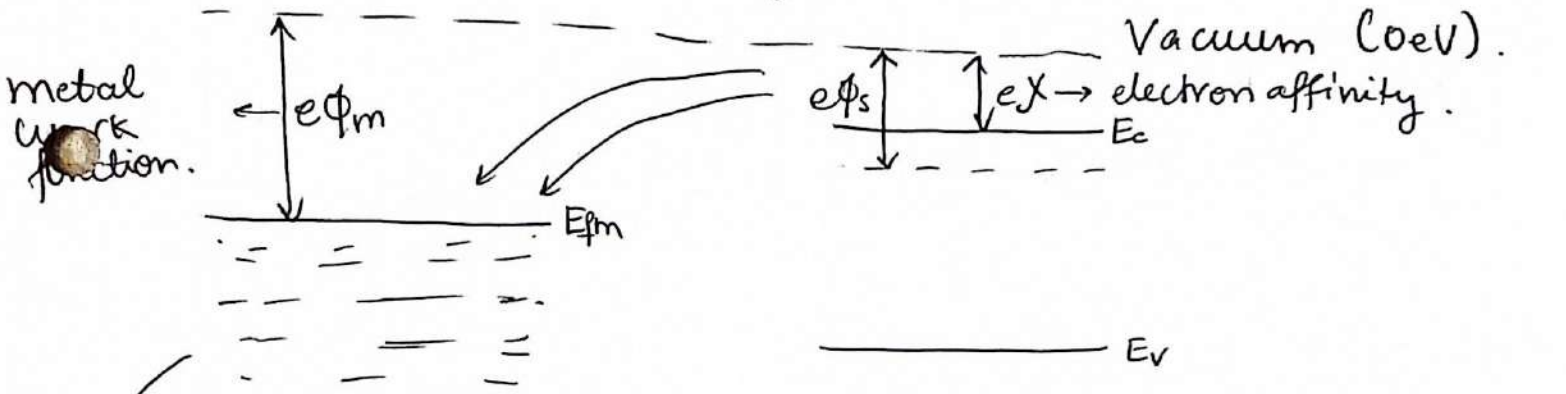
> There is always a n^+ & p^+ between metal & semiconductor.

> $n-n^+$ Junction



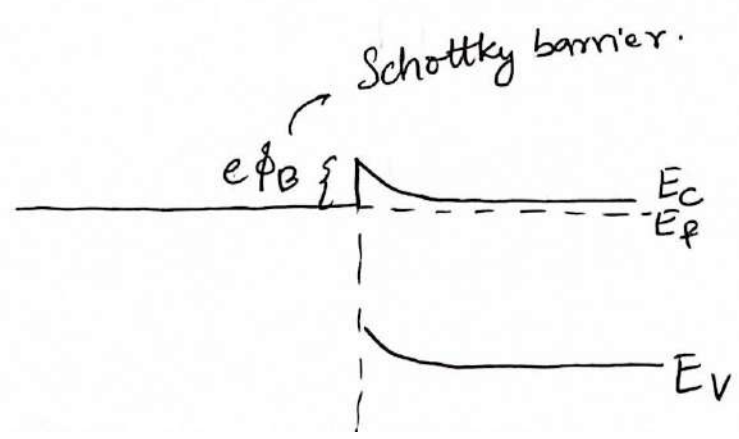
- > Under both forward & reverse bias conditions, the current ~~only~~ flows both ways since carrier concentration is high on both sides. ~~Slight~~ Slight imbalance due to the small potential barrier. Current flows mainly by one type of carrier.

> Schottky Junction. [Metals have overlapping bands or half filled bands, $\Rightarrow$ many vacant states that are easily accessible]



Typical values: $\sim 4.4\text{eV}$ for Al/Silver.
 $\sim 5\text{eV}$ for Au
 $\sim 5.7\text{eV}$ for Pt.

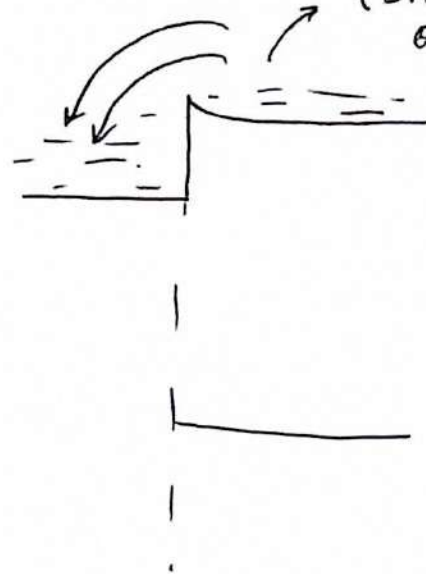
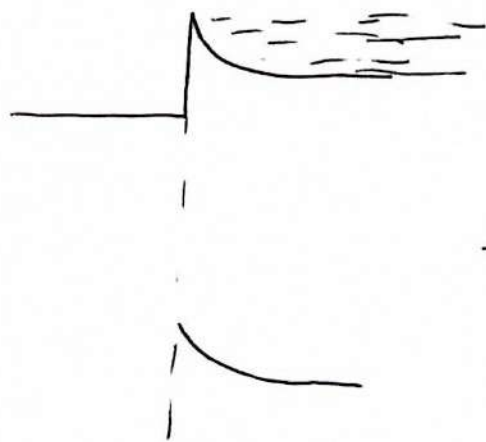
$e\phi \sim 4.0\text{eV}$ GaAs
 $\sim 4.0\text{eV}$ Si
 $\sim 4.3\text{eV}$ InP
 $\sim 4.1\text{eV}$ Ge.
 $\sim 4.9\text{eV}$ InAs



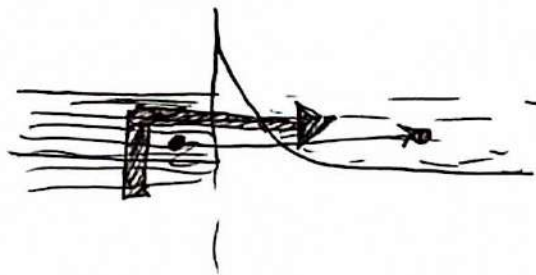
$$e\phi_B = e(\phi_m - \chi)$$

$\sim 0.6 - 0.8\text{eV}$ for nSi
 $\sim 0.7 - 0.9\text{eV}$ for nGaAs

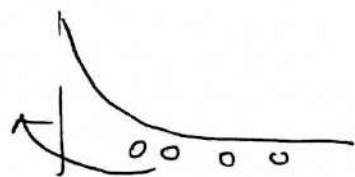
Forward bias



Reverse bias



(Also electron tunneling takes place i.e. evanescent coupling).

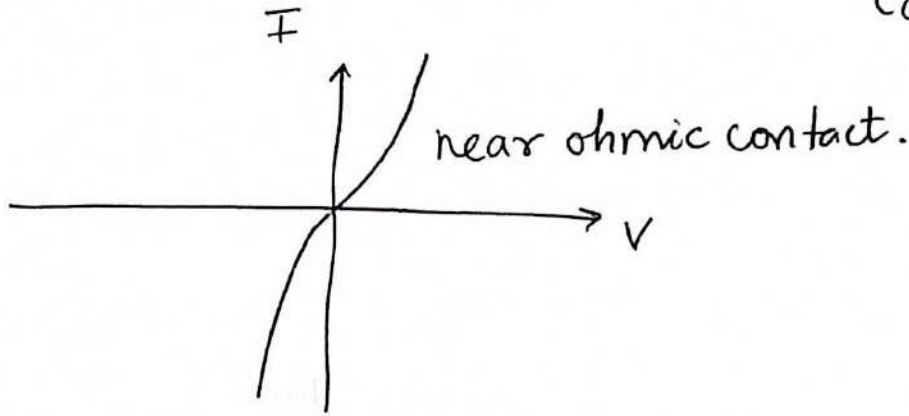


(Hole current is increased)

- > To increase tunneling current, we need to dope the s-c heavily. This is the reason for always using n^+ or p^+ contact with metal junctions!!
- > High doping $\Rightarrow$ high $P(x)$ & after 2 integrals $\mathcal{E}$ is very narrow.

> Forward current: Thermionic emission + Tunneling.
(dominant).

Reverse bias: Hole current + Tunneling.
(dominant)

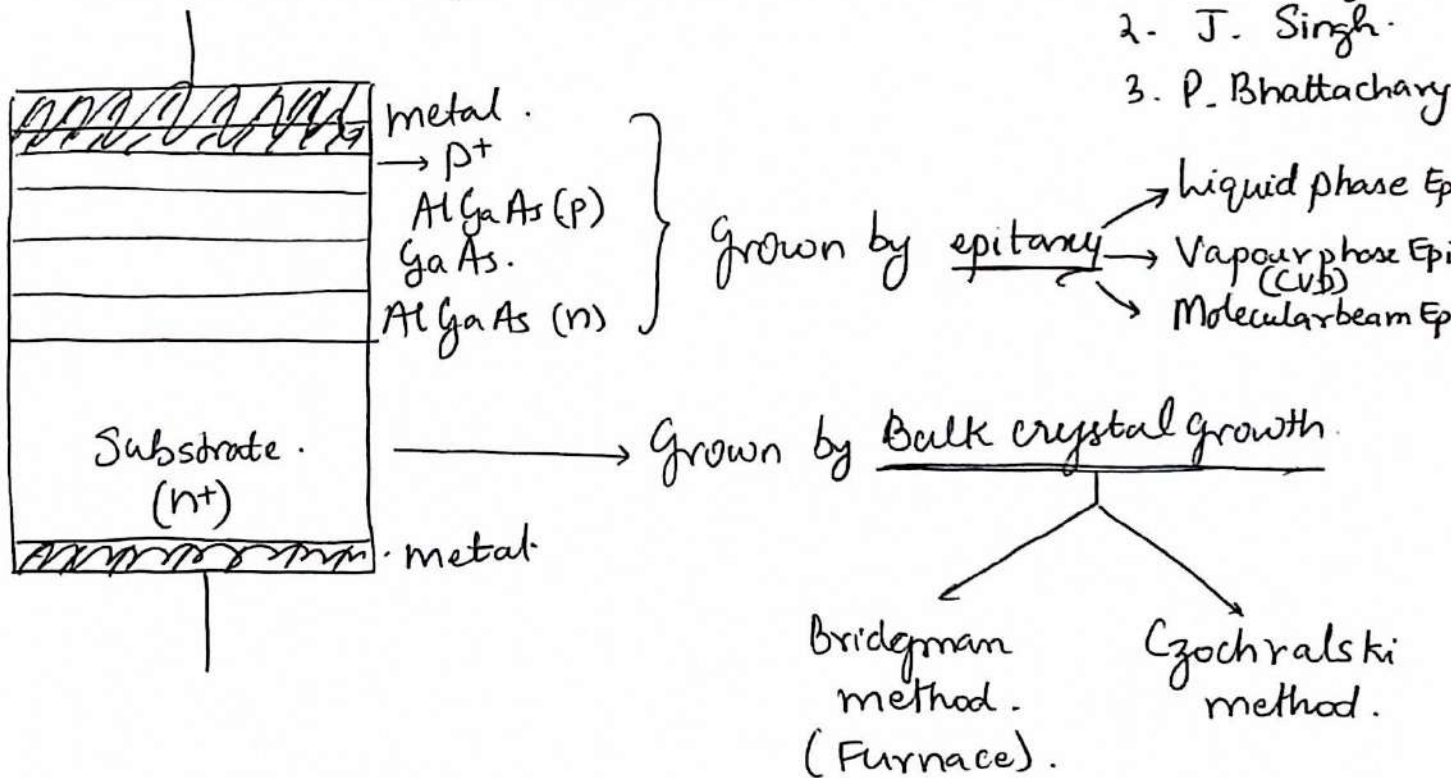


> low doping $\Rightarrow$ Rectifying contact

High doping $\Rightarrow$ Ohmic contact.

Lec 16 Fabrication of Heterojunction Devices.

1. S.M. Sze
2. J. Singh.
3. P. Bhattachary



Czochralski method.

- > Seed crystal is inside a melt & slowly pulled up.

Bridgman method.

- > polycrystalline boat is pulled through a temp gradient to create the ingot.
 - > Watch video for LPE, VPE(or CVD) & MBE.
-

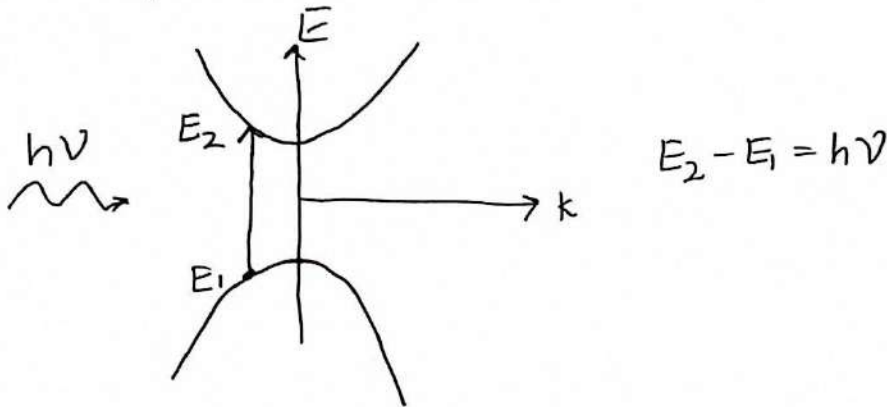
Lec 17 Interaction of Photons with Electrons & Holes in a Semiconductor.

49

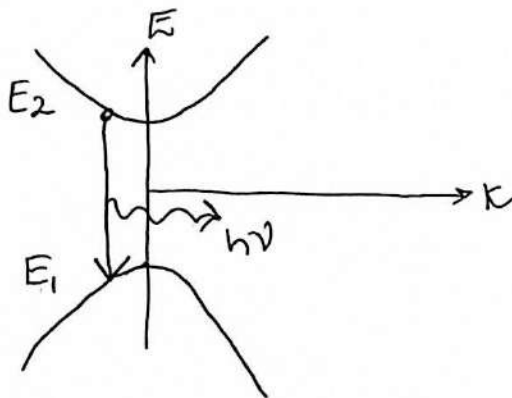
> Matter interacts with radiation via emission & absorption.

Spontaneous Stimulated.

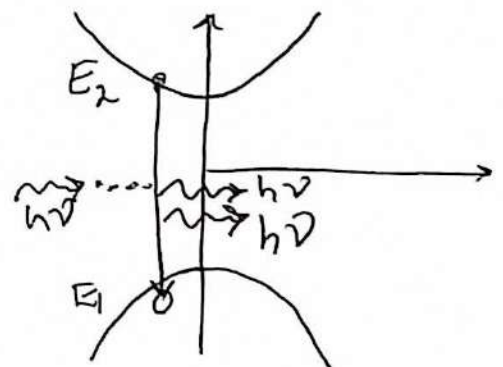
1) Absorption. (Photodetectors)



2) Spontaneous emission (LEDs)

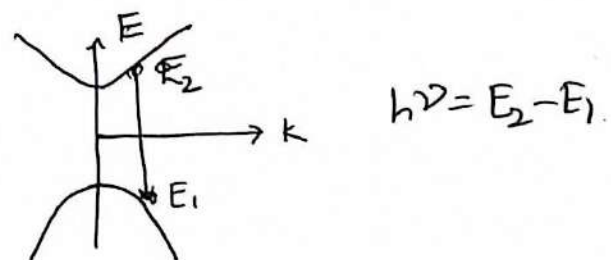


3) Stimulated emission. (Lasers)

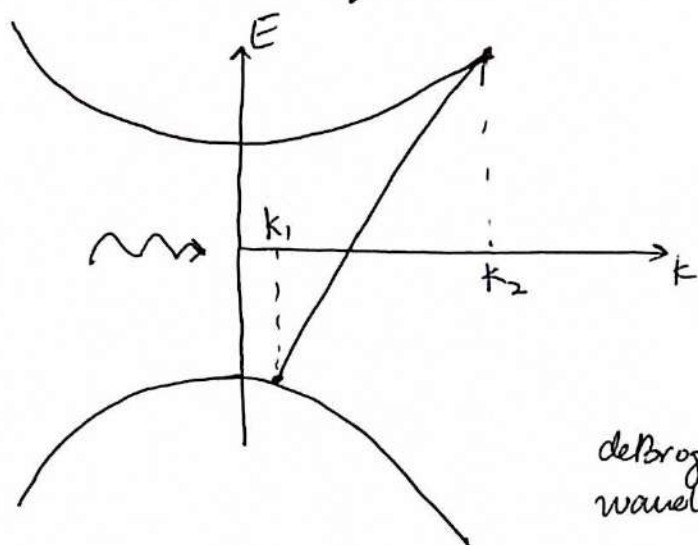


Basic laws

① Conservation of Energy.



2. Conservation of momentum



$$\hbar k_1 + \hbar k_{\text{photon}} = \hbar k_2$$

$$\Rightarrow k_1 + k_{\text{photon}} = k_2 \rightarrow \frac{2\pi}{\lambda_2}$$

deBroglie wavelength. $\leftarrow \frac{2\pi}{\lambda_1}$

assuming $\lambda \approx 1 \mu\text{m} = 10^4 \text{\AA}$

Edge of first Brillouin zone : $\frac{\pi}{a} = k$

$$\Rightarrow \frac{\pi}{L} < k_e < \frac{\pi}{a}$$

$\nwarrow$ $\uparrow$
 dim of S.C. $\quad$ 1mm $\quad$ 5\AA

Assuming $\lambda_e \approx 10 - 100 \text{\AA}$ (Typically)

$$\Rightarrow \lambda_e \ll \lambda_{\text{photon}} \Rightarrow k_e \gg k_{\text{photon}}$$

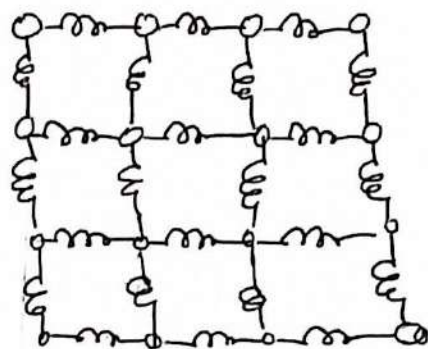
$$\Rightarrow \boxed{k_1 \approx k_2} \Rightarrow \text{transitions are vertical.}$$

What happens during Non-radiative transitions? (later)

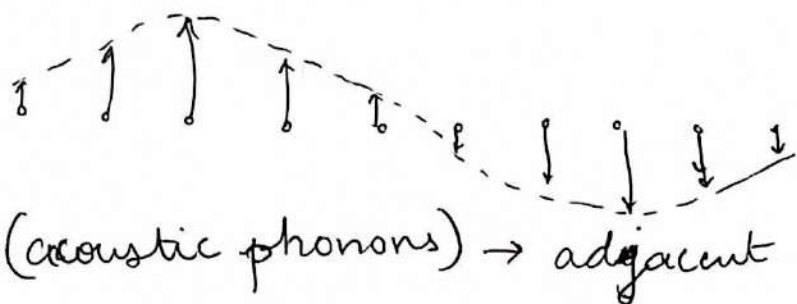
- Phonon-assisted radiative transitions can occur where the phonon can carry away the large Δk in oblique transitions.

Phonon : Quanta of lattice vibrations

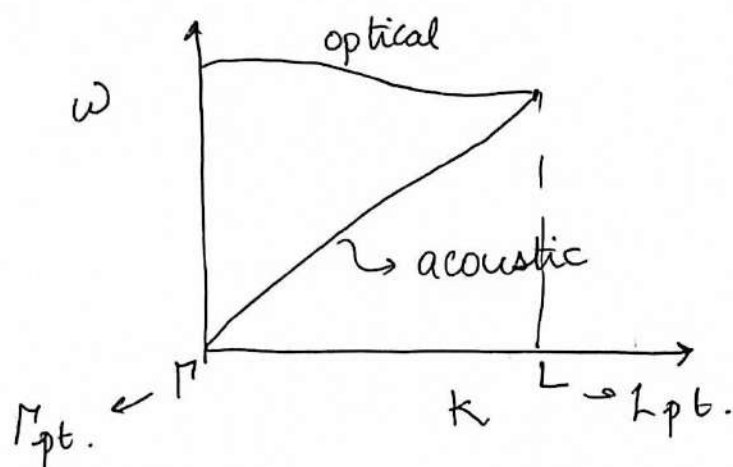
- Acoustic phonons
- Optical phonons.



(Optical phonons) → adjacent atoms are displaced in opposite direction. → Always high frequency.



(acoustic phonons) → adjacent atoms move together. → Can be from low to high.



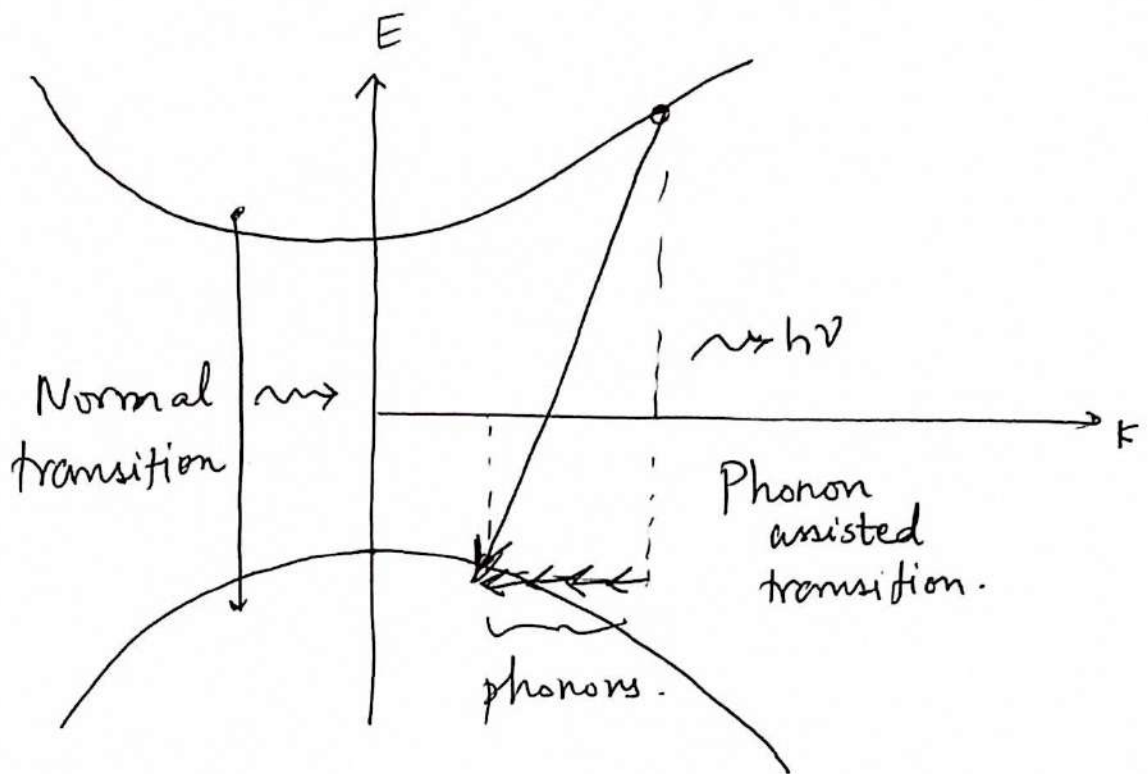
typical energies in s-c.

0.01 - 0.1 eV

- > Clearly K_{phonons} can be on the order of electrons.
- > But their energies are much lower.

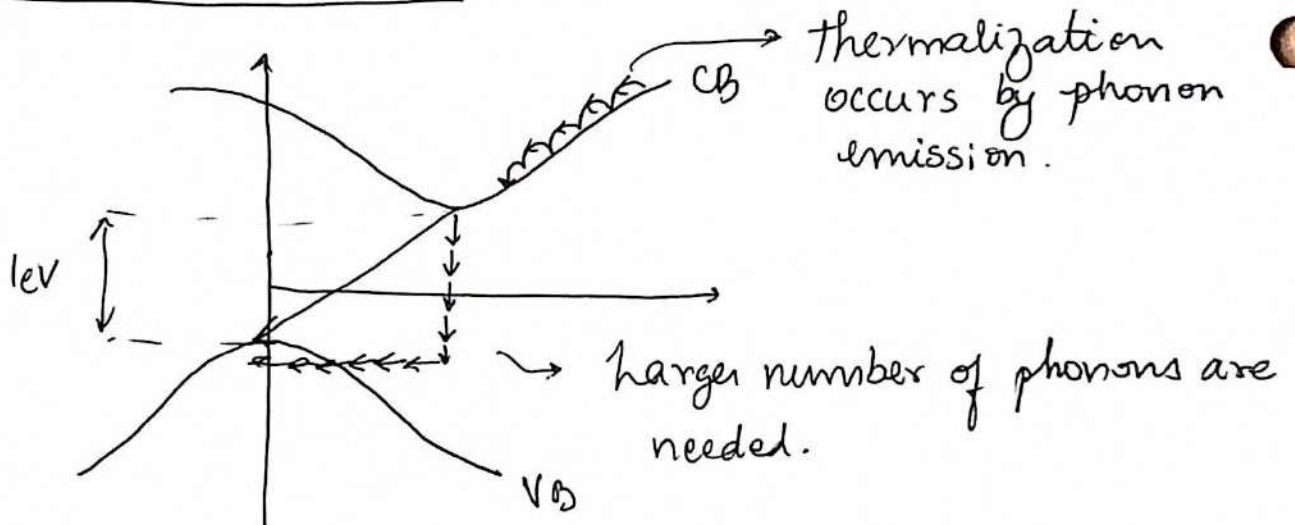
Therefore a combination of photon + phonon can allow for oblique transitions.

"Phonon assisted radiative transitions" → probability is lower due to...



Non-radiative Transitions.

Recall.
(Silicon is indirect B-G s-c)



- More common in indirect B-G s-c (like Si) since Δk needed is also large. Also, in this case they are quite frequent since photons are not participating. This is the main recomb. phenomena during forward bias, & can generate a lot of heat. IBGSC are still good for photon absorption devices since upward transitions have many...

Lec 18 : Optical Joint Density of States.

> What is the number of emissions per unit volume?

Recall,

$$n = \int n(E) dE = \int \rho_c(E) f(E) dE$$

carrier concentration. $\nearrow$ carrier density of e in CB $\nearrow$ density of states $\nwarrow$ probability of occupation.

$$P = \int p(E) dE = \int P_v(E) [1 - f(E)] dE.$$

Here the density of states $\rho_c(E)$ & $\rho_v(E)$ are independent. However, during photon interactions (emission or absorption), the probability of photon ~~ex~~ interaction depends on both ρ_c & ρ_v . Therefore, we need a joint density of states.

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c} \rightarrow \text{parabolic approx}$$

$$E_1 = E_v - \frac{\hbar^2 k^2}{2m_v}$$

$$\Rightarrow E_2 - E_1 = E_g + \frac{\hbar^2 k^2}{2} \underbrace{\left(\frac{1}{m_c} + \frac{1}{m_v} \right)}_{1 \rightarrow \text{reduced mass.}}$$

$$\Rightarrow h\nu = E_g + \frac{\hbar^2 k^2}{2m_r}$$

$$\Rightarrow k^2 = \frac{2m_r}{\hbar^2} (h\nu - E_g).$$

Recall

$$\Rightarrow E_2 = E_c + \frac{\hbar^2 k^2}{2m_c} = E_c + \frac{\hbar^2}{2m_c} \cdot \frac{2m_r}{\hbar^2} (h\nu - E_g)$$

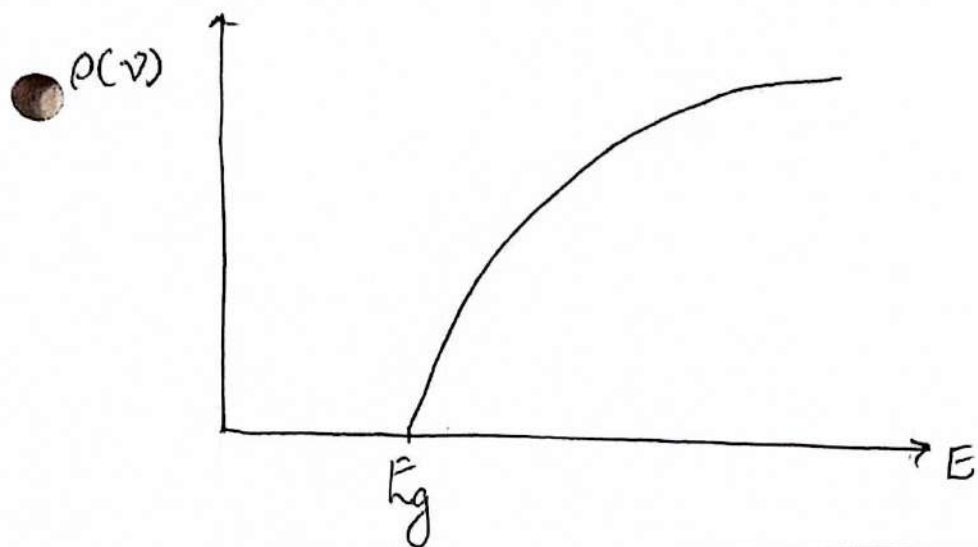
$$\Rightarrow E_2 = E_c + \frac{m_r}{m_c} (h\nu - E_g).$$

Since E_2 is directly proportion to ν (all other terms are constants),

we have $P_c(E_2) dE_2 = \underbrace{P(\nu) d\nu}_{\substack{\text{(per unit volume)} \\ \rightarrow \text{No. of states for photons of} \\ \text{energy b/w } h\nu \text{ \& } h(\nu+d\nu) \text{ to} \\ \text{interact with.}}}$

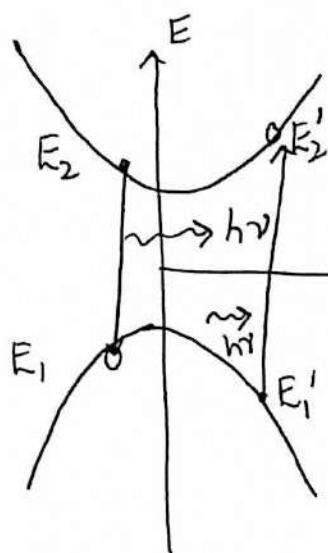
$$\begin{aligned} \Rightarrow P(\nu) &= P_c(E_2) \frac{dE_2}{d\nu} = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \underbrace{(E_2 - E_c)^{1/2}}_{\frac{m_r}{m_c} (h\nu - E_g)} \cdot \frac{h m_r}{m_c} \\ &= \frac{1}{\pi \hbar^2} (2m_c)^{3/2} \frac{m_r}{m_c} \left(\frac{m_r}{m_c} \right)^{1/2} (h\nu - E_g)^{1/2} \end{aligned}$$

$$\Rightarrow \boxed{P(\nu) = \frac{1}{\pi \hbar^2} (2m_r)^{3/2} (h\nu - E_g)^{1/2}.}$$



$$P(v) = \frac{1}{\pi \hbar^2} (2m_r)^{3/2} (\hbar v - E_g)^{1/2}$$

Probabilities of Emission and Absorption.



$$\hbar v = E_2 - E_1$$

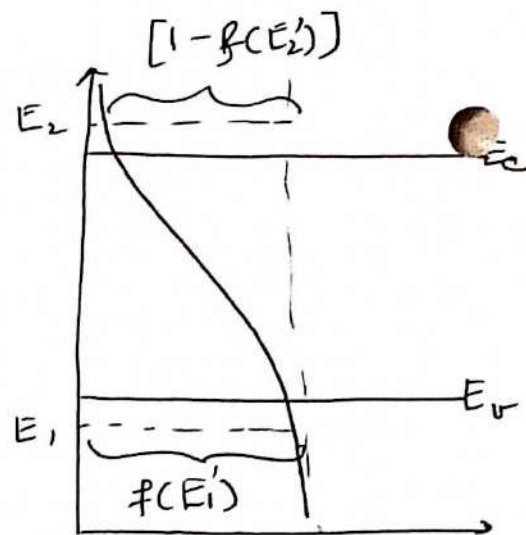
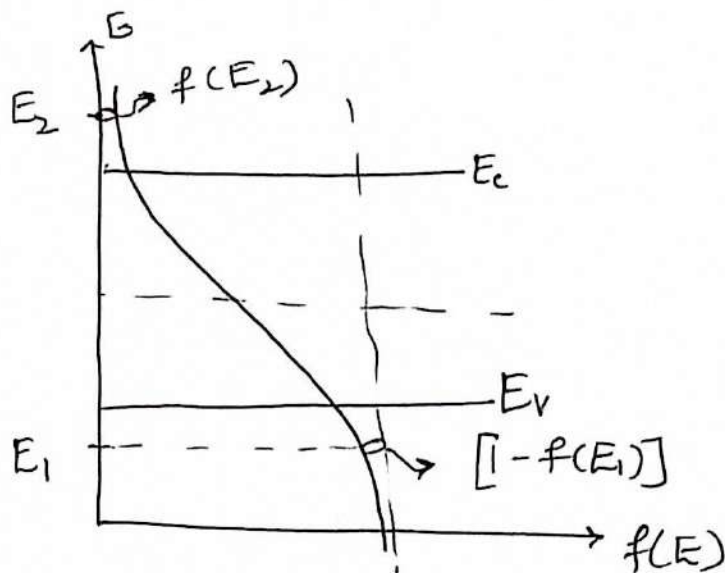
$$\left. \begin{aligned} \text{emission } P_e &= f(E_2) \cdot [1 - f(E_1)] \\ P_a &= f(E'_1) [1 - f(E'_2)] \end{aligned} \right\} \begin{array}{l} \text{Prob. of} \\ E_{2A} \end{array}$$

absorption

$$(\text{If } P_e > P_a \Rightarrow \text{emitter})$$

Case 1 : Thermal Equilibrium.

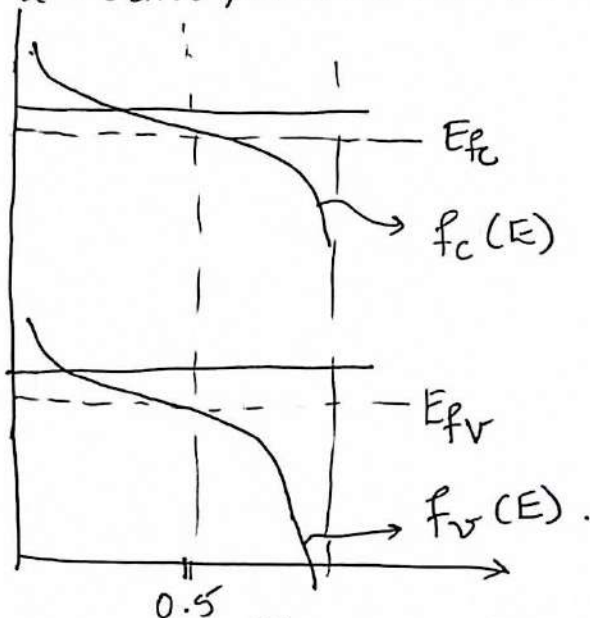
In this case, one Fermi function describes the prob. of occupation of an in both VB & CB.



> Clearly $P_e \ll P_a$. Therefore at Thermal Equilibrium it is impossible for emissions to occur.

Case 2): Quasi Equilibrium.

There are now 2 Fermi functions $f_c \neq f_v$.



$$P_e^{(w)} = \frac{1}{\left[e^{(E_2 - E_{fc})/kT} + 1 \right]} \cdot \left[\frac{e^{(E - E_{fv})/kT}}{1 + e^{(E - E_{fv})/kT}} \right]$$

$$P_a(\nu) = \frac{1}{1 + e^{(E_1 - E_{fv})/kT}} \cdot \frac{e^{(E_2 - E_{fc})/kT}}{[1 + e^{(E_2 - E_{fc})/kT}]}$$

For $P_e(\nu) > P_a(\nu)$ we need, $E_1 - E_{fv} > E_2 - E_{fc}$

$$\Rightarrow (E_{fc} - E_{fv}) > E_2 - E_1 = h\nu$$

$$\Rightarrow \boxed{E_{fc} - E_{fv} > E_g}$$

Requirement for
Emissions &
also for Lasers.

Lec 19 Rates of Emission and Absorption.

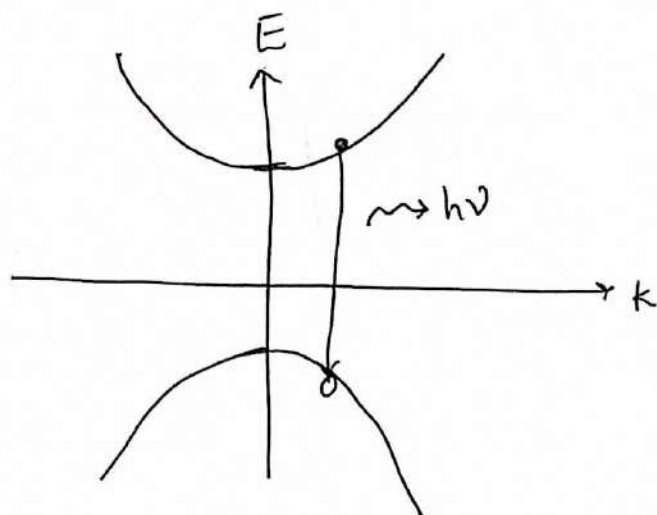
$\rho(\nu)$, $f_e(\nu)$, $f_a(\nu)$.
 optical joint density of states
 ↑ emission/absorption probability

$\Rightarrow \rho(\nu) f_e(\nu) \rightarrow$ no. of emission/unit volume/unit time

rate of spontaneous emission.

$$\Rightarrow r_{sp} \propto \rho(\nu) f_e(\nu)$$

$$\Rightarrow r_{sp} \propto A \rho(\nu) f_e(\nu) \quad \frac{1}{\tau_r} = \text{radiative recombination time.}$$



Total no. of ^{spontaneous} downward transition
 = radiative + non-radiative transitions.

$$\Rightarrow \text{Total rate constant } T$$

$$= A + S$$

$$= \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

$$\Rightarrow T = \frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

↑
 Spontaneous recombination time.

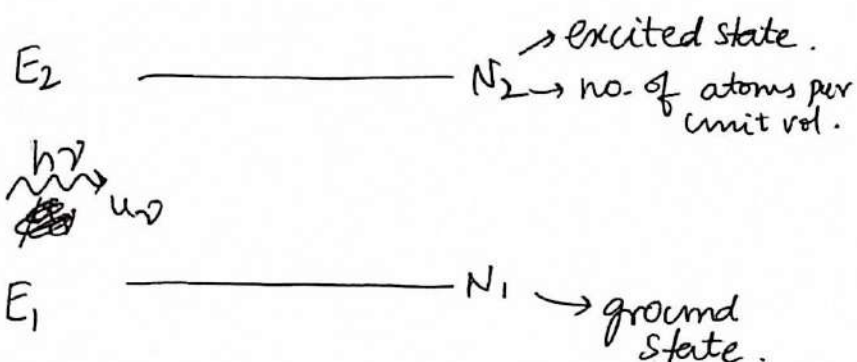
In DBQSC, $\tau_r \approx \tau_{nr}$.

$\Rightarrow$ 50% prob. of photon emission for each $\bar{e}$ transition.

In IDBQSC, $\tau_{nr} \ll \tau_r$

$\Rightarrow$ almost no radiative transitions.

Atomic system. (analogy)



$$\mu_{sp} \propto N_2$$

$$\Rightarrow \mu_{sp} = A N_2$$

If u_ν is the energy density
 i.e. $\frac{nh\nu}{V} = u_\nu$

$$\mu_{st} = B N_2 u_\nu$$

$$\mu_{ab} = B N_1 u_\nu$$

(59)

$g(\nu) \rightarrow$ atomic lineshape function.
(Strength of interaction at ν)

$$\Rightarrow \left. \begin{aligned} R_{sp}(\nu) &= A N_2 g(\nu) \\ R_{st}(\nu) &= B N_2 U_\nu g(\nu) \\ R_{ab}(\nu) &= B N_1 U_\nu g(\nu) \end{aligned} \right\} \begin{aligned} &\text{Here } A, B \text{ are the} \\ &\text{Einstein coefficients.} \\ &A = \frac{1}{t_{sp}} \rightarrow \text{spontaneous} \\ &\quad \text{emission} \\ &\quad \text{lifetime.} \end{aligned}$$

$$\boxed{\frac{A}{B} = \frac{8\pi h \nu^3}{(c/n)^3} \rightarrow \text{ref. ind.}}$$

Going back to ~~electronic~~ semiconductors.

$$R_{sp}(\nu) = A p(\nu) f_e(\nu)$$

$$R_{st}(\nu) = B p(\nu) f_e(\nu) U_\nu$$

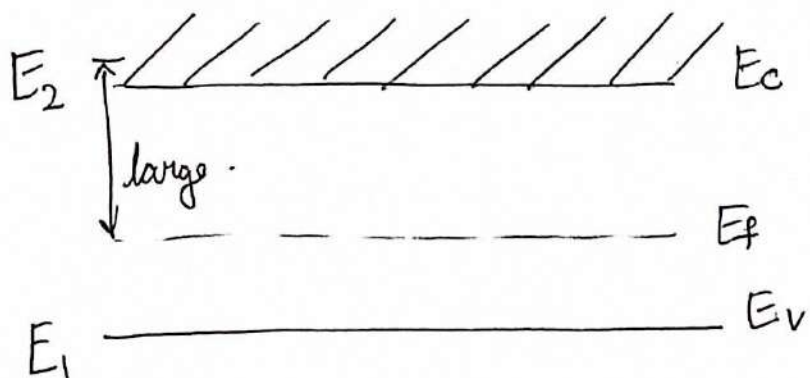
$$R_{ab}(\nu) = B p(\nu) f_a(\nu) U_\nu$$

$$\text{Where } A = \frac{1}{\tau_r} \times \boxed{\frac{A}{B} = \frac{8\pi h \nu^3}{(c/n)^3}}$$

Rate of Spontaneous Emission.

$$R_{sp}(\nu) = \frac{1}{\tau_r} \cdot \frac{1}{\pi \hbar^2} (2m_r)^{3/2} (h\nu - E_g)^{1/2} f_e(\nu).$$

$$\begin{aligned} \times f_e(\nu) \text{ at Thermal Equilibrium} &= f(E_2) [1 - f(E_1)] \\ &= \frac{1}{1 + e^{(E_1 - E_F)/KT}} \cdot \left[\frac{e^{E_1 - E_F/KT}}{1 + e^{(E_1 - E_F)/KT}} \right] \end{aligned}$$



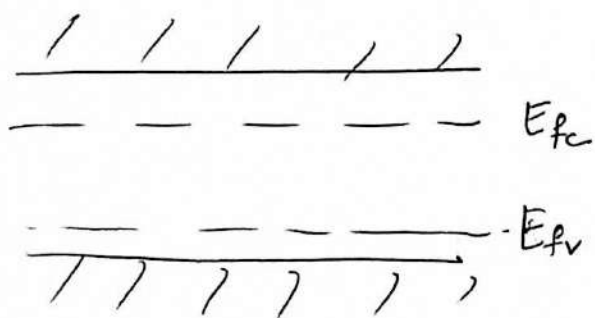
$$\Rightarrow f_e(\nu) \approx e^{-(E_2 - E_f)/kT} \cdot e^{(E_1 - E_f)/kT}$$

$$\approx e^{-(E_2 - E_1)/kT}$$

$$f_e(\nu) \approx e^{-h\nu/kT} \rightarrow \text{very small!} \Rightarrow \text{no emission!}$$

Thermal Eq.

At quasi equilibrium,



$$\Rightarrow f_e(\nu) = e^{-(E_2 - E_{fc})/kT} \cdot e^{(E_1 - E_{fv})/kT}$$

$$f_e(\nu) = e^{-h\nu/kT} \cdot e^{(E_{fc} - E_{fv})/kT}$$

large!

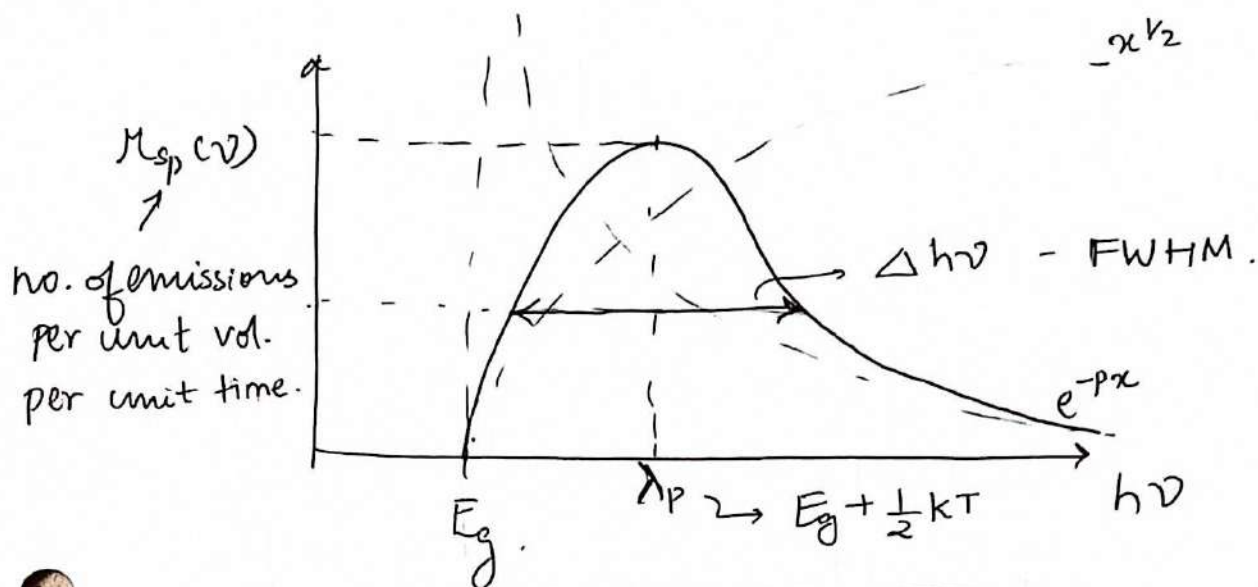
$\Rightarrow$ At Thermal equilibrium,

$$R_{sp}(\nu) = \frac{1}{C_r} \frac{1}{\pi \hbar^2} (2m_r)^{3/2} (h\nu - E_g)^{1/2} e^{-h\nu/kT}$$

$$= \frac{1}{C_r} \frac{1}{\pi \hbar^2} (2m_r)^{3/2} e^{-E_g/kT} \cdot (h\nu - E_g)^{1/2} e^{-(h\nu - E_g)/kT}$$

$$= D_0 (h\nu - E_g)^{1/2} e^{-(h\nu - E_g)/kT}$$

$$\{I_{sp} \approx D_0 x^{1/2} e^{-px}\}$$



$$\Delta\lambda \text{ [from } \Delta(h\nu)] = 1.45 kT \lambda_p^2$$

$\Delta\lambda \sim 2-3 \text{ nm}$ for Fabry Perot LDs.

$\Delta\lambda \sim 20 \text{ nm}$ for LEDs.

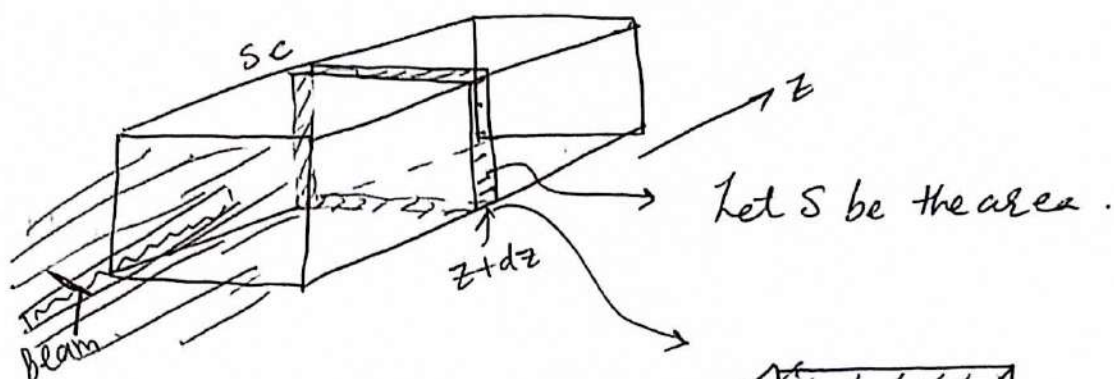
Lec 20 - Amplification by Stimulated Emission.

Recall,

$$I_{sp}(\nu) = A\rho(\nu)f_e(\nu)$$

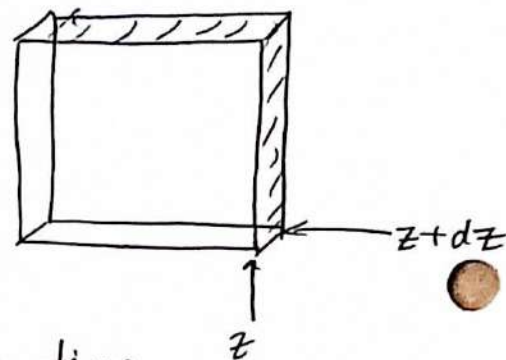
$$\sim D_0 (h\nu - E_g)^{1/2} e^{-(h\nu - E_g)/kT}$$

Rates of Stimulated Emission and Absorption.



$$R_{st}(\nu) = B \rho(\nu) f_e(\nu) u_\nu$$

$$R_{abs}(\nu) = B \rho(\nu) f_a(\nu) u_\nu$$



No. of stimulated emissions per unit time, in the volume element $Sdz = R_{st}(\nu) Sdz$.

⇒ Energy generated/unit time in the vol element $Sdz = R_{st}(\nu) Sdz h\nu$.

11/8 Energy absorbed/unit time in the vol element $Sdz = R_{ab}(\nu) Sdz \cdot h\nu$

⇒ Net energy generated $= (R_{st} - R_{ab}) Sdz \cdot h\nu$
①

(63)

● Incident irradiance $\rightarrow$ Energy flux / unit time.

Energy entering per unit time into the volume element is

$$I_\nu(z) S.$$

Energy leaving per unit time at $z+dz$ is

$$I_\nu(z+dz) S.$$

$$\Rightarrow \text{Net energy generated per unit time} = [I_\nu(z+dz) - I_\nu(z)] S.$$

L ②

● Equating ① & ②,

$$[I_\nu(z+dz) - I_\nu(z)] S = (M_{st} - M_{ab}) S dz \approx h\nu.$$

$$\Rightarrow I_\nu(z) + \frac{\partial I_\nu}{\partial z} dz$$

$$\Rightarrow \boxed{\frac{\partial I_\nu}{\partial z} = (M_{st} - M_{ab}) h\nu} \quad \text{--- ③}$$

$$= B\rho(\nu) u_\nu [f_e(\nu) - f_a(\nu)] h\nu$$

$$\hookrightarrow \frac{I_\nu}{\nu} \rightarrow \text{velocity}.$$

$$\times \frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$$

$$\Rightarrow B = \frac{\nu^3}{8\pi h\nu^3}$$

$$\Rightarrow \frac{\partial I_\nu}{\partial z} = \frac{\nu^3}{8\pi h \nu^3} \frac{\rho(\nu)}{\tau_r} \frac{I_\nu}{\nu} [f_e(\nu) - f_a(\nu)] h \nu.$$

$$\Rightarrow \frac{\partial I_\nu}{\partial z} = \underbrace{\frac{\nu^2}{8\pi \nu^2} \cdot \frac{\rho(\nu)}{\tau_r} \cdot [f_e(\nu) - f_a(\nu)]}_{\gamma_0(\nu)} \cdot I_\nu$$

$$\Rightarrow \boxed{I_\nu(z) = I_\nu(0) \cdot e^{\gamma_0(\nu)z}}$$

$$\Rightarrow \text{Gain} = e^{\underbrace{\gamma_0(\nu)L}_{\text{Gain coefficient!}}}$$

$$\gamma_0(\nu) = \frac{\nu^2}{8\pi \nu^2} \frac{\rho(\nu)}{\tau_r} [f_e(\nu) - f_a(\nu)] I_\nu$$

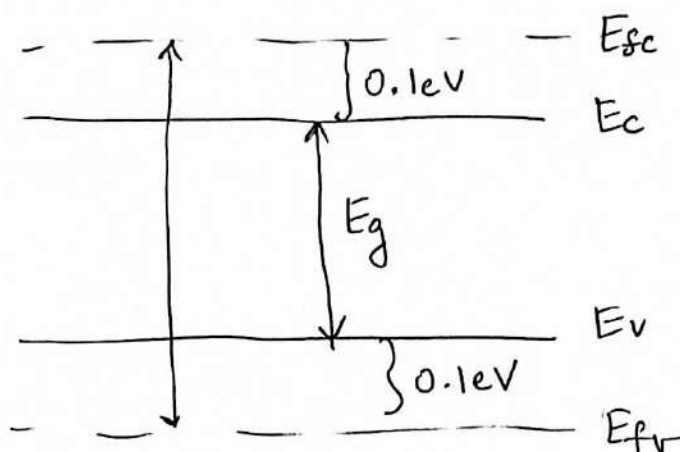
Gain exists when $f_e(\nu) > f_a(\nu)$ & recall that this

occurs when $\boxed{(E_{fc} - E_{fv}) > h\nu}$ $\rightarrow$ Analogous to population inversion.

If separation between quasi Fermi levels is larger than $h\nu$, we get gain! (Note that $E_{fc} > E_c$ & $E_{fv} < E_v$ is necessary implicitly)

lec 21 Semiconductor (laser) Amplifier

$$\gamma = \frac{(C/n)^2}{8\pi\nu^2} \frac{1}{\tau_r} \rho(\nu) \underbrace{[f_e(\nu) - f_a(\nu)]}_{\text{optical joint DOS.}}$$

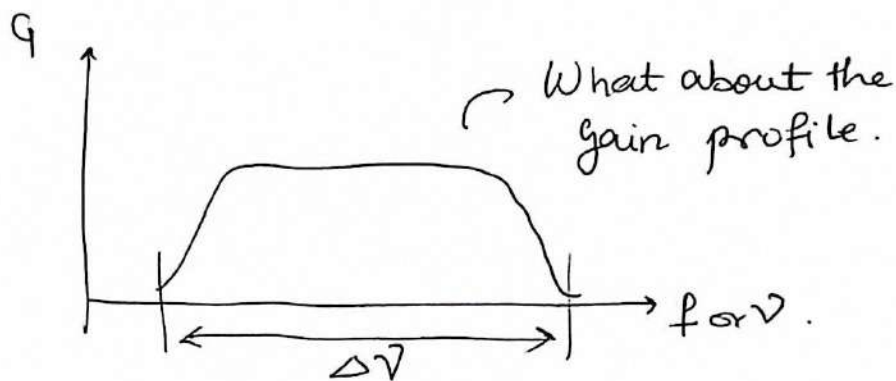


$$\Rightarrow \Delta E = h\Delta\nu = 0.2 \text{ eV}$$

$$\Rightarrow \Delta\nu = \frac{0.2 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 5 \times 10^{13} \text{ Hz.}$$

↑
Bandwidth.



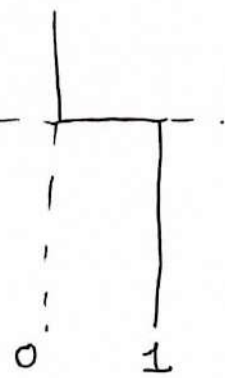
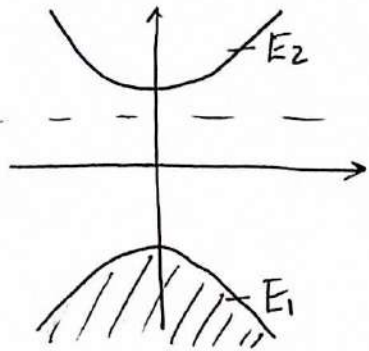
Expt. at OK

Case 1: Thermal Equilibrium.

$$\rho(\nu) = \frac{1}{\pi^2 h^3} (2m_r)^{3/2} (h\nu - E_g)^{1/2}$$

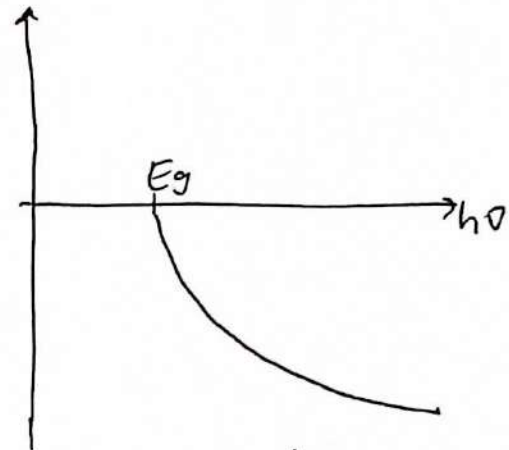
$$\Rightarrow \gamma = \left[\frac{(C/n)^2}{8\pi^2 \tau_r} (2m_r)^{3/2} \right] \frac{(h\nu - E_g)^{1/2}}{\nu^2} f_g(\nu) \quad \text{Fermi Inversion factor.}$$

$$\Rightarrow \gamma(\nu) = k \frac{(h\nu - E_g)^{1/2}}{\nu^2} f_g(\nu).$$



at $T=0K$
 $f(E_2)=0$
 $f(E_1)=1$
 $\Rightarrow f_g(\nu)=-1$

$\gamma(\nu)$ (gain coeff.)



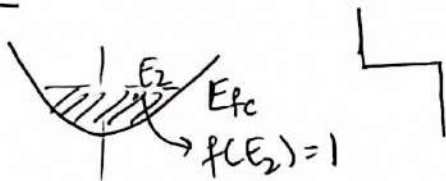
$\Rightarrow$ Loss.



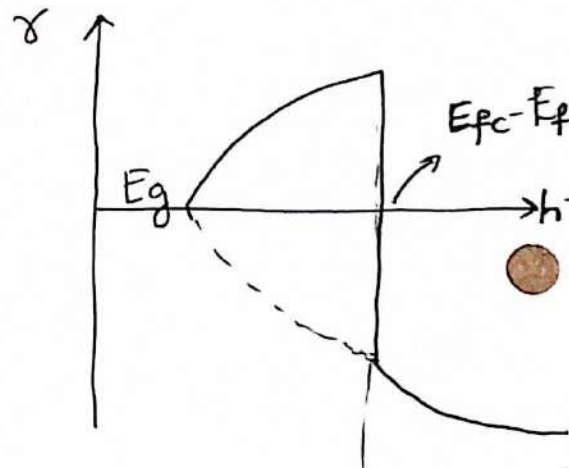
Typically $\Delta\nu$ is $0.1 \times$ value of $\nu \Rightarrow \nu^2$ is almost constant over $\Delta\nu$.

$\Rightarrow \gamma(\nu) \sim -k_0 (h\nu - E_g)^{1/2}$ (from $f_g(\nu)$)

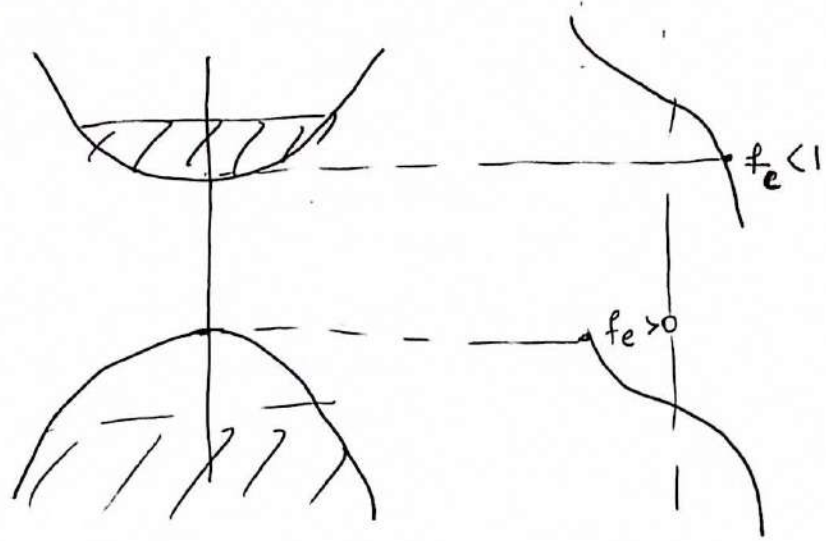
Case 2 Quasi Equilibrium (at $T=0$)



$\Rightarrow f_g(\nu)=1$

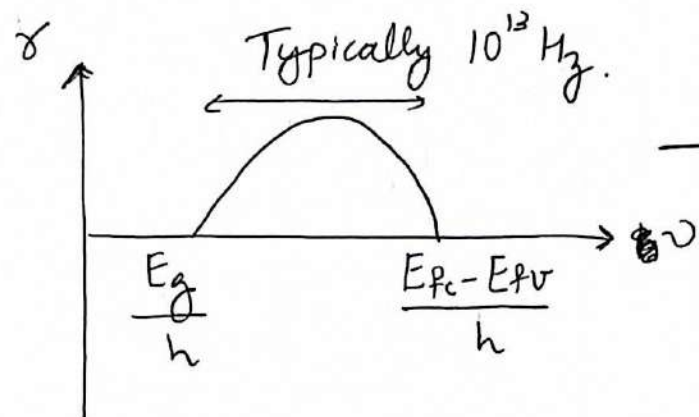
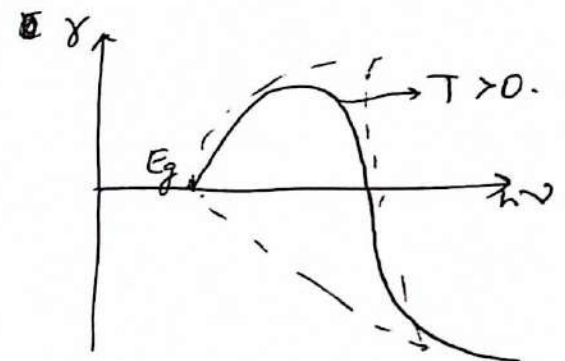


Case 3 Quasi Equilibrium at $T > 0K$



at $h\nu = E_{fc} - E_{fv}$

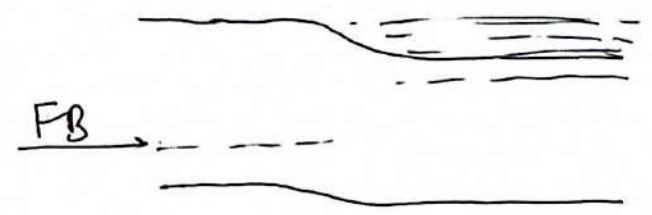
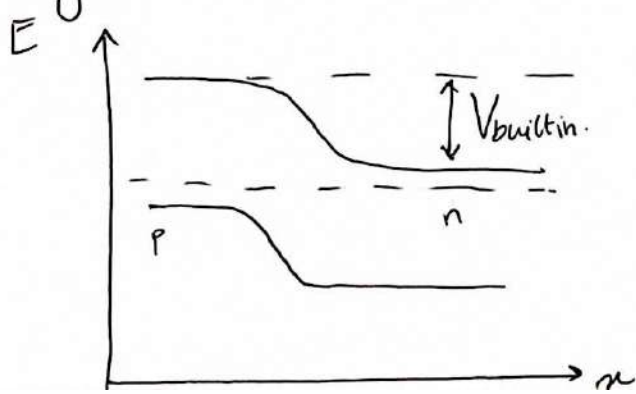
$f_g(\nu) = 0$



→ Gain profile is not flat!

→ How to maintain $E_{fc} - E_{fv} > E_g$, or how to pump an amplifier?

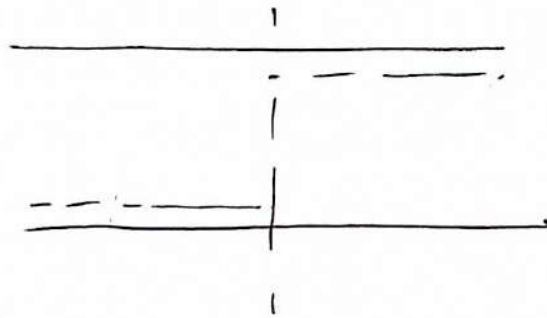
- By current injection in a forward biased p-n junction



> Even after heavy F-B note that $E_{fc} - E_{fv} < E_g$.

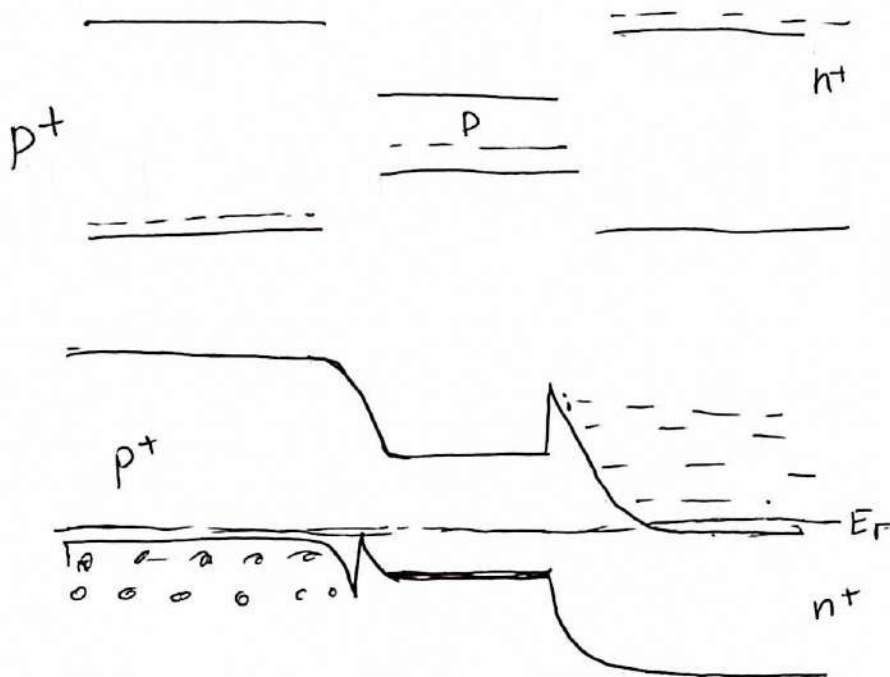
Extreme F-B :

Energy levels are back to where the original "separate" p & n were.

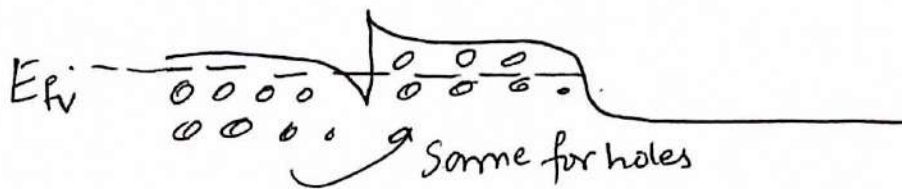
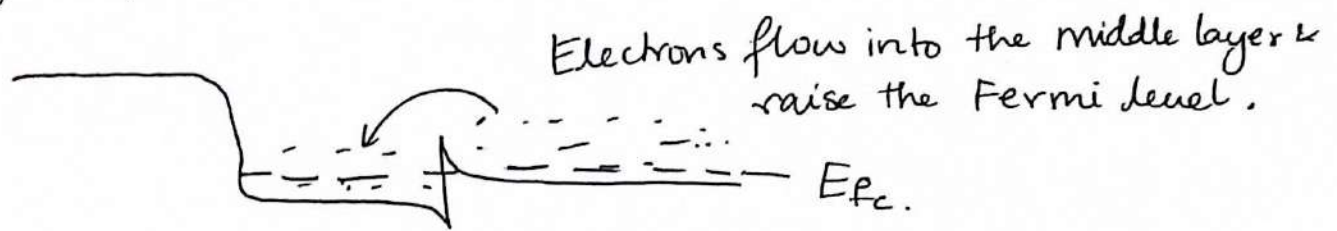


> Therefore if we start with a p & n that already have $E_f > E_c$ & $E_f < E_v$ (i.e. highly doped "degenerate" semiconductors) there is a possibility under extreme F-B to achieve $E_{fc} - E_{fv} > E_g$. But it is not practical.

> Use of Double Hetero Structures for Amplification.



After forward biasing,



$$E_{fc} - E_{fv} > E_g$$

Also $\Delta n \rightarrow$ very large.
 $\Delta p \rightarrow$ very large.

Recall,

$$E_{fc} = E_c + kT \left[\ln \frac{n}{N_c} + \frac{1}{\sqrt{8}} \frac{n}{N_c} \right]$$

$$E_{fv} = E_v - kT \left[\ln \frac{p}{N_v} + \frac{1}{\sqrt{8}} \frac{p}{N_v} \right]$$

$\Rightarrow E_{fc} \uparrow \uparrow \quad E_{fv} \downarrow \downarrow$

Lec 22 Absorption Spectrum of Semiconductors.

Recall,

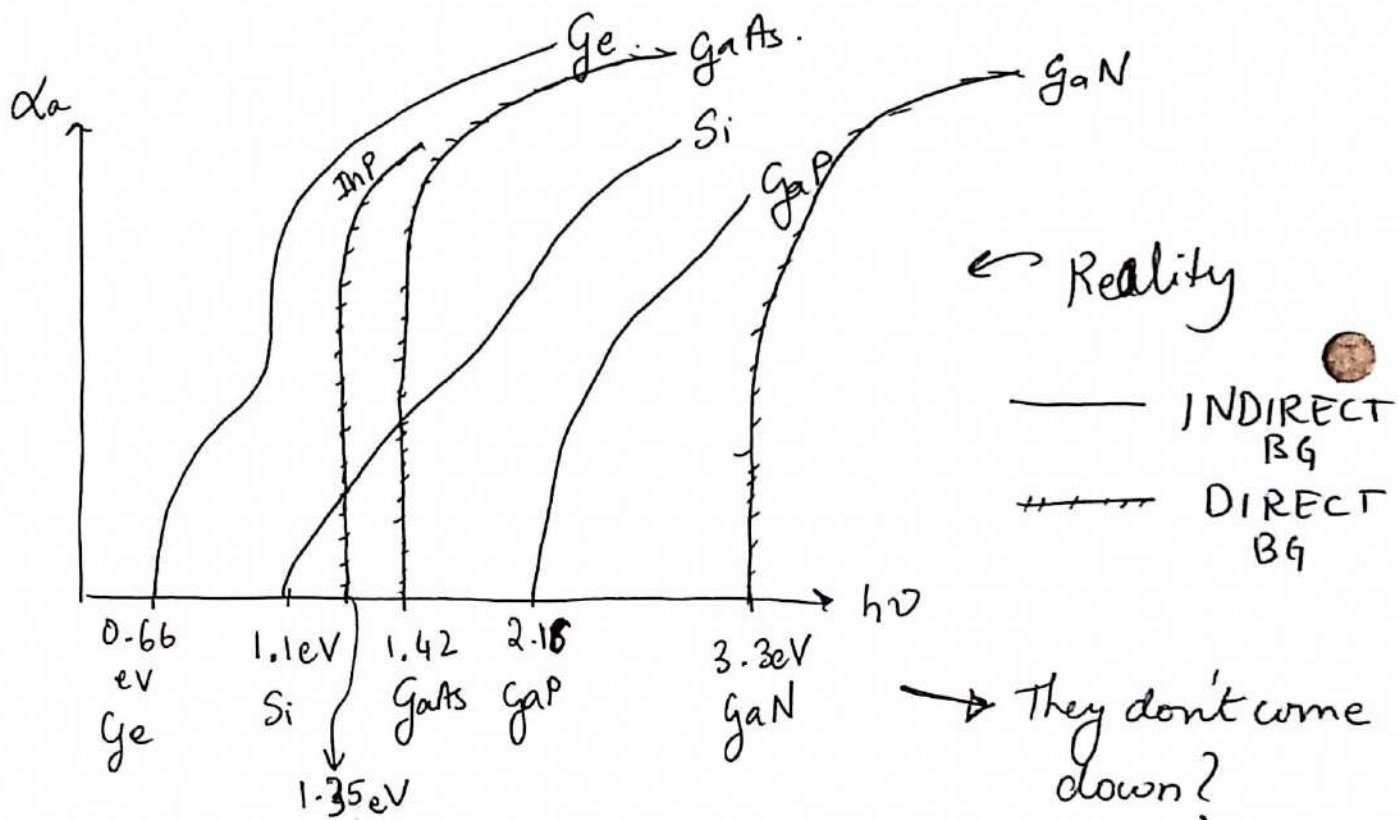
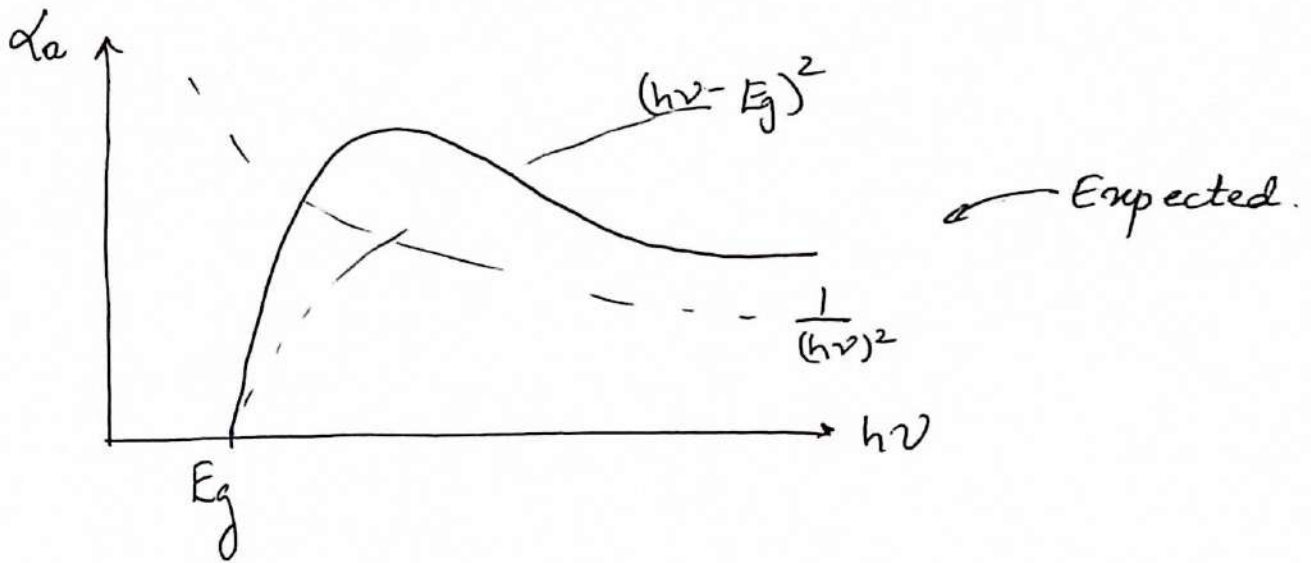
$$\gamma = \frac{(q_n)^2}{8\pi v^2} \cdot \frac{1}{\epsilon_r} (2m_r)^{3/2} \cdot \frac{1}{\pi \hbar^2} (h\nu - E_g)^{1/2} f_g(\nu).$$

If $f_g(\nu) < 0$,

$\gamma = -\alpha_a \Rightarrow$

$$\alpha_a = \frac{(q_n)^2}{2\epsilon_r} (2m_r)^{3/2} f_0(\nu) \frac{(h\nu - E_g)^{1/2}}{(h\nu)^2}$$

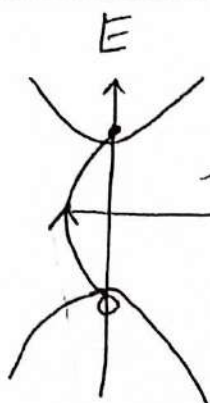
attenuation coefficient.



> Because of parabolic approx. & the assumption of DBG SC. There are also many other effects

- Intraband transitions. (low energy)
- Excitonic transitions.
- Phonon transitions.

Excitonic Transitions



If this e-h pair are close in space, they form a single localized entity due to the Coulomb force.

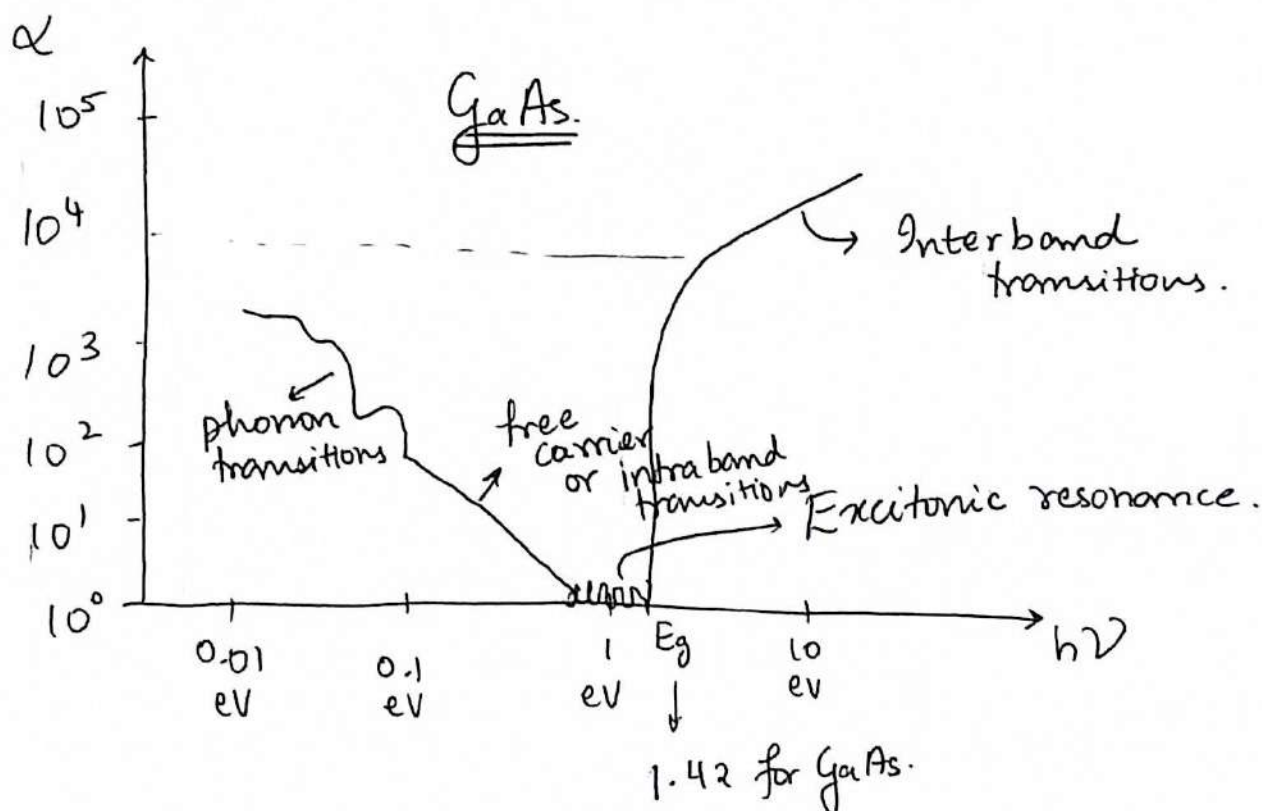
> This is called an exciton.

> They have low binding energy $\approx 10 \text{ meV}$.

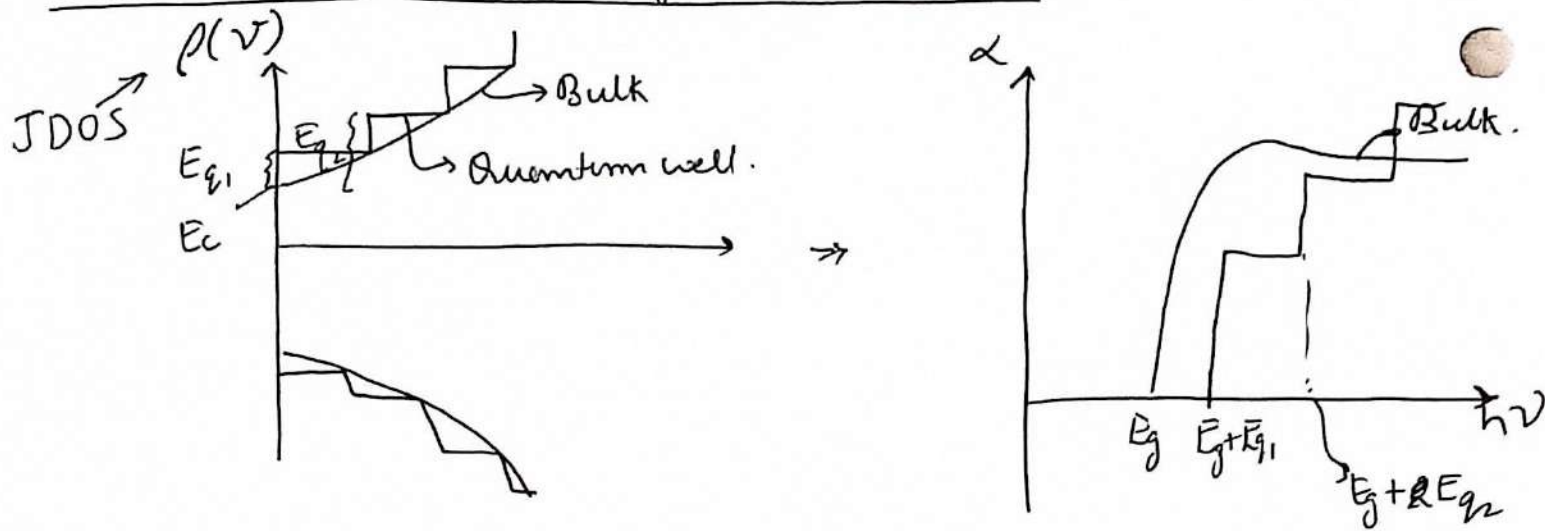
(H atom for example has a binding energy of $> 10 \text{ eV}$)

> Excitons don't usually occur in S-C at room temp since $kT \approx 25 \text{ meV}$.

> Exciton transitions have slightly lower energy than E_g .



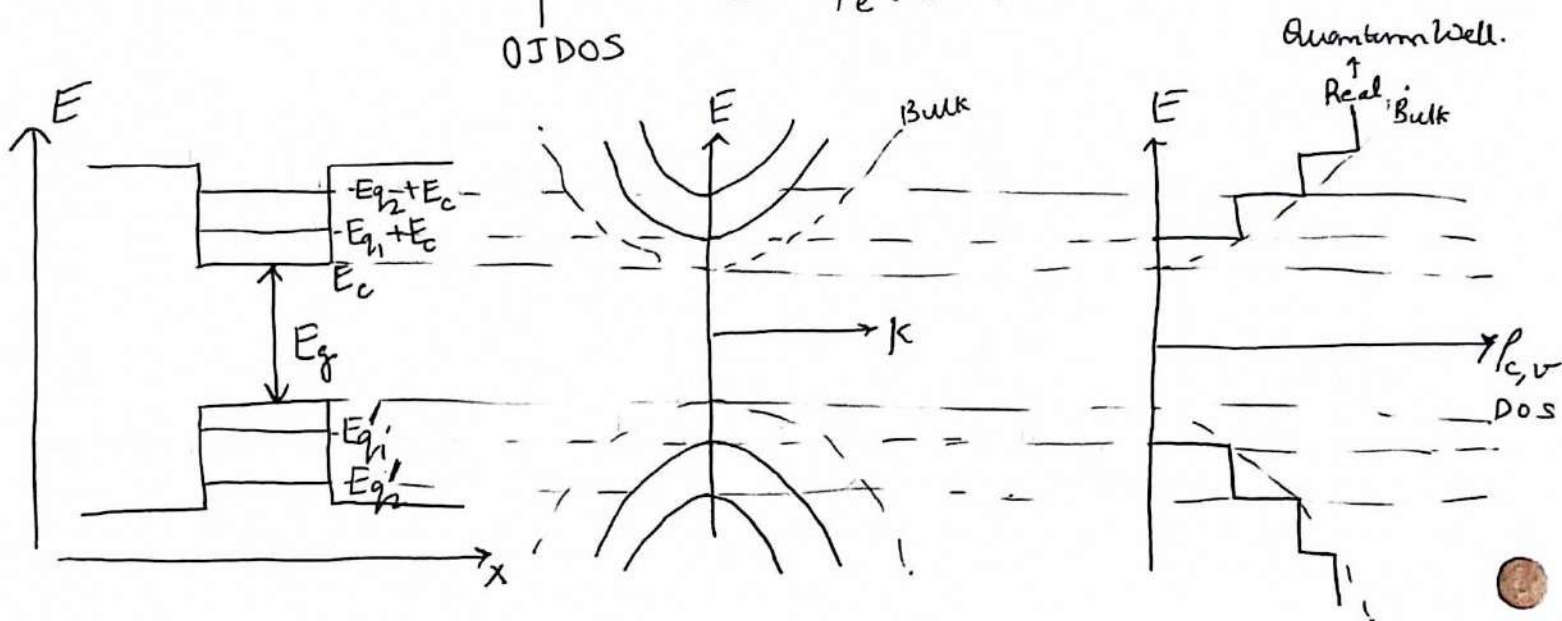
> Quantum Well absorption spectrum



→ Abrupt transitions in absorption spectra due to quantized energy levels & quantized JDOS.

Lec 23 Gain and Absorption Spectrum of Quantum Well Structures.

$$\gamma = \frac{(e/n)^2}{8\pi\gamma^2} \frac{1}{\tau_r} \underbrace{\rho(\nu)}_{\text{JDOS}} \underbrace{f_g(\nu)}_{f_e(\nu) - f_a(\nu)}$$



$$\Rightarrow \rho_c = 0 \quad E_c < E < E_c + E_{q1}$$

$$= \frac{m_c}{\pi \hbar^2 L_z} \quad E_c + E_{q1} < E < E_c + E_{q2}$$

$$= \frac{2m_c}{\pi \hbar^2 L_z} \quad E_{q2} + E_c < E < E_c + E_{q2}$$

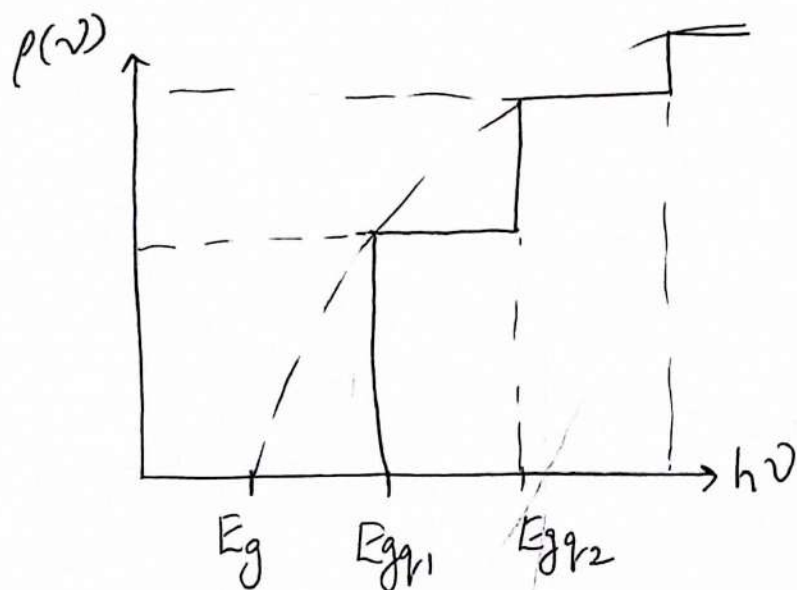
$$E_{qq1} = E_g + E_{q1} + E'_{q1} \leq h\nu \text{ for photon interactions.}$$

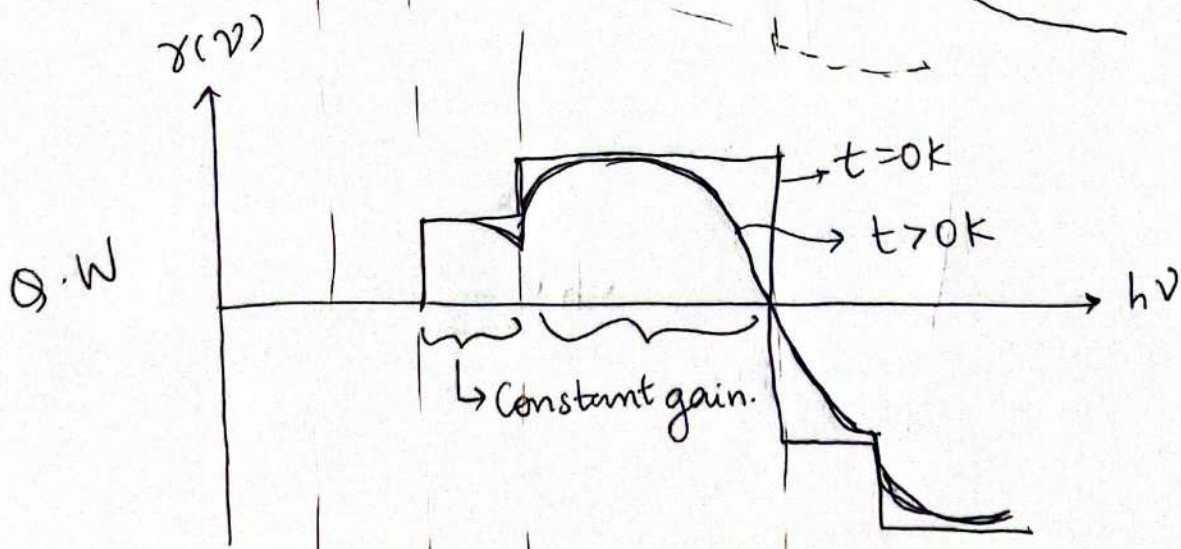
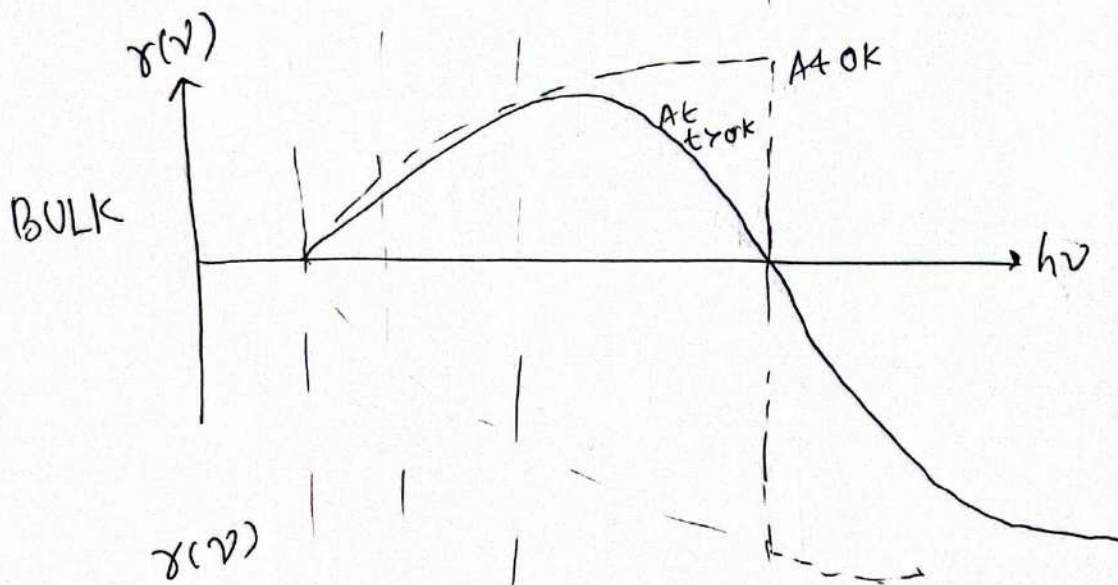
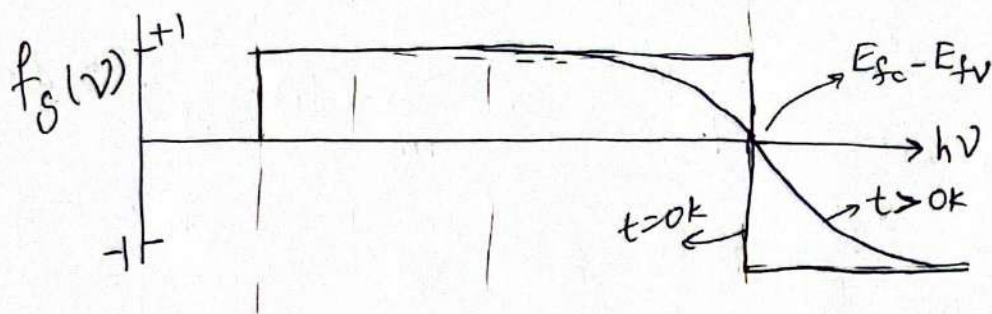
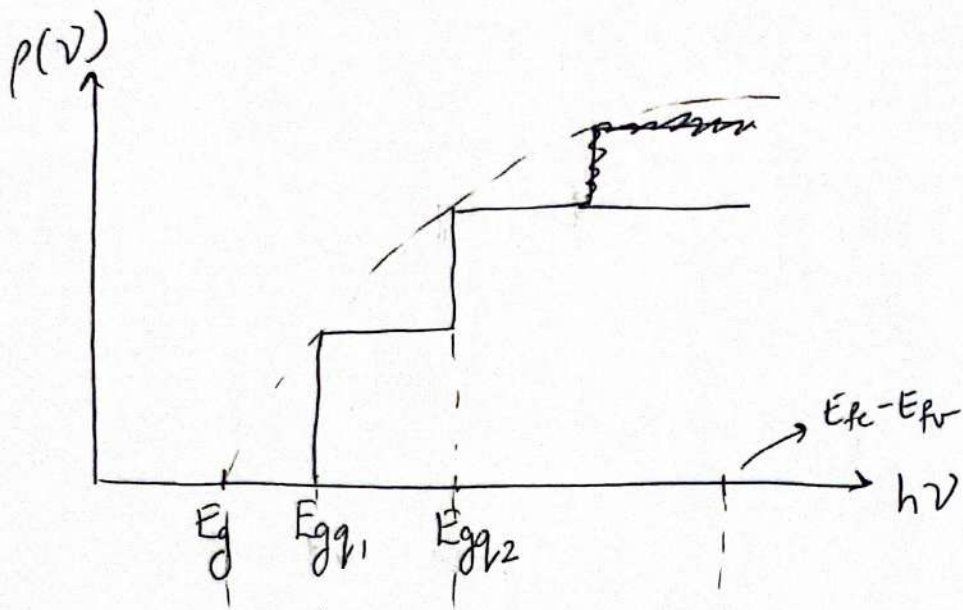
$$\rho(\nu) = \rho_c(E_2) \cdot \frac{dE_2}{d\nu}, \quad E_2 = E_c + \frac{m_r}{m_c}(h\nu - E_g) \rightarrow \text{Bulk.}$$

$$= \frac{m_c}{\pi \hbar^2 L_z} \times \frac{m_r}{m_c} \cdot h. \quad = E_c + E_{q1} + \frac{m_r}{m_c}(h\nu - E_{qq1}).$$

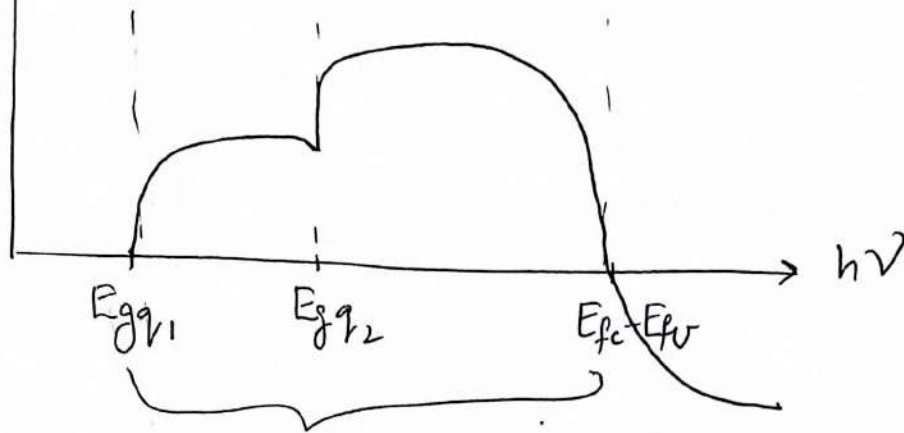
$\rightarrow \text{Q-Well}$

$$= \frac{2m_r}{\hbar L_z} \quad \text{for } E_{qq1} < h\nu < E_{qq2}.$$



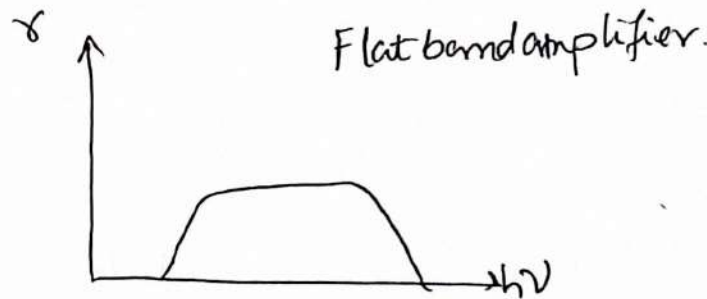


Redrawing,
 $\gamma(\nu)$



Amplification bandwidth.

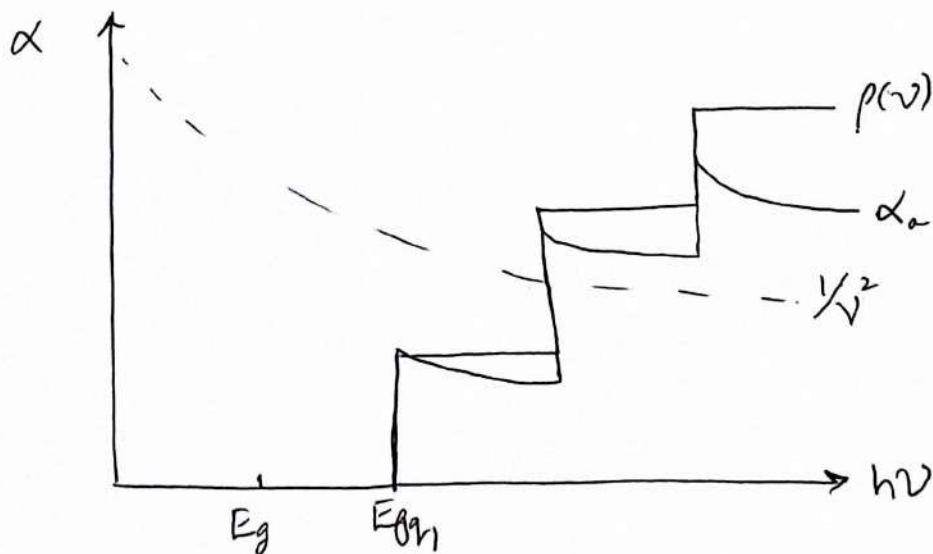
$$\text{If } E_{fc} - E_{fr} = E_{g2}$$

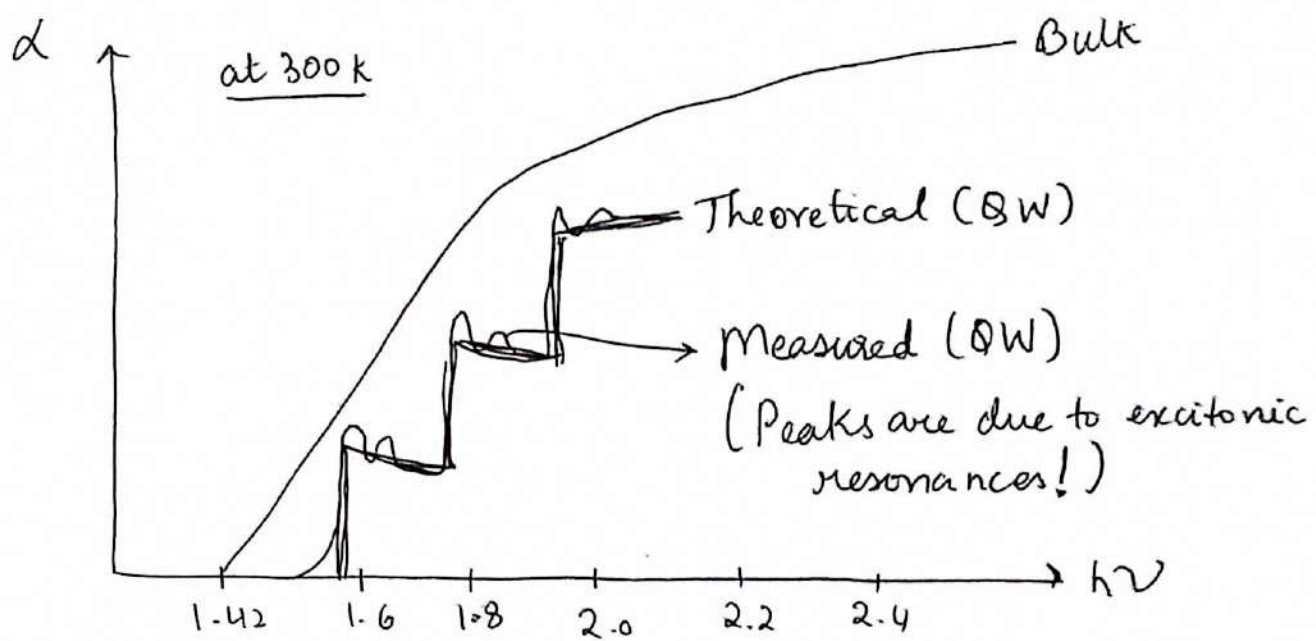


Flat band amplifier.

Loss spectrum.

$$\alpha_a \approx \frac{(Y_n)^2}{8\pi \tilde{\epsilon}_r} \cdot \frac{\rho(\nu)}{\nu^2}$$



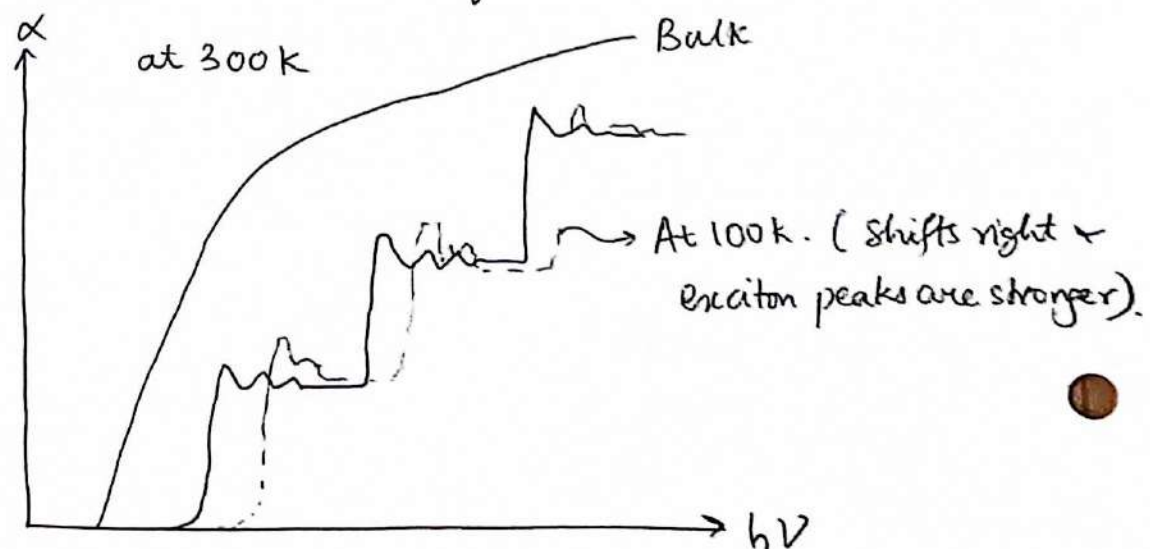


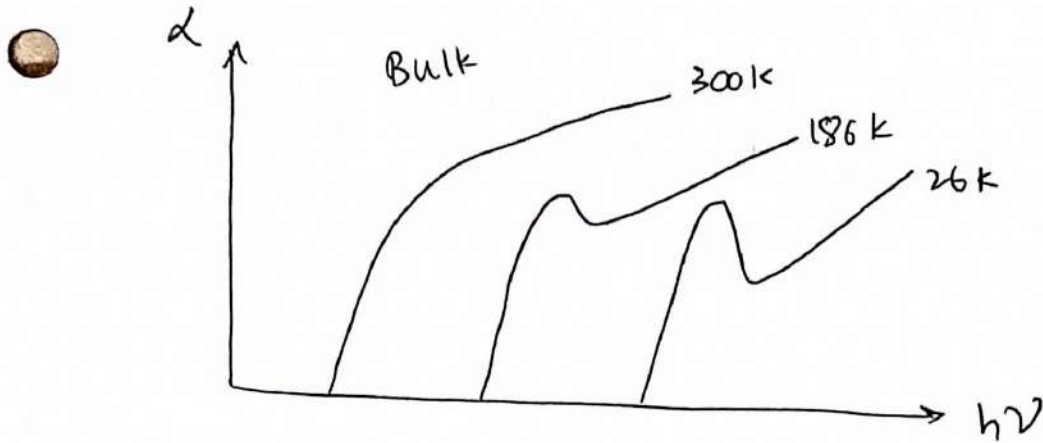
Lec 24 Electro Absorption Modulator

Quantum Well Modulators.

↳ Quantum Confined Stark Effect (QCSE)

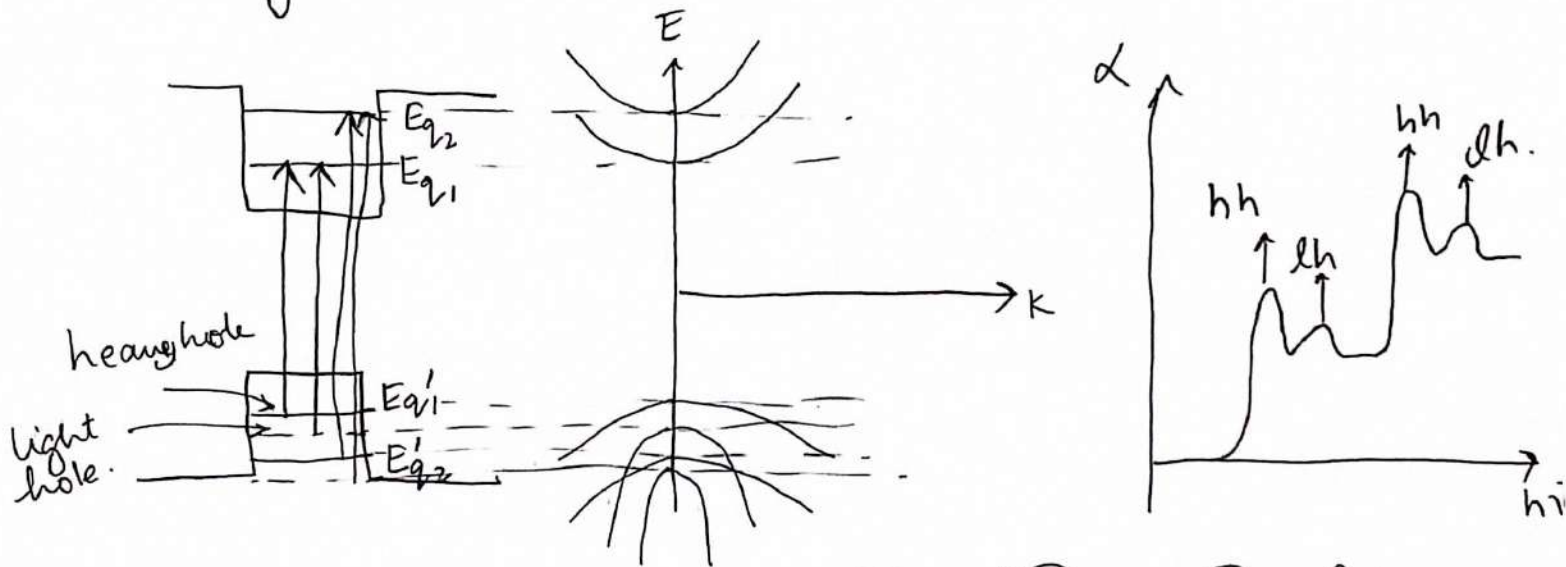
Shift in absorption spectrum due to external electric field. Originally in absorption spectrum of H atom





> Binding energy ~~of~~ of excitons is $\approx 4-5 \text{ meV}$. kT at room temperature is $\approx 25 \text{ meV}$. So at lower temperature excitons can exist stably.

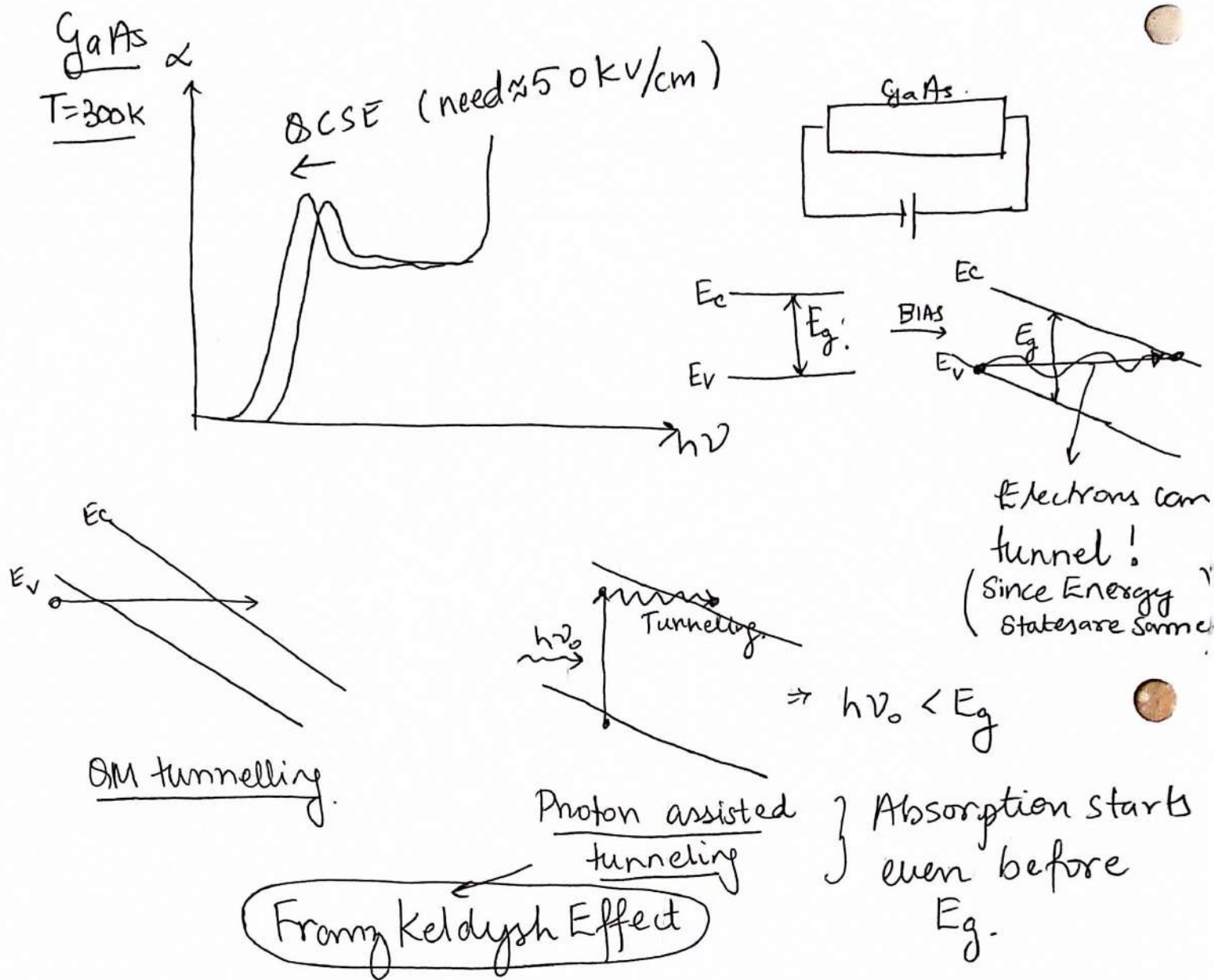
> Why do excitons exist in QW at 300 K?



> Note that excitonic energy in QW is higher due to the quantized state.

> In a quantum well the electron & hole are confined, $\Rightarrow$ they "line-up" strongly which leads to strong coupling & higher binding energy . BE is 3-4 times higher.

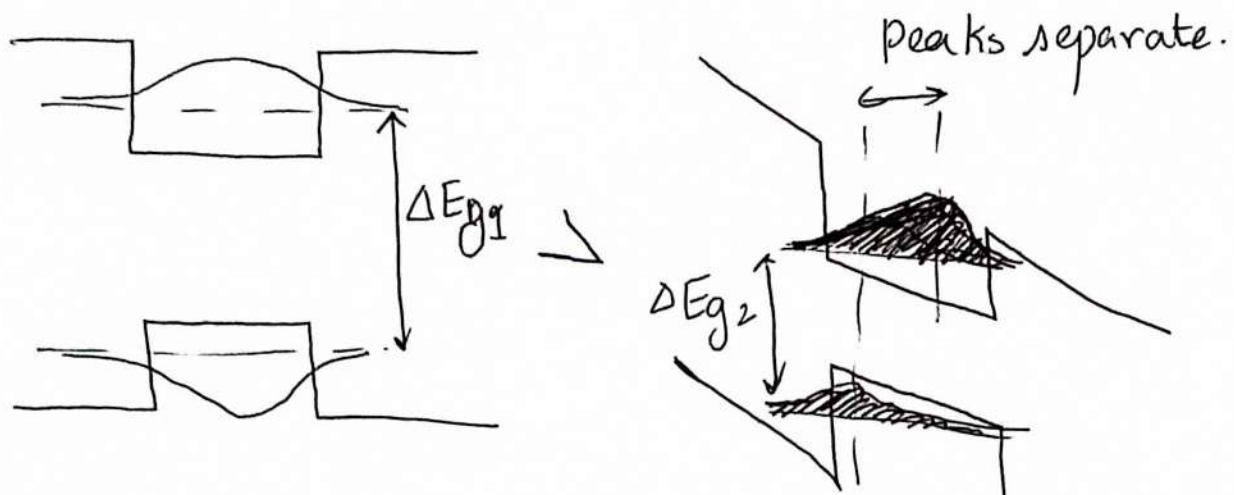
Quantum Confined Stark Effect



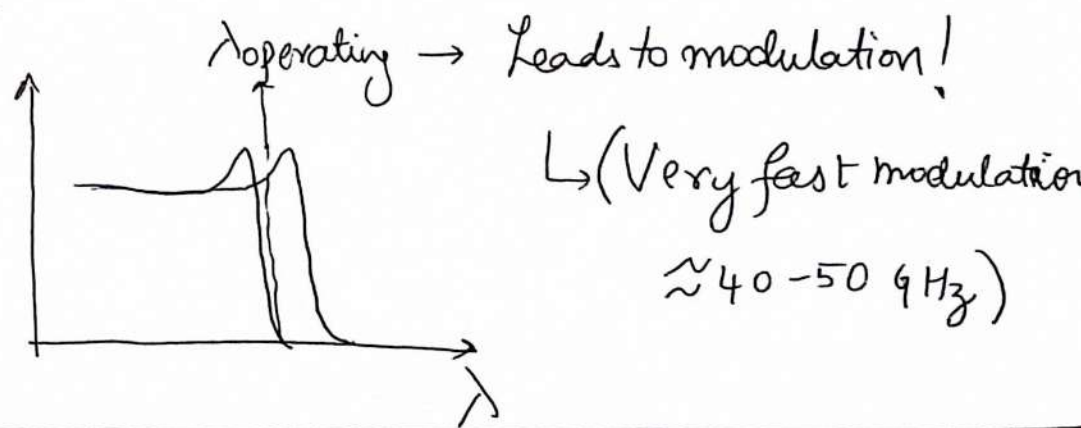
This was made possible because of band tilting.

→ Absorption edge shifts down in Energy (i.e. Red Shift of the absorption edge) → Photon Assisted Tunneling!

→ In a quantum well the same thing happens and is called QCSE!

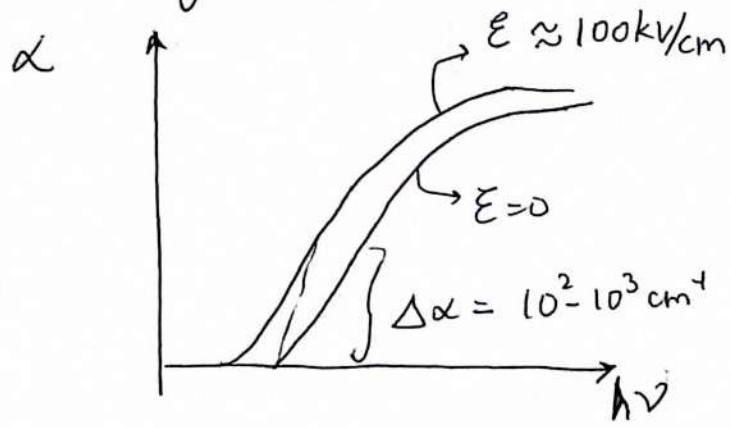


> Note further that exciton is still held in place despite the applied E-field due to the confinement by the well.

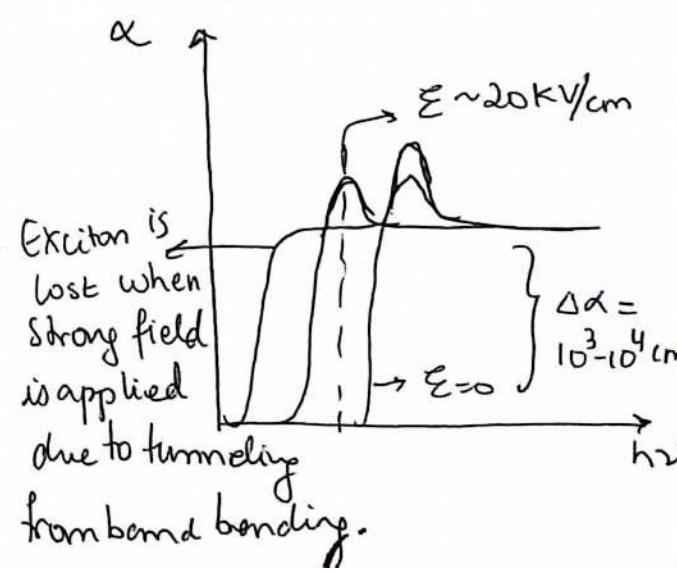


Lec 25 - Electro Absorption Modulator - II

Fromz Keldysh Effect (Bulk)



QCSE



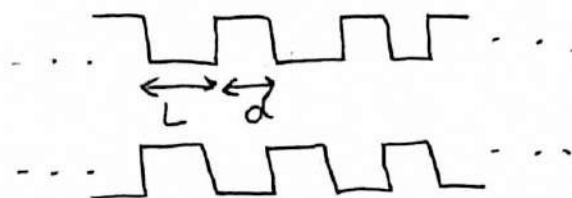
$\epsilon = 20 \text{ kV/cm} = 2 \times 10^4 \text{ V/cm} = 2 \text{ V}/\mu\text{m}.$

~~Reverse biased~~

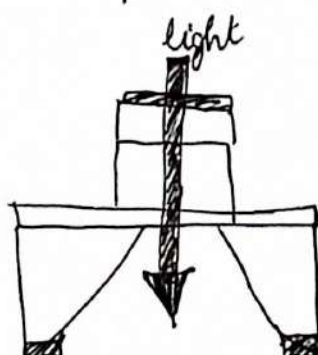
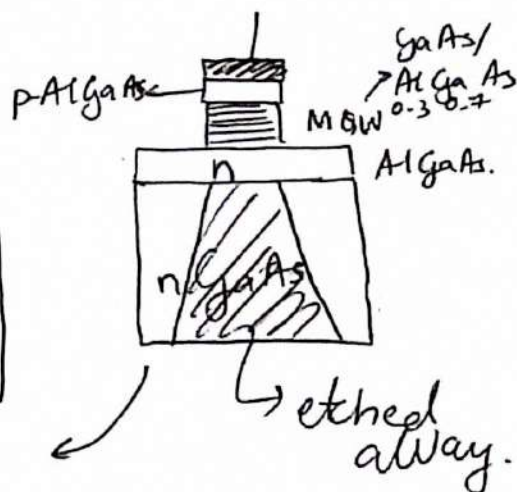
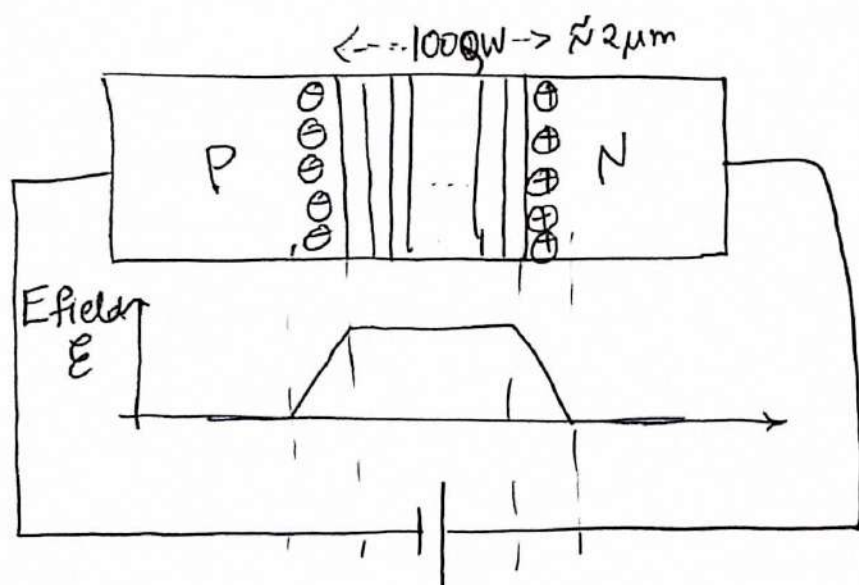
> How many quantum wells are needed for significant attenuation? About ~~at~~ 100.

$d \sim 100 \text{ \AA} \text{ \& } L \sim 100 \text{ \AA}$

$\Rightarrow$ 50 ~ 100 wells are needed.



Reverse biased PIN M&W $\rightarrow$ multiple

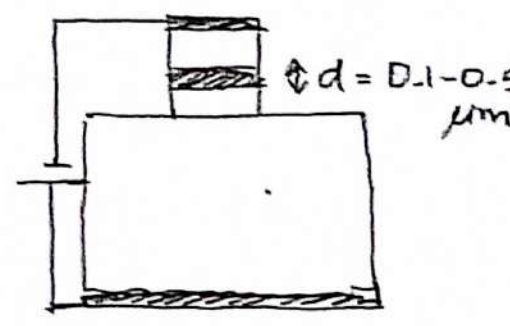
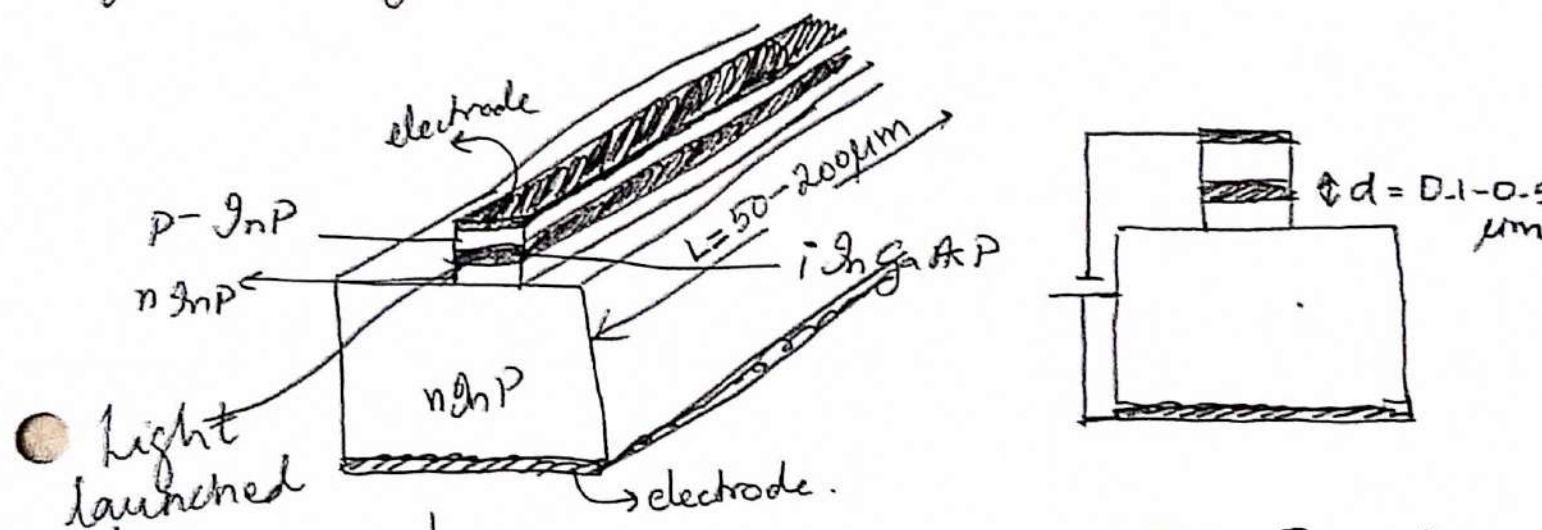


$\lambda_p \sim 800 \text{ nm}.$

Extinction Ratio or Modulation index

$$E-R = \frac{I_{out}(E_p)}{I_{out}(E=0)} = e^{-\Delta\alpha L}$$

If L is large, even Franz Keldysh effect can be used.



↳ Ridged waveguide. Uses Franz Keldysh effect.

↳ Waveguide configuration of EAM:

$$\alpha_{eff} = \alpha_a \Gamma$$

↳ Confinement factor (How much of the mode is inside the absorbing region)

> Requires only 1-2 volts since height is small.

Lec 26 - Semiconductor Optical Amplifiers (SOA)

Amplifier Characteristics

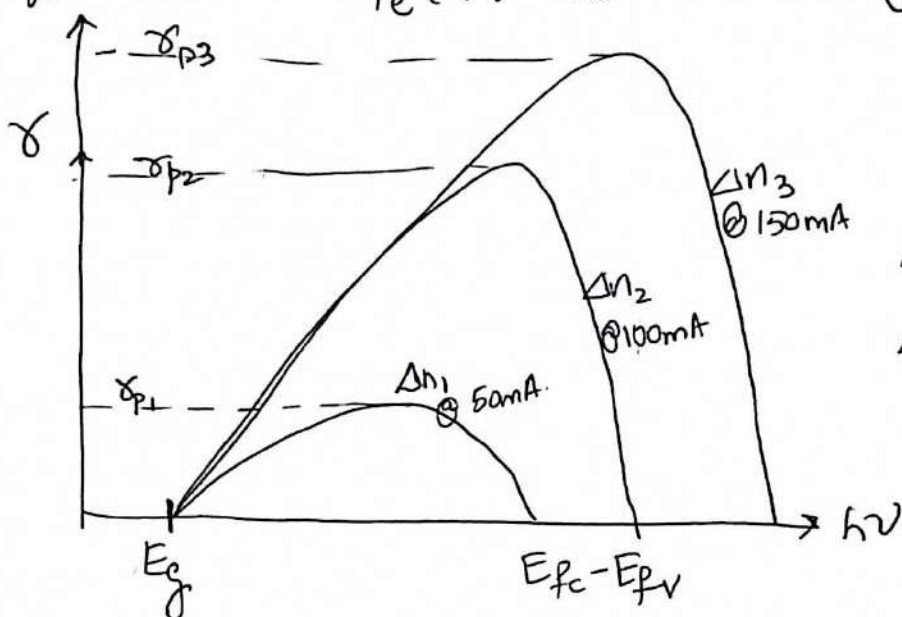
- Gain characteristics
 - Noise
 - Polarization dependent gain
 - ∴ etc.
- Peak gain
 - Gain spectrum
 - BW
 - P_{sat}

Recap: $\gamma_0(\nu) = \frac{(C_n)^2}{8\pi \nu^2 \tau} P(\nu) [f_e(\nu) - f_a(\nu)]$

small signal gain coefficient.

$$f_e(\nu) - f_a(\nu) > 0 \Leftrightarrow (E_{fc} - E_{fv}) > h\nu$$

→ Forward Bias —
P-n heterojunction.



$\Delta n \rightarrow$ excess carrier conc.

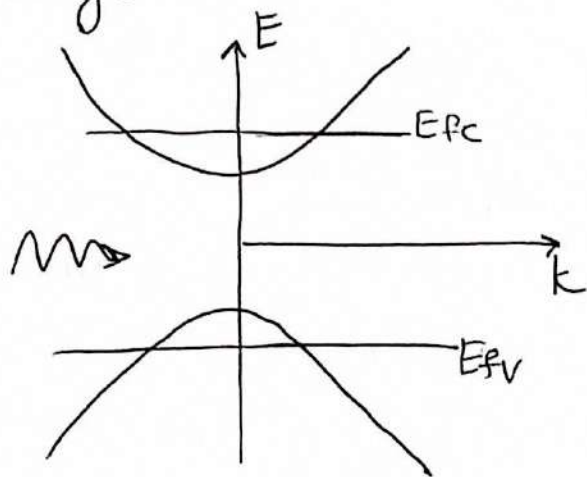
$$\Delta n_3 > \Delta n_2 > \Delta n_1$$

Saturated Gain Coefficient

small signal $\gamma_0 = \frac{\gamma_p}{1 + 4\pi^2 \tau^2 (\nu - \nu_p)^2} \rightarrow$ homogeneously broadened line shape function.

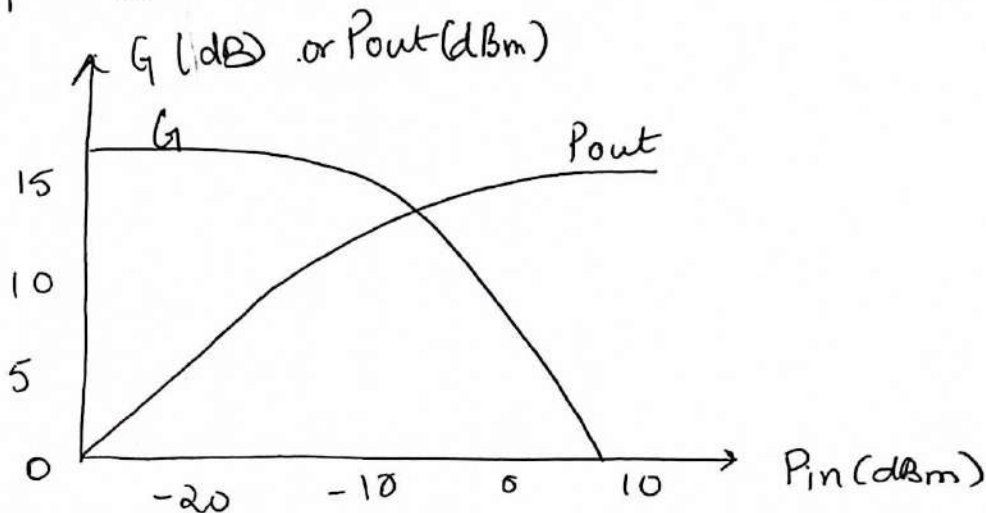
large signal $\gamma(\nu) = \frac{\gamma_p}{1 + 4\pi^2 \tau^2 (\nu - \nu_p)^2 + P_r/P_s} \rightarrow$ Saturation power.

Why does



> When large Input power is used, a lot of electrons are brought down & $E_{fc} \downarrow$ & $E_{fv} \uparrow$.
At low power, pumping can replenish the lost carriers. This is the reason for saturation.

$$G = e^{\gamma_0 L}$$



$$NF = 10 \log \frac{SNR_{op}}{SNR_{i/p}}$$

Typical numbers (SOA)

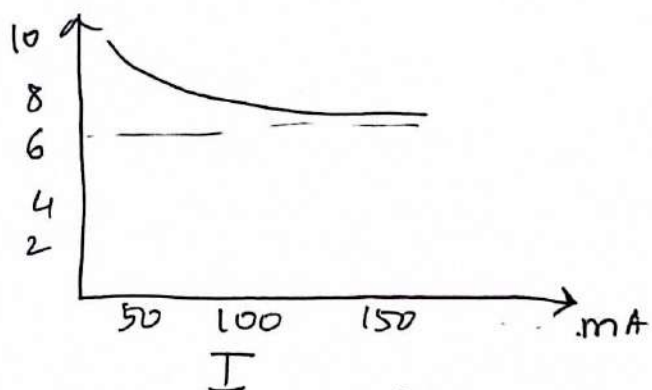
Gain: 20-30 dB

BW: 50-100 nm

P_{sat} : ~10 dBm

NF: ~6 dB

vs



EDFA (long range communication)

Gain: 30-50 dB

BW: 30-40 nm

P_{sat} : > 20 dBm

NF: ~3 dB - 3.5 dB

[All optical switching & logic gates & wavelength converters]

Lec 27 Semiconductor Light Sources.

Electroluminescence. (Injection electroluminescence)

Thermal Equilibrium

$$n_0 = n_i + N_D \quad \& \quad p_0 = p_i + N_A. \quad \} \text{ carrier concentrations.}$$

$$G_0 \propto n_0 p_0 \rightarrow \text{rate of generation.}$$

$$R_0 \propto n_0 p_0 \rightarrow \text{rate of recombination.}$$

$$= \mu n_0 p_0$$

↳ rate constant. (depends on material)

$$\text{At Thermal equilibrium, } G_0 = R_0 = \mu n_0 p_0.$$

Assuming some bias is now added. ∴

$R \rightarrow$ rate of injection.

$\Delta n, \Delta p \rightarrow$ excess carrier concentration.

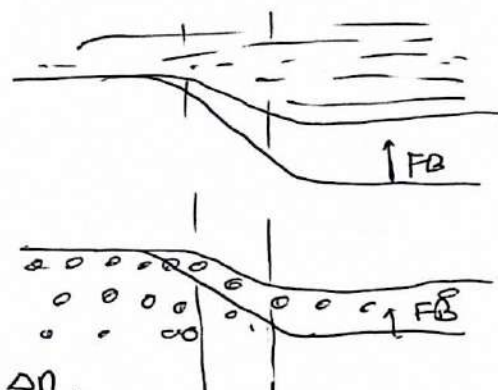
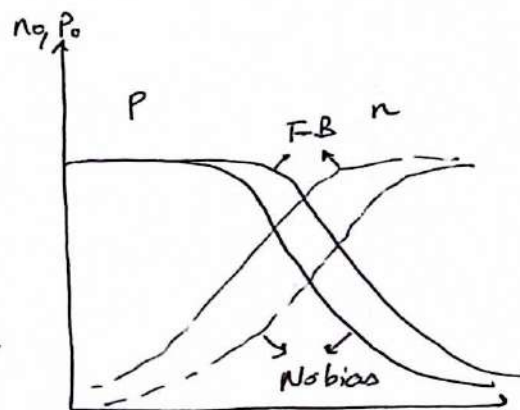
$\& \Delta n = \Delta p$ since e-h come in pairs.

$$\& n = n_0 + \Delta n \quad ; \quad p = p_0 + \Delta p.$$

$$\Rightarrow (G_0 + R) \propto np = \mu np$$

$$= \mu (n_0 + \Delta n) (p_0 + \Delta p)$$

$$= \mu n_0 p_0 + \mu (n_0 + p_0 + \Delta n) \Delta n //$$



or $R = \underbrace{\mu (n_0 + p_0 + \Delta n)}_{\tau} \Delta n$. per c.c. $\Rightarrow$ μ has unit cc/s .
 per cc per sec.

$\Rightarrow R = \tau^{-1} \Delta n$ where $\tau = \frac{1}{\mu (n_0 + p_0 + \Delta n)} \stackrel{\text{if } \Delta n \ll n_0, p_0}{\approx} \frac{1}{\mu (n_0 + p_0)}$

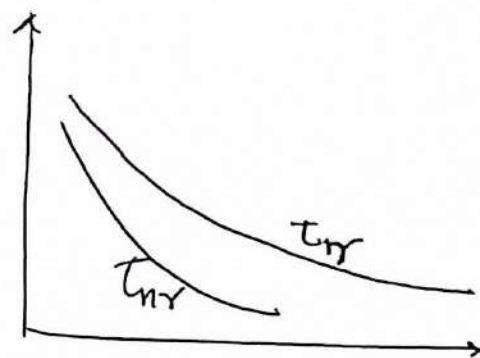
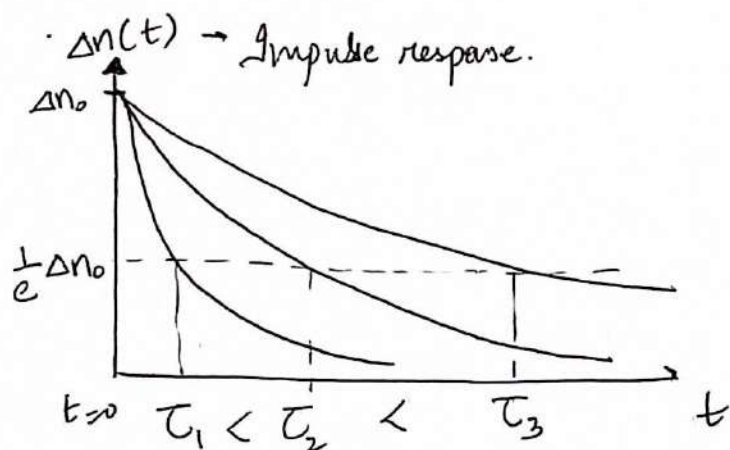
$\Rightarrow R = \frac{\Delta n}{\tau}$

τ $\rightarrow$ excess carrier recombination time.

$\mu \rightarrow$ rate constant of recombination $\begin{cases} \rightarrow \text{radiative} \\ \rightarrow \text{non-radiative} \end{cases}$

$\Rightarrow \mu = \mu_r + \mu_{nr}$

$\Rightarrow \text{Rate of Recomb} = \mu_r n p + \mu_{nr} n p$ } let $\tau_r \propto \frac{1}{\mu_r}$ & $\tau_{nr} \propto \frac{1}{\mu_{nr}}$



	τ_r	τ_{nr}
Si	10ms	100ns
GaAs	100ns	100ps
GaN	10ns	0.1ns

Small lifetime $\tau \Rightarrow$ it is dominant!

$\Rightarrow$ Si is a poor radiator.

Internal Quantum Efficiency (η_i)

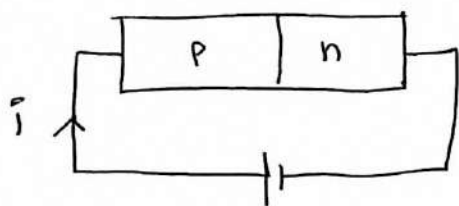
$$\eta_i = \frac{\text{radiative recombinations per sec}}{\text{Total recombinations per sec}}$$

$$= \frac{R_r}{R_r + R_{nr}} = \frac{\tau}{\tau_r}$$

$$\eta_i \text{ of GaAs} \cong 0.5$$

$$\eta_i \text{ of Si} \cong 10^{-5}$$

In general direct bandgap materials have $\eta_i \approx 0.2-0.9$ & indirect are very low.



$$R = \frac{\Delta n}{\tau} \quad \left\{ \begin{array}{l} \text{no. of recombinations per unit time} \\ \text{per unit vol.} \end{array} \right.$$

$$i = \text{charge / time.}$$

$\Rightarrow$ no. of carriers per unit time.

Rate of recombination of carriers = Rate of injection of carriers.
 $\hookrightarrow R V = (i/e) \quad \left\{ \begin{array}{l} \text{volume.} \end{array} \right.$

$$\Rightarrow \frac{i}{e} = \frac{\Delta n}{\tau} \cdot V$$

What is the optical power generated?

$n_{ph} \cdot h\nu \rightarrow$ total power generated.
 $\hookrightarrow$ no. of photons.

$V \cdot R \rightarrow$ rate of recombinations.

$\eta_i \cancel{VR} \rightarrow$ no. of photons generated

$\Rightarrow \eta_i R V h\nu \rightarrow$ optical power generated.

$$P_{gen}^{opt} = \eta_i \left(\frac{i}{e} \right) h\nu$$

$$\Rightarrow \boxed{P_{gen}^{opt} = \eta_i \left(\frac{h\nu}{e} \right) i} \rightarrow \text{power vs. applied current!}$$

Lec 28 LED 1 - Device structure & Parameters.

Let $\eta_e \rightarrow$ extraction efficiency.

$$\Rightarrow P_{out} = \underbrace{\eta_e \eta_i}_{\eta_{ext}} \left(\frac{i}{e} \right) h\nu$$

$\eta_{ext} \rightarrow$ external efficiency.

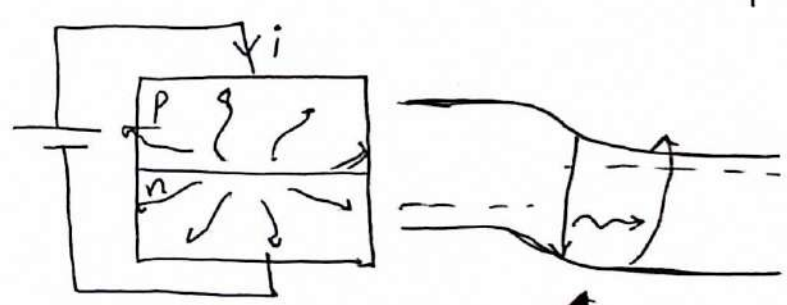
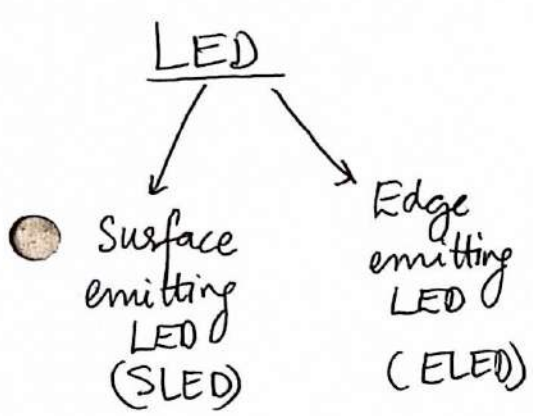
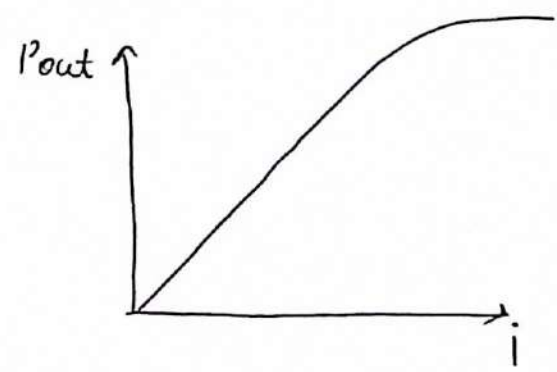
$$\Rightarrow \boxed{P_{out} = \eta_{ext} \left(\frac{i}{e} \right) h\nu}$$

$$P_{out} = \eta_{ext} \left(\frac{i}{e} \right) h \frac{c}{\lambda}$$

$$= \eta_{ext} \left(\frac{hc}{e} \right) \frac{i}{\lambda}$$

$$P_{out} = \eta_{ext} \left(\frac{1.24}{\lambda} \right) i$$

$$\Rightarrow P_{out} \propto i$$



> Some light is reflected back due to air-dielectric interface.

$$R = |\alpha|^2 = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$= \left(\frac{3.6 - 1.0}{3.6 + 1.0} \right)^2$$

$$\approx 0.32$$

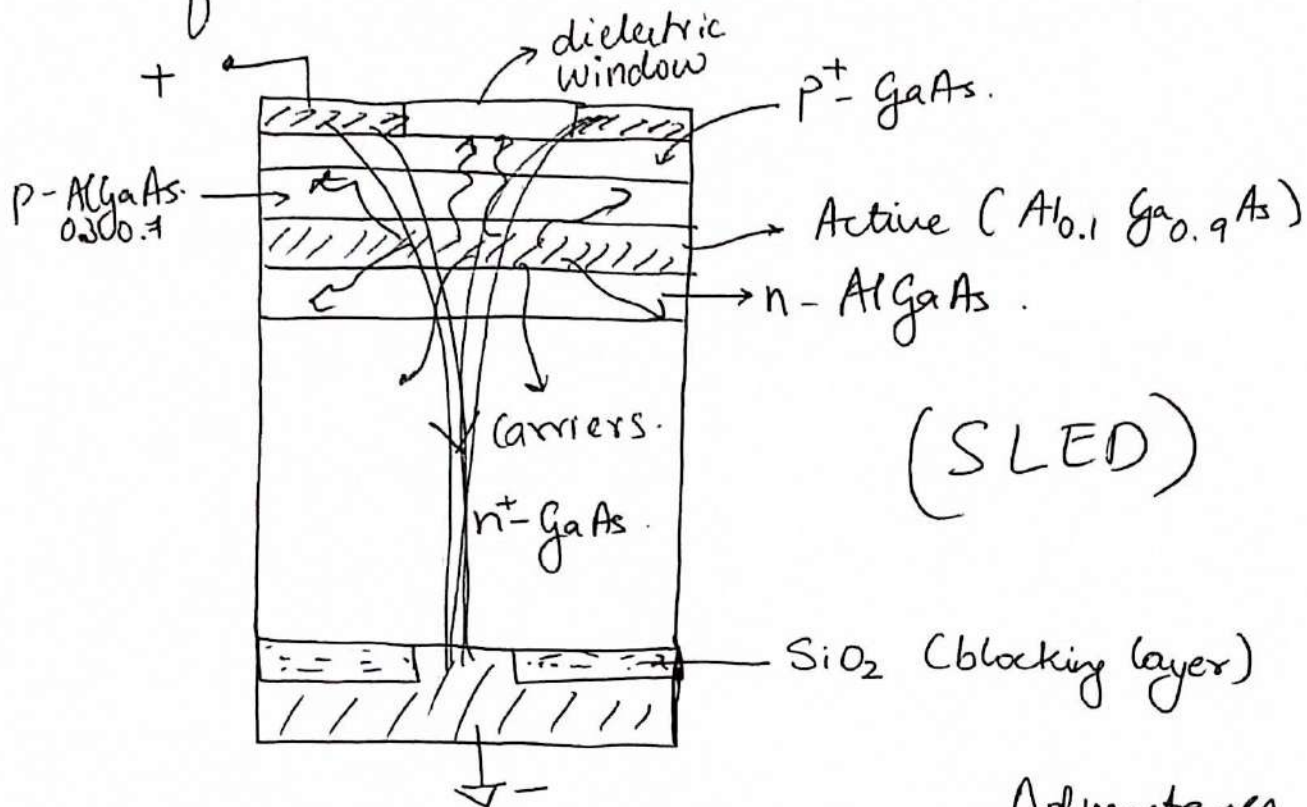
↑
reflection
coeff
for Power
at normal
incidence

Sources of loss

- ① > Reabsorption of generated photons.
- ② > Reflection loss at S-C-air interface.
- ③ > Total internal reflection loss.

Typically, $\eta_e \approx 0.2 - 0.3$
How to reduce? Double Hetero Structures //

Use of Double Hetero Structures to mitigate losses.



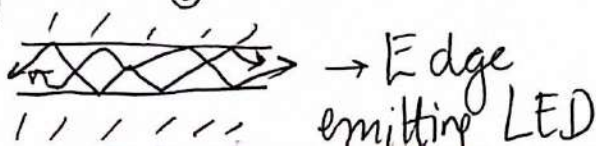
Advantages

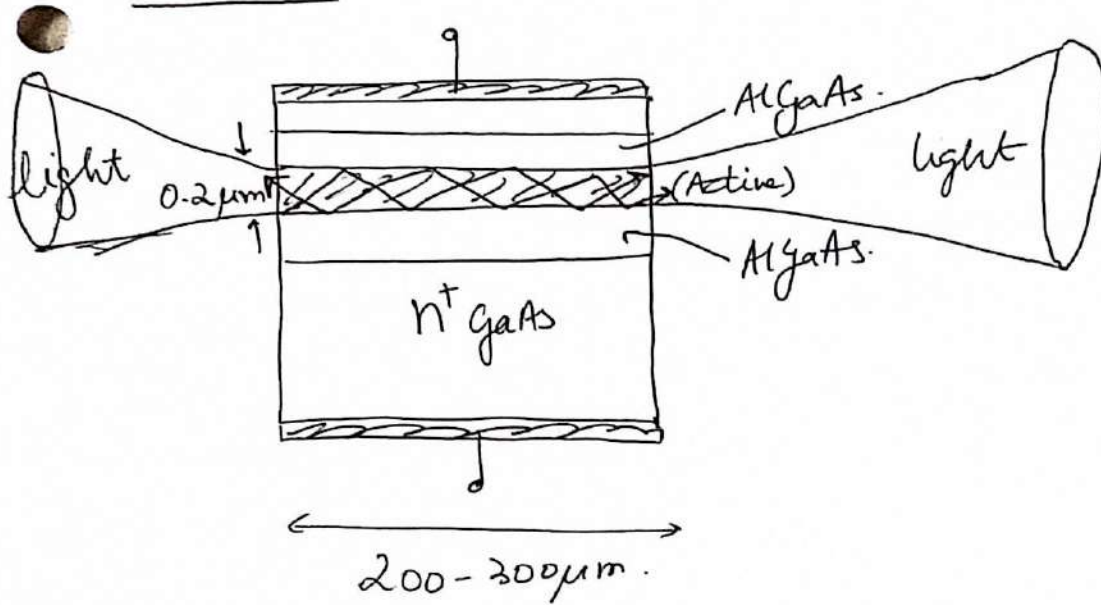
① Avoids re-absorption losses since bandgap outside active region is higher.

② Carrier confinement causes very high charge concentration.

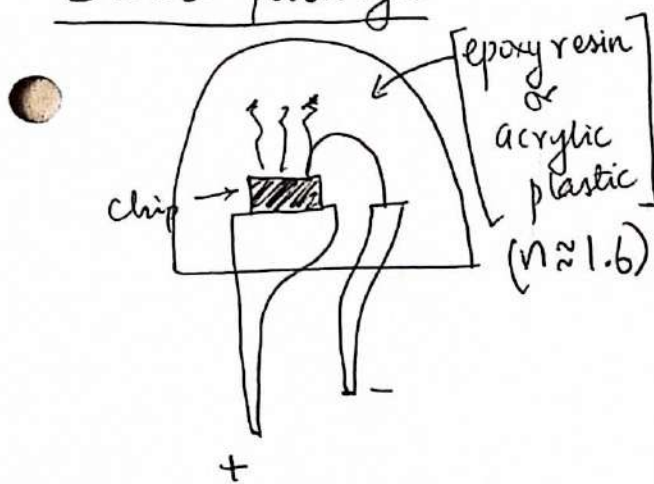
③ Optical confinement (not relevant for surface emitting LED)

Refractive index is higher when bandgap is ~~high~~ low!!



ELED

Higher directionality in output light compared to SLED.

SLED package

1) To protect the chip & reduce reflection losses.

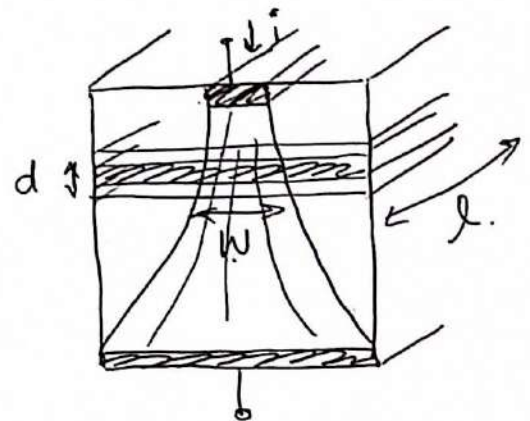
> Bragg reflectors in n^+ GaAs can reflect light up.
> Texturizing the surface can reduce Total internal reflection

Lec 29 LED II - Device Characteristics

$$\Delta n = \frac{(i/e) \tau}{l \times w \times d} \approx \frac{i}{l \times w} = J$$

↑
current density.

$$\Rightarrow \Delta n = \frac{J \tau}{l d}$$



LED device characteristics

1. ^{current power} $i-P$ & ^{voltage.} $i-V$ characteristic

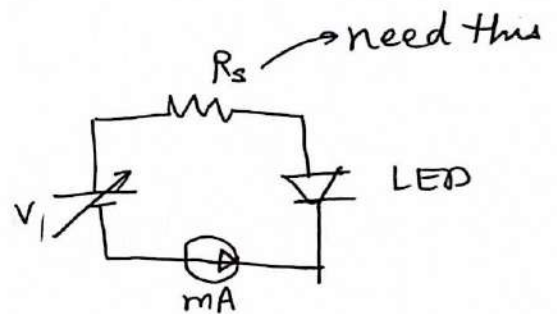
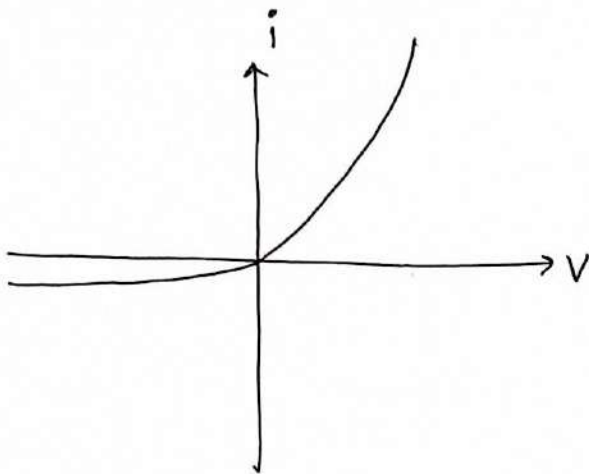
aka $L-I$ characteristic
light current

2. Wavelength spectrum (spectral distribution)

3. Emission pattern (angular spectrum or radiation pattern)

4. Modulation bandwidth.

1. $I-V$ characteristics

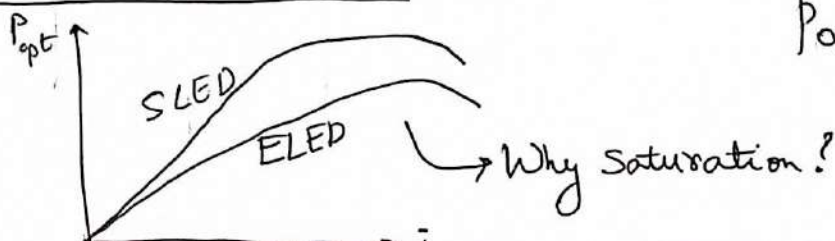


R_s is needed to ensure current does not exceed I_{Fmax} → data sheet!

max supply voltage.

$$\frac{V_{max}}{I_{Fmax}} = R_s$$

$L-I$ characteristic



$$P_{out} = \eta_{ext} \left(\frac{i}{e} \right) h\nu$$

Why saturation occurs?

1. Joule heating $i^2 R$

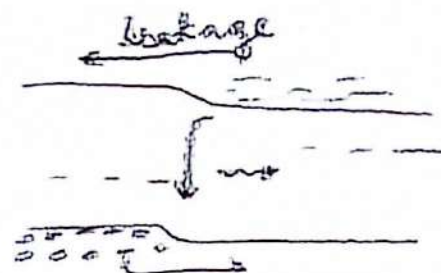
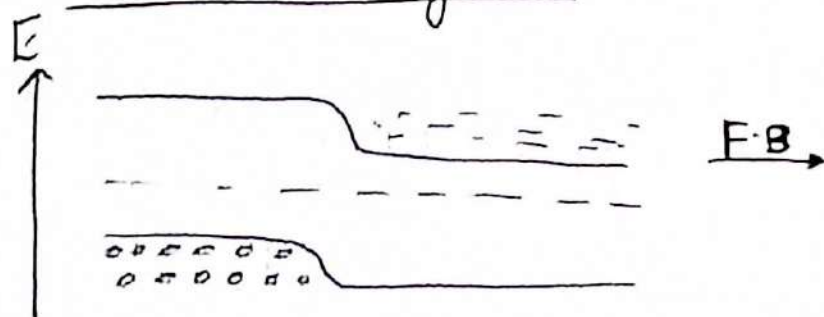
As temp $\uparrow$ η_i (internal quantum efficiency) decreases
Since too many phonons exist.

2. Carrier leakage loss

3. Stimulated emission loss.

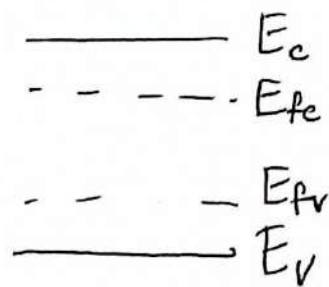
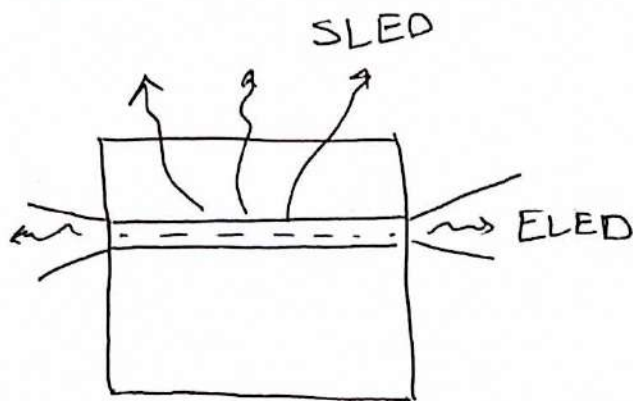
● Lec 30 - LED III - Output Characteristics

2. Carrier leakage loss



Instead of recombining in the depletion region, they pass by the barrier & go directly to the electrode

3. Stimulated Emission Loss



As $FB \uparrow$, $E_c - E_{Fv} \uparrow \Rightarrow$ stimulated emission starts increasing

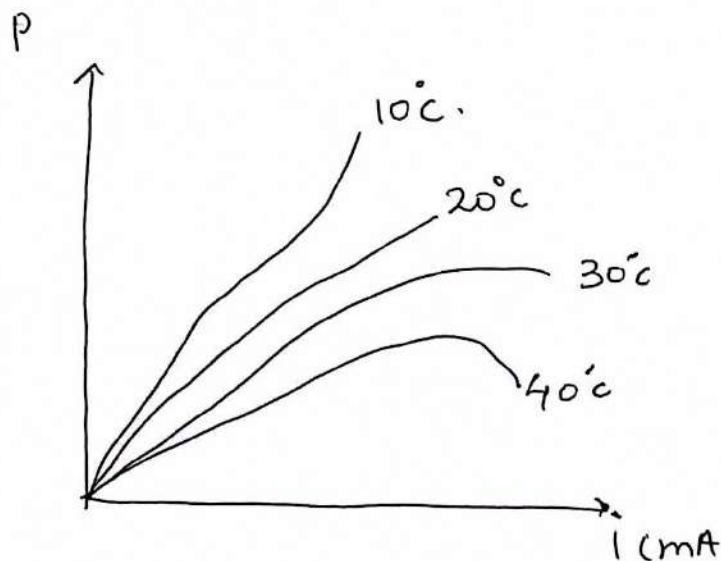
$\Rightarrow$ stimulated emission occurs along ELED direction.

$\Rightarrow$ it is a loss for SLED & gain for ELED.

> However, even for ELED, the 1.) & 2). loss mechanisms dominate at room temp.

> At lower temperatures it is possible to see "gain" in ELEDs. Such diodes are called SLDs.

$\hookrightarrow$ Superluminescent diodes.

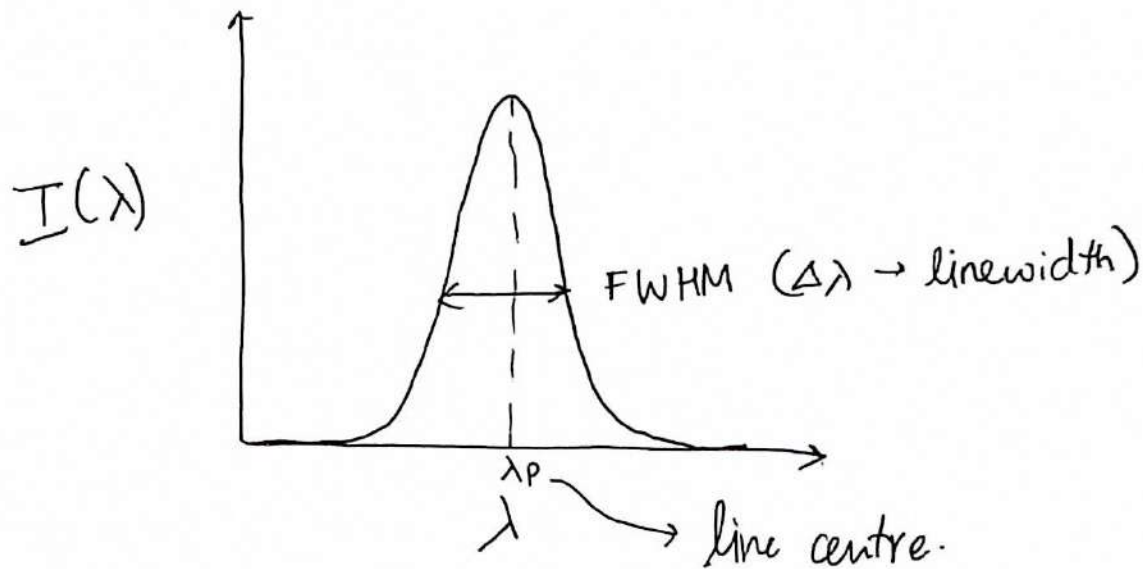


$\downarrow$
ELEDs that are much longer to increase stimulated gain.

Lec 30 LED-III - Output Characteristics (contd.)

(95)

Wavelength Spectrum (Spectral distribution)



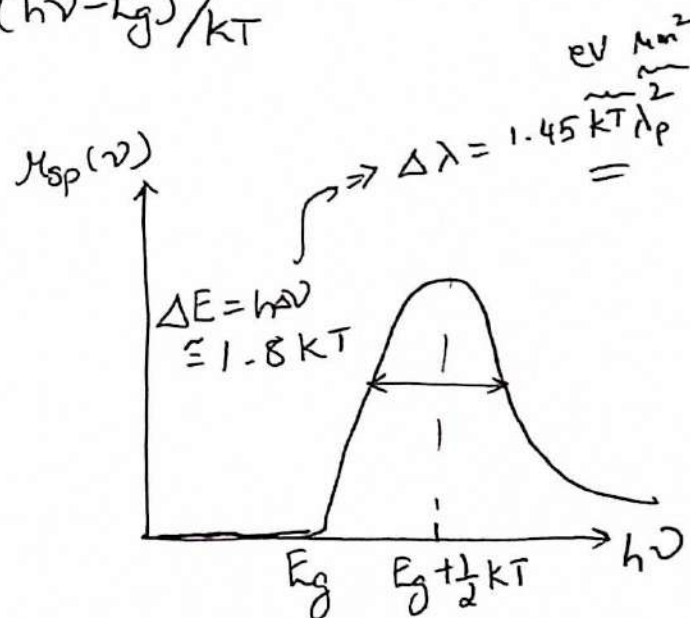
Recall,

$$M_{sp}(\nu) = D_0 (h\nu - E_g)^{1/2} \cdot e^{-(h\nu - E_g)/kT}$$

$$= D_0 x^{1/2} e^{-ax}$$

$$h\nu_p = E_g + \frac{1}{2}kT$$

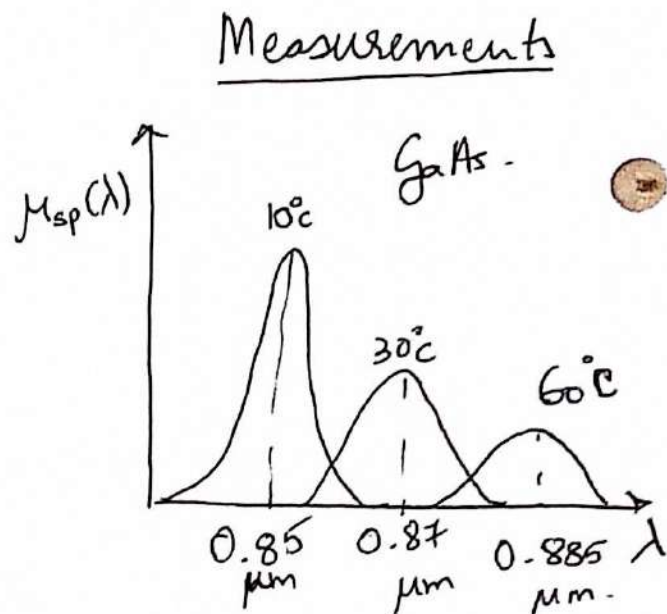
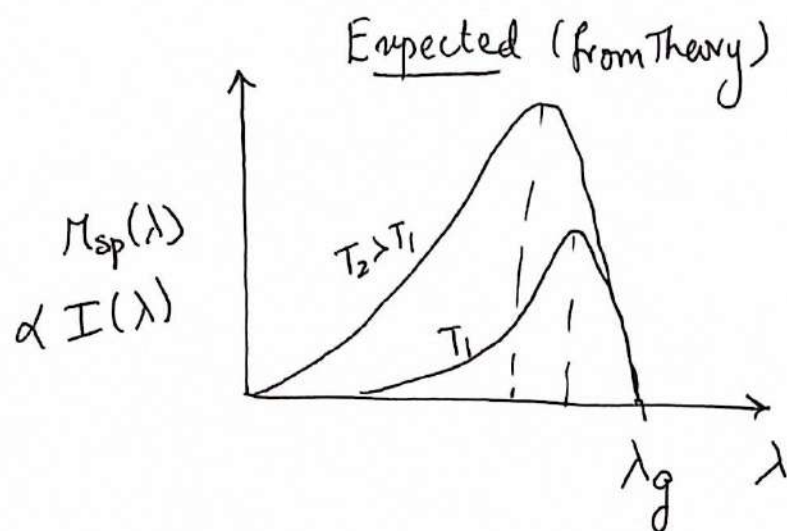
$$\Delta\lambda \approx 1.45 kT \lambda_p^2$$



Example: $kT \approx 0.025$ eV & let $\lambda_p = 800$ nm.

$$\Rightarrow \Delta\lambda = 1.45 \times 0.025 \times 0.64 \approx 0.023 \mu\text{m}$$

Typically $\lambda \sim 0.4 - 0.7 \mu\text{m}$ in visible.



> peak goes up in λ instead of lower.

> Also peak goes down in M_{sp} instead of higher.

Why the discrepancy?

1) > Bandgap is temperature dependant.

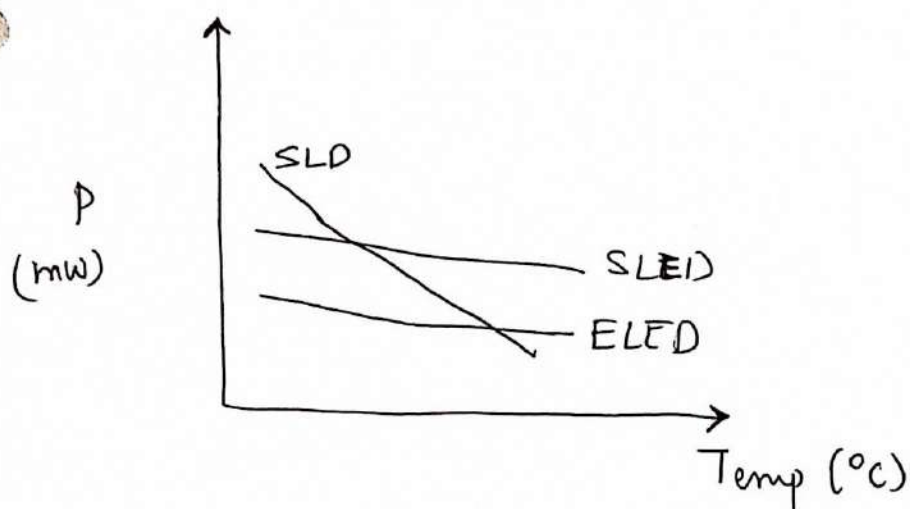
	<u>0K</u>	<u>300K</u>
<u>GaAs:</u>	1.52eV	1.42eV

Empirical formula: $E_g(T) = 1.519 - 5.405 \times 10^{-4} \frac{T^2}{(T+204)}$

for GaAs

2) > Peak decreases since η_i goes down exponentially with T .
due to increase in phonon transitions.

Lec 31

1) L-I characteristics (review)3) Angular distribution (Radiation pattern)

$$I(\theta) = I_0 (\cos \theta)^m \text{ - LED.}$$

$$m = 1, 2, 3, \dots$$

Planar LEDs (no encapsulation)

~~Typically~~
$$I(\theta) = I_0 \cos \theta \rightarrow \text{Lambertian distribution.}$$

($\Delta\theta = 120^\circ$)

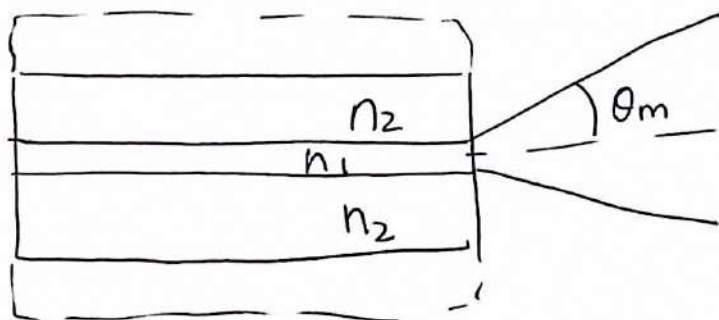
> Typically m is between 2 & 3.

> m should be low for display LEDs for wide angle viewing.

> ELEDs have higher directivity.

Numerical aperture.

$$\sin \theta_m = N.A. = \sqrt{n_1^2 - n_2^2}$$

Typically $\Delta\theta \approx 30^\circ$.

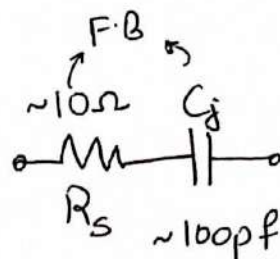
HPBW in elevation

$\Delta\theta$ in azimuth is higher in ELEDs.
 $\approx 120^\circ$

4. Modulation Bandwidth

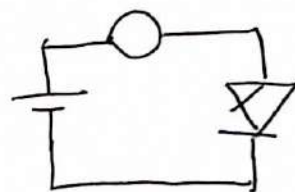
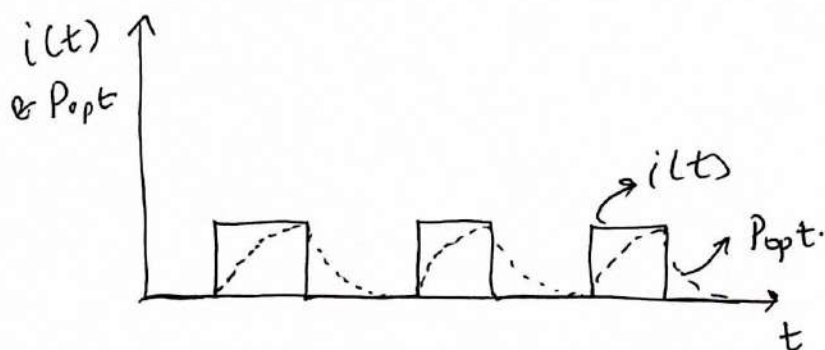
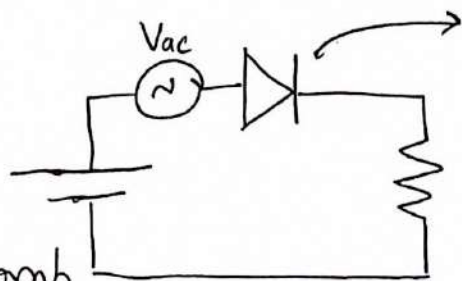
- Carrier recombination time.
- RC time constant

$$f_{BW} = \frac{1}{2\pi RC}$$



$$\Rightarrow f_c \approx 100 \text{ MHz}$$

1) Carrier recomb



Due to finite recombination time, P_{opt} is limited.

$$\tau \approx 10^{-6} - 10^{-9} \text{ s.}$$

display

communication.

$$i(\omega) = \frac{i_0}{(1 + \omega^2 \tau^2)^{1/2}}$$

current in LED

LED-V Materials & applications

Responsivity

$$R = \frac{P_{opt}}{i} = \eta_{ext} \frac{1.24}{\lambda} \quad \text{Watt/Amp}$$

Typical numbers: $50 \mu\text{W}/\text{mA}$.

$\Rightarrow 100\text{mA} \rightarrow 5\text{mW}$

Light conversion efficiency or wall-plug efficiency

$$\eta = \frac{P_{opt}}{P_{elec}} = \eta_{ext} \frac{i \cdot \frac{1.24}{\lambda (\mu\text{m})}}{iV} = \eta_{ext} \frac{1.24}{V\lambda}$$

$iV \rightarrow$ voltage across diode.

internal quantum

η_i

η_e

external extraction

0.3-0.6

0.1-0.3

external

η_{ext}

0.03-0.2

wall-plug

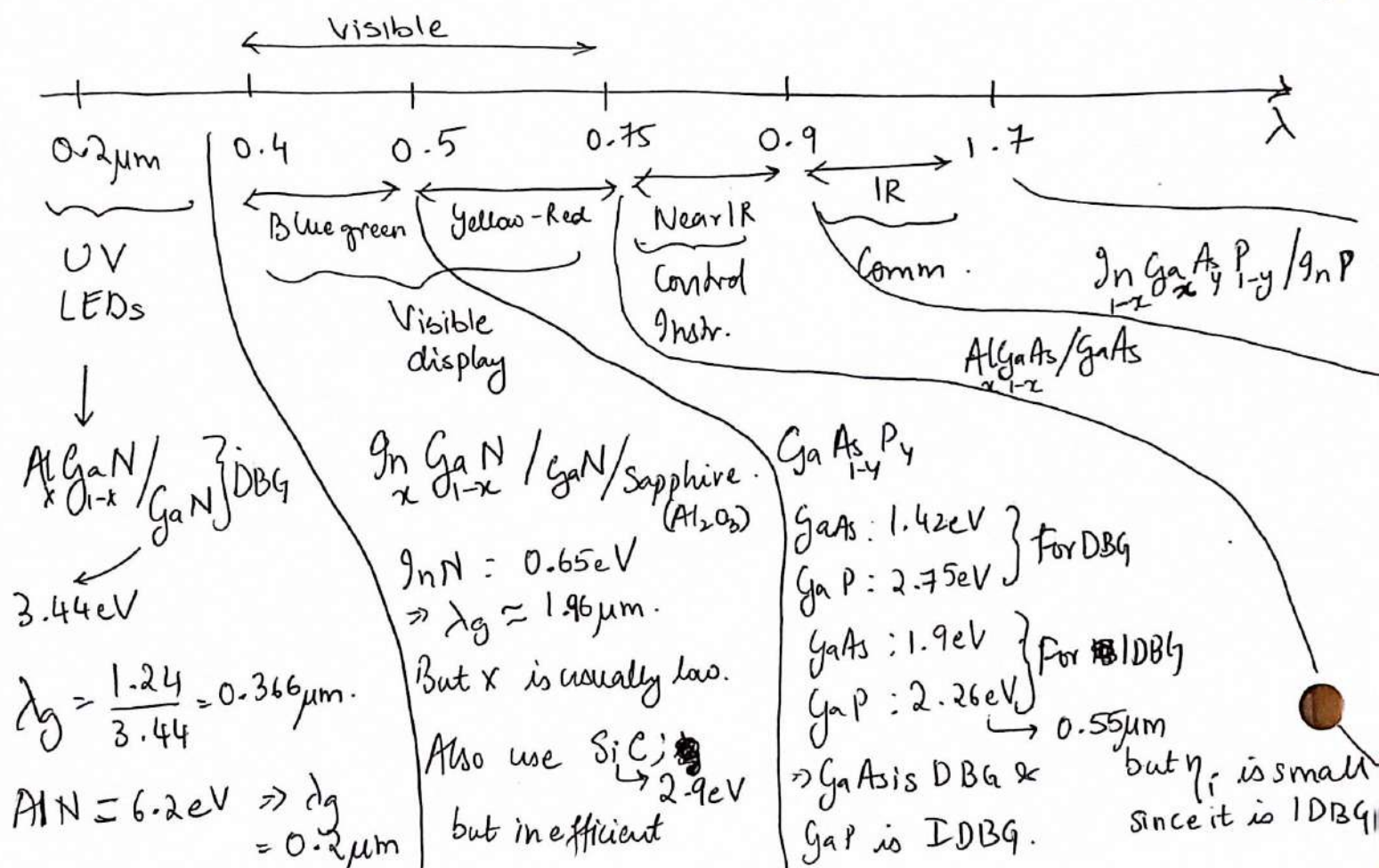
η_{wp}

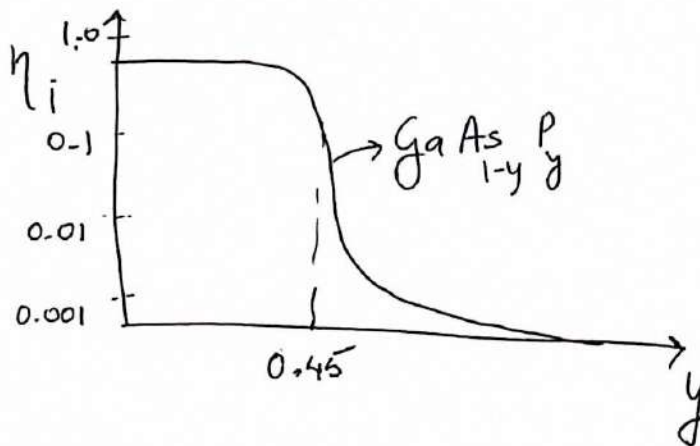
0.02-0.2

} Typical numbers.

Applications

- 1) Display.
- 2) Communication (low data rate MM fiber & cheap).
(upto 100 Mbps)
- 3) UV LEDs for purification/decontamination of water.
 - Security
 - Epoxy resin curing.
- 4) Control & instrumentation. (remote controls etc.)
(opto isolators).
- 5) White light LEDs.

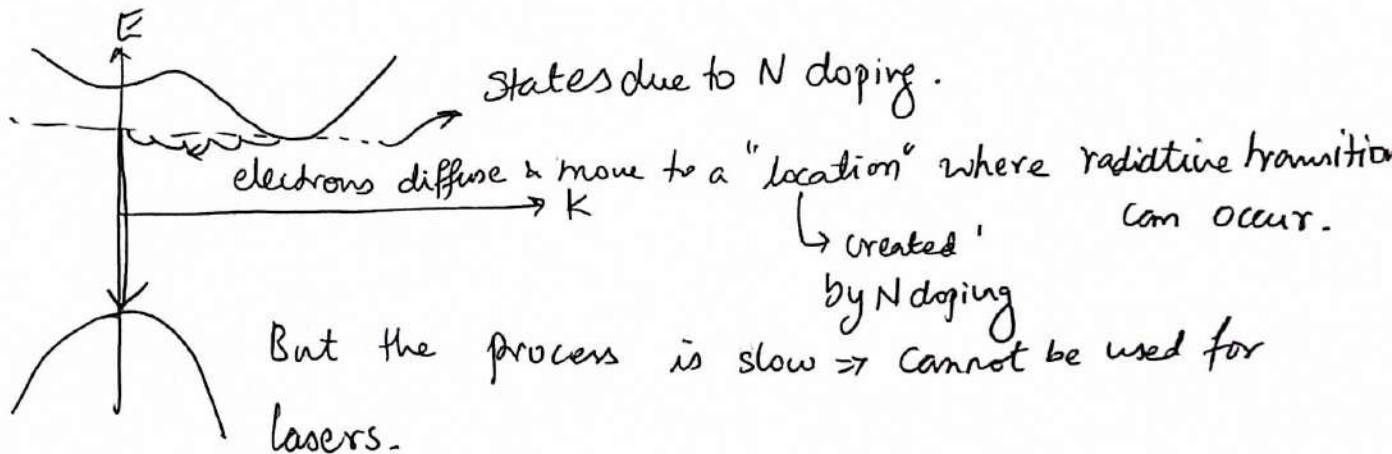




How to make $0.55\mu\text{m}$ LED without dropping η_i ?

Is-electronic centers

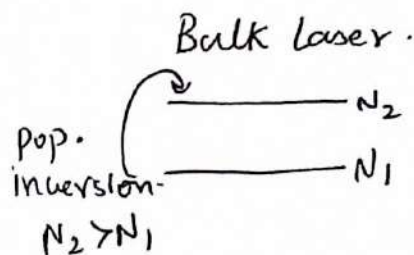
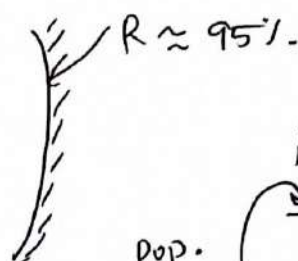
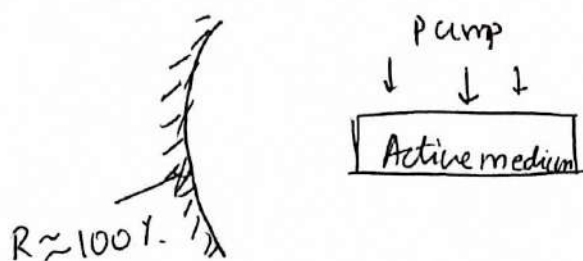
$\text{GaP} : \text{N}$



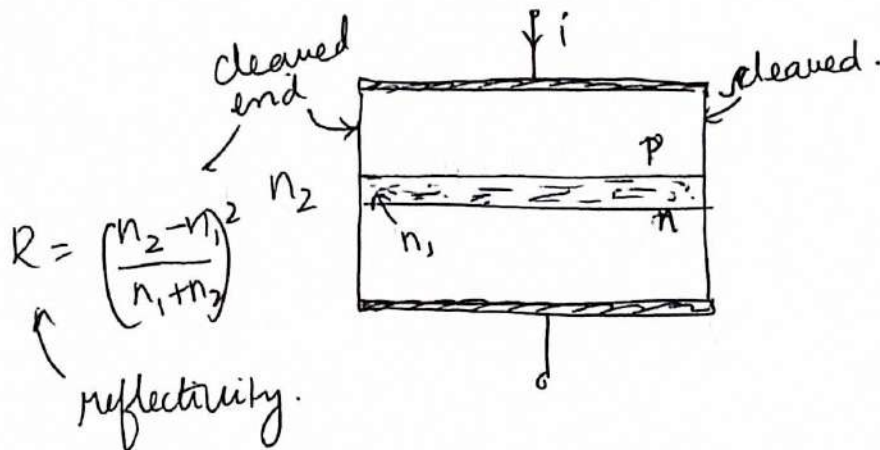
Lec 33 Laser Basics

3 components of a Laser.

- 1) Gain medium
- 2) Pump or power supply
- 3) Optical Feedback unit.



Semiconductor laser.



$$\Delta n = \frac{(i/e)\tau}{l \times w \times d}$$

$$E_{fc} - E_{fv} > E_g$$

> If the reflecting mirrors are spherical, the modes inside are Hermite-Gauss or Laguerre-Gauss. Fundamental mode is Gaussian TEM₀₀.

↖ Bulk laser.

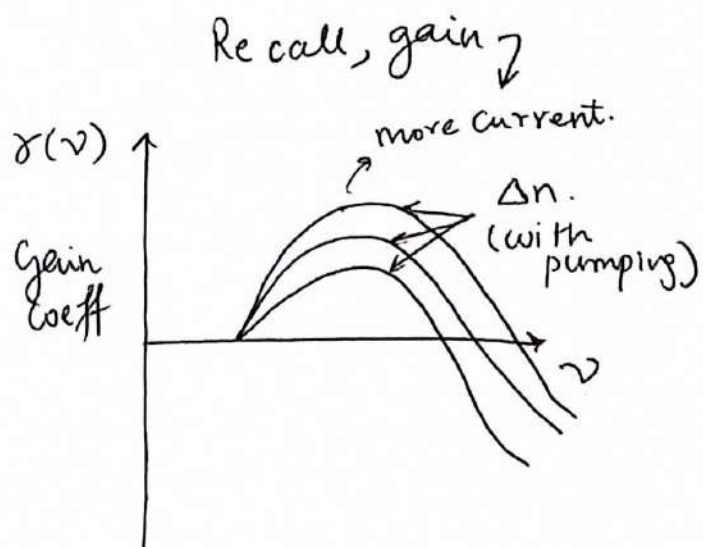
Laser dynamics (steady state).

$$\text{Gain} = \text{loss}$$

Gain in active medium loss in resonator.

loss mechanisms

- > Reflectivity of mirrors.
- > Scattering loss inside medium.
- > Diffraction losses due to finite beam extent.

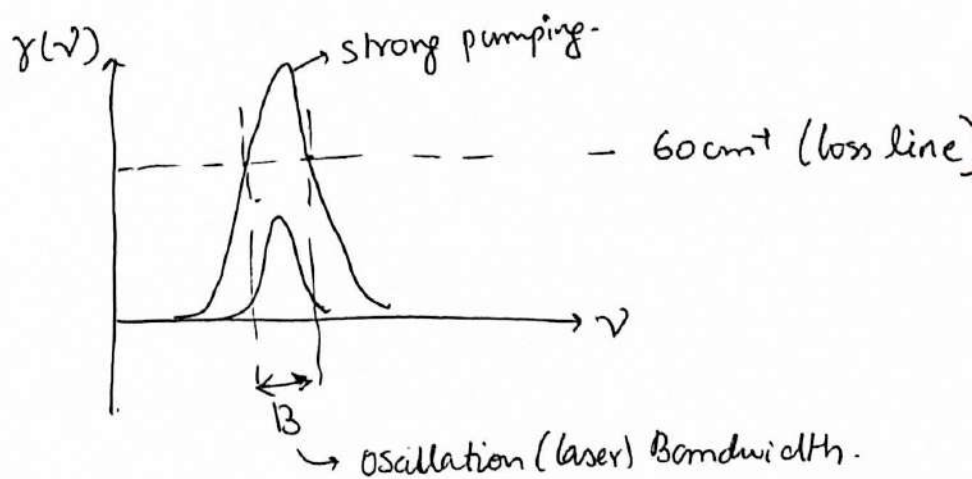


$$\alpha_{\text{loss coeff}} = \alpha_s + \alpha_m = \alpha_s + \frac{1}{2l} \ln\left(\frac{1}{R_1 R_2}\right)$$

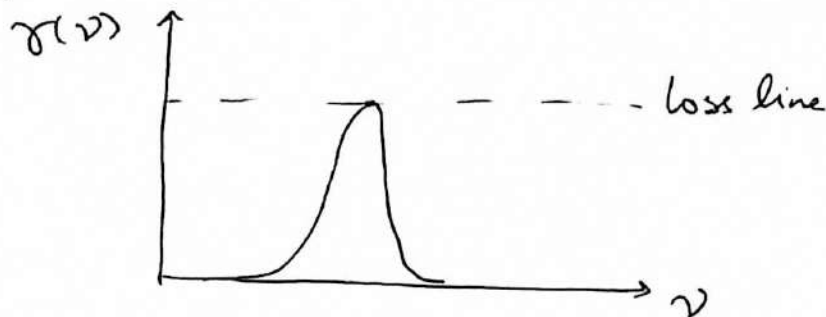
α_s (Scattering) α_m (mirrors)

$10-50 \text{ cm}^{-1}$ $\approx 300 \mu\text{m}$ 38 cm^{-1} 0.32

$$\Rightarrow \alpha \approx 60 \text{ cm}^{-1} \Rightarrow g(v) = \alpha_r \text{ for laser action.}$$



> To ensure $\text{gain} = \text{loss}$, the laser action changes the gain profile to be:



> Many dynamic mechanisms contribute to this steady state condition.

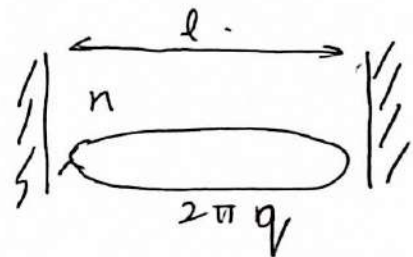
Optical Resonator

Longitudinal modes

↓
Resonance frequencies

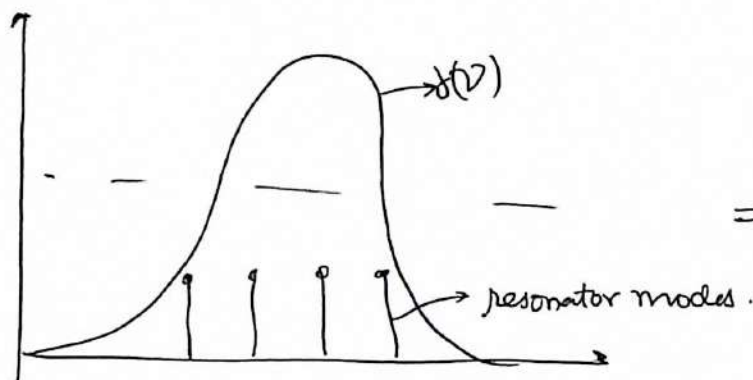
Transverse modes.

↓
Field distribution.

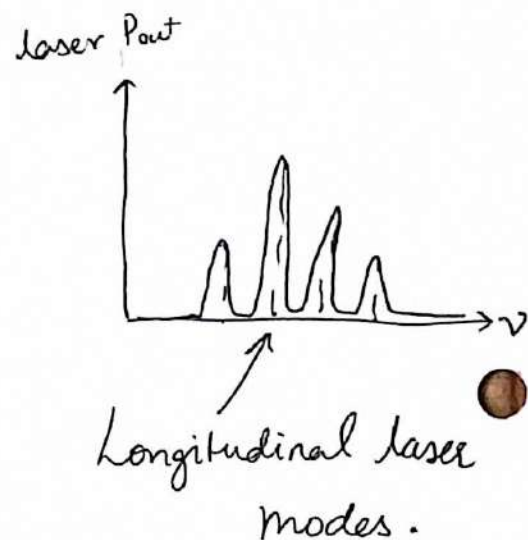


$$\Rightarrow k_{on} \cdot 2l = q \cdot 2\pi$$

$$\Rightarrow \boxed{\nu_q = \frac{q \cdot c}{2nl}}$$



$\Rightarrow$



Lec 34

Semiconductor Lasers - I Device Structures (shown in PPTs)

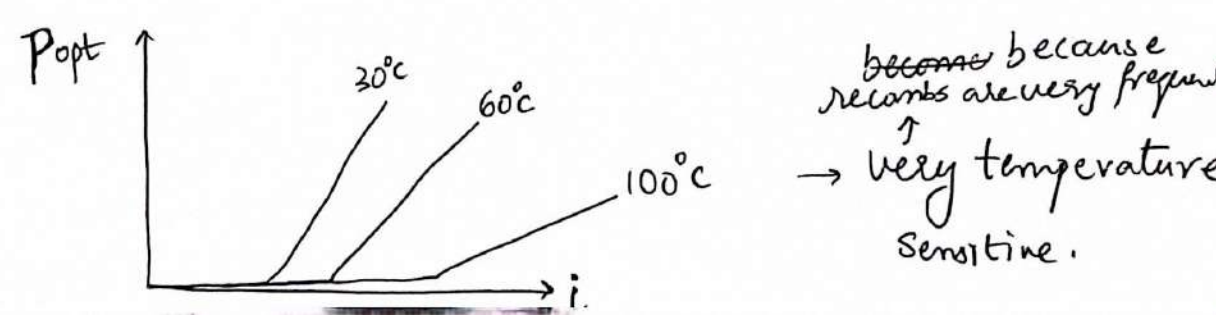
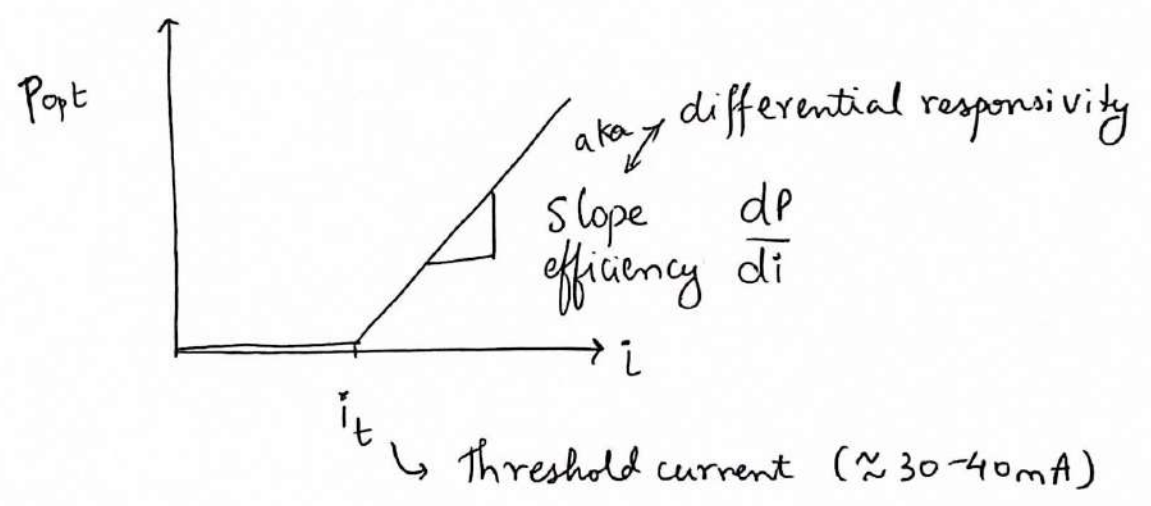
- > Semiconductor lasers can be "directly modulated" (i.e. modulate power supply), atleast upto few GHz.
- > Basic structure
 - > Forward bias p-n junction of a direct bandgap S-c.

$$\gamma(\nu) = \frac{(q/n)^2}{8\pi\nu^2} \frac{1}{\hbar^2\tau} (h\nu - E_g)^{1/2} [f_c(E_2) - f_v(E_1)]$$

$h\nu = E_2 - E_1$

(slides are used for remaining lecture).

[Gain guided vs. Index guided lasers]



Lec 35 Laser II - Output Characteristics

Recap: $\gamma_p = \alpha_a \left(\frac{\Delta n}{\Delta n_T} - 1 \right)$

Lasing conditions: $\text{gain} = \text{loss}$.

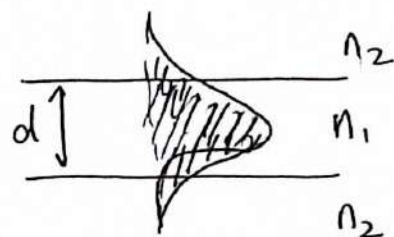
$$\Rightarrow \gamma_{pt} = \alpha_r.$$

$$\Rightarrow \alpha_r = \alpha_a \left(\frac{\Delta n}{\Delta n_T} - 1 \right) = \alpha_a \left(\frac{J_t}{J_T} - 1 \right)$$

$$\text{or } J_t = J_T \left(1 + \frac{\alpha_r}{\alpha_a} \right)$$

Confinement Factor Γ

$\Gamma \rightarrow$ Fractional energy in the active region. Only this experiences gain.



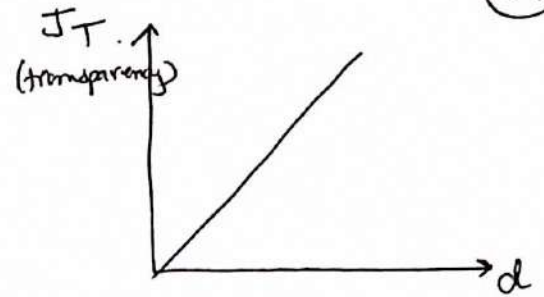
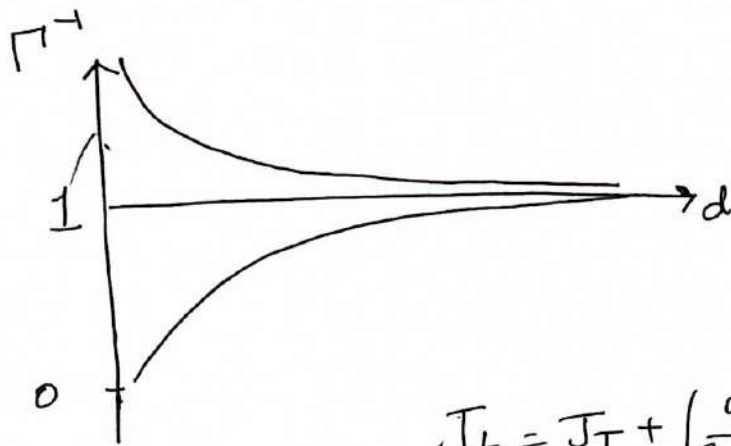
$$\Rightarrow \text{Cavity gain} = \Gamma \gamma_p$$

$\Rightarrow$ Really we have $\Gamma \gamma_p = \alpha_r$.

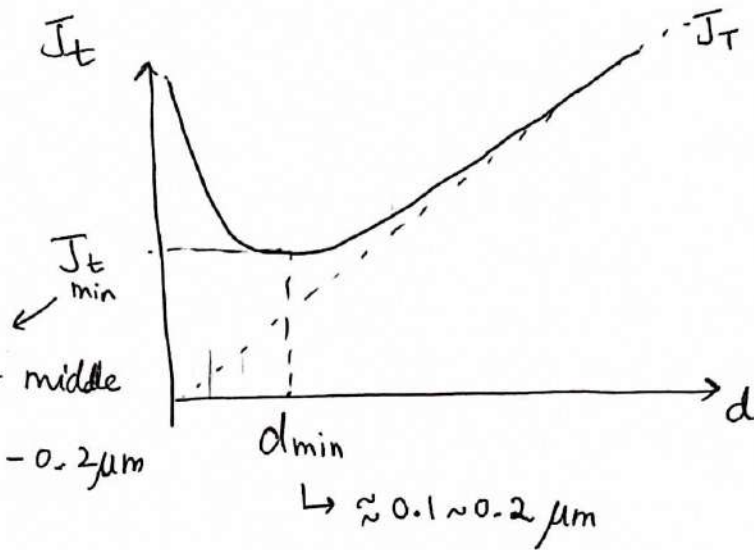
$$\Rightarrow J_t = J_T \left(1 + \frac{\alpha_r}{\Gamma \alpha_a} \right)$$

Threshold

$V = \frac{2\pi}{\lambda} d \sqrt{n_1^2 - n_2^2}$ if d is small, evanescent energy $\uparrow$
 $\nearrow$ V -number $\Rightarrow \Gamma \downarrow$ which is bad.



$$J_t = J_T + \left(\frac{d_r}{da} \right) J_T$$



This is the reason for middle layer to have $\approx 0.1 - 0.2 \mu\text{m}$ thickness.

Spectral linewidths

$\Delta\lambda \approx 20 - 40 \text{ nm}$ for LEDs

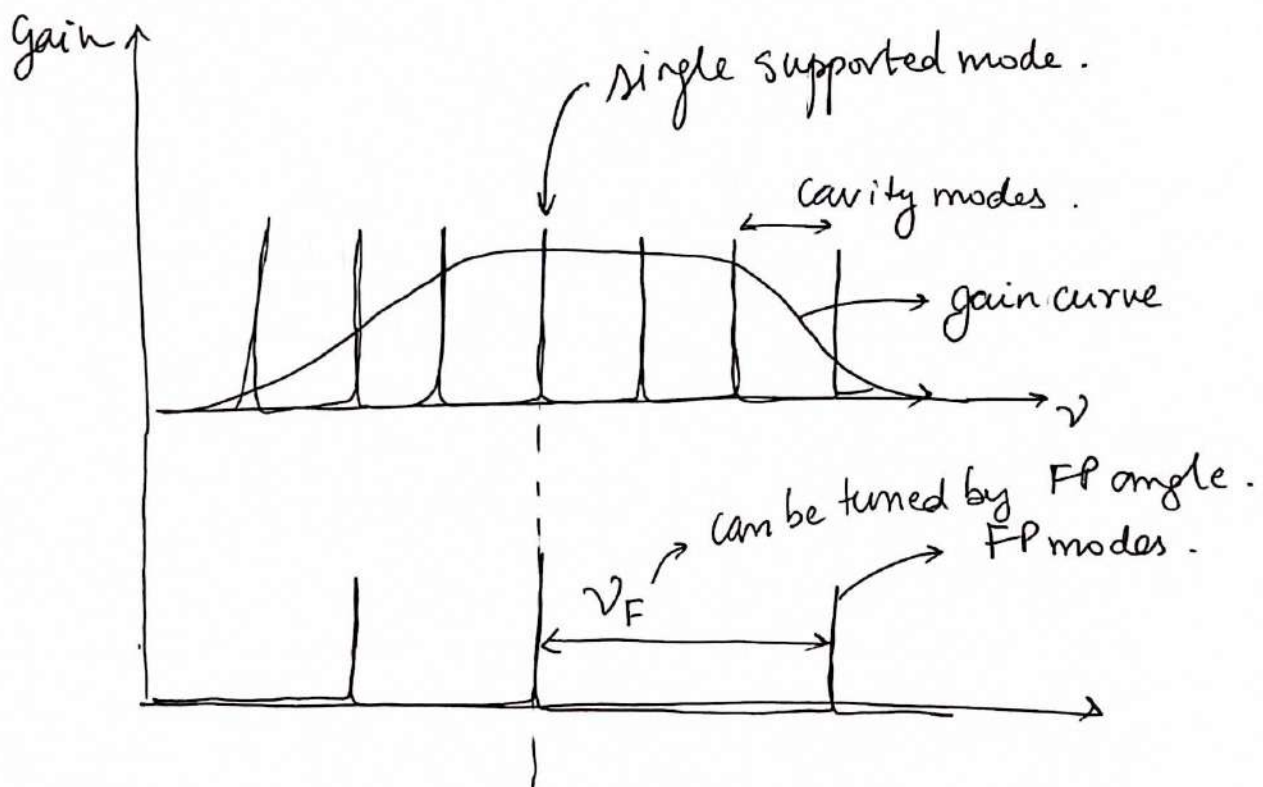
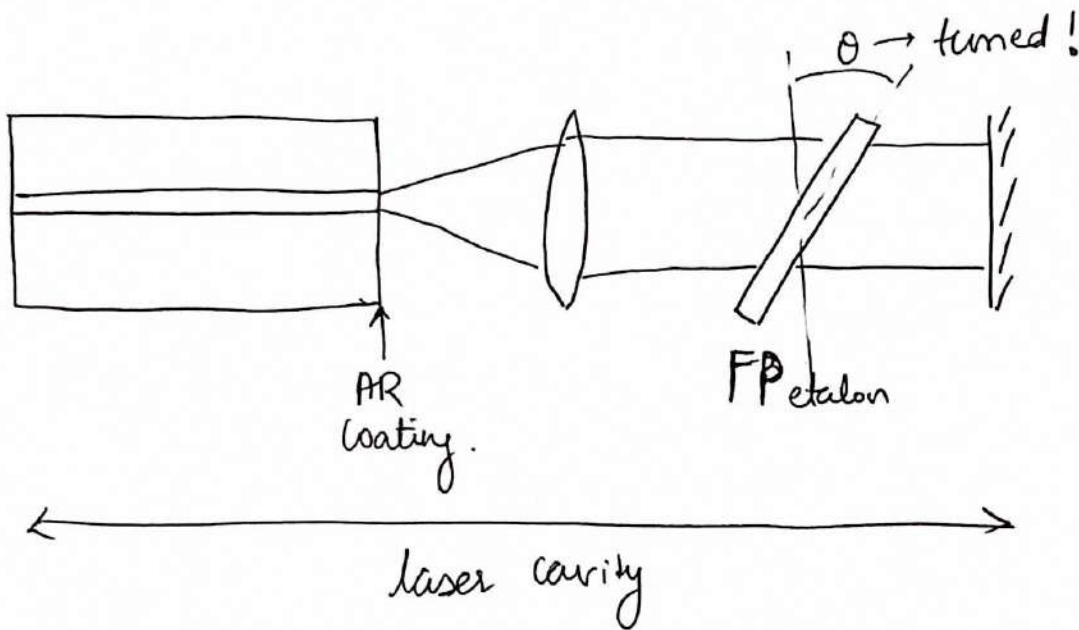
$2 - 3 \text{ nm}$ for F.P. laser diodes

$\leq 0.1 \text{ nm}$ for DFB lasers & VCSELs.
(distributed feedback)

lec 36 - Laser III - Single Frequency Lasers

Fabry Perot Etalon (ECL - external cavity laser).

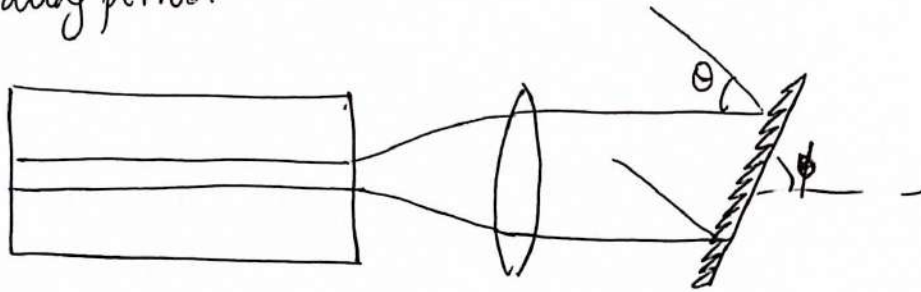
$$\nu_F = \frac{c}{2nl} \rightarrow \text{Free spectral range or freq. spacing b/w modes.}$$



> ECH can also be made using a grating.

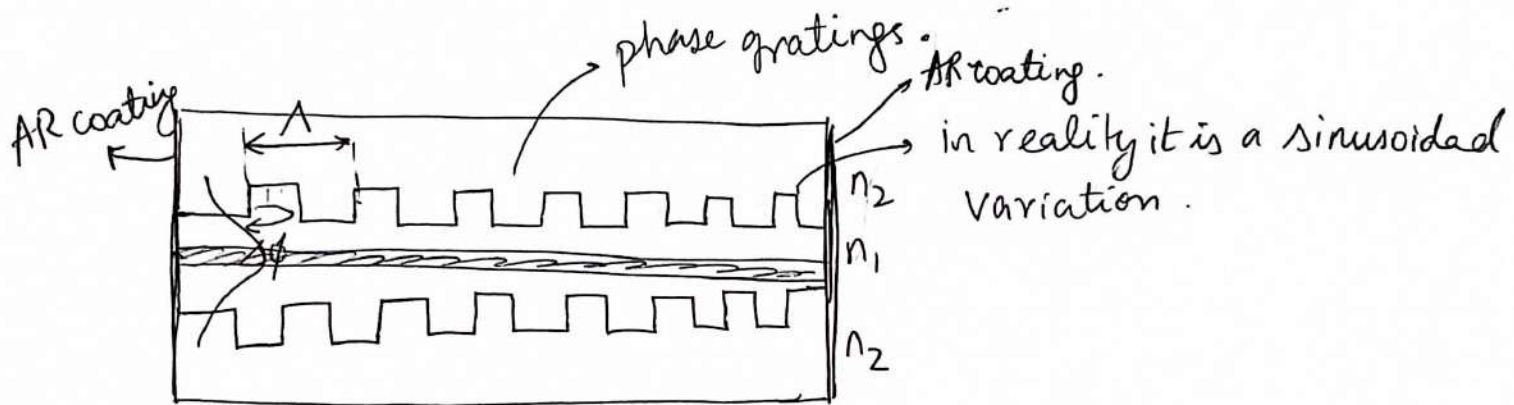
$$2d \sin \theta = n\lambda$$

grating period.



at some λ , get full reflection.

Distributed feedback laser (DFB) (Fixed frequency lasers)

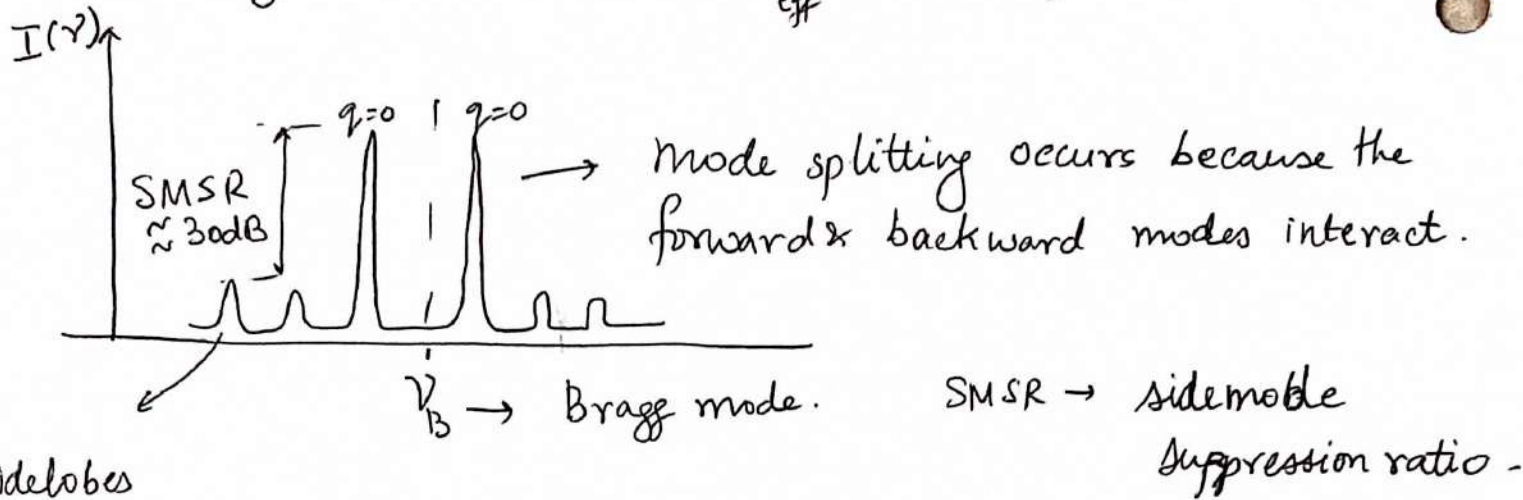


$$\Delta\phi = 2k_0 n \frac{\Lambda}{2} = \pi$$

not 2π since π happens already at on interface.

$$\Rightarrow \lambda_B = 2n \frac{\Lambda}{n_{\text{effective}}}$$

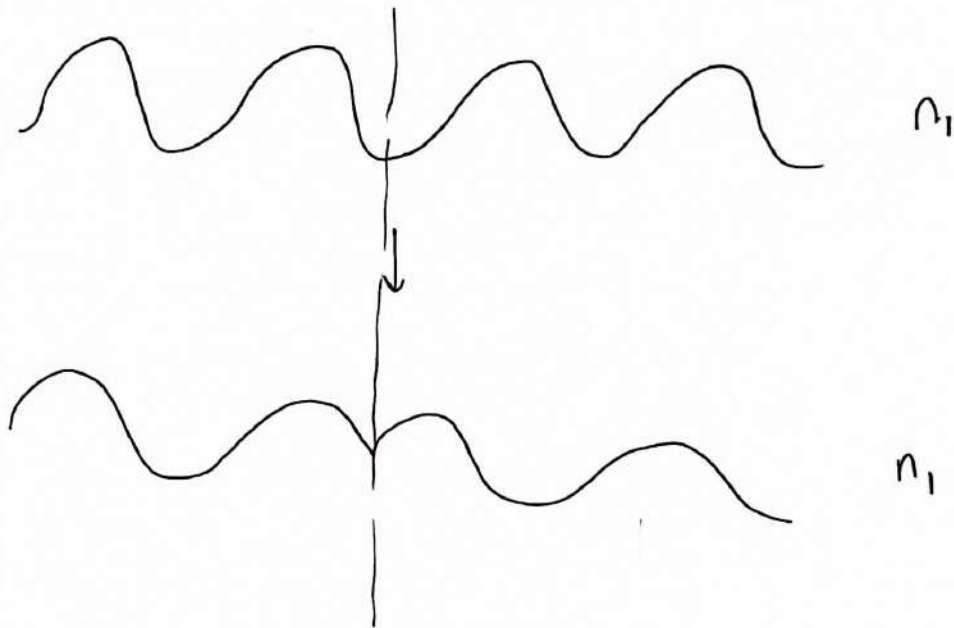
In reality
$$\nu_L = \nu_B \pm \left(\frac{c}{2n_{\text{eff}}L} \right) \left(q + \frac{1}{2} \right)$$

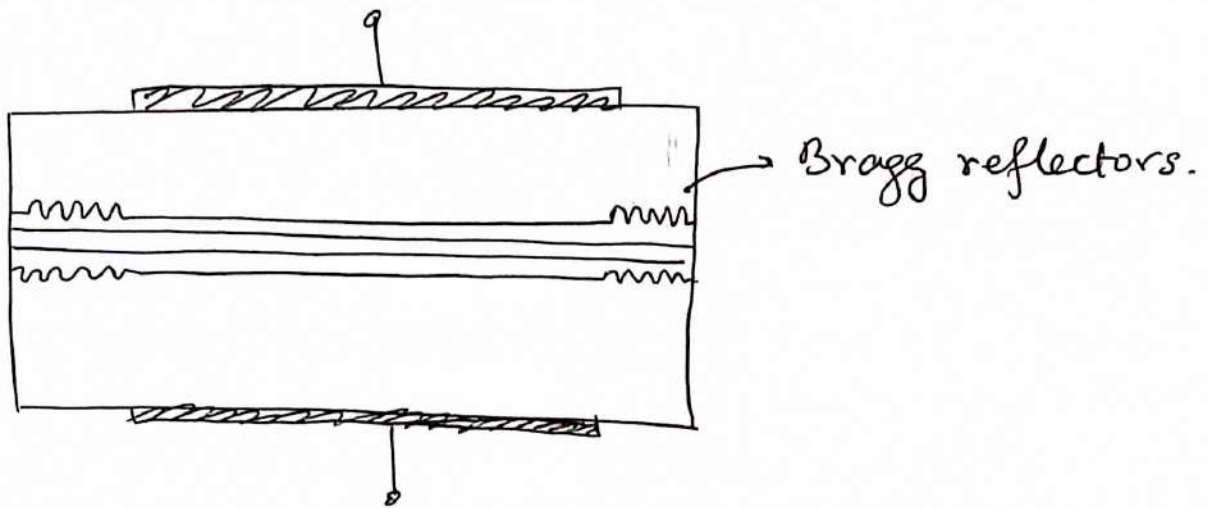


Sidelobes

because it is not a perfect sinusoidal grating.

> The degeneracy can be broken by introducing a $1/4$ shift in the cavity. This shift is introduced at the center of the grating & causes a strong reflection at ν_B .



DBRDistributed Bragg Reflector laser

> DBR lasers are much more stable than DFB. Why?

$$n_{\text{eff}} = n_r - i n_i \rightarrow n_i < 0 \Rightarrow \text{gain. } \{n_r \& n_i \text{ are related by Kramers Kronig}\}$$

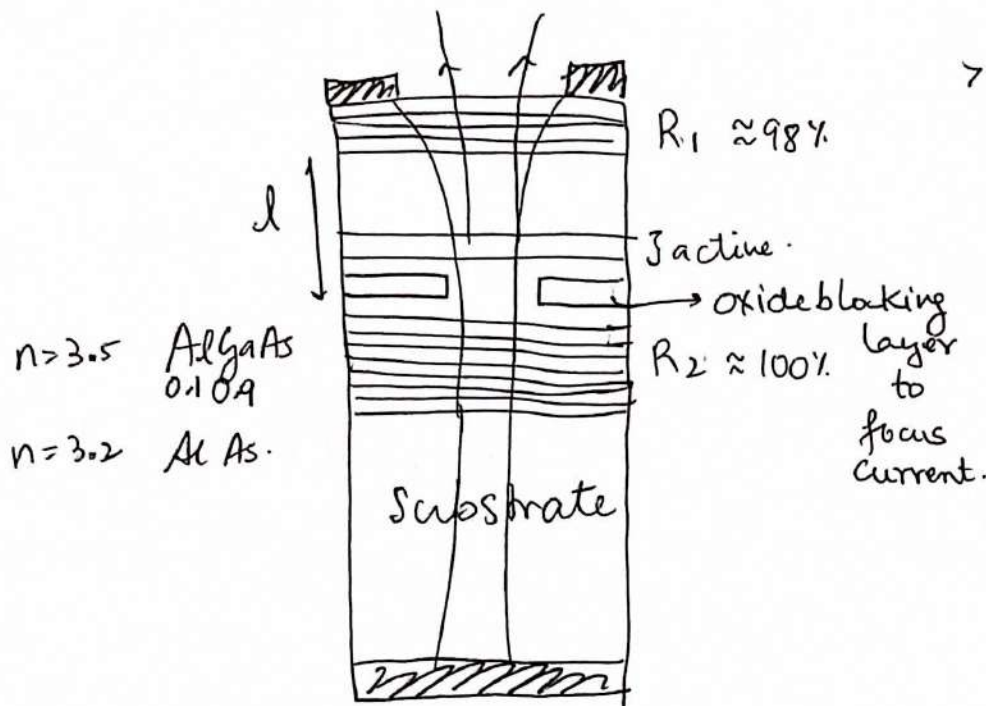
↳ The imaginary part is a function of electrical current since it depends on medium gain.

> So n_{eff} is a function of applied current $\Rightarrow \lambda_B = 2 n_{\text{eff}} \Lambda$

how depends on current.

> In DBR, current does not flow through reflecting part.

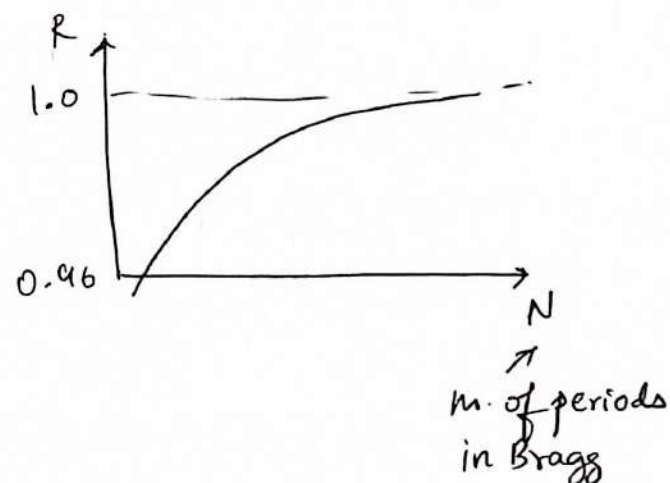
lec 37 VCSEL: Vertical Cavity Surface Emitting Lasers. (Vertical DBR laser).



- > Useful for array of lasers.
- > High density data interconnects in computer chips.
- > Can be directly modulated up to mm GHz

When $R \approx 0.9 \rightarrow l = 285 \mu\text{m}$
 $R \approx 0.99 \rightarrow l \approx 10 \mu\text{m}$
 $R \approx 0.999 \rightarrow l \approx 2.5 \mu\text{m}$
 $R \approx 0.9998 \rightarrow l \approx 0.2 \mu\text{m}$

$$l \approx \frac{1}{(\alpha - \alpha_s)} \ln\left(\frac{1}{R}\right)$$

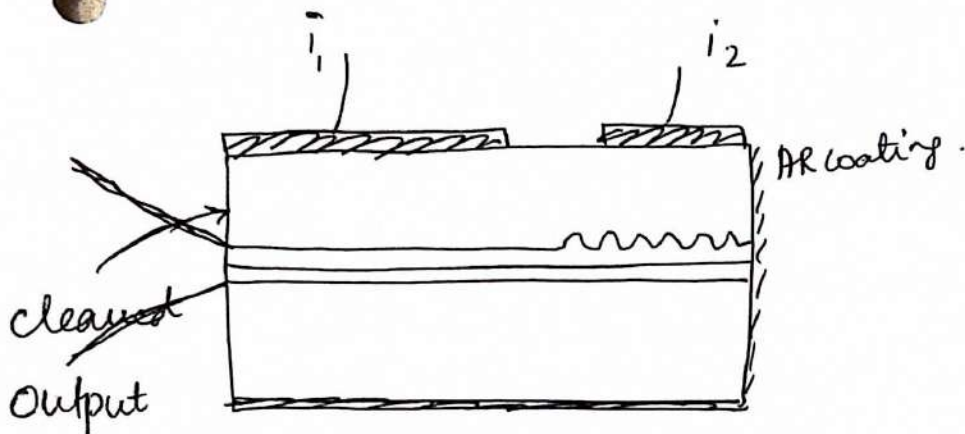


> $l \approx 10 \mu\text{m} \Rightarrow \mathcal{V}_f$ is large & therefore only one mode exists!

> Output rad patter is also symmetric.

> Controlling transverse modes is very difficult $\rightarrow$ Current research.

Tunable laser based on DBR structure.



i_1 controls laser power.

i_2 controls laser frequency.

$$\nu_q = \frac{c}{\lambda_B} = \frac{c}{2n_{eff}\Lambda}$$

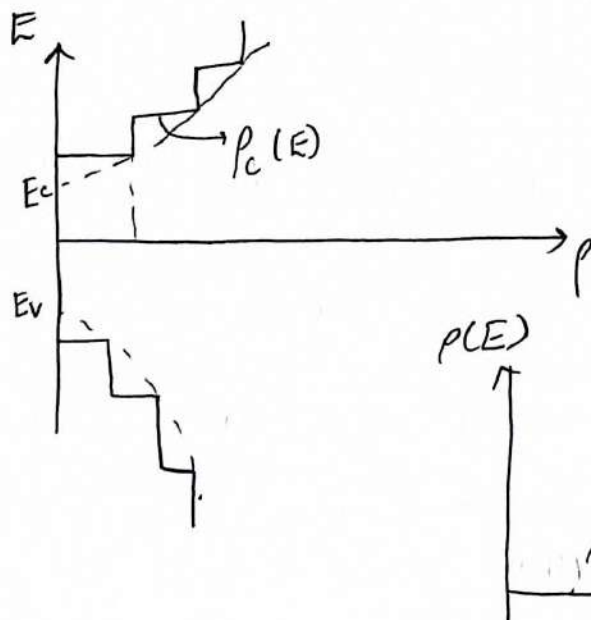
Tunable range $\approx 5-10\text{nm}$

> ECL tuning range $\approx 50\text{nm}$.

> DFB lasers can be tuned by temperature ($\approx 1-3\text{nm}$).
 $\Delta\lambda \rightarrow (0.1\text{nm}/^\circ\text{C})$

Lec 38 : Quantum Well Lasers.

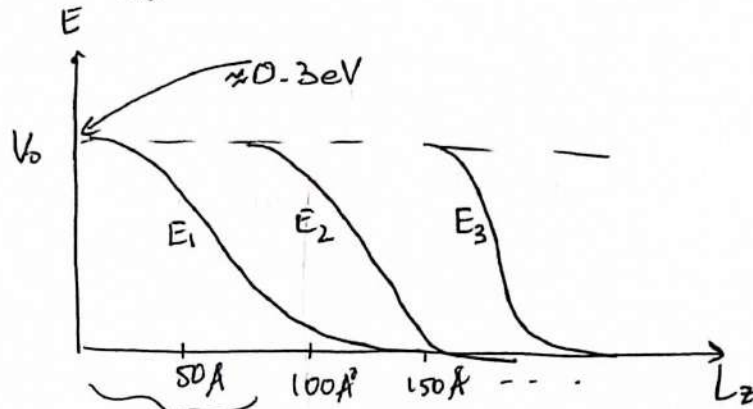
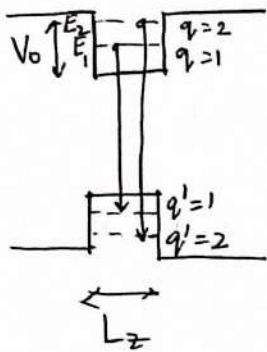
Recall from Lec 6, Density of states in Quantum Wells.



$$p_c(E) = \frac{m_c}{\pi \hbar^2 L_z}$$

$$p_v(E) = \frac{m_v}{\pi \hbar^2 L_z}$$

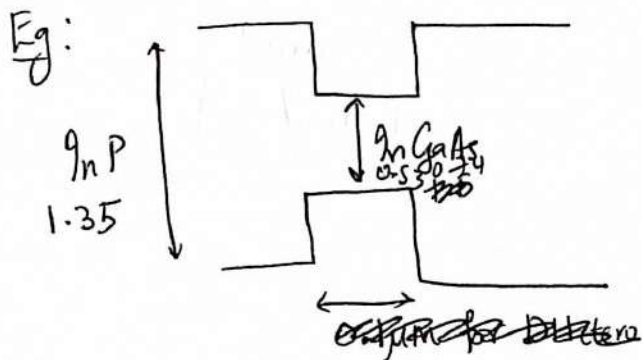
In Lec 12 & 13, → Energy states in Quantum Well Structures.



→ Therefore control of λ_{opt} can be more easily achieved by changing L_z .

→ only one mode allowed

→ Effective bandgap changes with L_z !



$$\begin{aligned} & \text{InGaAs} / \text{InP} \rightarrow \Delta E_g = 0.61 \text{ eV} \\ & 0.74 \text{ eV} \quad 1.35 \text{ eV} \\ & \Delta E_c \approx 0.4 \text{ eV} \quad \Delta E_v \approx 0.21 \text{ eV} \end{aligned}$$

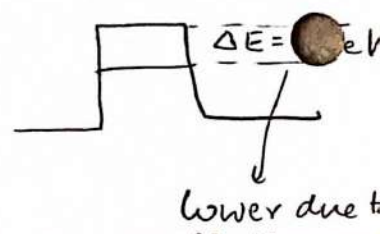
QWL: Quantum Well Laser is a Sc Double Hetero laser, where the thickness of active layer $\leq$ de Broglie wavelength of electrons.

$$\lambda_g (\text{InGaAs}) = \frac{1.24}{E_g} = \frac{1.24}{0.74} \approx 1.67 \mu\text{m. at } L_z = 1000 \text{ nm}$$

However when QWL dimensions are used,

$$\Rightarrow \lambda_g = \frac{1.24}{0.8} = \underline{\underline{1.55 \mu\text{m}}} \quad \text{when } L_z \approx 100 \text{ nm.}$$

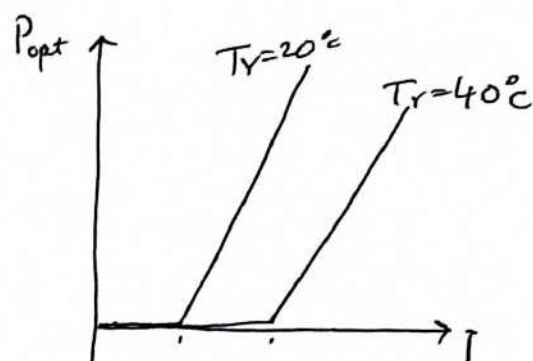
$$\text{at } 80 \text{ nm, } \lambda_g \approx \underline{\underline{1.46 \mu\text{m}}}$$



Lower due to

Advantages of QWL

- lower thresholds
 - less temperature sensitivity.
- $$I_t(T) = I_0(T_r) e^{+\frac{(T-T_r)}{T_0}}$$
- ↳ reference.



$T_0 \sim 140 K$ for GaAs FP lasers.

$T_0 \sim 400 K$ for QW lasers.

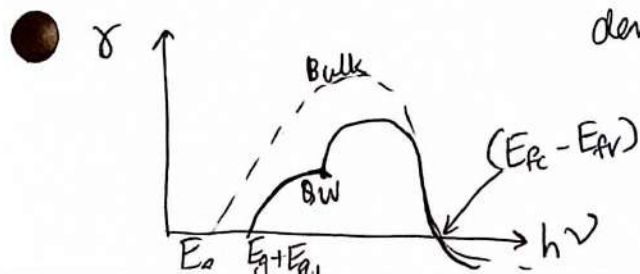
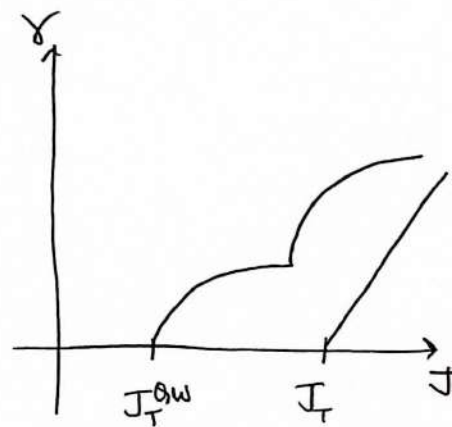
Why is this so?

- > Since only one (or few) modes exist; phonon transitions are less likely to occur. Therefore carrier distribution is much more stable.
- > Furthermore, the number of allowed nonradiative transitions are fewer (across band).

Why Low Threshold?

Recall, $\Delta n_T = \frac{J_T \tau}{ed}$ or $J_T = \frac{\Delta n_T ed}{\tau}$.

$$\gamma = \alpha_a \left(\frac{J}{J_T} - 1 \right) = \frac{\lambda^2}{8\pi\tau} \underbrace{(\nu) f_g(\nu)}_{\text{density of states}}$$

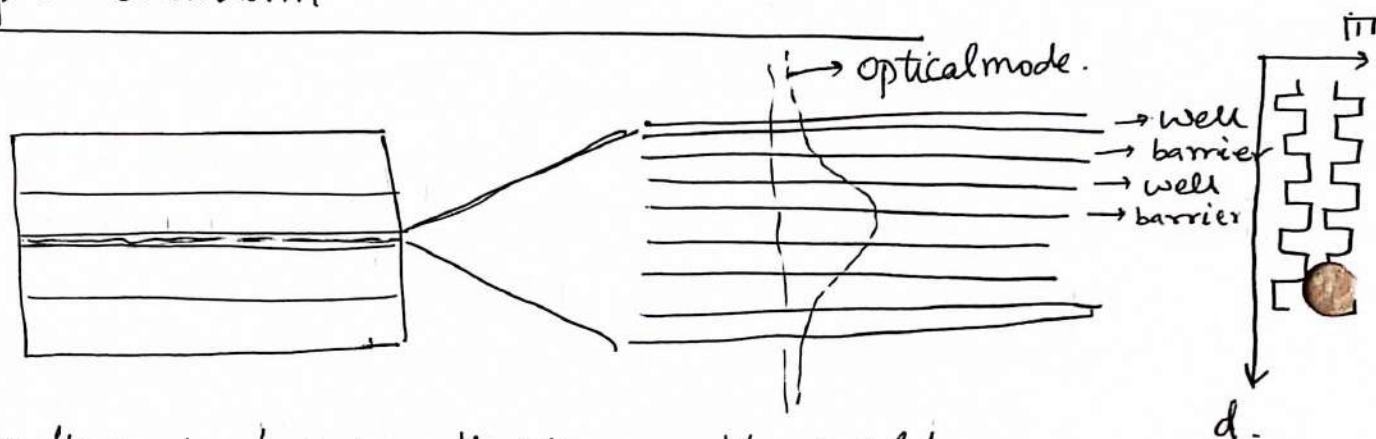


Confinement Factor (Γ) $\rightarrow$ discussed earlier. (amt. of energy in core).

$$i_t = i_T \left(1 + \frac{\alpha_r}{\Gamma \alpha_a} \right) \quad > \text{In QWL } \Gamma \text{ is lower. } (\approx 0.4)$$

Despite $\Gamma \downarrow$, gain is larger at lower currents & therefore threshold is low.

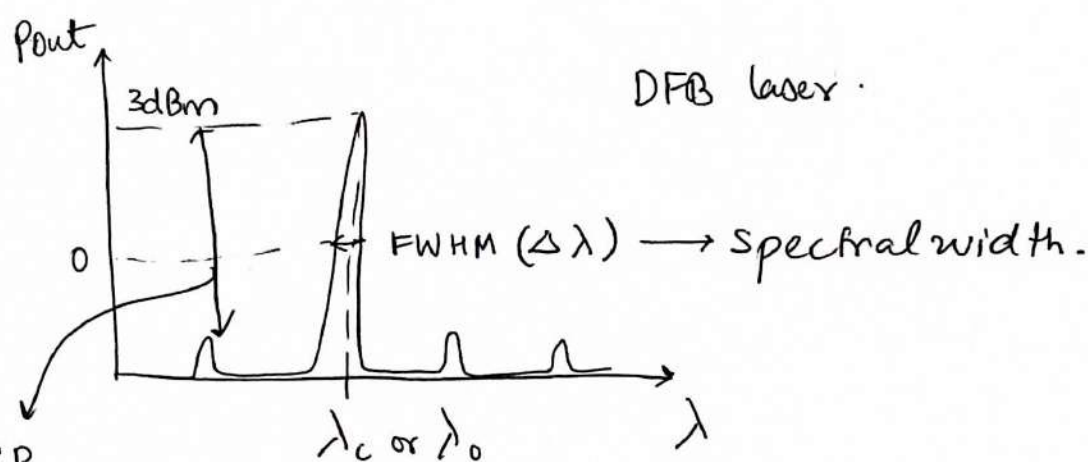
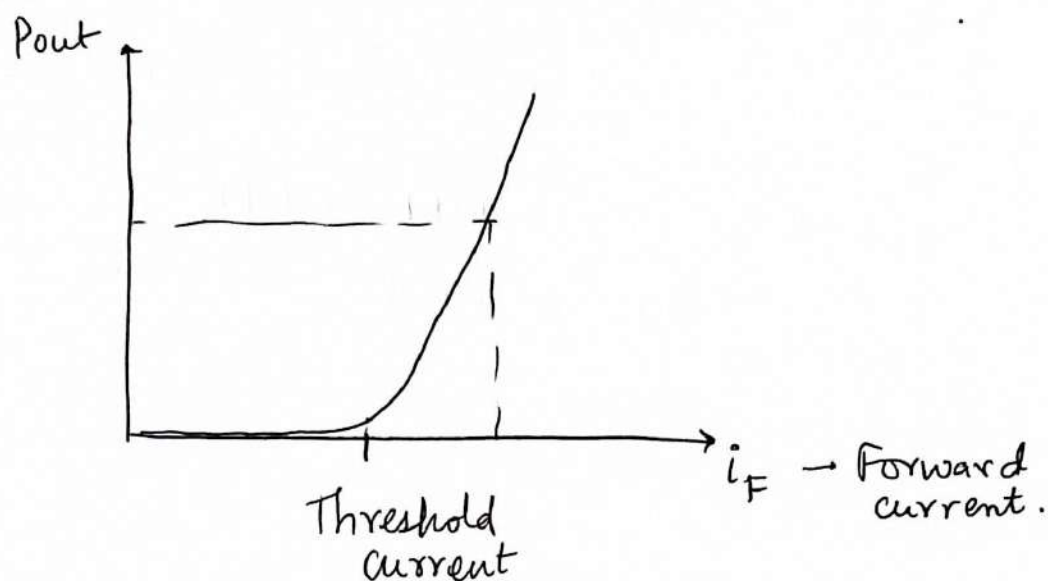
Multiple Quantum Well (MQW) lasers.



- > With sufficient barrier thickness, the neighbouring electron wells do not interact.
- > A single optical mode propagates in all the wells & therefore γ is high (gain $\uparrow\uparrow$) $\Rightarrow$ higher optical output power.

Lec 39 : Practical Laser Diodes and Handling.

● Characteristics (Data sheet)



SMSR
(sidemode
suppression
ratio)

> Monitor current $\rightarrow$ current in monitor photodiode.

> Thermistor TCR $\rightarrow$ Temperature coefficient of resistance.

● TEC Power $\rightarrow$ Thermo electric cooler power.

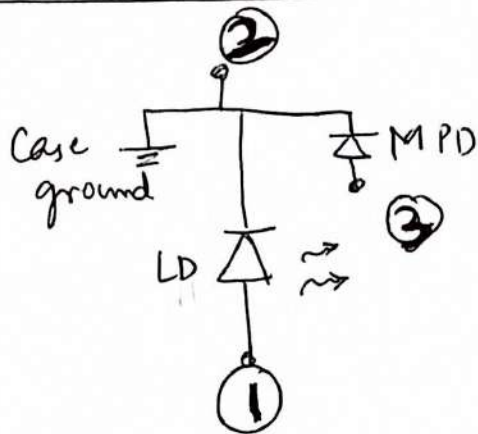


used to
control
TEC to
maintain
constant T.

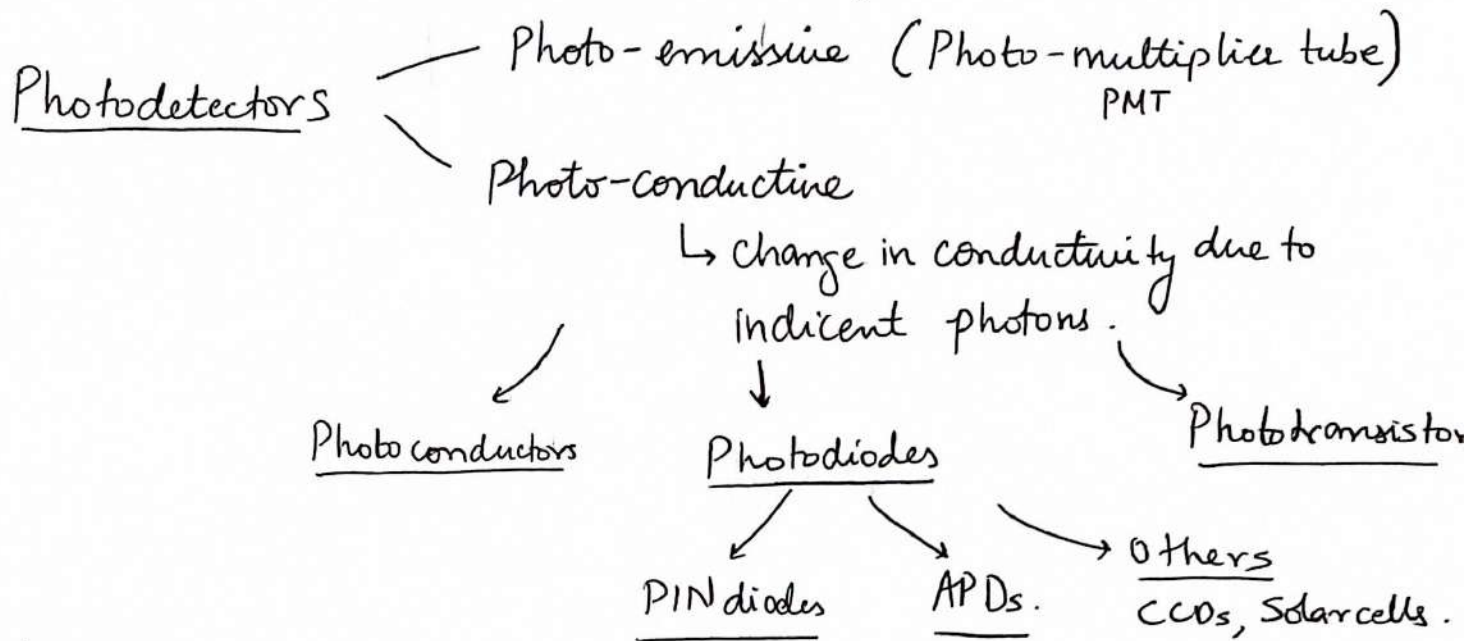
Device Packages

- Free space output (TO-3 Window can & SOT-148 window can)
- Fiber pigtail (14 Pin DIL or DIP package & 14 Pin Butterfly package).

TO 3 & SOT 148



Lec 40 General Characteristics of Photodetectors.

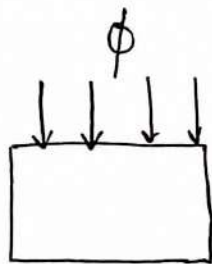


Thermopiles

Temp change with incident photon.

General Characteristics of Photodetectors.

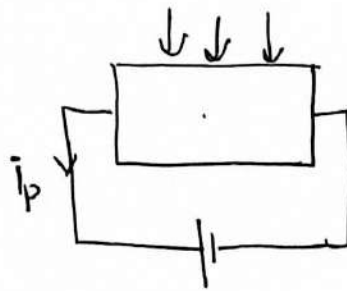
- 1) $\eta \rightarrow$ quantum efficiency
- 2) $R \rightarrow$ responsivity
- 3) $t_r \rightarrow$ rise time / impulse response / bandwidth.
- 4) $P_{\text{Noise}} \rightarrow$ Noise power / Dark current.



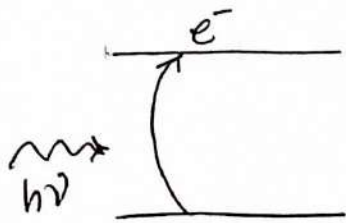
$$\phi = \text{photon flux} = \frac{P_{\text{opt}}}{h\nu}$$

$$\eta = \frac{\text{Carrier flux generated which contribute to the photocurrent}}{\text{Incident photon flux}}$$

$$\eta = \frac{(i_p/e)}{(P/h\nu)}$$



$$0 < \eta < 1$$



"Generation of 1 e-h pair in the semiconductor is equivalent to 1 charge moving in the external circuit." $\rightarrow$ Raman's theorem?

Responsivity

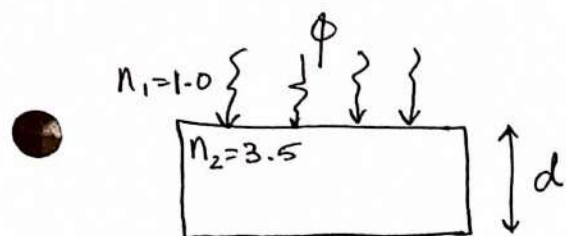
For LEDs, $R = \frac{P_{opt}}{i} \left(\frac{W}{A} \right)$

For Photodetectors, $R = \frac{i_p}{P_{opt}} \left(\frac{A}{W} \right)$

$$\Rightarrow R = \frac{i_p}{P_{opt}} = \eta \cdot \frac{e}{h\nu} = \eta \cdot \frac{\lambda (\mu m)}{1.24}$$

$$\Rightarrow \boxed{R = \eta \cdot \frac{\lambda (\mu m)}{1.24}}$$

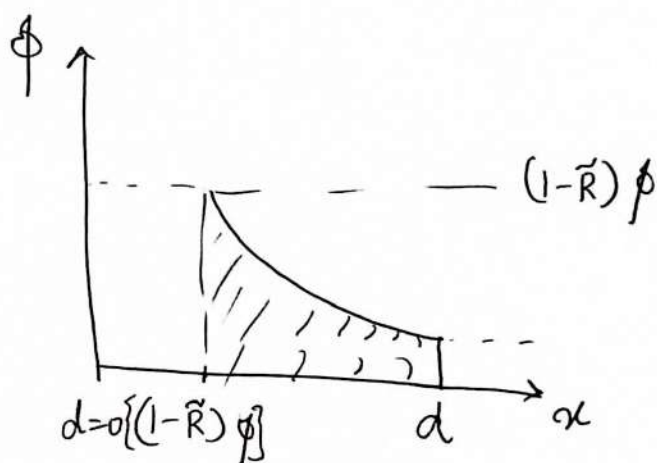
> $\eta = ?$ & how to maximize it?



$\tilde{R} \rightarrow$ reflectivity due to air-dielectric interface.

photon flux entering detector : $\phi(1-\tilde{R})$

Inside the medium photon flux decays exponentially due to absorption.



fractional
 $\wedge$ number of photons absorbed = $\phi(1-R)(1-e^{-\alpha d})$

> If $\tilde{\gamma}$ is the fractional number of photons that lead to electrical current. It has 2 components: i) photons lost to phonons or trapped states ii) generated e-h pairs may recombine immediately.

> If N photons are incident, $\tilde{\gamma}N$ photons contribute to current in external circuit.

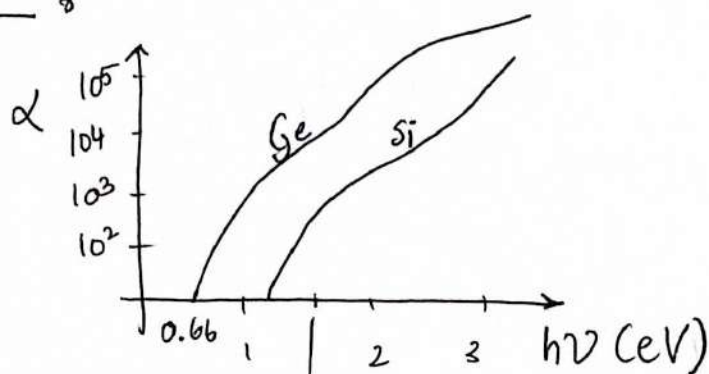
$$\Rightarrow \phi \rightarrow \underbrace{\phi(1-R)(1-e^{-\alpha d})}_{\eta} \zeta$$

$$\Rightarrow \boxed{\eta = (1-R)(1-e^{-\alpha d}) \zeta}$$

How to maximize η now?

$R=0$? AR coating.

$\alpha d \rightarrow \infty$?



Si $\xrightarrow{E_g}$ 1.1 eV

Ge $\rightarrow$ 0.66 eV

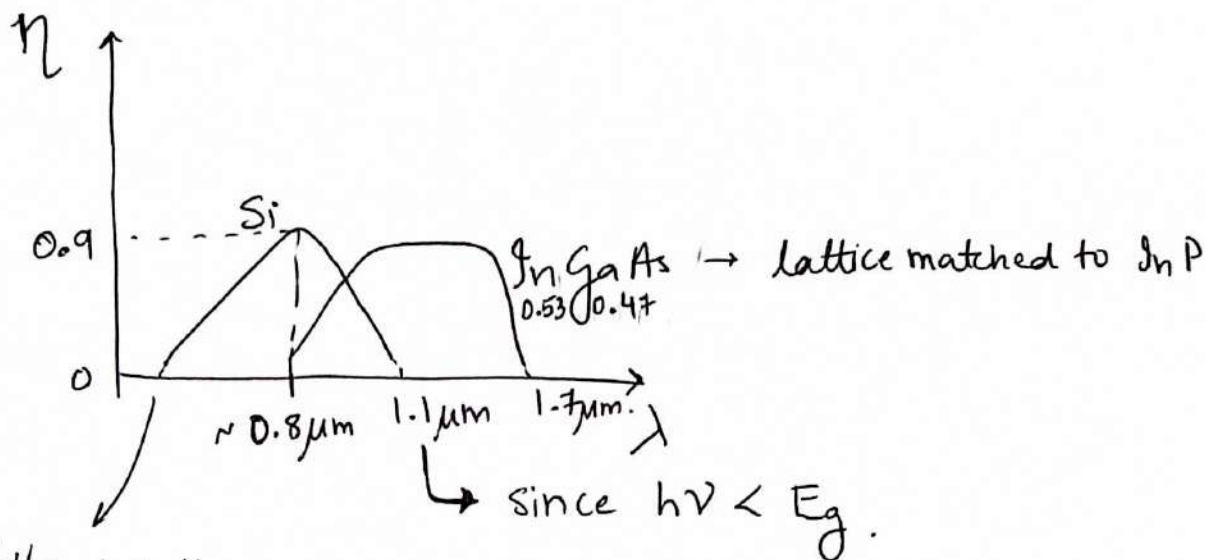
$\rightarrow$ 1.5 eV AlGaAs/GaAs laser.

Choose d based on α to maximize η .

$\zeta \rightarrow 1$?

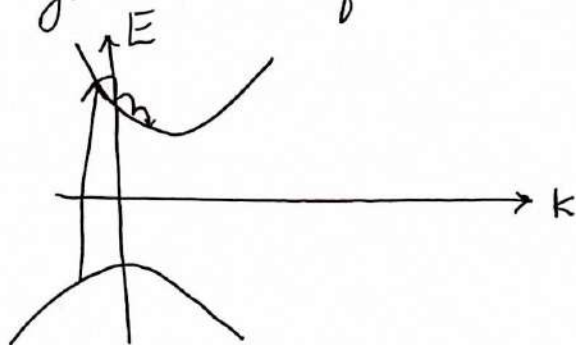
- 1) > Bias needs to be applied to pull e-h pairs into circuit.
 - > PN junction should be reverse biased.
- 2) > Make sure no defects exist $\rightarrow$ very careful fabrication.

Lec 41 Responsivity and Impulse Response



Why does it drop here?

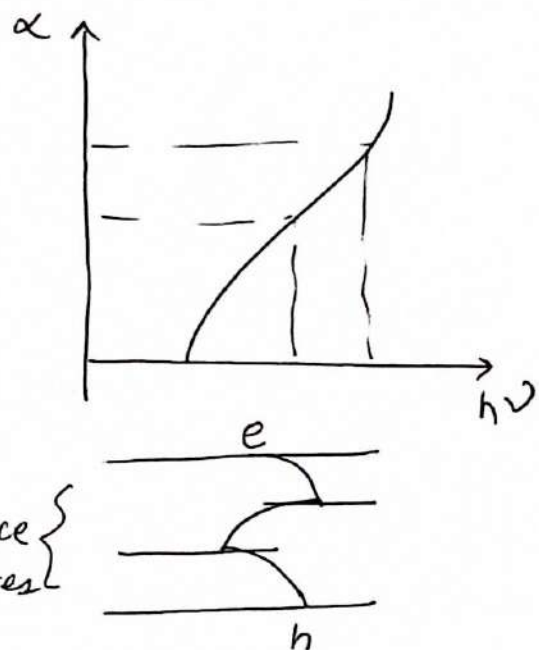
1) Generation of hot carriers.

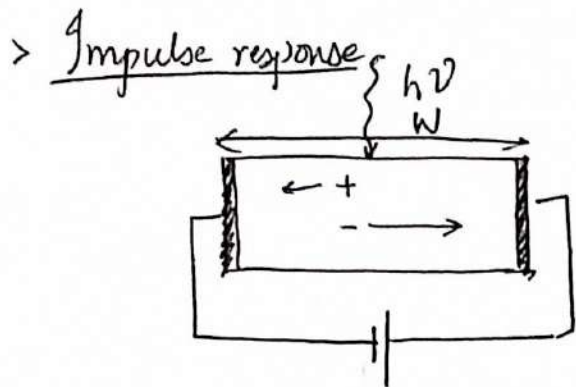
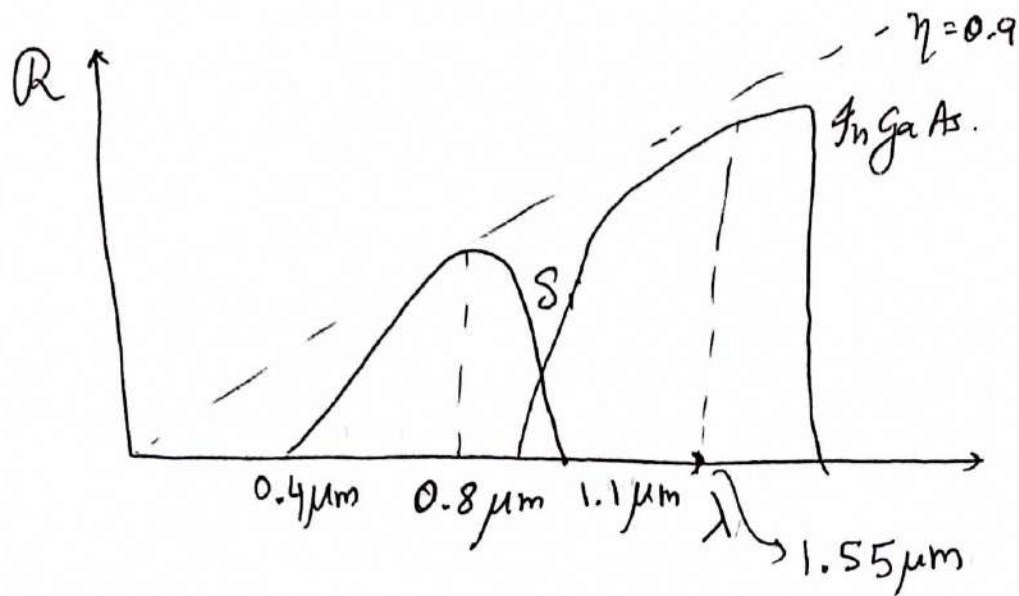


These hot carriers quickly thermalize & generate a large number of phonons which facilitate e-h recombinations, reducing η .

2) Absorption near the surface.

Since α is large, most absorption occurs near surface. However, surfaces have dangling bonds & surface states which facilitate e-h recombinations.





$i_h(t)$ & $i_e(t)$
(hole) (electron)

$$i_e(t) = - \frac{(-e) v_e}{W} = \frac{e v_e}{W}$$

$$i_h(t) = - \frac{e(-v_h)}{W} = \frac{e v_h}{W}$$

$$\Rightarrow i(t) = \frac{e(v_e + v_h)}{W}$$

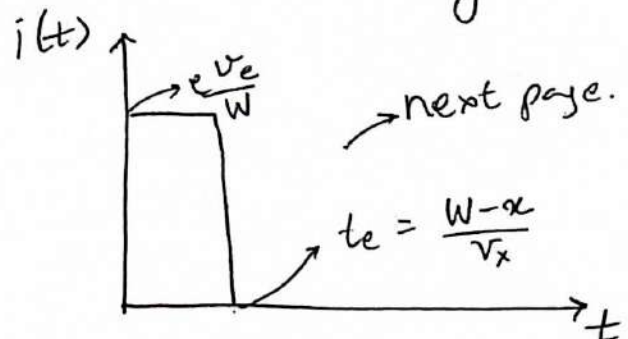
Ramo's Theorem

$$i(t) = - \frac{q}{W} v(t)$$

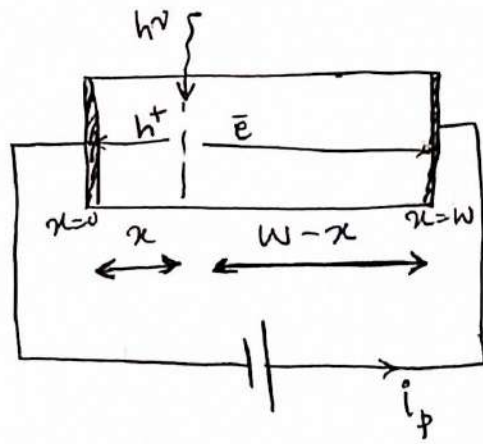
↳ instantaneous velocity of carrier.

$$v = \mu E \rightarrow \text{E-field}$$

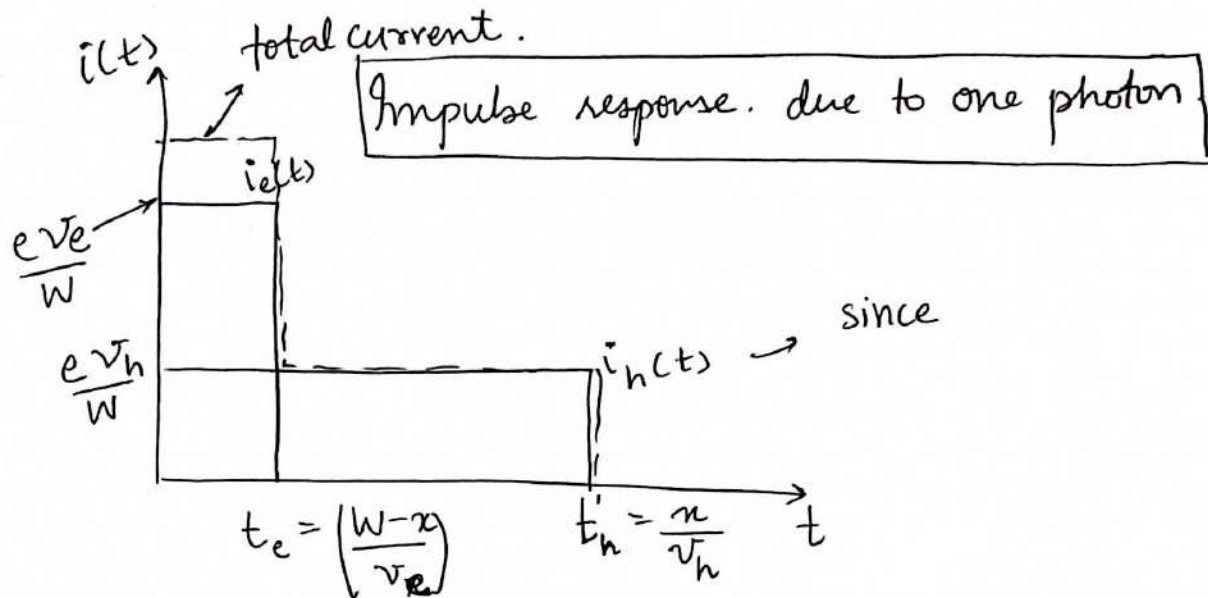
↑
mobility



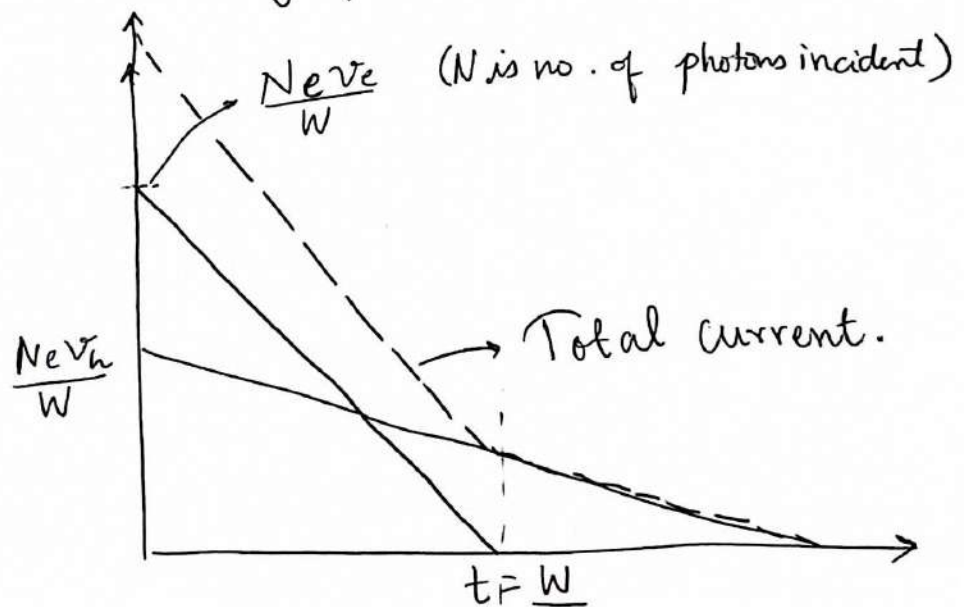
Material	μ_e	μ_h	(cm ² /Vs)
Si	1500	450	
Ge	3900	1900	
GaAs	8500	400	
InP	4600	150	



> When e-h pair is generated, current in external circuit persists until the electron & hole are absorbed into the electrodes.

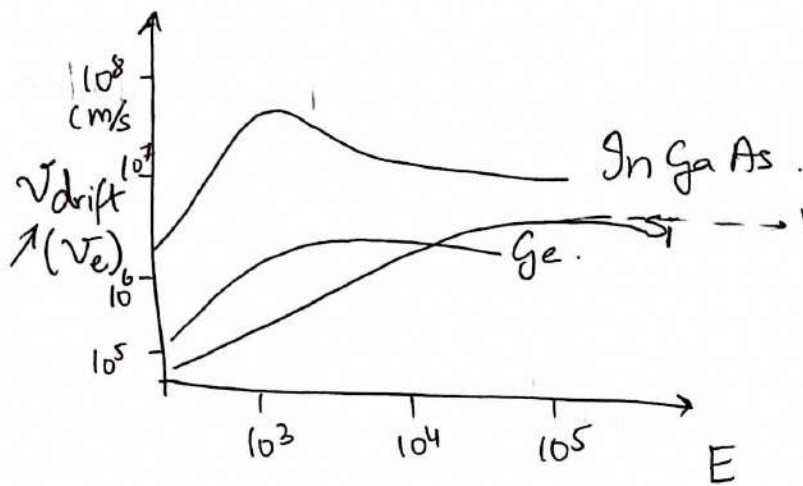
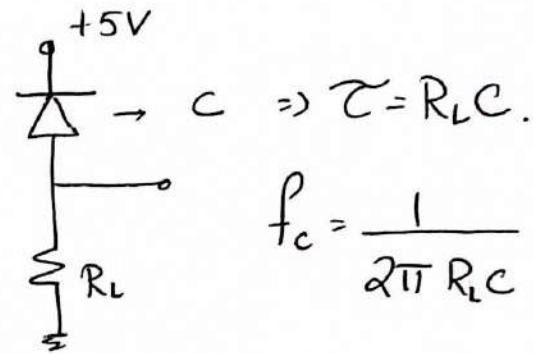


> What happens when many photons are incident all over the sc?



Impulse response is determined by

- 1) Transit time of carriers
- 2) RC time constant of circuit.



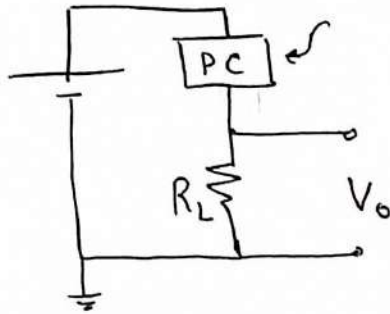
$V_{sat} \rightarrow$ Apply E-field at V_{sat} !

Lec 42 Photo conductors.

aka photoresistors
or light dependent resistors (LDRs).

> Conductivity of a semiconductor changes with incident photon flux.

> $J = J_{\text{dark}} + J_{\text{photo}}$



$\Rightarrow J = \sigma E$; σ - conductivity

$= \rho \mu E$ ρ - chargedensity

$J = ne\mu E$ μ - mobility

n - carrier conc.

If n_0 & p_0 are the concentrations when $\phi = 0$, then

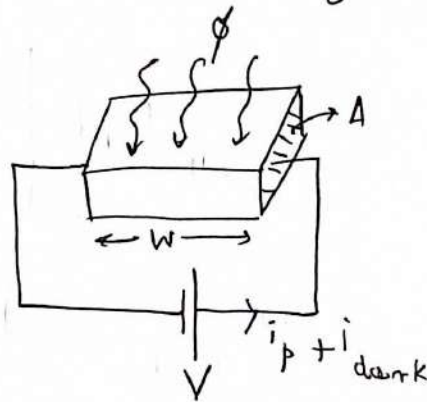
$\sigma = (n_0 \mu_e + p_0 \mu_h) e$. Or $J_{\text{dark}} = (n_0 \mu_e + p_0 \mu_h) e E$.

If $\Delta n \rightarrow$ excess carrier concentration of electrons due to ϕ .

$\Delta p = \Delta n$

$\Rightarrow J_{\text{photo}} = (\Delta n \mu_e + \Delta p \mu_h) e E$

$= (\mu_e + \mu_h) \Delta n e E$



photon flux

If $R \rightarrow$ rate of generation of excess carriers/unit volume.

$$R = \frac{\eta \phi}{WA} \quad \text{but recall } R = \frac{\Delta n}{\tau} \rightarrow \text{excess carrier recomb. time.}$$

↑
rate of recomb.

$$\Rightarrow \Delta n = \frac{\eta \phi \tau}{WA}$$

$$J_{\text{photo}} = \frac{\eta \phi \tau e}{WA} (\mu_e + \mu_h) E.$$

$$\Rightarrow i_p = J_{\text{photo}} A = \frac{\eta \phi \tau}{W} e (v_e + v_h)$$

$$\Rightarrow i_p = \eta \phi \tau e \left(\frac{1}{t_e} + \frac{1}{t_h} \right) \quad \text{transit time across } W.$$

$$\phi = \frac{P_{\text{opt}}}{h\nu} = \frac{P_{\text{opt}}}{hc} \lambda.$$

$$\phi_e = \frac{P_{\text{opt}}}{\left(\frac{hc}{e}\right)} \lambda = \frac{P_{\text{opt}}}{1.24} \lambda \text{ (}\mu\text{m)}.$$

$$\Rightarrow i_p = \eta \left(\frac{\tau}{t} \right) \frac{\lambda}{1.24} P_{\text{opt}}$$

$$\Rightarrow R = \frac{i_p}{P_{\text{opt}}} = \eta \frac{\lambda}{1.24} \left(\frac{\tau}{t} \right) \quad \text{Amp/watt}$$

carrier recombination time. $(10^{-12} - 10^{-3} \text{ s})$

Responsivity of a photoconductor.

usually $\left(\frac{\tau}{t}\right)$ from 10^{-4} to 10^5

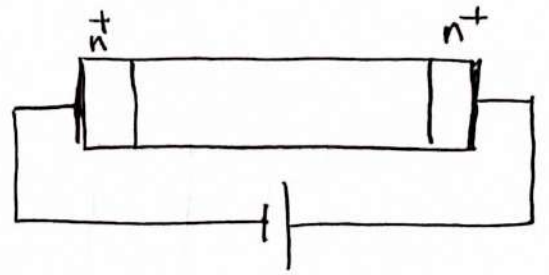
$$\frac{1}{t} = \frac{1}{t_e} + \frac{1}{t_h}$$

$$\frac{1}{t} \approx \frac{1}{t_e}$$

transit time. (if $W = 1 \text{ mm}$, $v_e \approx 10^6 \text{ cm/s}$)
 $\Rightarrow \approx 10^{-8} \text{ s}$

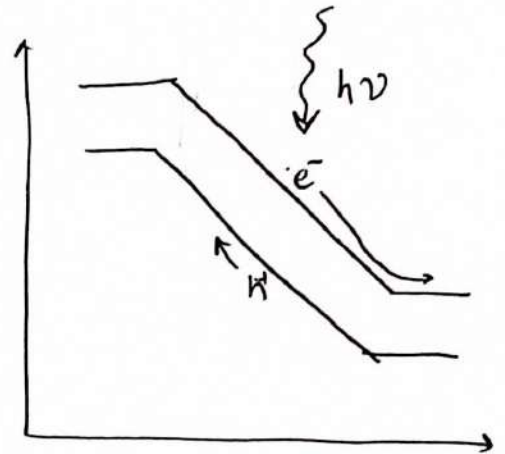
Where is this gain coming from?

- > When a single photon is incident, an electron hole pair is generated.



The electron moves quickly & is absorbed at the contact. The hole takes longer.

~~As the hole is in the way~~ But to maintain charge neutrality electrons are sequentially pumped into the S.C until either the hole is

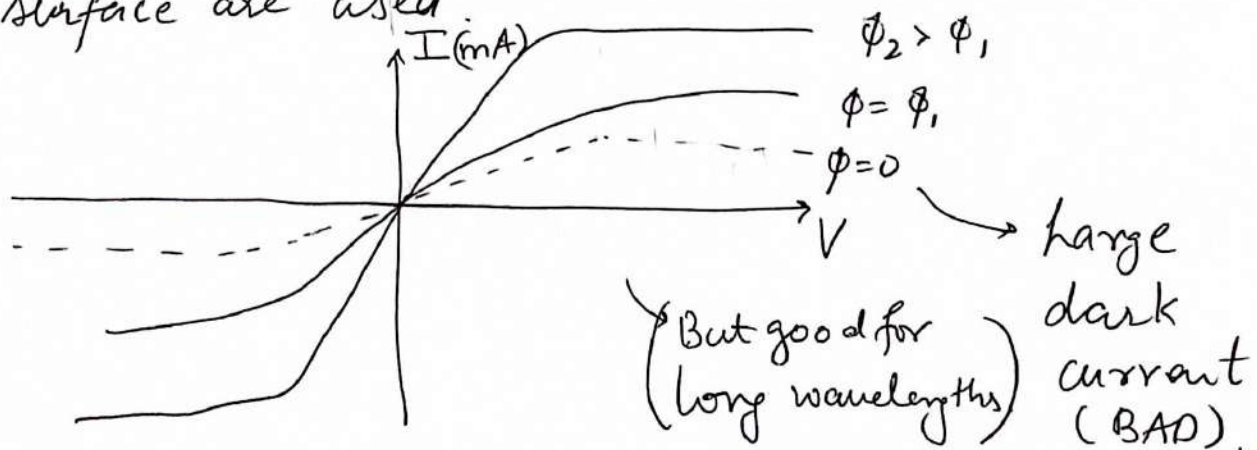


absorbed or until e^-h^+ annihilate. From Raman's Theorem

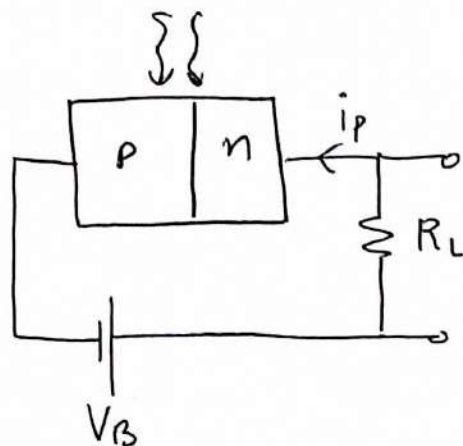
We can conclude that a persistent current flows as long as the hole exists, with just 1 photon!!

- > Impulse response is long though. Gain-bandwidth product remains constant.

- > For high speed applications, interdigitated electrodes on the surface are used.



Lec 43 Semiconductor Photodiodes



Why PIN?

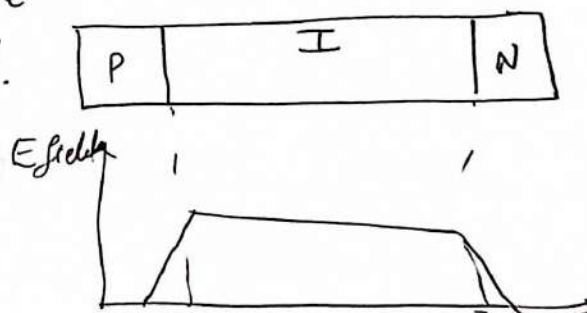
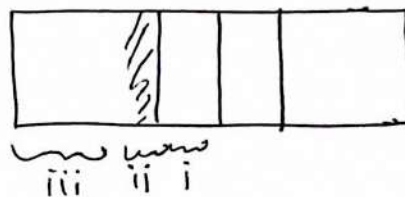
Three regions

- i) Depletion region (drift current)
- ii) Diffusion region (edge of depletion)
- iii) Away ~~from~~ from depletion region.

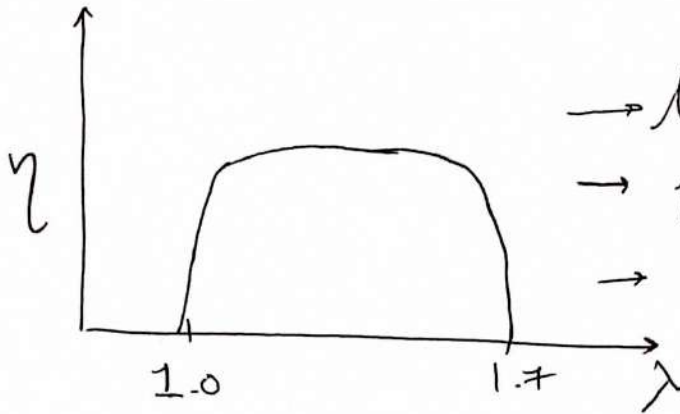
> Field only exists in i).

> Carriers generated in iii & ii are not swept away, only generate noise & not current.

> In PIN diode, region i) is enhanced! Also junction capacitance is reduced. Also more area for light to be captured.



→ All optical comms use $\text{In}_{0.53}\text{Ga}_{0.47}\text{As} / \text{InP}$.



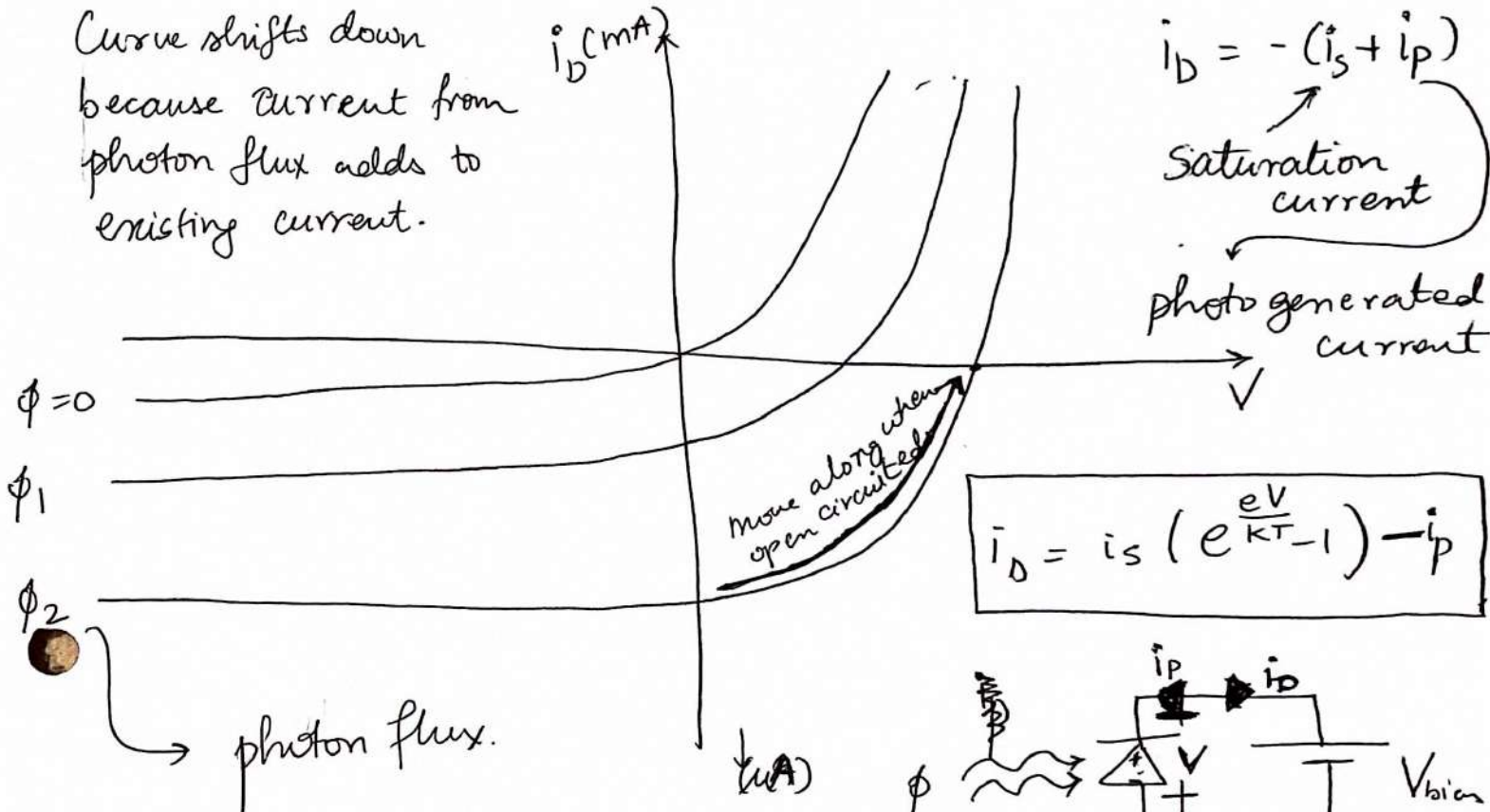
- large responsivity.
- low loss at 1.55 μm window.
- Has very large V_{sat} & high mobility μ_e .

⇒ Very sharp impulse response.

→ At 32:00 show a plot of η vs λ for Si & InGaAs.

Reverse Current Characteristics (★)

Curve shifts down because current from photon flux adds to existing current.



$$i_D = -(i_s + i_p)$$

↑
Saturation current

photo generated current

$$i_D = i_s (e^{\frac{eV}{kT}} - 1) - i_p$$

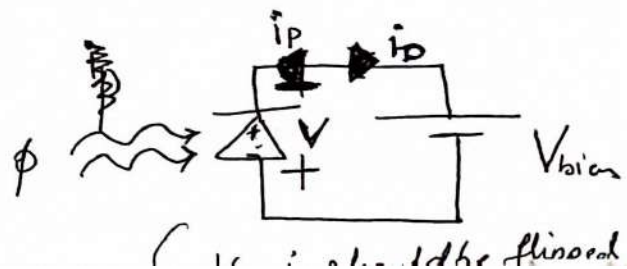


Photo voltaic

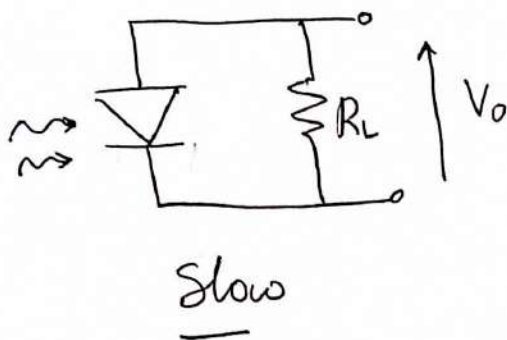
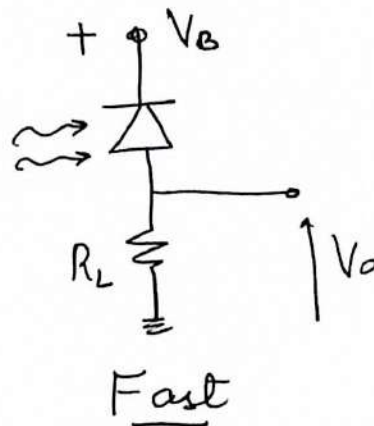


Photo conductive.



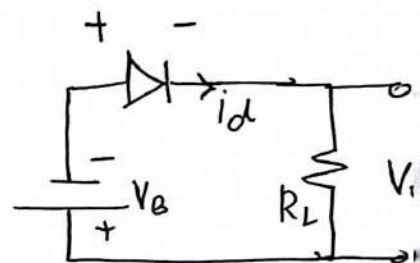
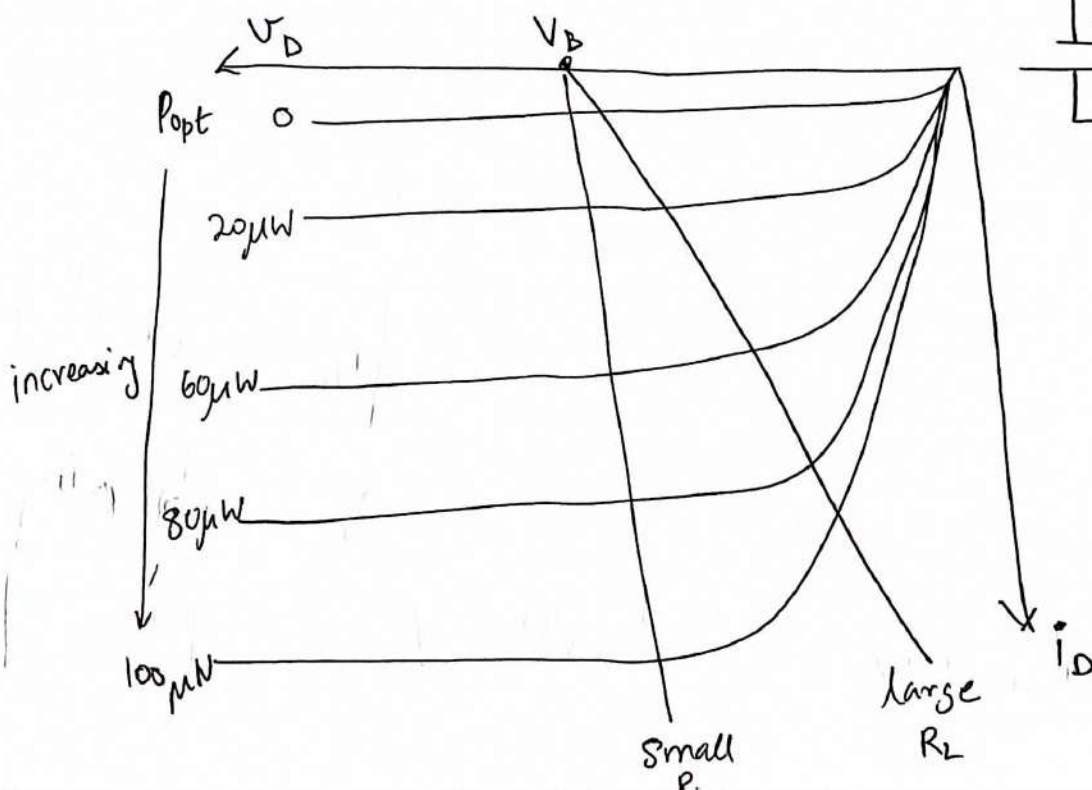
Characteristics

> Dynamic Range $\times$ Linearity $\times$ Sensitivity.

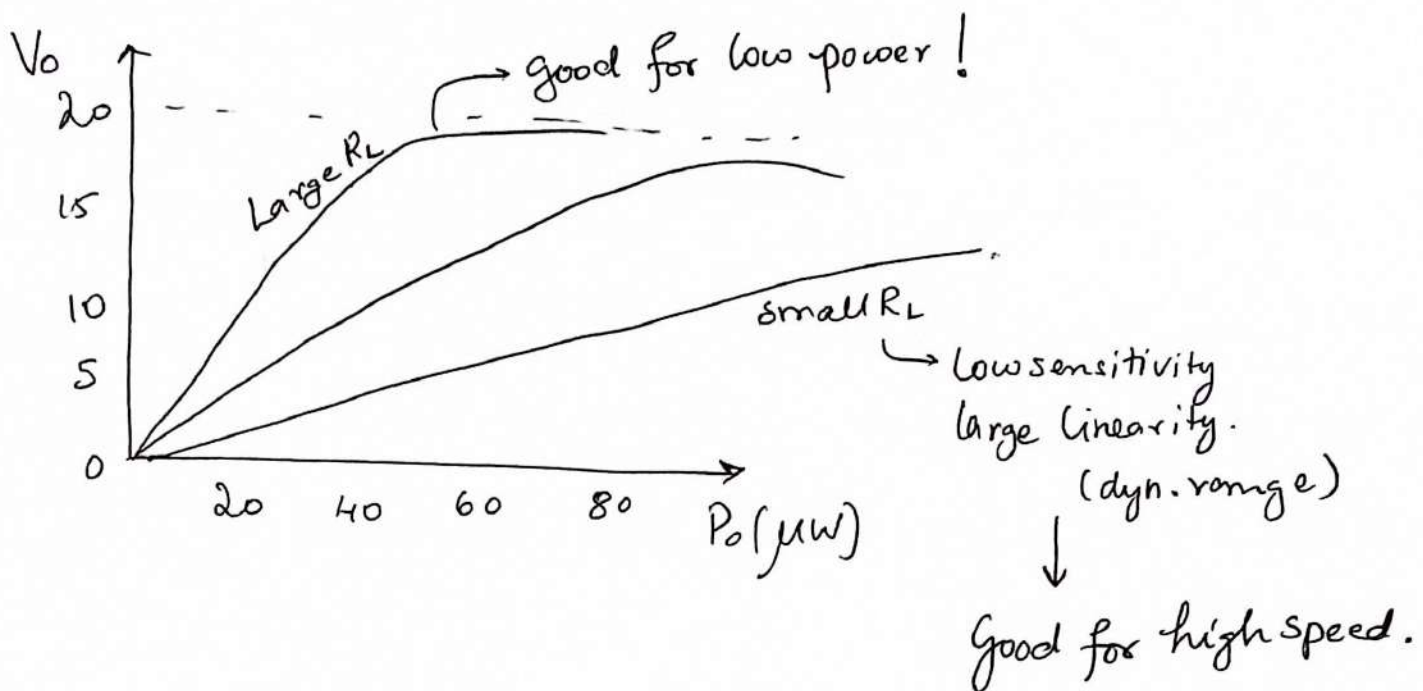
$P_{out\ max} - P_{out\ min.}$

$\rightarrow (V_{out\ vs.\ P_{opt}})$

load line behaviour



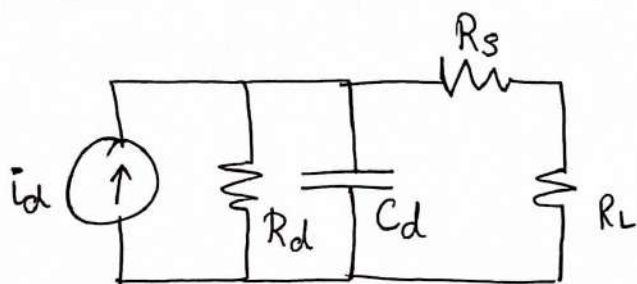
Linearity vs. Sensitivity.



$$BW = \frac{1}{2\pi R_L C_d}$$

lec 44 Avalanche Photo Diodes (APDs)

Equivalent Circuit of Photodiodes



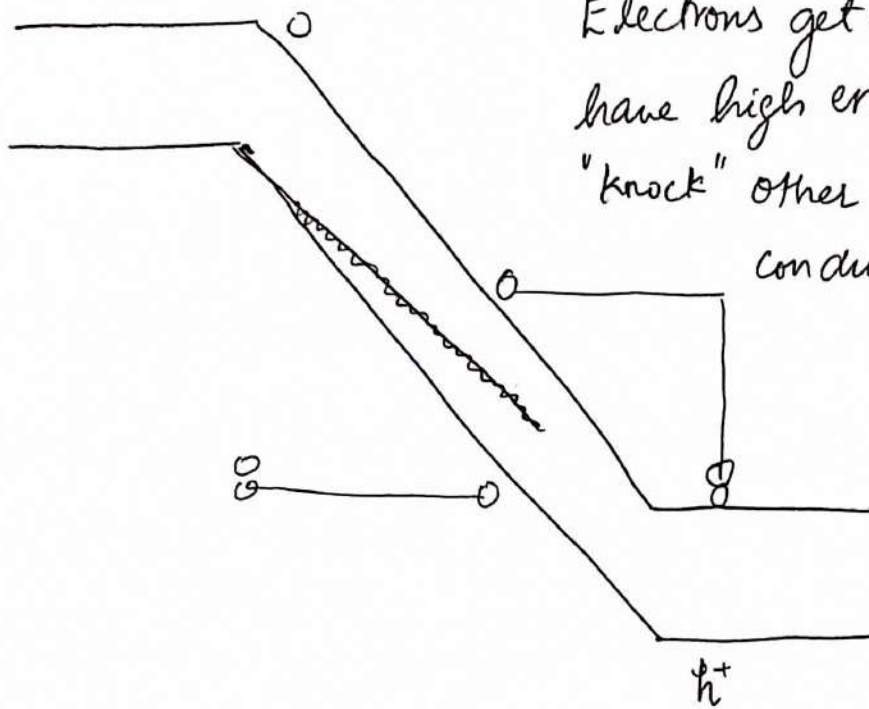
$$t_r = 2.19 R_L C_d$$

$$f_{3dB} = \frac{0.35}{t_r} \approx \frac{1}{2\pi R_L C_d}$$

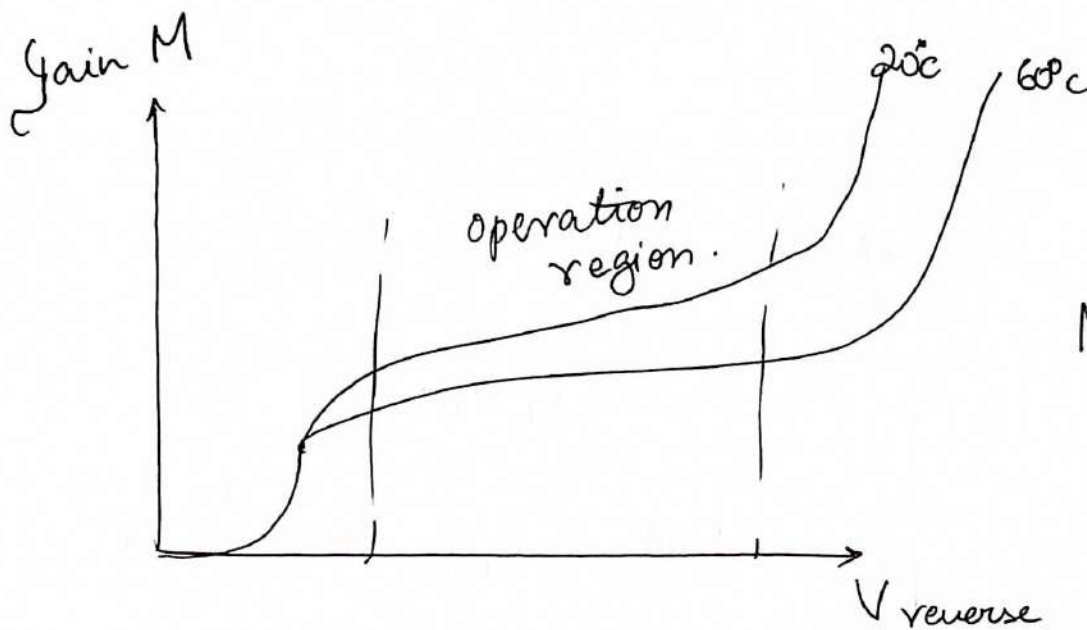
> APDs $\rightarrow$ PIN with large reverse bias.

- > Built in device gain due to avalanche multiplication.
- > High sensitivity, more noisy.
- > High current gain.

Carrier multiplication.



Electrons get accelerated & have high energy, which then "knock" other electrons into conduction band & so on.



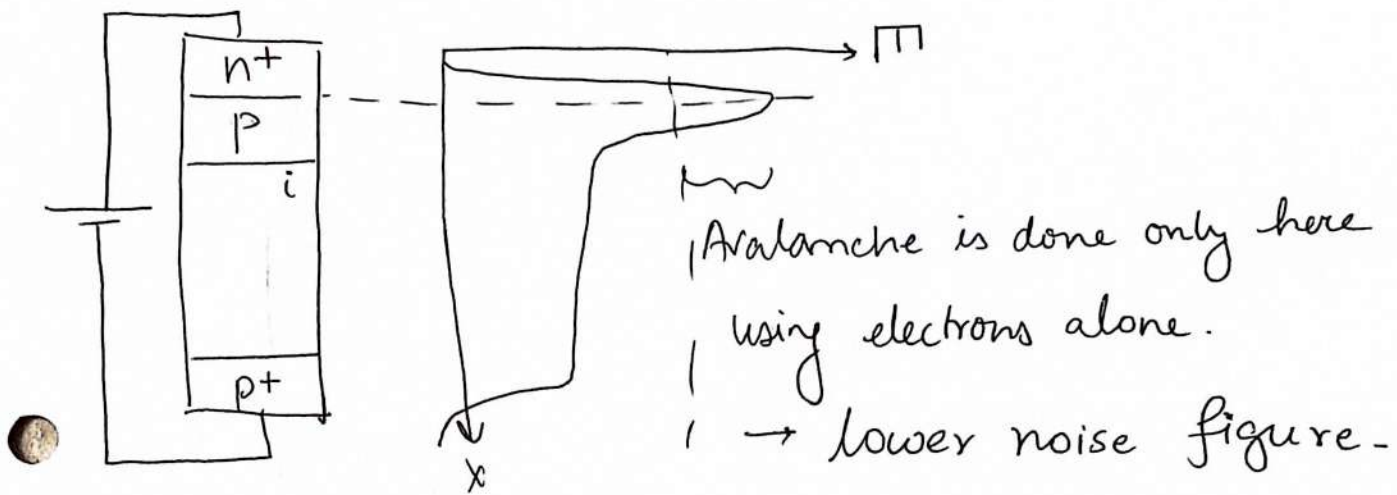
$$M = \frac{i_M}{i_P}$$

$$\alpha = \frac{1}{(Z_{int})_{av}} \rightarrow \text{impact ionization coeff.}$$

$\hookrightarrow$ avg. dist b/w 2 successive collisions

● In silicon, $\frac{\alpha_e}{\alpha_h} \sim 30$ for $E \sim 10^5$ V/cm.

⇒ Reach through APD.



$M \rightarrow$ gain in circuit
current vs. photocurrent.

→ Faster device too since avalanche process does not continue through the entire device length. Impulse response is faster!

Lec 45 Other Photodetectors.

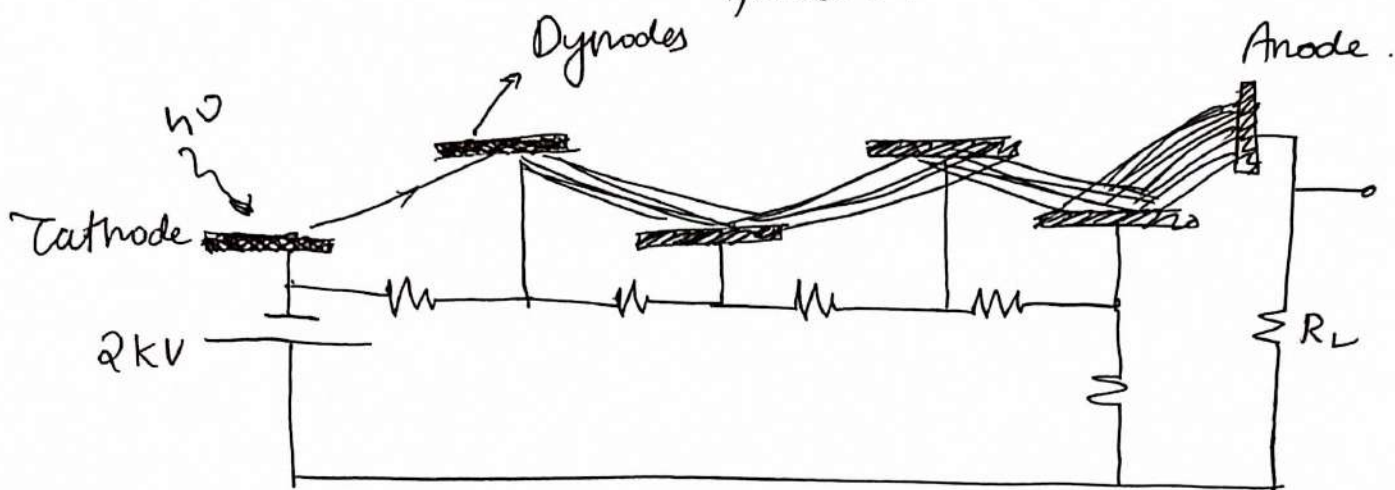
- 1) PMT
- 2) Phototransistors
- 3) Solar cells
- 4) QWIP
- 5) Thermal detectors.

Photomultiplier tube.

Basic principle ∴ Photoelectric effect + avalanche multiplication

$$h\nu = e\phi_m + K_{in} E_n.$$

work function



> One incident electron generates 4-6 scattered electrons.

$$\Rightarrow G = \delta^{N \rightarrow \text{no. of dynodes}}$$

$$G \approx 10^7$$

$$\text{If } P_{opt} = 1 \mu W$$

$$\eta = \frac{i/e}{P_{opt}/h\nu} \Rightarrow i = \eta \frac{P_{opt} e}{h\nu}$$

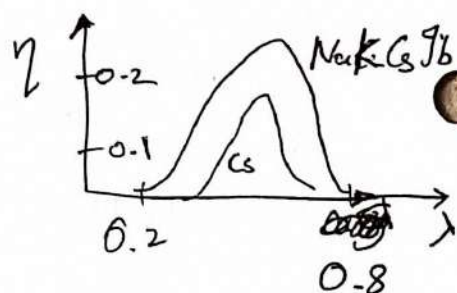
$$\Rightarrow i = \eta P_{opt} \frac{\lambda (\mu m)}{1.24}$$

$$\Rightarrow i \approx 10^{-13} A \rightarrow \text{primary current.}$$

Photo cathode ∴ $W = e\phi_m < h\nu$

Cesium $\approx 2eV \Rightarrow \lambda_g < 620nm$ com work!

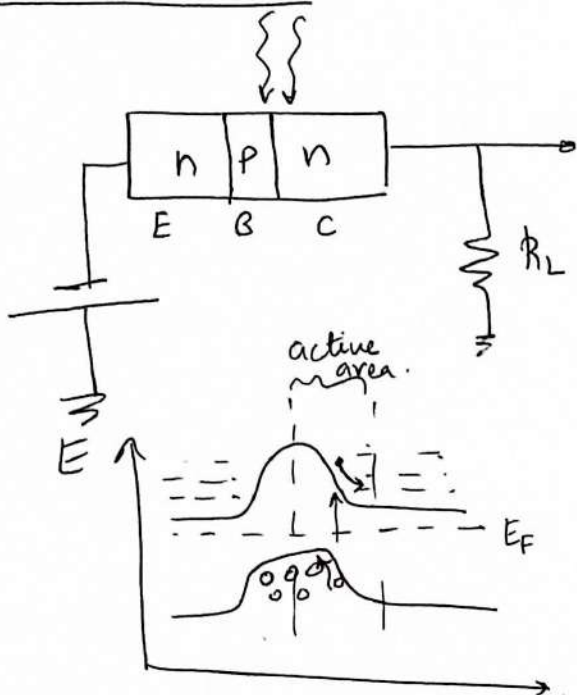
NaKCsIb $\approx 1.42eV$
 $\Rightarrow \lambda_g \approx 0.88 \mu m.$



$$G = \frac{i_{load}}{i_p} = 10^7 \Rightarrow i_{load} = 10^{-6} A = 1 \mu A$$

At $R_L = 10^6 \Omega \Rightarrow V = \underline{1V} \rightarrow \underline{\text{Super sensitive!}}$

Phototransistor

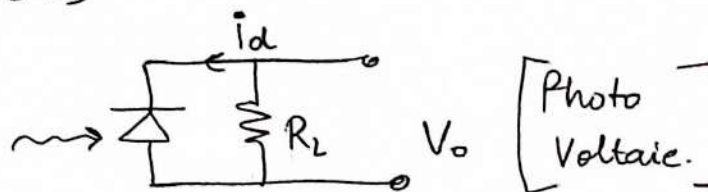
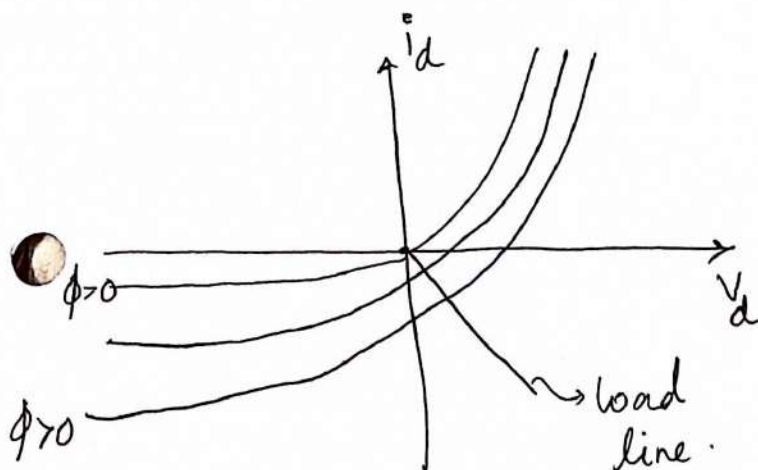


barrier is lowered when e-h pairs are created by incident photons.

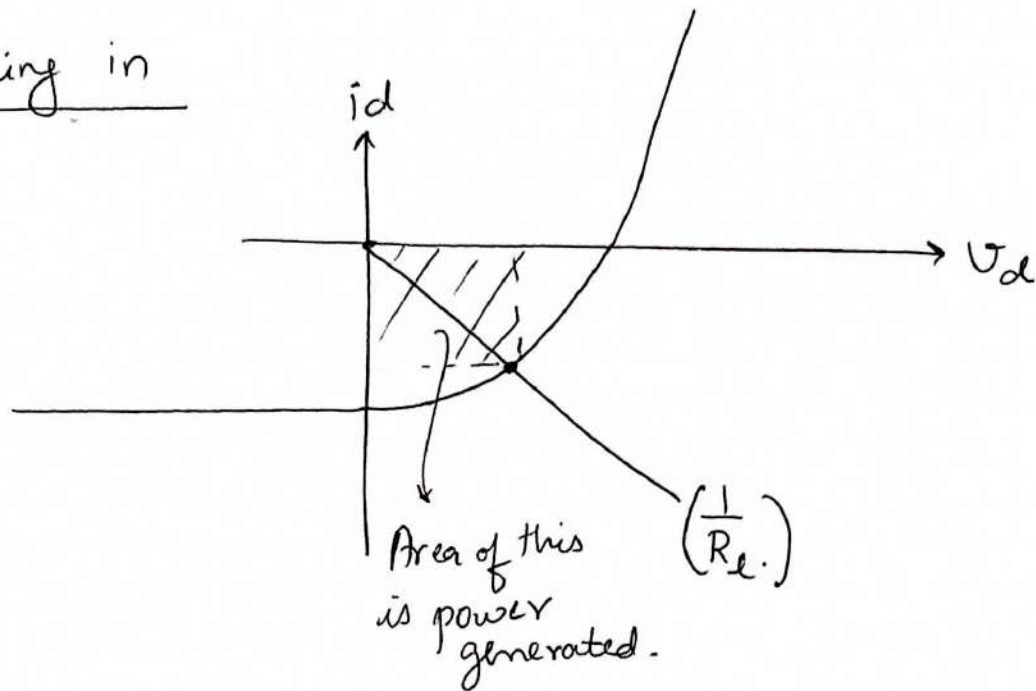
Charge accumulation in the base limits the speed.

Gain is due to barrier lowering.

Solar Cell (PN or PIN diode)

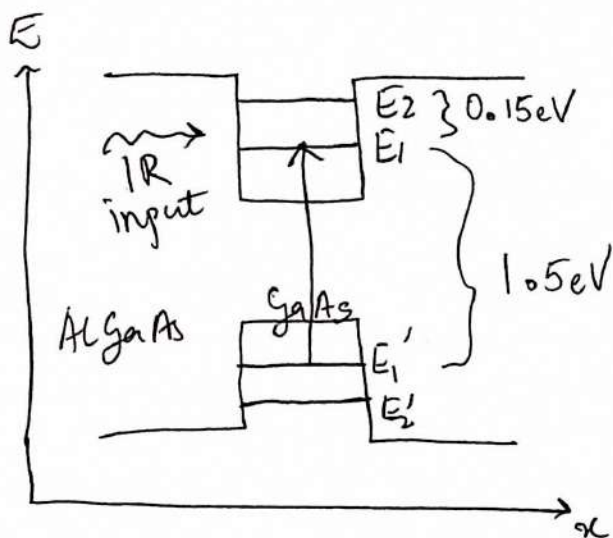


Zooming in

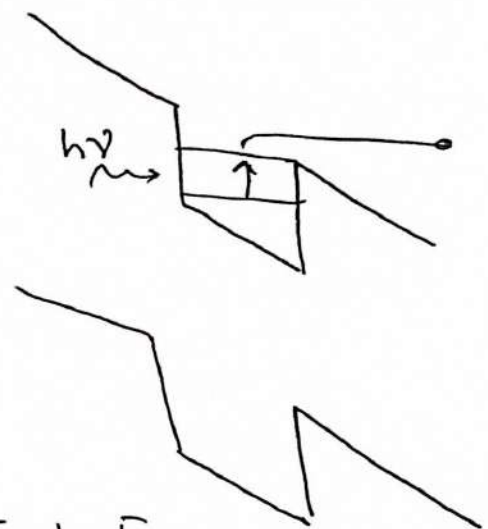


QWIP

Quantum Well Infrared Photodetectors.



Applying
E-field.



Incident photon moves e from E_1 to E_2

which tunnels or escapes out to generate a current.

low energy input photons are used for the intraband transitions. Typically multiple quantum wells are used.