EECS 522 - RFIC

Channel capacity : Max. data rate. Independent of latency and modulation.

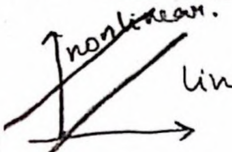
AWGN channels : Capacity ∝ B.W

$$C = BW \log_2(SNR+1)$$

↳ If SNR↑×4, we could use 4 levels, ⇒ 2 extra bits ... hence the log.

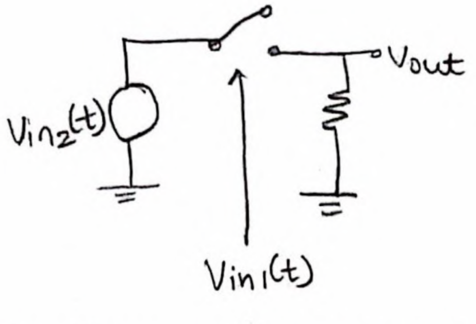
Basic concepts

① **Linearity** :- Linear combination of inputs produces the same linear combination at the outputs.
Graphically:- Straight line through the origin.



② **Time invariance** :- Response does not change with time.

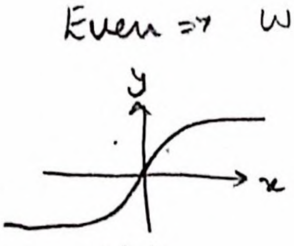
→ Linear ⇒ may change the frequency content of input
LTI ⇒ Don't. Although magnitudes of each freq. component may vary.



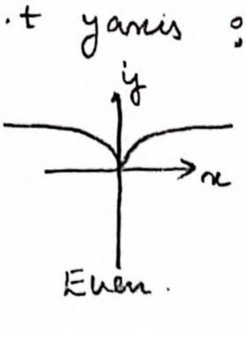
→ V_{in1} to Vout is nonlinear, V_{in2} to Vout is linear.
→ Both are time variant.
→ In circuits with switches, the system must be well defined to draw conclusions.

③ **Memory/memoryless** :- Output depends only on current input ⇒ Memoryless
L, C networks have memory.

④ **Symmetry** :- Odd ⇒ w.r.t origin : $x(t) \to y(t) \Rightarrow -x(t) \to -y(t)$
Even ⇒ w.r.t y axis : $x(t) \to y(t) \Rightarrow -x(t) \to y(t)$



Odd



Even.

LTI: $y(t) = x(t) * h(t)$

LTV: $y(t) = x(t) * h(t, \tau)$

Nonlinear: Approximated using the Volterra series.

Units in RF

① **power** $\Big|_{dBm} = 10 \log (\text{power in mW})$

$0dB = 1mW$
$3dB = 2mW$
$-3dB = 0.5mW$

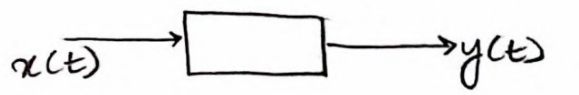
What $V_{pk\text{-}pk}$ is necessary to deliver 1mW (0dBm) to a 50Ω load? $V_{pp} = 632mV$

$$\Rightarrow \boxed{0dBm \leftrightarrow 1mW \leftrightarrow 632mV @ 50\Omega}$$

② **Ratios** $\Big|_{dB}$ $\quad \frac{V}{V} = 20 \log \frac{V_1}{V_2} \quad ; \quad \frac{P}{P} = 10 \log \frac{P_1}{P_2}$

$\frac{V}{V}$	dB
1	0
2	6
3.1	10
4	12
10	20

Nonlinearity Effects



$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) \rightarrow$ 3rd degree of nonlinearity

Ⓘ **Harmonic generation** → Assuming alphas are constant ⇒ Not so nonlinear

If $x(t) = A \cos \omega t$

$$y(t) = \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t$$

$$= \underbrace{\frac{\alpha_2 A^2}{2}}_{0th} + \underbrace{\left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t}_{\text{fundamental}} + \underbrace{\frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t}_{\text{Higher order harmonics}}$$

Observations

→ ① Differential pair (or any diff treatment) gets rid of the even harmonics including 0 since they are even functions!

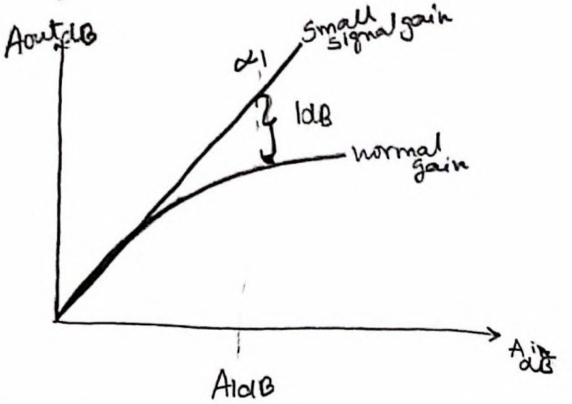
→ ② Amplitude of n^{th} harmonic has A^n and above terms.
⇒ n^{th} harmonic $\approx \propto A^n$ when $A \ll 1$ also $\alpha_1 \gg \alpha_2 \, \alpha_3 \rightarrow$ Not so nonlinear

(II) Gain compression

→ Small signal gain = α_1 since $A \ll 1$

Regular gain @ fundamental = $\alpha_1 + \frac{3}{4}\alpha_3 A^2$ where α_3 is usually negative

⇒ as A ↑, gain @ fundamental drops.



⇒ Gain is a function of Ain.

→ A1dB is the input value for which gain falls by 1dB.

→ $20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{1dB}^2\right| = 20\log|\alpha_1| - 1dB$

$$\boxed{\Rightarrow A_{1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}}$$

(III) Blocking / Desensitization

→ A strong tone at some frequency (ω_2) can lower the signal amplitude of interest @ ω_1.

$$x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$$

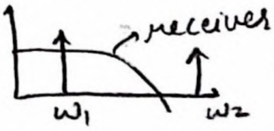
$$y(t) \cong \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right)A_1\cos\omega_1 t \quad \text{when } A_2 \gg A_1$$

⇒ Gain α_1 is densitized by A_2

Blocking ⇒ gain is 0.

→ Can ω_1 and ω_2 be arbitrary? No since the receiver itself has a B.W.

In this case alphas are fns of frequency and are not constant.



⇒ No blocking

(IV) Cross modulation

→ If there is an interfering signal A_2 which is modulated. It's modulation leaks into our desired signal A_1.

If $A_2 \xrightarrow[A.M]{} A_2(1+m\cos\omega_m t).\cos\omega_2 t$

$$\Rightarrow y(t) = \left[\alpha_1 A_1 + \frac{3}{2}\alpha_3 A_2^2 A_1\left(1+\frac{m^2}{2}+\frac{m^2}{2}\cos 2\omega_m t + 2m\cos\omega_m t\right)\right]\cos\omega_1 t$$

→ Now the signal is amplitude modulated at ω_m and $2\omega_m$.

(V) Intermodulation

→ 2 input signals at ω_1 and ω_2 produce outputs at frequencies other than the harmonics of ω_1 and ω_2.

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

⇒ $y(t)$ has the following terms (ignored DC and higher harmonics)

$$\text{at } \omega = \omega_1 \pm \omega_2 : \alpha_2 A_1 A_2 \cos(\omega_1+\omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2)t$$

rd order
$$\begin{cases} \text{at } \omega = 2\omega_1 \pm \omega_2 : \dfrac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1+\omega_2)t + \dfrac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1-\omega_2)t \\ \text{at } \omega = \omega_1 \pm 2\omega_2 : \dfrac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2+\omega_1)t + \dfrac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2-\omega_1)t \end{cases}$$

$$\text{at } \omega = \omega_1, \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t \Rightarrow \alpha_1 + 3\alpha_3\left(\frac{A_1^2}{4} + \frac{A_2^2}{2}\right)$$

is the gain of signal 1 ie $A_1 \cos\omega_1 t$

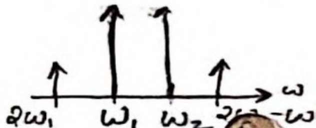
$$\left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t \Rightarrow \alpha_1 + 3\alpha_3\left(\frac{A_1^2}{2} + \frac{A_2^2}{4}\right)$$

is gain of 2nd signal.

→ The signals we care about are at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ because they are too close to ω_1 and ω_2 to filter them out.

How do we characterize it? Two tone test!

Use $A_1 = A_2 = A$, The ratio of the o/p 3rd order products ($2\omega_1 - \omega_2$ & $2\omega_2 - \omega_1$) to $\alpha_1 A$ (small signal gain) gives the IM distortion.



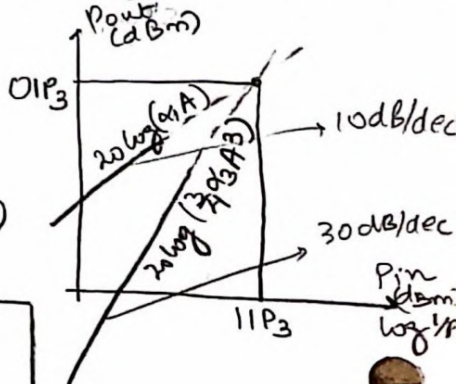
*→ This affects only the amplitude of our signal of interest. So can we just use phase modulation? No, because it affects the zero crossing points (and hence the stability of the system).

→ Third intercept point (IP_3)

When A is small gain is $\simeq \alpha_1$. When A ↑, gain ↑ ∝ A (ignoring gain compression) Whereas third order IM terms ↑ ∝ A^3

→ IP_3:- Input amplitude where the fundamental freq. amplitude (to the first order approximation) is equal to the 3rd order IM terms ($2\omega_1 - \omega_2$ & $2\omega_2 - \omega_1$) is called IP_3.

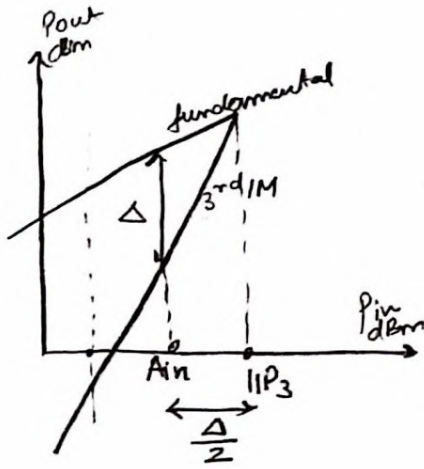
$$|\alpha_1| A_{IP_3} = \frac{3}{4}|\alpha_3| A^3 \Rightarrow \boxed{A_{IP_3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}}$$



→ In reality gain compression sets in much before A_{IP_3}. So extrapolate linearly from small signal gain on a log-log plot.

How do we measure IP3 on a circuit?

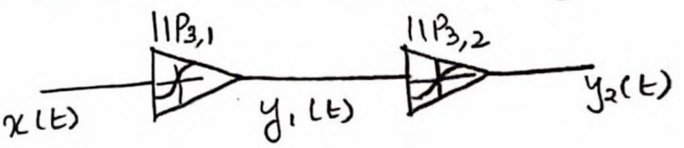
$$\frac{A_{\omega_1,\omega_2}}{A_{IM3}} = \frac{|\alpha_1|\,A_{in}}{\frac{3}{4}|\alpha_3|\,A_{in}^3} = \frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|\cdot\frac{1}{A_{in}^2} = \frac{A_{IP_3}{}^2}{A_{in}^2}$$



$$\Rightarrow IIP_{3\,dB} = \frac{1}{2}\left(20\log A_{\omega_1,\omega_2} - 20\log A_{IM_3}\right) + 20\log A_{in}$$

$$\Rightarrow \boxed{IIP_{3\,dB} = \frac{1}{2}\left(\text{Diff. of fundamental \& IM3}\right) + \text{Input}_{dB}\,(A_{in})}$$

(VI) Cascaded nonlinear stages



$$\begin{cases} y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) \\ y_2(t)\ \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t) \end{cases}$$

Solve for $y_2(t)$ in terms of $x(t)$. and use two tone test $x(t) = A\cos\omega_1(t) + A\cos\omega_2 t$

You get,

$$y(t) = \underbrace{\alpha_1\beta_1 A\left(\cos\omega_1 t + \cos\omega_2 t\right)}_{\text{fundamental}} + \underbrace{\left(\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3\right)\left(\tfrac{3}{4}A^3\right)\left(\cos(2\omega_1-\omega_2)t + \cos(2\omega_2-\omega)t\right)}_{\text{3rd IM}} + \dots$$

$$\Rightarrow A_{IP_3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|} \xrightarrow[\text{it}]{\text{rewriting}} \frac{1}{A_{IP_3}^2} = \underbrace{\frac{1}{A_{IP_{3,1}}^2}}_{\text{Stage ①}} + \frac{3\alpha_2\beta_2}{2\beta_1} + \underbrace{\frac{\alpha_1^2}{A_{IP_{3,2}}^2}}_{\text{Stage ②}}$$

→ Notice, $A_{IP3,2}$ is scaled down by gain of stage① so we want $A_{IP3,2}$ to be high to begin with. Intuitively, outputs of stage① are 'large signals' so we want $A_{IP3,2}$ to be large so that it can handle them.

→ The term $\frac{3\alpha_2\beta_2}{2\beta_1}$ comes from $\omega_1-\omega_2, 2\omega_1$ & $2\omega_2$ generated by stage①. It can be filtered so we are left with $\frac{1}{A_{IP_3}^2} = \frac{1}{A_{IP_{3,1}}^2} + \frac{\alpha_1^2}{A_{IP_{3,2}}^2}$

→ Generalizing,

$$\frac{1}{A_{IP_3}^2} = \frac{1}{A_{IP_{3,1}}^2} + \frac{\alpha_1^2}{A_{IP_{3,2}}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP_{3,3}}^2} + \frac{\alpha_1^2\beta_1^2\gamma_1^2}{A_{IP_{3,4}}^2} + \dots$$

→ Later stages should be more linear!

RF System parameters

Receiver specifications

① Noise ≈ Sensitivity (Also depends on modulation and BER)

② Quality factor (Image rejection)

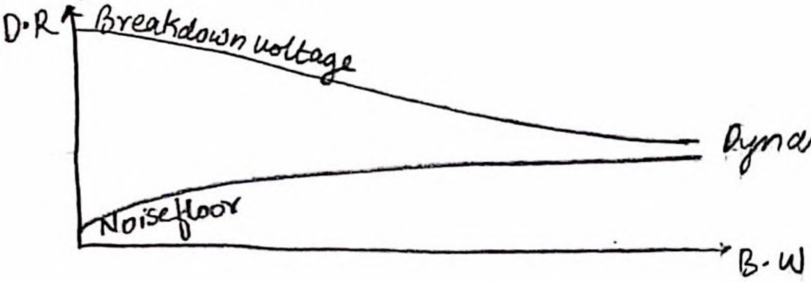
③ Linearity (max i/p signal) → Set by breakdown voltage.

① &③ give dynamic range!

④ P_{dc} - dc power consumption

⑤ Gain

Dynamic Range



Dynamic range ↓ with data rate.

$$\begin{tikzcd} \text{Speed} \arrow[r, leftrightarrow] \arrow[d] & \text{Dynamic range} \arrow[d] \\ f_T \text{ cutoff} & V_{BR} \text{ breakdown} \end{tikzcd}$$

$\boxed{f_T \times V_{BR} = JFM}$ → Johnson figure of merit $[\frac{MV}{s}]$ $\boxed{JFM = \frac{E_{br} \cdot V_{sat}}{2\pi}}$

E_{br} :- Breakdown field

V_{sat} :- saturation velocity.

$JFM_{Si} = 1$ → normalised to Si

$JFM_{GaAs} = 2.7$

$JFM_{SiC} = 20$

$JFM_{GaN} = 27.5$ (Costly)

Transmitter specifications

① P_{dc} → DC power consumption → Keep low

② P_{out} → Transmitted power → Keep high.

③ Power spectral density mask

④ Noise (Want to send pure signal)

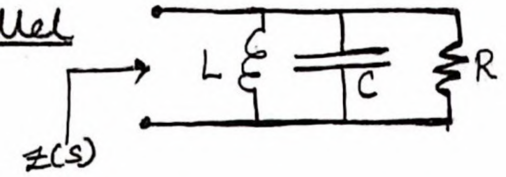
⑤ Error vector magnitude.

Passive Structures

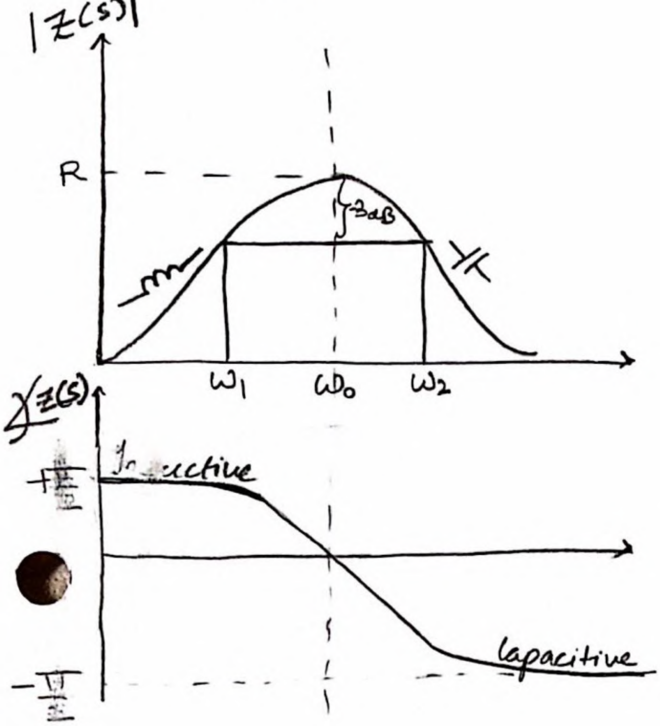
→ Can handle higher frequencies
→ Higher efficiency at high frequencies.

RLC circuits.

(I) Parallel



$$Z(S) = \frac{1}{C} \cdot \frac{S}{S^2 + \underbrace{\frac{1}{RC}}_{\frac{\omega_0}{Q}} S + \underbrace{\frac{1}{LC}}_{\omega_0^2}}$$



$\underset{-3dB}{BW} = \left.\omega_2 - \omega_1\right\}$ $|Z| = R - 3dB$ → Solve for 2 roo
*find $\omega_1 - \omega_2$

$\Rightarrow \boxed{BW = \frac{\omega_0}{Q}}$ At a given ω_0 $Q\uparrow \Rightarrow BW\downarrow$

$$\measuredangle Z = \frac{\pi}{2} - \tan^{-1}\left(\frac{\frac{\omega\omega_0}{Q}}{-\omega^2+\omega_0^2}\right) \Rightarrow \frac{d\measuredangle Z}{d\omega} = \frac{2Q}{\omega_0} \text{ at } \omega =$$

$$\boxed{Q = \frac{\omega_0}{2} \cdot \left|\frac{d}{d\omega} \measuredangle Z(j\omega_0)\right|}$$ High Q ⇒ Steep Slope in phase!

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad \boxed{Q = \frac{R}{\omega_0 L} = \omega_0 RC = \frac{R}{\sqrt{\frac{L}{C}}}}$$

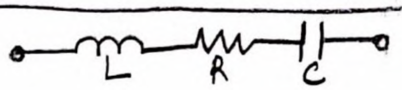
Large R ⇒ Ideal LC ⇒ Q↑

Less current through R

→ Large C ⇒ Low X ⇒ current flows more easily ⇒ High Q.
→ Large L ⇒ High X ⇒ opposes currents ⇒ Low Q.
⇓ More current through R

Wo occurs early for both C↑ & L↑

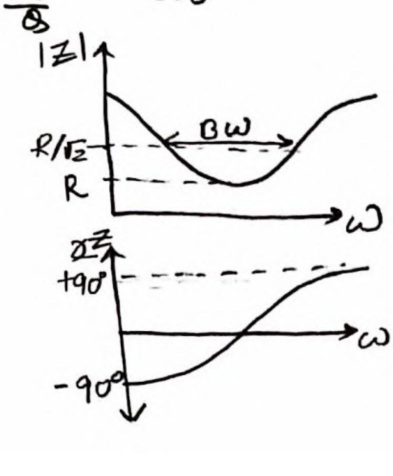
(II) Series



$$Z(S) = \frac{L}{S}\left(S^2 + \underbrace{\frac{R}{L}}_{\frac{\omega_0}{Q}} S + \underbrace{\frac{1}{LC}}_{\omega_0^2}\right)$$

$\omega_0 = \frac{1}{\sqrt{LC}}$; $BW = \frac{\omega_0}{Q}$; $Q = \frac{\omega_0}{2}\left|\frac{d}{d\omega}\measuredangle Z(j\omega)\right|$

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{\sqrt{\frac{L}{C}}}{R}}$$



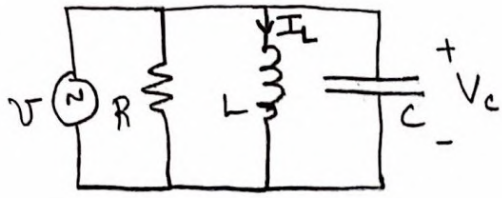
$C\uparrow \Rightarrow X_C\downarrow \Rightarrow I\uparrow \Rightarrow$ more dissipation through R.

$L\uparrow \Rightarrow X_L\uparrow \Rightarrow I\downarrow \Rightarrow$ less dissipation through R

$R\to 0 \Rightarrow$ Ideal LC $\Rightarrow Q\to\infty$.

Essence of ω_0 and Q

ω_0 Take



Let $V = V_0 \cos \omega_0 t$

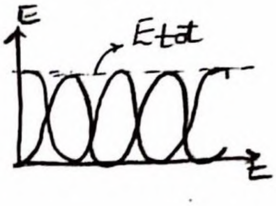
$$E_c(t) = \frac{1}{2} C V_c^2 = \frac{1}{2} C V_0^2 \cos^2 \omega_0 t$$

$$E_L(t) = \frac{1}{2} L I_L^2 = \frac{1}{2} L \left[\frac{V_0 \sin \omega_0 t}{L \omega_0}\right]^2$$

$$= \frac{1}{2} C V_0^2 \sin^2 \omega_0 t$$

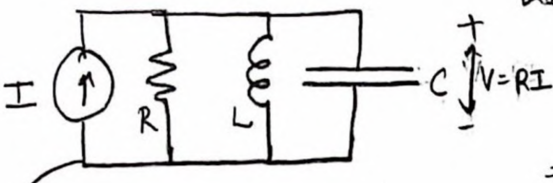
$\Rightarrow$ $\boxed{E_c(t) + E_L(t) = \frac{1}{2} C V_0^2}$ $\Rightarrow$ Total energy in LC is a constant!

The energies are out of phase



→ Only when $\omega = \omega_0$. "Therefore, ω_0 is the frequency at which the total energy in LC is constant and oscillates between Ctotal & Ltotal"

Let's take



Let $I = I_0 \cos \omega_0 t$

$$|Z_L| = |Z_c| \Rightarrow |I_L| = |I_c| = \frac{RI}{L\omega_0} = IR\sqrt{\frac{C}{L}}$$

$\Rightarrow$ $\boxed{|I_L| = |I_c| = I\,Q}$ Could destroy components at high Q

Q $V = RI \Rightarrow V_0 = RI_0 \Rightarrow E_{\text{tot stored}} = \frac{1}{2} C V_0^2 = \frac{1}{2} C (RI_0)^2$, Here V_0 is peak value!

$$E_{\text{tot dissipated in one time period}} = \frac{1}{2} R I^2 T = \frac{1}{2} \frac{V^2}{R} T$$

$$\Rightarrow \frac{E_{\text{tot stored}}}{E_{\text{tot dissipated in one } T}} = \frac{RC}{T} = \frac{RC \cdot \omega_0}{2\pi} = \frac{Q}{2\pi}$$

$$\Rightarrow \boxed{Q = 2\pi \frac{E_{\text{total stored in one period}}}{E_{\text{dissipated in one period}}} = \omega_0 \cdot \frac{\text{Stored energy}}{\text{Dissipated power}}}$$

This is the fundamental definition of Q applicable at all frequencies!

→ Impulse response $v(t) \propto e^{-t/2RC} \propto e^{\left(\frac{-t}{T_0}\right)\left(\frac{\pi}{Q}\right)}$ $\Rightarrow$ At each period amplitude drops by $e^{-\pi/Q}$ $\Rightarrow$ High Q $\Rightarrow$ Sustain longer!

→ A system could have multiple ω_0s & Q.

→ Q is defined for any frequency.

→ 1nH & 1pF $\Rightarrow$ $f_0 \approx 5$ GHz

→ Q is roughly the no. of cycles of ringing.

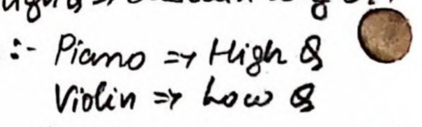
$$Q = \omega_0 \frac{E_{tot}}{P_{avg}}$$

$$Q = \frac{\omega_0}{BW} = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right|$$

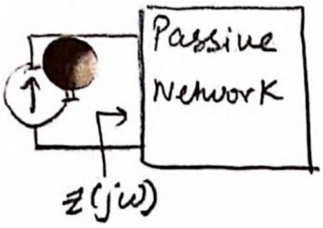
$$Q = \frac{Im(Z(\omega))}{Re(Z(\omega))}$$

Ex:- Piano $\Rightarrow$ High Q

Violin $\Rightarrow$ Low Q



General Theorems for RLC



$$Z(j\omega) = R(j\omega) + j\,X(j\omega)$$

$$= \frac{2\,P_{dissipated} + 4j\omega\left[E_M(\omega) - E_E(\omega)\right]}{|I|^2}$$

$E_M(\omega)$, $E_E(\omega)$ are avg stored M & E energies.

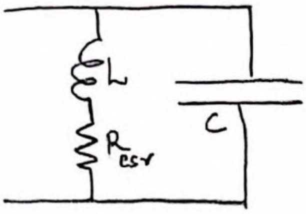
@ ω_0 :- $E_M(\omega) = E_E(\omega)$

$$\Rightarrow \boxed{X(j\omega) = 0}$$

For Q :-

Q :- $Q = \frac{Im\{Z\}}{Re\{Z\}}$ {Under certain conditons}

→



$$\boxed{\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}}$$

$$Z(j\omega_r) = R(j\omega_r) = \frac{L}{RC} = \frac{\left(\sqrt{\frac{L}{C}}\right)^2}{R} = \frac{(L\omega_0)^2}{R} = \frac{1}{R(C\omega_0)^2}$$

Where, $\omega_0 = \frac{1}{\sqrt{LC}}$ ⇒(no R!)

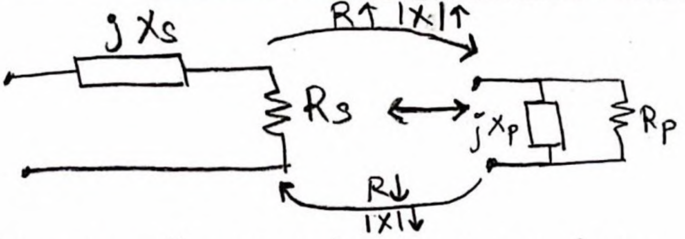
→ At resonance the circuit behaves as a resistor

$$R_p = \frac{\left(\sqrt{\frac{L}{C}}\right)^2}{R}.$$

If R is small ⇒ $R \ll \omega L$

$$\boxed{Q = \frac{\omega_0 L}{R}}$$

→ Series ↔ Parallel transformations.



$$R_p = \frac{R_s^2 + X_s^2}{R_s} \qquad R_s = R_p \cdot \frac{X_p^2}{R_p^2 + X_p^2}$$

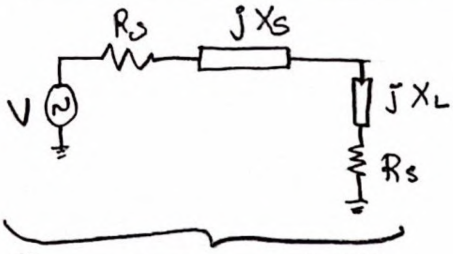
$$X_p = \frac{R_s^2 + X_s^2}{X_s} \qquad X_s = X_p \cdot \frac{R_p^2}{R_p^2 + X_p^2}$$

→ These equations are true at all frequencies however, R, X could become functio of frequency.

- The signs of X_p, X_s remain same ⇒ Caps remain Caps & inductors remain inducto

→ Also, $R_p = R_s(1+Q^2)$; $L_p = L_s\left(1+\frac{1}{Q^2}\right)$; $C_s = C_p\left[1+\frac{1}{Q^2}\right]$; $R_s = R_p\left[\frac{1}{1+Q^2}\right.$

Maximum Power Transfer.



For maximum power

$Z_L = Z_s^*$

P_{RL} = power delivered to the load = $\frac{V_{RL}^2}{2R_L}$

$$P_{R_L} = \frac{V^2}{2} \cdot \frac{R_L}{(R_L+R_s)^2+(X_s+X_L)^2}$$

To maximize $P_{RL} \Rightarrow \frac{dP_{RL}}{dR_L} = 0$

$\Rightarrow R_s = R_L$ & $\frac{dP_{RL}}{dX_L} = 0 \Rightarrow X_s = -X_L$

P_s = power at source

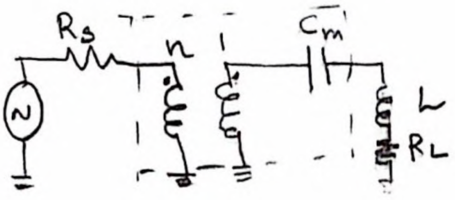
$$P_s = \frac{V^2}{4R_s}$$

P_{RLmax} = max. power available at load

$$R_{Lmax} = \frac{V^2}{8R_s}$$

Matching.

→ Transformers :-



→ C_m resonates with L at $\omega_{operating}$

→ $n^2 = \frac{R_s}{R_L}$

→ Transformers are lossy, large & have poor coupling ($k<1$).

→ L-match

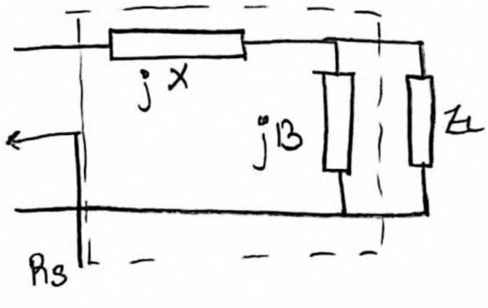
→ We can transform the impedance looking at load using an LC network that is also not lossy.

① → Find the Q that gives the desired R_p, R_s based on $R_p = R_s(1+Q^2)$.

② → Change Q by adding an inductor or capacitor (to the desired Q)

③ → Find L/C to resonate with X_L.

Downconverter ($R_s < R_L$)



$$B = \frac{X_L \pm \sqrt{\frac{R_L}{R_s}}\sqrt{R_L^2+X_L^2-R_L R_s}}{R_L^2+X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L R_s}{R_L} - \frac{R_s}{B R_L}$$

$X = \omega L$ or $-\frac{1}{\omega C}$

$B = \frac{-1}{\omega L}$ or ωC

X ⇒ +ve ⇒ inductor.

B ⇒ +ve ⇒ capacitor.

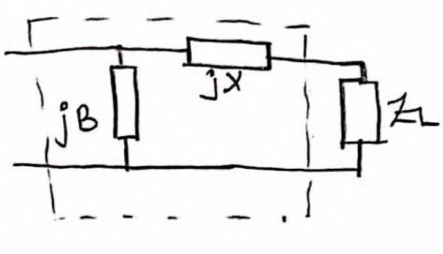
X, B ⇒ +ve ⇒ low pass.

X, B ⇒ -ve ⇒ High pass.

→ Prefer lowpass! It is easie to make inductors b/w 2 nodes & caps b/w a node & ground. Also these give higher Qs.

→ Also choose based on DC bias.

Upconverter ($R_s > R_L$)



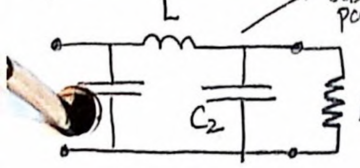
$$X = \pm\sqrt{R_L(R_s-R_L)} - X_L$$

$$B = \pm\sqrt{\frac{R_s-R_L}{R_s}}$$

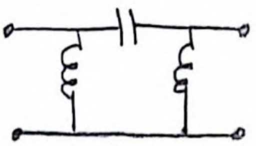
Pi π match.

Better, we can absorb inductor's parasitic caps in C_1, C_2 } Also $C_1 C_2$ make it robust to inductor parasitics.

π match gives extra degree of freedom to design for B.W.



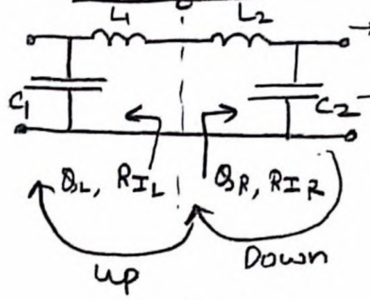
Lowpass



High pass.

→

Analysis



→ $R_{I_L} = R_{I_R} = R_I$ if n/w is matched.

→ $Q = Q_L + Q_R$

Design equation

$$\boxed{Q = \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_P}{R_I} - 1}}$$

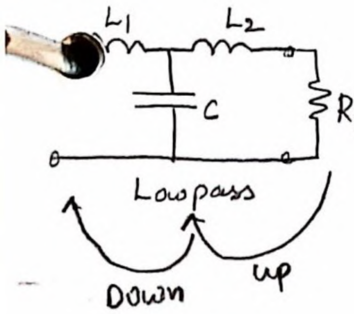
$$\longrightarrow \underline{RHS} \quad Q_R \sqrt{\frac{R_P}{R_I} - 1} = \frac{\omega_0 L_2}{R_I} \;;\quad \underline{LHS} \quad Q_L = \sqrt{\frac{R_{in}}{R_I} - 1}$$

} Design RHS, LHS as 2 L matches w.r.t R_I.

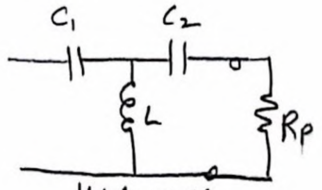
A more approximate approach

$$R_I = \frac{\left(\sqrt{R_{in}} + \sqrt{R_P}\right)^2}{Q^2} \;;\quad C_1 = \frac{Q_L}{\omega_0 R_{in}} \;;\quad C_2 = \frac{Q_R}{\omega_0 R_P} \;;\quad L = \frac{Q R_I}{\omega_0}$$

T-match



Lowpass



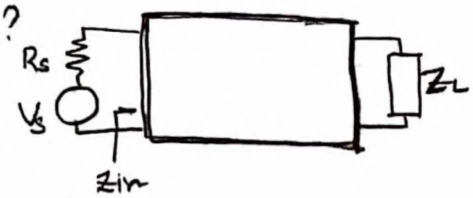
High pass

$\Rightarrow Q = Q_L + Q_R$

$$Q = \sqrt{\frac{R_T}{R_{in}} - 1} + \sqrt{\frac{R_I}{R_L} - 1}$$

→ For lossless matching networks, matching from one side guarantees matching from other side

→ How well can we design the matching network?



$$\Gamma = \frac{Z_{in} - R_s}{Z_{in} + R_s}$$

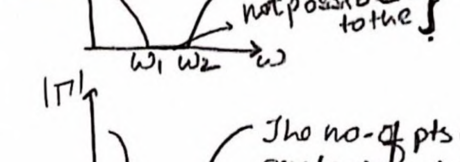
Reflection coeff.

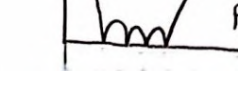
$\Gamma = 0 \Rightarrow$ perfect match

$|\Gamma| = 1 \Rightarrow$ perfect mismatch.

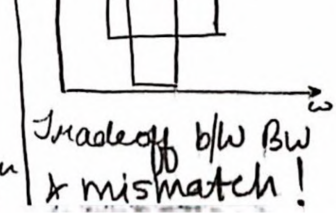
→ We could match at one frequency but how much Bandwidth would we get?

→ It is limited by the Bode-Fano limit. $\int_0^\infty \ln \frac{1}{|\Gamma(j\omega)|} d\omega < \frac{\pi}{RC}$ for parallel RC

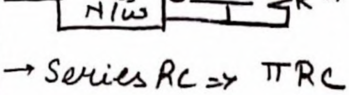




The no. of pts of perfect match is given by the integral



Tradeoff b/w BW & mismatch!



$\Rightarrow$ The limit is $\frac{\pi}{RC}$

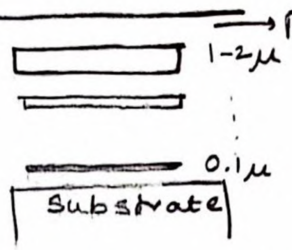
→ Series RC $\Rightarrow \pi RC$

→ Parallel RL $\Rightarrow \frac{\pi L}{R}$

→ Series RL $\Rightarrow \pi R$

Integrated Passives on CMOS

→ CMOS has 5-12 metal layers



Metals: Al → Low ρ
→ Easy manufacture
→ ↑ Adherance to SiO_2

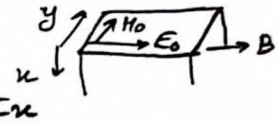
Cu → Better condu[illegible]ity
→ Hard manufactu
→ Better electro-migration.

Electromigration: How much current a metal can carry per unit width. Very strong E fields move Ions & change the lattice structure forming holes which ↑ E fields ⇒ positive feedback!

Effect ① Penetration of E fields.:

Wave equation inside a dielectric: $\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)\vec{E}$. $\frac{\sigma}{\omega\epsilon}$ → loss tangent

Assuming $\sigma \gg \omega\epsilon \Rightarrow$ loss is very small $\Rightarrow \nabla^2 E = j\omega\mu(\sigma\vec{E})$ $\{\vec{E} = E_z$



How does $\vec{E}$ field change with x? $\frac{d^2E}{dx^2} = j\omega\mu\sigma E_z = \tau^2 E_z \Rightarrow E_z = C_1 e^{-\tau x} + \underset{0}{\cancel{C_2}} e^{+\tau x}$ since $E_z \not\to \infty$

$E_z(x=0) = E_0$

$\Rightarrow \boxed{E_z = E_0 e^{-\tau x}} \Rightarrow$ Exponential decay inside the metal.

$\tau = \sqrt{j\omega\mu\sigma} = \frac{1+j}{\delta}$ where $\boxed{\delta = \sqrt{\frac{1}{\pi f \mu\sigma}}}$ Skin depth: Depth at which E_0 drops by e.

Function of material and frequency!

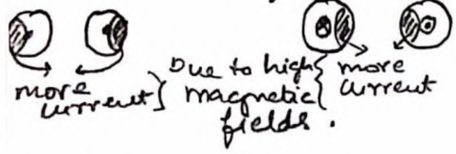
→ At high frequencies thick metals are pretty useless.

f	Cu	Al
1GHz	2μ	2.6μ
10G	0.65μ	0.82μ
100G	0.2μ	0.26μ

} δ values!

Effect ② Current flows where M field is stronger.

Proximity effects:- Two adjacent wires!



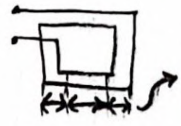
} → Signals could be coupled
→ Resistance also ↑.

Capacitors: ① Lateral field caps are better: Better control & tolerance, ↓ substrate coupling
② Fractal caps for ↑ area but very hard to simulate/model.
③ MIM cap: High ε insulator. Low breakdown voltage. Cap. density is high ≈ $\frac{1fF}{\mu m^2}$
④ Lateral field vertical bars:- Highest density!

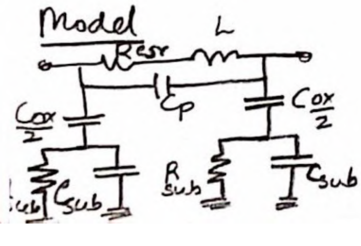
Inductors → $L \propto \frac{1}{\sqrt{W}} \Rightarrow$ Thinner ⇒ L ↑ ; $0.2nH \leq \underset{(spiral)}{L_{on\ chip}} \leq 20nH$. ; $\geq 20nH \Rightarrow$ Bondwire
$\leq 0.2nH \Rightarrow$ Tx. lines.

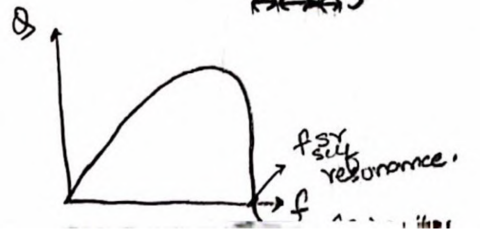
→ Hollow spirals since inner loops have ↓ φ flux & high loss.

→ Rule of Thumb:



Keep these three distances same for good Q&L

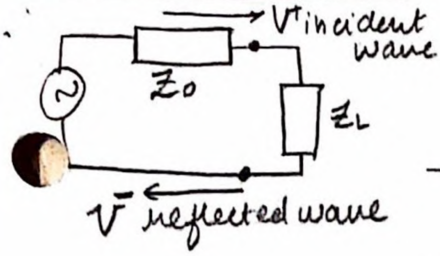




→ Shield patterns are used to preve[illegible] substrate coupling and break eddy currents in the shield. Also ↑ Q.

→ Floating shields are better but harder t[illegible] model.

Smith Charts



$$\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

S.C

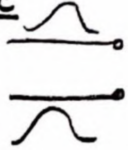
180° phase shift since boundary condition is $V=0$.

→ $Z_L = Z_0 \Rightarrow \Gamma = 0$ matched

→ $Z_L = 0 \Rightarrow \Gamma = -1$

→ $Z_L = \infty \Rightarrow \Gamma = +1$

O.C



0° phase shift since $I=0$

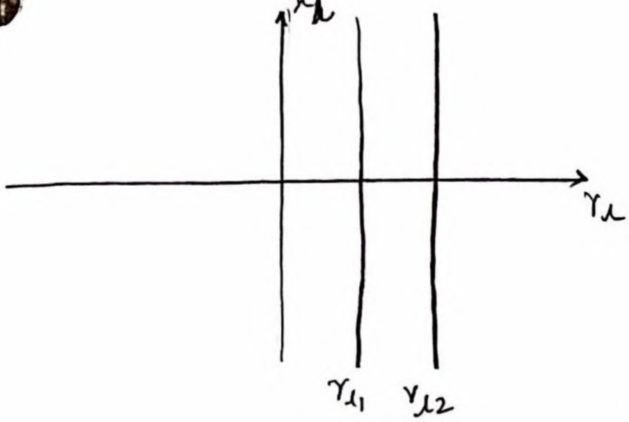
$$\Gamma = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{z_l - 1}{z_l + 1}$$

Smith chart is a graphical representation of this bilinear transform.

$\Gamma = \Gamma_r + j\,\Gamma_{im}$, $z_l = r_l + j\,x_l$ $\Rightarrow$ $\Gamma_r = \dfrac{r_l^2 + x_l^2 - 1}{(r_l+1)^2 + x_l^2}$ $\quad r_l = \dfrac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 - \Gamma_i^2}$

$$\Gamma_i = \frac{2x_l}{(r_l+1)^2 + x_l^2} \qquad x_l = \frac{2\Gamma_i}{(1-\Gamma_r)^2 - \Gamma_i^2}$$

→ Constant r_l & eliminate x_l $\Rightarrow \left(\Gamma_r - \dfrac{r_l}{1+r_l}\right)^2 + \Gamma_i^2 = \left(\dfrac{1}{1+r_l}\right)^2$ → circle.



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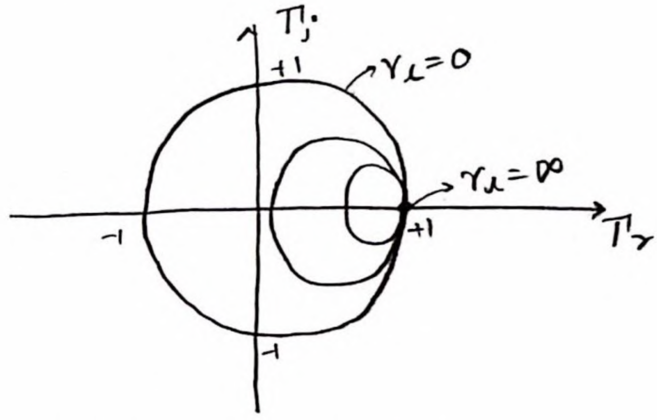
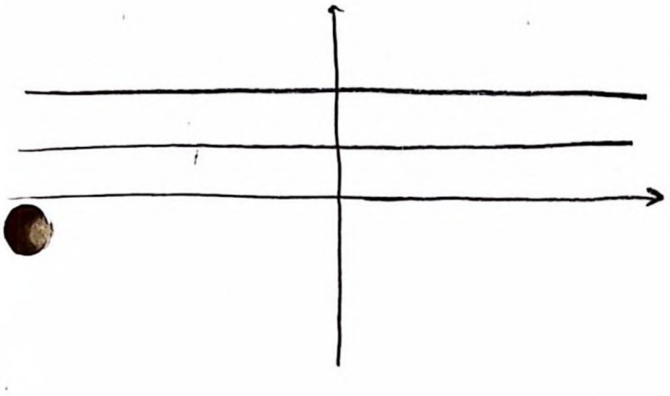


→ Constant x_l & eliminate r_l $\Rightarrow (\Gamma_r - 1)^2 + \left(\Gamma_i - \dfrac{1}{x_l}\right)^2 = \left(\dfrac{1}{x_l}\right)^2$



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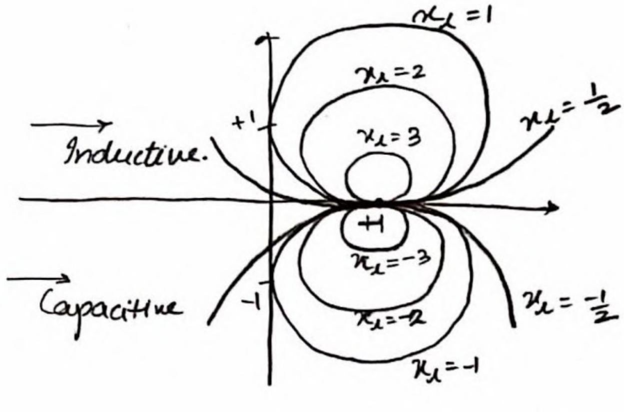
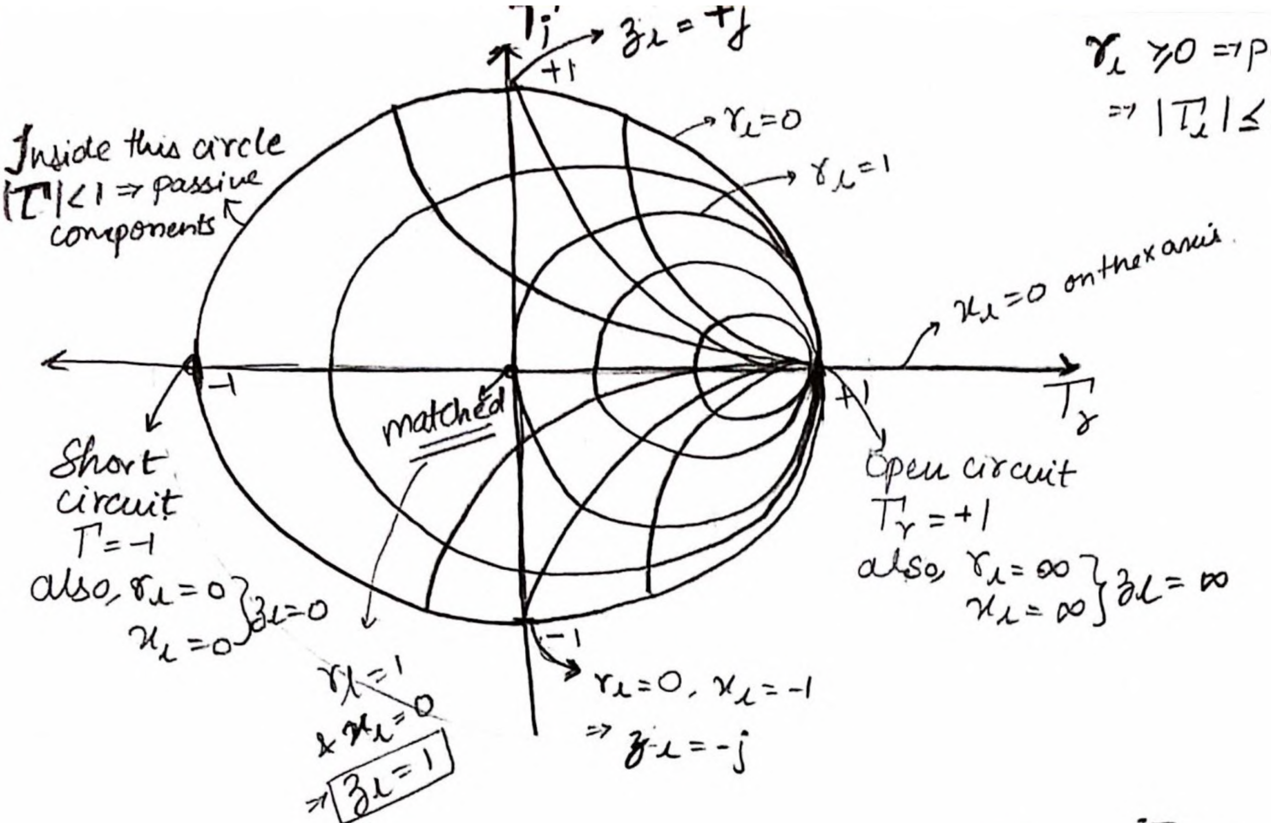


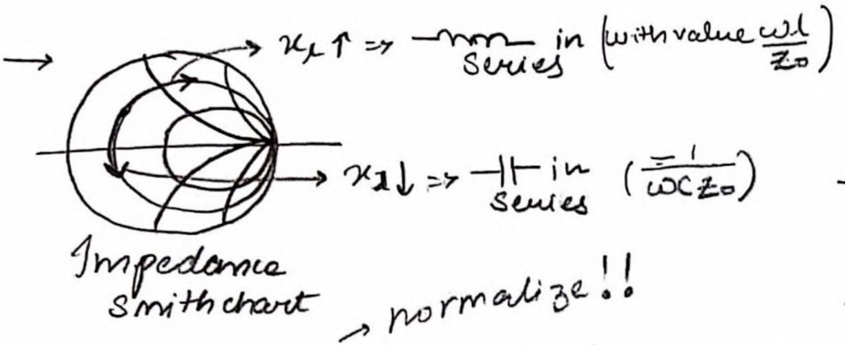
$r_l \geq 0 \Rightarrow$ passive load.
$\Rightarrow |\Gamma_l| \leq 1$ } inside unit circle.



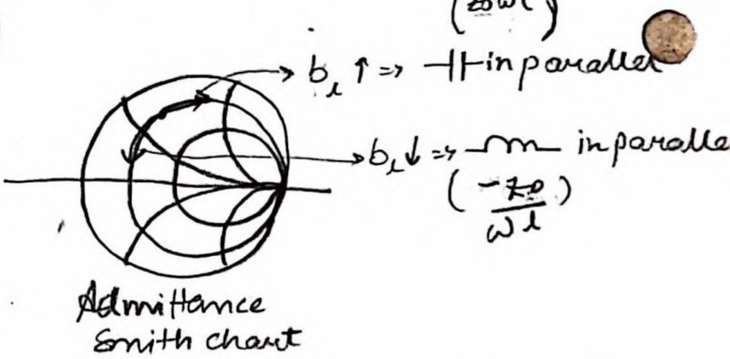
→ Smith chart for admittance $y = \frac{1}{z}$: $\Gamma = e^{j\pi} \frac{y-1}{y+1}$ flipped ↔ & ↕
↳ mirror w.r.t origin.

→ To find admittance find impedance and mirror w.r.t origin.

→



Impedance Smith chart



Admittance Smith chart

→ Find where z_l (→ normalize!!) is on the chart & move to the centre by moving on constant r_l lines and getting to the $r_l = 1$ circle.

→ $Z_l = r_l + j\, x_l$

$y_l = g_l + j\, b_l$

$x_l = j\omega L$ (or) $\frac{-j}{\omega C}$

$b_l = j\omega C$ (or) $-\frac{j}{\omega L}$

x_l +ve ⇒ inductor in series ; x_l −ve ⇒ capacitor in series.

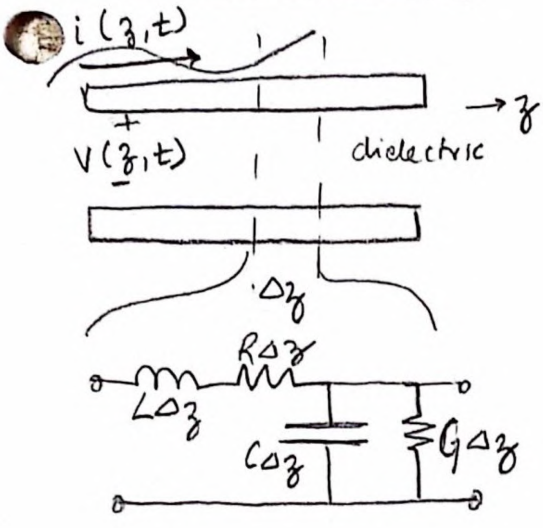
b_l +ve ⇒ capacitor in parallel ; b_l −ve ⇒ inductor in parallel.

$C_{parallel} = \frac{b}{2\pi f Z_0}$ $\qquad C_{series} = \frac{-1}{2\pi f Z_0 x}$

$L_{parallel} = \frac{-Z_0}{2\pi f b}$ $\qquad L_{series} = \frac{x Z_0}{2\pi f}$

S - parameters

Transmission lines



$$V(z) = \overset{\text{forward wave}}{V_0^+ e^{-\gamma z}} + \overset{\text{backward wave}}{V_0^- e^{+\gamma z}}$$

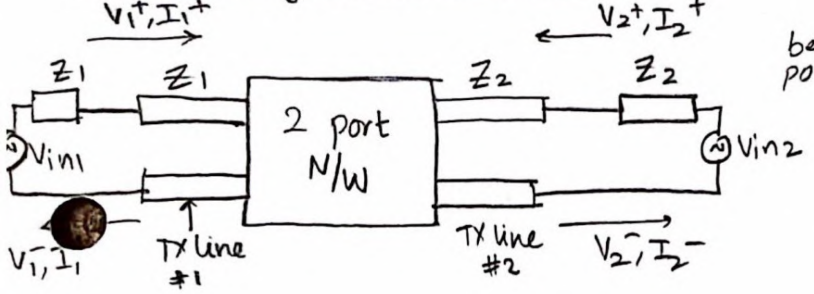
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = Z_0 = \frac{R+j\omega L}{\gamma}$$

↑ characteristic impedance.

Scattering parameters



because power →
$$\frac{V_1^-}{\sqrt{Z_1}} = S_{11}\frac{V_1^+}{\sqrt{Z_1}} + S_{12}\frac{V_2^+}{\sqrt{Z_2}}$$

$$\frac{V_2^-}{\sqrt{Z_2}} = S_{21}\frac{V_1^+}{\sqrt{Z_1}} + S_{22}\frac{V_2^+}{\sqrt{Z_2}}$$

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Oftenly used notation: $a_1 \triangleq \frac{V_1^+}{\sqrt{Z_1}}$; $b_1 \triangleq \frac{V_1^-}{\sqrt{Z_1}}$

$a_2 \triangleq \frac{V_2^+}{\sqrt{Z_2}}$; $b_2 \triangleq \frac{V_2^-}{\sqrt{Z_2}}$

$$\rightarrow S_{11} = \frac{V_1^-}{V_1^+}\Bigg|_{\substack{V_2^+=0 \\ (Vin_2=0)}} = \Gamma_{L1}$$

V_2^- maybe ≠0.

→ S_{11} is a measure of i/p mismatch.

$$\rightarrow S_{21} = \sqrt{\frac{Z_1}{Z_2}}\,\frac{V_2^-}{V_1^+}\Bigg|_{\substack{V_2^+=0 \\ Vin_2=0}}$$

* S_{21} is a measure of gain from P① to P②

$$\rightarrow S_{22} = \frac{V_2^-}{V_2^+}\Bigg|_{\substack{V_1^+=0 \\ Vin_1=0}} = \Gamma_{L2}$$ {Output mismatch

$$\rightarrow S_{12} = \sqrt{\frac{Z_2}{Z_1}}\,\frac{V_1^-}{V_2^+}\Bigg|_{\substack{V_1^+=0 \\ Vin_1=0}}$$ {Reverse gain}

Eg:- Amplier that is matched!

$$S = \begin{bmatrix} 0 & \text{small} \\ 3.16\angle\phi & 0 \end{bmatrix}$$

small → reverse gain

φ → due to delay, some phase is added.

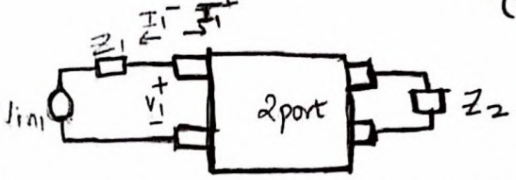
→ How can we measure from value of sources?

$$\left.\begin{aligned} V_1 &= V_1^+ + V_1^- \\ I_1 &= I_1^+ + I_1^- = \frac{1}{Z_1}(V_1^+ - V_1^-) \end{aligned}\right\} \text{Tx-L equations} \Rightarrow V_1^+ = \frac{1}{2}(V_1 + ZI_1)$$

$$\Rightarrow V_1^+ = \frac{1}{2} V_{in1}$$

$$\Rightarrow S_{21} = 2\sqrt{\frac{Z_1}{Z_2}} \cdot \frac{V_2^-}{V_{in1}} \Bigg|_{\substack{V_2^+=0 \\ (V_{in2}=0)}}$$

$$S_{12} = 2\sqrt{\frac{Z_2}{Z_1}} \cdot \frac{V_1^-}{V_{in2}} \Bigg|_{\substack{V_1^+=0 \\ (V_{in1}=0)}}$$



But what is the physical meaning of S parameters?

Available source power: $P_{av} = \frac{|V_{in1}|^2}{4Z_1} = |a_1|^2$ since $a_1 \triangleq \frac{V_1^+}{\sqrt{Z_1}} = \frac{V_{in1}}{2\sqrt{Z_1}}$

Average power delivered to P1: $P \triangleq \frac{1}{2} Re\{V_1 I_1^*\} \Rightarrow P_1 = \frac{1}{2} Re\{|a_1|^2 - |b_1|^2 + (a_1^* b_1 - a_1 b_1^*)\}$

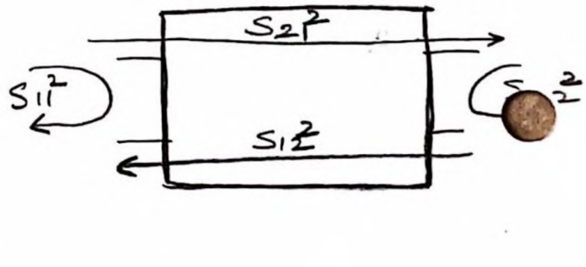
$$\Rightarrow P_1 = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2 \Rightarrow \boxed{\underbrace{\frac{1}{2}|b_1|^2}_{\text{Reflected power}} = \underbrace{\frac{P_{av}}{2}}_{\text{max. power delivered to load when matched}} - P_1}$$

→ Power that is actually delivered

$$\Rightarrow S_{11}^2 = \frac{|b_1|^2}{|a_1|^2} = \frac{\text{Reflected power at P1}}{\text{Max. available source power that can be delivered to a matched load at P1}}$$

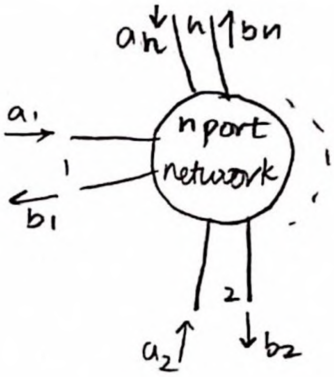
$$S_{21} = \frac{b_2}{a_1}\Bigg|_{\substack{a_2=0 \\ V_2^+=0}} \Rightarrow \frac{\frac{\overset{\leftrightarrow V_2^-}{V_2}}{\sqrt{Z_2}}}{\frac{V_{in1}}{2\sqrt{Z_1}}}\Bigg|_{\substack{a_2=0 \\ V_2^+=0}} \Rightarrow |S_{21}| = \frac{\frac{|V_2|^2}{2Z_2}}{\frac{1}{2}\frac{|V_{in}|^2}{4Z_1}}$$

$$\Rightarrow |S_{21}|^2 = \frac{\text{power delivered to load at P2}}{\text{Max available power at the source that can be delivered to a matched load}}$$



→ S_{22}, S_{12} are vice versa.

Properties of S parameters



① P_i = Total power delivered to port i.

$$\underbrace{P_i}_{\text{power delivered}} = \frac{1}{2}(\underbrace{a_i^* a_i}_{\text{power incident}} - \underbrace{b_i^* b_i}_{\text{power reflected}})$$

Total power delivered to n port network

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \frac{1}{2}(a_i^* a_i - b_i^* b_i)$$

$$P_{total} = \frac{1}{2}[a]^T[a]^* - \frac{1}{2}[b]^T[b]^* \quad \text{where } [a] = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}; [b] = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

We know $[b] = \underset{n\times n}{[S]}[a]$

$$\Rightarrow P_{tot} = \frac{1}{2}[a]^T[a^*] - \frac{1}{2}[a]^T[S]^T[S]^*[a]^* = \frac{1}{2}[a]^T\left\{\overset{\text{unit matrix}}{\underset{n\times n}{[U]}} - [S]^T[S]^*\right\}[a$$

⇒ If network is lossless $P_{tot} = 0$

$$\Rightarrow \boxed{[S]^T[S]^* = [U]}$$

For lossless networks, S is a unitary matrix.

In two port:

$$|S_{11}|^2 + |S_{21}|^2 = 1$$ → all power goes from P1 to P2

$$|S_{12}|^2 + |S_{22}|^2 = 1$$ ← all pwr goes from P2 to P1

$$S_{11}S_{12}^* + S_{21}S_{22}^* = 0$$

All 3 conditions must be Satisfied

② For a reciprocal network: $[S]^T = [S] \Rightarrow S_{12} = S_{22}$ $[\nearrow]$

③ For a matched network $S_{ii} = 0 \Rightarrow \begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix}$

Note:- A 3 port network cannot be matched, lossless & reciprocal at once.

Useful trig identities.

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$$

$$\cos^3 A = \frac{3}{4}\cos A + \frac{1}{4}\cos 3A$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin^3 A = \frac{3}{4}\sin A - \frac{\sin 3A}{4}$$

Noise

Math prelims

mean $\overline{x} = E\{x\} = \int_{-\infty}^{\infty} x \cdot f_x(x)\,dx = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,dt$

$$\overline{x^2} = E\{x^2\} = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx \quad \lim_{\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)\,dt$$

Variance

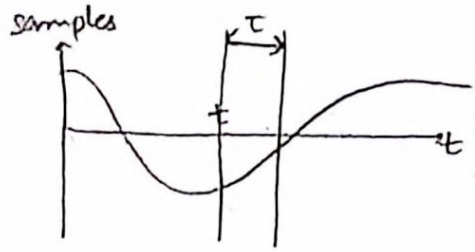
$$\sigma_x^2 = E\{(x-\overline{x})^2\} = \overline{x^2} - \overline{x}^2$$

Stationary :- Statistical properties do not change with time

Ergodic :- Ensemble average = Time average.

Autocorrelation

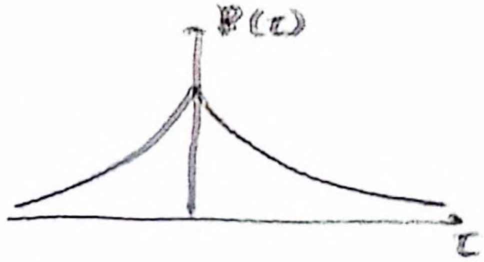
$$R(\tau) = \int_{-\infty}^{\infty} x^*(t)\,x(t+\tau)\,dt$$



→ move this window from -∞ to ∞ and see how similar the two points are.

→ For a stationary R.P it is only a fn of τ

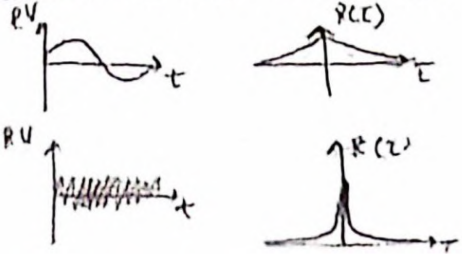
For a random variable or stationary R.P



→ $R(\tau) = E\{x^*(t)\,x(t+\tau)\}$

→ Wideband R.V ⇒ narrow $R(\tau)$ since it quickly becomes dissimilar as τ increases.

→ |||ly Narrowband R.V ⇒ wide $R(\tau)$



Power Spectral Density.

$$S_x(f) = F \cdot T\left\{ R_x(T) \right\} \longrightarrow \frac{\overline{V^2}}{\Delta f} \text{ or } \frac{\overline{i^2}}{\Delta f}$$

→ $\int_{-\infty}^{\infty} S_x(f)\,df$ gives total power • Note $\underset{\text{accounts for -ve freq}}{2}\int_{f_1}^{f_2} S_x(f)\,df$ gives total power in band f_1 to f_2.

→ For LTI Systems

$$\boxed{S_y(f) = S_x(f)\,|H(j\omega)|^2}$$

> From fluctuation dissipation & Equipartition theorems:

$\boxed{S_v(f) = 4KTR}$ - Thermal noise of a resistor

Noise correlation

Correlation factor

$$\rho_{xy} = \frac{\overset{\to\ \text{cross correlation}}{E\{x y^*\}}}{\sqrt{\underbrace{E\{|x^2|\}}_{\text{power of } x}\underbrace{E\{|y^2|\}}_{\text{power of } y}}} = \frac{E\{x y^*\}}{|x|\,|y|} \qquad \because \sqrt{E\{|x^2(t)|\}} = |x(t)|_{rms}$$

$$\underset{\text{uncorrelated}}{0} \le \rho \le \underset{\text{fully correlated}}{1}$$

If $\rho_{xy} = 0 \Rightarrow P_{tot} = P_x + P_y = \frac{V_x^2}{R} + \frac{V_y^2}{R}$

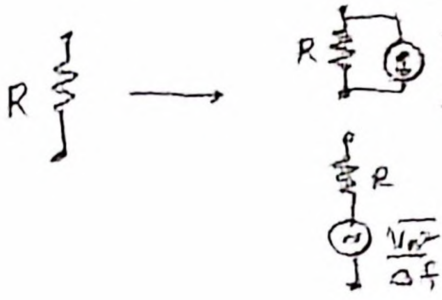
If $\rho_{xy} = 1 \Rightarrow P_{tot} \neq P_x + P_y$

$$V_{tot} = V_x + V_y$$

$$\Rightarrow P_{tot} = \frac{(V_x + V_y)^2}{R}$$

Noise Types

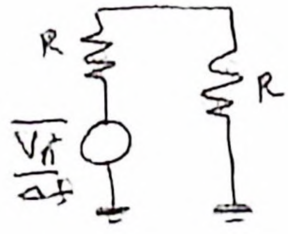
① Thermal Noise



$$\frac{\overline{I_n^2}}{\Delta f} = \frac{4KT}{R}$$

$$\frac{\overline{V_n^2}}{\Delta f} = 4KTR$$

Example ① Available noise power

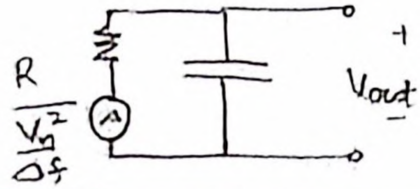


$$\Rightarrow \underset{1Hz}{P} = \frac{\overline{V_n^2}}{4R} = \frac{4KTR}{4R} = \underline{\underline{KT}}$$

$$= \underline{\underline{4\times10^{-21}}} \ \frac{W}{Hz}$$

$$P_{avail} = -174 \ dBm/Hz$$

Example ②



$$V_{out} = V_n \frac{1}{RSC+1}$$

$$\frac{\overline{V_{out}^2}}{\Delta f} = \frac{\overline{V_n^2}}{\Delta f}\left|\frac{1}{RSC+1}\right|^2$$

$$\overline{V_n^2}\left[\frac{1}{1+R^2C^2\omega^2}\right]$$

$$\frac{\overline{V_{out}^2}}{\Delta f} = \frac{4KTR}{1+R^2C^2\omega^2}$$

$$\Rightarrow \overline{V_{out}^2} = \int_0^{+\infty}\frac{4KTR}{1+R^2C^2\omega^2} = \underline{\underline{\frac{KT}{C}}} \quad \text{Independent of } R$$

Solution ②

Equipartition theorem.

$$\frac{1}{2}KT = \frac{1}{2}C\overline{V_{out}^2}$$

$$\Rightarrow \overline{V_{out}^2} = \frac{KT}{C}$$

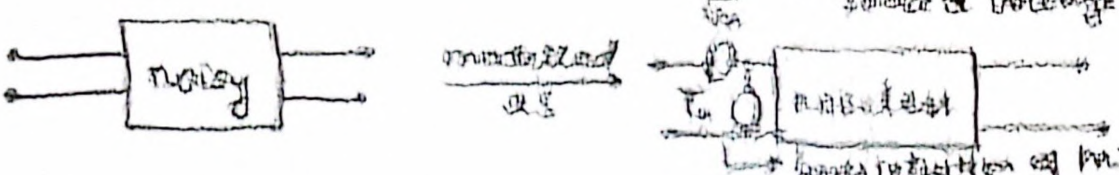
Avg. charge on capacitor

$$Q_n = \sqrt{\frac{KT}{C}}\,C = \sqrt{KTC}$$

- Lowpass ⇒ As R↑, BW↓ ⇒ total power is constant
  $\boxed{\frac{\overline{V_{out}^2}}{\Delta f}}$
- In reality thermal noise is white upto $f \le \frac{KT}{h}$. Beyond we need to use the Bose Einstein distribution

Noise in 2 port

→ Need both to account for i/p being current source & voltage source.



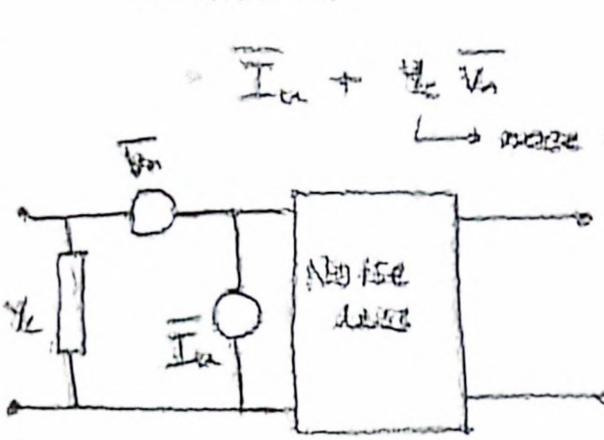
↳ gives indication of minimum signal needed at i/p

> To fully describe noise in circuit we need 4 parameters

$|V_n|$, $|I_n|$ and Re & Im part of $\rho_{V_n I_n}$

> Let $\overline{I_n} = \overline{I_u} + \overline{I_c}$ → fully correlated
($\overline{I_u}$ ↓ uncorrelated)

$= \overline{I_u} + Y_c \overline{V_n}$
↳ noise correlation admittance (Assuming linearity).



$$\rho_{V_n I_n} = \frac{\overline{I_n V_n^*}}{|\overline{I_n}||\overline{V_n}|}$$

$$Y_c = \frac{\overline{I_n V_n^*}}{|\overline{V_n}|^2}$$

$$\left[\because \overline{I_n V_n^*} = \overset{0}{\cancel{\overline{I_u V_n^*}}} + \overline{V_n}\,\overline{V_n^*} \cdot Y_c \quad \text{or } Y_c = \frac{\overline{I_n V_n^*}}{|V_n|^2} \right]$$

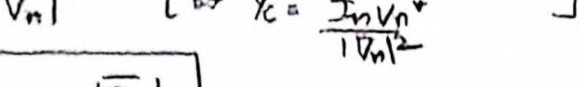
$$\boxed{Y_c = \rho_{V_n I_n} \frac{|\overline{I_n}|}{|\overline{V_n}|}}$$

Noise Factor (F) → how much noise is circuit adding.

$$F = \frac{SNR_{i/p}}{SNR_{o/p}} = \frac{S_i/N_i}{GS_i / (GN_i + N_a)} = 1 + \frac{N_a}{GN_i} \geq 1 \quad — ①$$

$$F = \frac{\text{noise power at o/p (total)}}{\text{noise power at o/p due to input}} = F_1 + \frac{F_2 - 1}{G_1} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

for cascaded stages.



G is power gain

∴ First stage should be LNA

Noise Figure (NF)

$NF = 10 \log F \geq 0 dB$

Noise temperature (T_e)

→ Na (noise of amplifier) is modelled as a resistor at temperature Te

$$N_a = (K T_e \Delta f) G.$$

$$\Rightarrow T_e = \frac{N_a}{K G \Delta f} \quad — ②$$

From ① and ② $F = 1 + \frac{T_e}{T}$

$$(F-1)\underset{\substack{\uparrow \\ \text{actual} \\ \text{temp.}}}{T} = \underset{\substack{\uparrow \\ \text{Equivalent} \\ \text{noise temp.}}}{T_e}$$

Noise measure (Noise vs. gain)

$$\underset{\substack{\swarrow \\ \text{Low} \Rightarrow \text{good.}}}{M} = \frac{F-1}{1-\frac{1}{G}}$$

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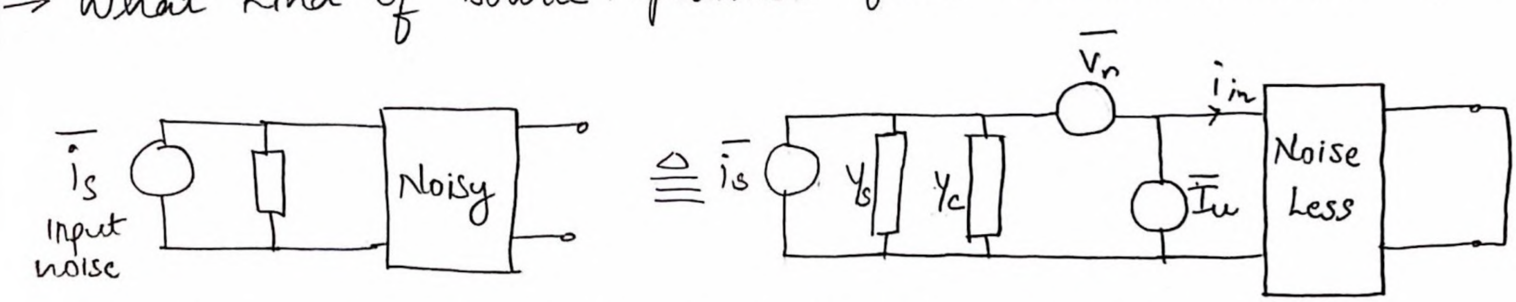
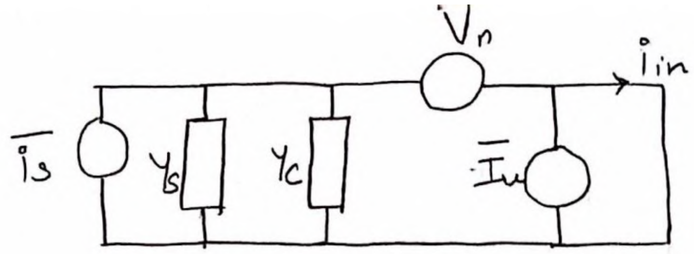
* Noise matching *

→ What kind of source impedance gives minimum noise at O/P.



→ Short O/P since noise is independant of Zout

→ Circuit is noiseless so we can ignore it for our noise analysis



$$F = \frac{\overline{i_{in}^2}_{total}}{\overline{i_{in}^2}_{\,Yp}} = \frac{\overline{|i_s + I_u + (Y_c+Y_s)V_n|^2}}{\overline{i_s^2}}$$

$$= \frac{i_s^2 + I_u^2 + (Y_c+Y_s)^2 V_n^2 + 0 + 0 + 0}{\overline{i_s^2}}$$

$$\boxed{F = 1 + \frac{\overline{I_u^2} + |Y_s+Y_c|^2\,\overline{V_n^2}}{\overline{i_s^2}}}$$

Equivalent noise impedances

$\overline{V_n^2} = 4KTR_n\Delta f \longrightarrow$ defined as a source for V_n

$\overline{I_u^2} = 4KTG_u\Delta f \longrightarrow$ source for I_u

$\overline{i_s^2} = 4KTG_s\Delta f \longrightarrow$ real part of $Y_s = G_s + jB_s$

$Y_s = G_s + jB_s$

$Y_c = G_c + jB_c$

$$\Rightarrow \quad F = 1 + \frac{G_u + |Y_c+Y_s|^2 R_n}{G_s}$$

$$F = 1 + \frac{G_u + \left[(G_c+G_s)^2 + (B_c+B_s)^2\right]R_n}{G_s}$$

→ Find Y_s optimum to minimize F.

Set, $\frac{dF}{dB_s} = 0$ & $\frac{dF}{dG_s} = 0$ and solve.

$\Rightarrow$ $B_S = -B_C = B_{Sopt}$.

$$G_S = \sqrt{\frac{G_u}{R_n} + G_c^2} = G_{Sopt}$$

if uncorrelated

$$\boxed{F_{min} = 1 + 2R_n(G_{Sopt} + G_c)} \boxed{= 1 + 2\sqrt{R_n G_u}}$$

$$\boxed{F = F_{min} + \frac{R_n}{G_S}\left[(G_S - G_{Sopt})^2 + (B_S - B_{Sopt})^2\right]}$$

R_n = Input referred equivalent noise resistance for $\overline{V_n}$

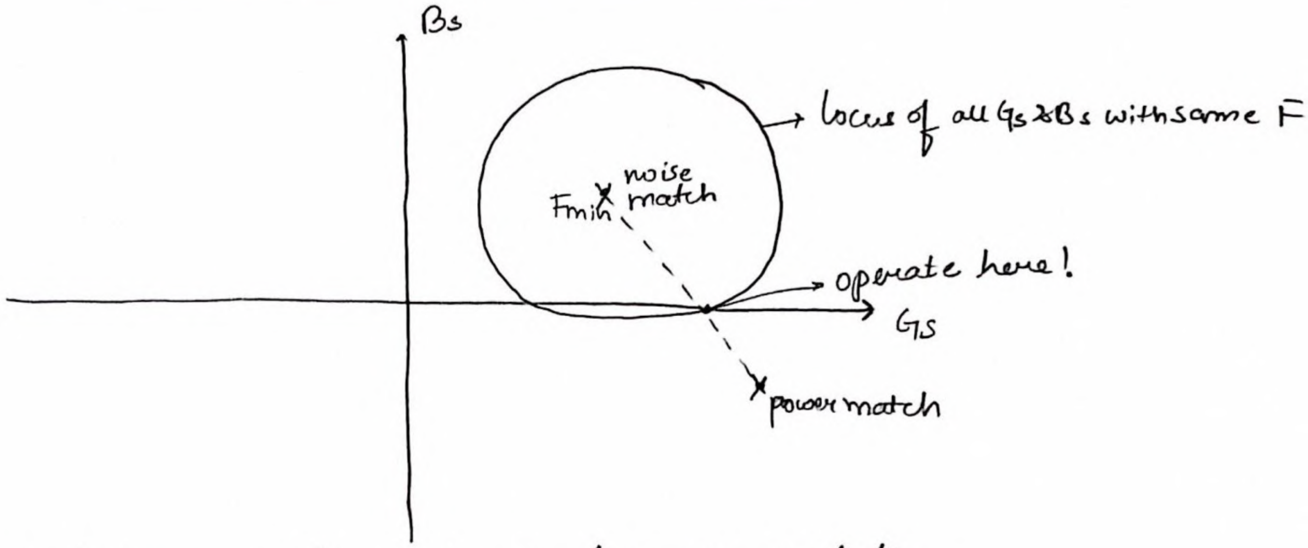
B_S = Source susceptance

B_C = i/p correlated susceptance.

G_S = Source conductance

G_u = uncorrelated i/p conductance

G_c = correlated i/p conductance.



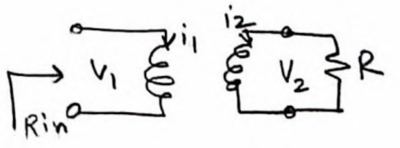
→ Tradeoff between noise match and power match.

→ There is an optimum input Y_s admittance to get best noise factor!

$$R_n = \frac{\overline{V_n^2}/\Delta f}{4kT}$$

$$G_u = \frac{\overline{I_u^2}/\Delta f}{4kT}$$

Note on transformer matching



$$\frac{|V_1|}{|V_2|} = \frac{1}{n} \qquad \frac{|I_1|}{|I_2|} = n$$

$$\Rightarrow \boxed{R_{in} = \frac{R}{n^2}}$$

Noise sources in MOSFET

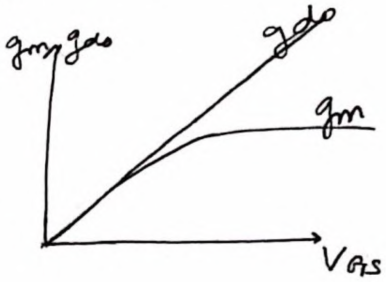
1) Thermal noise of channel :

$$\boxed{\overline{i_{nd}^2} = 4KT\gamma g_{do} \Delta f}$$

at the drain

$$g_{do} = \left.\frac{\partial I_d}{\partial V_{gs}}\right|_{V_{DS}=0} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})$$

conductance of channel



$g_{do} = g_m$ for long channel devices.

$g_{do} = \frac{g_m}{\alpha}$ for short channel where $\alpha \leq 1$

→ g_m & g_{do} are fundamentally different.

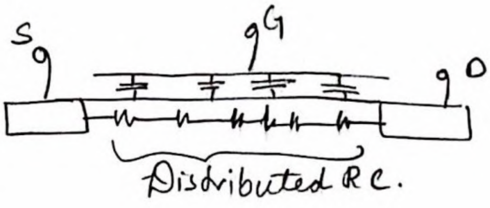
→ γ models variation due to CLM 

$\gamma = 1$ ($V_{DS}=0$) channel is flat.

$\gamma = \frac{2}{3}$ (saturation & long channel)

$\gamma = 2-6$ (saturation & short channel) since charge moves in a discrete lattice & tunneling occurs.

2) Gate noise :



$$g_g = \frac{\omega^2 C_{gs}^2}{5 g_{do}}$$

$$\Rightarrow \boxed{\overline{i_{ng}^2} = 4KT\delta g_g \Delta f} \qquad \delta = 2\gamma$$

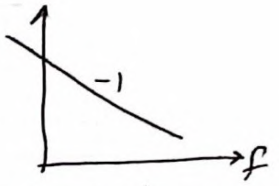
→ g_g & g_{do} are correlated ⇒ correlation factor 'c'

$$c = \frac{\overline{i_{ng}\, i_{nd}^*}}{|\overline{i_{ng}}|\,|\overline{i_{nd}}|} \simeq \underline{\underline{-j\,0.395}}$$

3) Flicker noise (not much in BJT)

$$\overline{i_{nd}^2} = \frac{k}{f} \cdot \frac{g_m^2}{WL\, C_{ox}^2} \Delta f$$ not white

↳ Empirical model



power law.

4) Shot noise (more in BJT not MOSFET)

→ Discrete transitions across barrier when current flows.

→ It is a poisson R.V

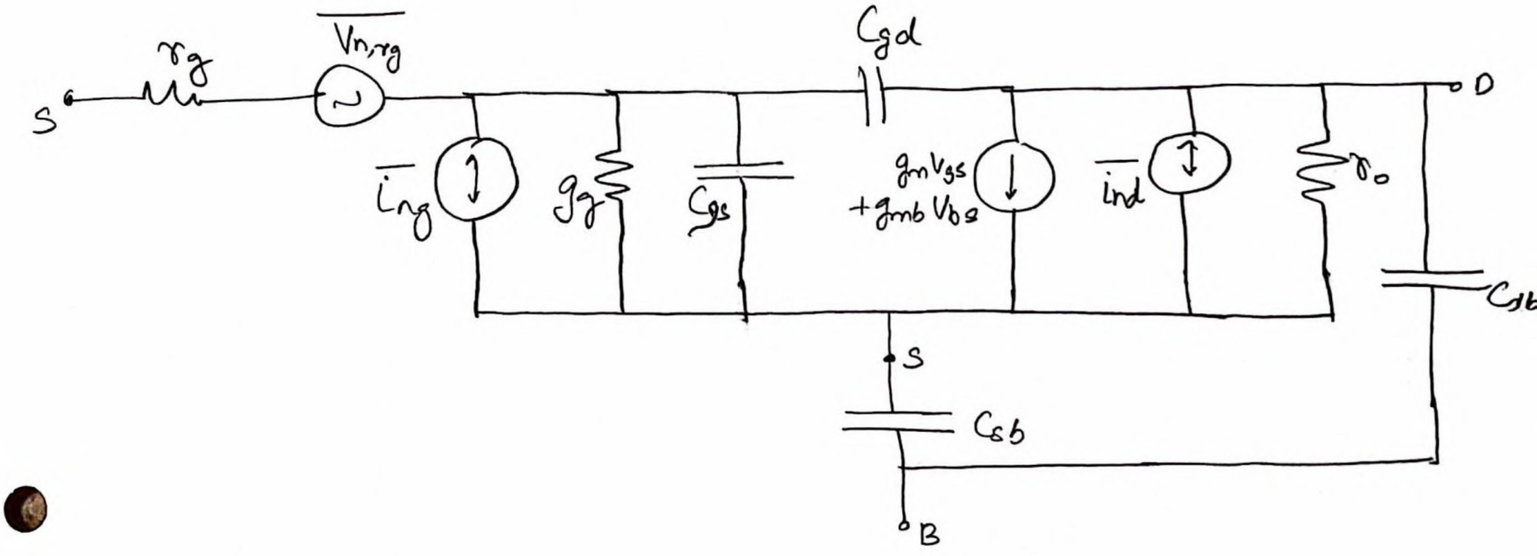
$$S_i(0) = 2\overset{\text{rate of events}}{\lambda}\; \underset{\text{weight of one event}}{|\varphi(0)|^2}$$

$$= 2\left(\frac{I}{q}\right) q^2$$

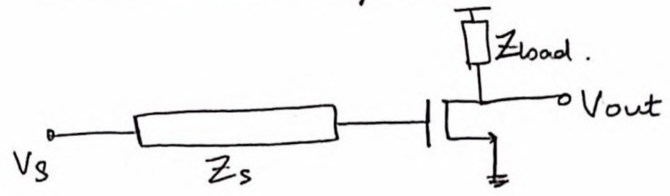
$$= 2Iq$$

$$\Rightarrow \boxed{\overline{i_n^2} = 2qI\Delta f}$$

Noise model of MOSFET

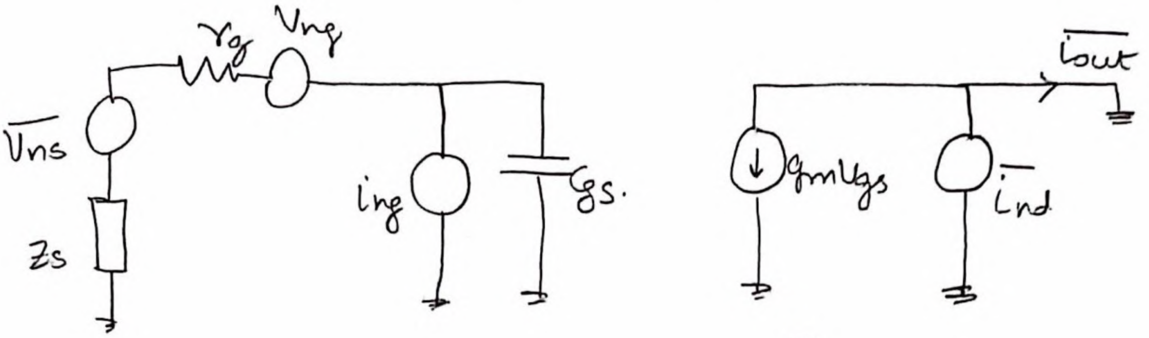


Noise example



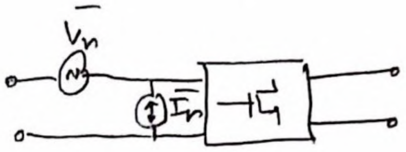
Find: $\overline{V_n}$, $\overline{I_n}$, Y_c

Solution : Ignoring C_{gd}, r_o, Z_L, g_g



To find $\overline{V_n}$: Short input

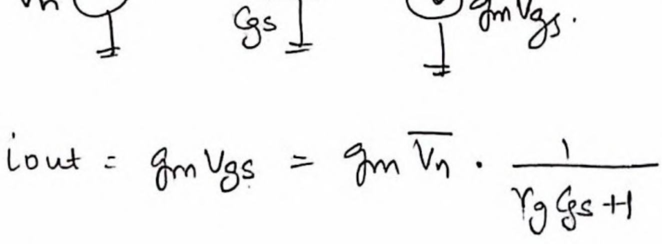
To find $\overline{I_n}$: Open input



$V_n = ?$

$$i_{out} = \overline{i_{nd}} + g_m v_{gs} = \overline{i_{nd}} + g_m \overline{i_{ng}} \cdot \frac{r_g}{r_g C_{gs} s+1} + g_m \overline{V_{ng}} \frac{1}{r_g C_{gs} s+1} \quad ①$$

Replace all noise sources by $\overline{V_n}$



$$i_{out} = g_m V_{gs} = g_m \overline{V_n} \cdot \frac{1}{r_g C_{gs}+1} \quad ②$$

① = ② ⇒

$$\Rightarrow \overline{V_n} = \frac{r_g C_{gs} S+1}{g_m} + \overline{i_{nd}} + r_g \overline{i_{ng}} + \overline{V_{ng}}$$

III^ly $\overline{I_n} = \overline{i_{ng}} + \frac{S C_{gs}}{g_m} \overline{i_{nd}}$

Note :- While finding $\overline{V_n}$, $\overline{I_n}$ use mean values of noise sources.
For noise factor use 2nd moment (power)

But, $\overline{V_n}$, $\overline{I_n}$ are correlated

$$\overline{I_n} = \overline{I_u} + Y_c \overline{V_n}$$

$$\overline{I_n \cdot V_n^*} = 0 + Y_c |\overline{V_n}|^2$$

$$\Rightarrow Y_c = \frac{\overline{\overline{I_n}\,\overline{V_n^*}}}{|\overline{V_n}|^2}$$

Assume $Y_g = 0$ (make it easier)

$$\overline{V_n}^* = \frac{1}{g_m} \overline{i_{nd}}^* \quad \left\{ \begin{array}{l} \text{since } \overline{V_n V_n^*} = |V_n|^2 = \frac{1}{g_m^2} |i_{nd}|^2 \\ \Rightarrow \overline{V_n} = \frac{1}{g_m} \overline{i_{nd}} \end{array} \right.$$

$$\overline{I_n} = \frac{sC_{gs}}{g_m} \overline{i_{nd}} + \overline{i_{ng}}$$

$$\Rightarrow \overline{I_n V_n^*} = \overline{\left(\frac{sC_{gs}}{g_m} \overline{i_{nd}} + \overline{i_{ng}}\right)\left(\frac{1}{g_m} \overline{i_{nd}^*}\right)}$$

$$= \frac{sC_{gs}}{g_m^2} \overline{|i_{nd}|^2} + \frac{1}{g_m} \overline{i_{ng}\, i_{nd}^*}$$

$$Y_c = \frac{\overline{I_n V_n^*}}{|V_n|^2} = \frac{\frac{sC_{gs}}{g_m^2} \overline{i_{nd}^2} + \frac{1}{g_m} \overline{i_{ng}\, i_{nd}^*}}{\frac{1}{g_m^2} \cdot \overline{i_{nd}^2}}$$

$$= sC_{gs} + g_m \frac{\overline{i_{ng}\, i_{nd}^*}}{\overline{i_{nd}^2}}$$

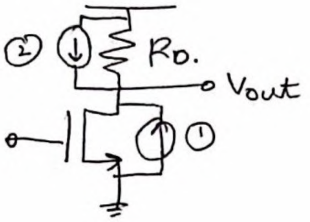
$$= sC_{gs} + g_m \frac{\overline{i_{ng}\, i_{nd}^{*}}}{\sqrt{\overline{i_{nd}^2}}\,\sqrt{\overline{i_{nd}^2}}} \cdot \sqrt{\frac{\overline{i_{ng}^2}}{\overline{i_{ng}^2}}}$$

$$= sC_{gs} + g_m \cdot c \cdot \sqrt{\frac{\overline{i_{ng}^2}}{\overline{i_{nd}^2}}} \quad \text{where } c = \frac{\overline{i_{ng}\, i_{nd}^{T}}}{\overline{i_g}\ \overline{i_{nd}}}$$

$$\Rightarrow Y_c = sC_{gs} + g_m . c \sqrt{\frac{\delta g_g}{\gamma g_{do}}}$$

$$\boxed{Y_c = j\omega C_{gs} \left[1 - \frac{g_m}{g_{do}} |c| \sqrt{\frac{\delta}{5\gamma}} \right]}$$

Example :



$\gamma = \frac{2}{3}$

$\overline{i_{ng}} = 0$

$g_g = 0$

$g_m - g_{do}$

Find $\overline{V_n}$? **Method ①**

$$\frac{\overline{V_{n\,out}^2}}{\Delta f} = \left(4KT.\frac{2}{3} g_m + \frac{K}{C_{ox}WL} \frac{1}{f} g_m^2 + \frac{4KT}{R_D} \right)$$

$$\frac{\overline{V_{n_{in}}^2}}{\Delta f} = \frac{\frac{\overline{V_{n_{out}}^2}}{\Delta f}}{A_v^2} = \frac{\overline{V_{n,out}^2}}{\Delta f (g_m R_D)^2}$$

$$\frac{\overline{V_{n_{in}}^2}}{\Delta f} = 4KT \frac{2}{3g_m} + \frac{K}{C_{ox}WL} \cdot \frac{1}{f} + \frac{4KT}{g_m^2 R_D}$$

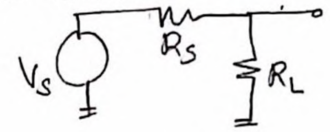
Method ② From previous example.

$$\overline{V_n^2} = \frac{1}{g_m^2} \overline{i_{nd}^2} = \frac{1}{g_m^2} \cdot 4KT.\frac{2}{3} g_m + \frac{k}{C_{ox}WLf} g_m^2 + \frac{1KT}{R_D}$$

(Assuming $g_m = g_{do}$)

$$\overline{V_n^2} = \frac{4KT.2}{3g_m} + \frac{k}{C_{ox}WLf} + \frac{4KT}{g_m^2 R_D}$$

Example :

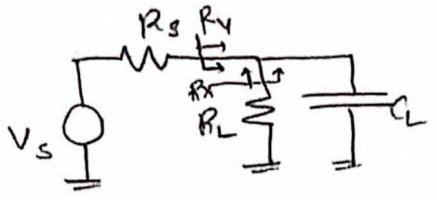


Noisefactor F?

$$\overline{V_{n,tot}^2}_{out} = \overline{V_{n_{RS}}^2} \left(\frac{R_L}{R_L + R_S} \right)^2 + \overline{V_{n_{R_L}}^2} \left(\frac{R_S}{R_S + R_L} \right)^2 \ \& \ \overline{V_{n\,out}^2}_{(i/p)} = \overline{V_{n_{R_S}}^2} \left(\frac{R_L}{R_L + R_S} \right)^2$$

$$\Rightarrow \boxed{F = 1 + \frac{R_S}{R_L}}$$

Example:



$F = ?$

$$R_X = R_S \parallel \frac{1}{SC_L} = \frac{R_S}{1+R_S SC_L}$$

$$R_Y = \frac{R_L}{1+SR_LC_L}$$

$$\frac{\overline{V_{out}^2}}{\Delta f}_1 = \left|\frac{R_X}{R_X+R_L}\right|^2 \overline{V_{nL}^2} \quad -①$$

$$\frac{\overline{V_{out}^2}}{\Delta f}_2 = \left|\frac{R_Y}{R_Y+R_S}\right|^2 \overline{V_{nS}^2}$$

From ①:

$$\frac{R_X}{R_L+R_X} = \frac{\frac{R_S}{1+SR_SC_L}}{\frac{R_S}{1+SR_SC_L}+R_L} = \frac{R_S}{(R_S+R_L)+SR_LR_SC_L}$$

$$\Rightarrow \left|\frac{R_X}{R_L+R_X}\right|^2 = \left(\frac{R_S}{R_S+R_L}\right)^2 \left|\frac{1}{1+S\frac{R_LR_SC_L}{R_S+R_L}}\right|^2 = \left(\frac{R_S}{R_S+R_L}\right)^2 \left\{\frac{1}{1+\omega^2\left(\frac{R_SR_LC}{R_S+R_L}\right.}\right.$$

$$\Rightarrow \frac{\overline{V_{out}^2}}{\Delta f}_1 = \left(\frac{R_S}{R_S+R_L}\right)^2 \left\{\frac{1}{1+f^2\left(\frac{2\pi R_SR_LC_L}{R_S+R_L}\right)^2}\right\} 4KTR_L$$

$$\boxed{\int\frac{1}{1+a^2x^2} = \frac{\tan^{-1}ax}{a}}$$

$$\Rightarrow \overline{V_{out_1}^2} = 4KTR_L \cdot \left(\frac{R_S}{R_S+R_L}\right)^2 \int_0^\infty \left\{\frac{1}{1+f^2\left(\frac{2\pi R_SR_LC_L}{R_S+R_L}\right)^2}\right\}df$$

$$= \cancel{4}KT\cancel{R_L}\left(\frac{R_S\cancel{R_S}}{\cancel{R_S+R_L}\,(R_S+R_L)}\right)^{\cancel{2}} \cdot \frac{\cancel{R_S+R_L}}{\cancel{2\pi}\,\cancel{R_S}\,\cancel{R_L}\,C_L} \cdot \frac{\cancel{\pi}}{\cancel{2}}$$

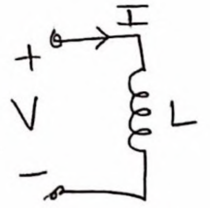
$$\overline{V_{out_1}^2}_{(Load)} = \frac{KT}{C_L}\left[\frac{R_S}{R_S+R_L}\right]$$

$$\overline{N_{out_2}^2} = \frac{KT}{C_L}\left[\frac{R_L}{R_L+R_S}\right]$$

$$NF = \frac{\frac{\cancel{KT}}{\cancel{C_L}}}{\frac{\cancel{KT}}{\cancel{C_L}}\left[\frac{R_L}{R_S+R_L}\right]} = \boxed{1+\frac{R_S}{R_L}}$$

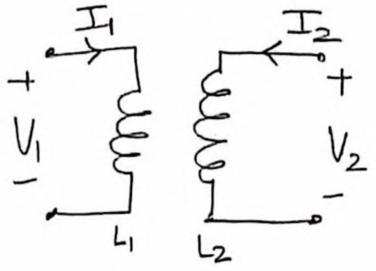
same as before

Quick note of passive (inductors) for midterm



$\phi = LI$

$V = sLI$



$\phi_1 = L_1 I_1 + M I_2$

$\phi_2 = M I_1 + L_2 I_2$

$$\Rightarrow [\phi] = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} [I]$$

$V_1 = sL_1 I_1 + sMI_2$

$V_2 = sMI_1 + sL_2 I_2$

$$\text{coupling factor} = \frac{M}{\sqrt{L_1 L_2}} = K$$

$= 1 \Rightarrow$ perfect coupling.

$$\Rightarrow \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = s \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

How to find NF?

Step① Small signal model.

Step② Draw out all noise currentsources.

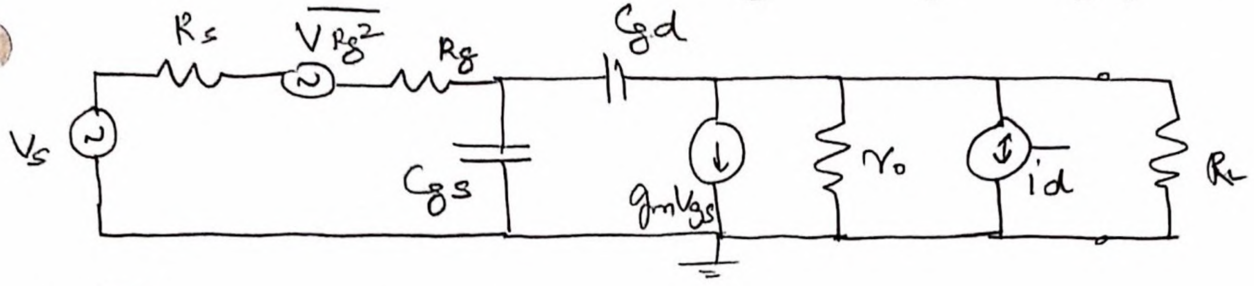
Step③ Short the output since NF is independent of load (It is a ratio) $\frac{I_{tot}^2 Z_{load}}{I_{np}^2 Z_{load}}$

Step④ Find contribution of each independent noise source (null all other independent sources)

Step⑤ Add powers to find $\overline{i_{out,tot}^2}$ & add amplitudes if correlated.

Step⑥ $NF = \frac{tot\ \overline{I^2}}{}$.

• FET Noise Factor {using a different style of solving for NF}



R_s: $\overline{V_s^2} = 4KTR_s\Delta f$

R_g: $\overline{V_g^2} = 4KTR_g\Delta f$

R_{ch}: $\overline{i_d^2} = 4KT\gamma g_{d0}\Delta f$

R_L: $\overline{i_L^2} = 4KTG_L\Delta f$

Input noise is drawn as voltages since we can just multiply them by gm and they would show up at o/p at low frequencies since Cgs would be open.

At low freq :- Cgs is open => $\overline{V_s}$ & $\overline{V_g}$ show up as ($\times g_m$) at $\overline{i_{out}}$.

$$\Rightarrow \overline{i_o^2} = \overline{i_d^2} + \overline{i_L^2} + \left(\overline{v_g^2} + \overline{v_s^2}\right)g_m^2$$

$$\Rightarrow F = 1 + \frac{\overline{V_g^2}}{\overline{V_s^2}} + \frac{\overline{i_d^2} + \overline{i_L^2}}{g_m^2\,\overline{v_s^2}}$$

$$\boxed{F = 1 + \frac{R_g}{R_s} + \frac{g_{d0}\gamma + G_L}{R_s g_m^2}}$$

$g_m = g_{d0}$ (long channel)

$$\Rightarrow \boxed{F = 1 + \frac{R_g}{R_s} + \frac{\gamma}{g_m R_s} + \frac{G_L G_s}{g_m^2}}$$

Higher frequencies: We know, $g_m = \frac{i_{out}}{V_{gs}}$ (since id shows directly at iout since source is gr

Let $G_m = \frac{V_{gs}}{V_s}$

$$\Rightarrow \frac{i_{out}}{V_s} = \frac{i_{out}}{V_{gs}} \times G_m = g_m \cdot G_m.$$

⇒ To refer the input noise sources directly to output current scale g_m by G_m which is just a voltage divider!

$$G_m = \frac{1/j\omega C_{gs}}{1/j\omega C_{gs} + R_s + R_g} = \frac{1}{1 + j\omega C_{gs}(R_s + R_g)}$$

$$\Rightarrow |G_m|^{-2} = 1 + \omega^2 C_{gs}^2 (R_s + R_g)^2$$

From earlier, $F = 1 + \frac{R_g}{R_s} + \frac{g_{d0}\gamma + G_L}{R_s \underbrace{g_m^2}_{\rightarrow \text{scale this.}}}$

$$F = 1 + \frac{R_g}{R_s} + \frac{g_{d0}\gamma + G_L}{R_s g_m^2} \left[|G_m|^{-2} \right]$$

$$\Rightarrow F = 1 + \frac{R_g}{R_s} + \frac{g_{d0}\gamma + G_L}{R_s g_m^2} \left(1 + \omega^2 C_{gs}^2 (R_s + R_g)^2 \right).$$

Assume $\underset{\Rightarrow (R_L \text{ is large})}{G_L \rightarrow 0}$ and $1 + \omega^2 C_{gs}^2 (R_s + R_g)^2 \rightarrow \omega^2 C_{gs} (R_s + R_g)^2$

$$F = 1 + \frac{R_g}{R_s} + \frac{\gamma}{\alpha g_m R_s} \cdot \left(\omega^2 C_{gs}^2 (R_s + R_g)^2 \right) \quad \left(\because \frac{g_m}{g_{d0}} = \alpha \right)$$

Assume $R_s \gg R_g$ $\Rightarrow$ $F = 1 + \frac{\gamma}{\alpha g_m \cancel{R_s}} \left\{ \omega^2 C_{gs}^2 R_s^{\cancel{2}} \right\}$

$$\omega_T^2 = \frac{g_m^2}{C_{gs}^2}$$

$$\Rightarrow \boxed{F = 1 + \frac{\gamma}{\alpha} \left(\frac{\omega}{\omega_T} \right)^2 g_m R_s}$$

MOSFET NOISE - REVIEW/SUMMARY

$Y_c = G_c + jB_c$

$Y_s = G_s + jB_s$

$\overline{I_n} = \overline{I_u} + Y_c \overline{V_n}$

$\alpha = \frac{g_m}{g_{do}} = 1$ (for long channel)

$\gamma = \frac{2}{3}$ for saturation & long channel

$\left(\frac{\overline{i_{nd}^2}}{\Delta f} = 4KT\gamma g_{do} \right.$ channel thermal noise$\left. \right)$

$\delta = 2\gamma$

$\left(\frac{\overline{i_{ng}^2}}{\Delta f} = 4KT\delta g_g \right).$

$\xrightarrow{*} G_C \simeq 0$

$\xrightarrow{*} B_c = \omega C_{gs} \left[1 - \alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right] \longrightarrow$ from Y_c

$\xrightarrow{*} R_n = \frac{\gamma}{\alpha} \cdot \frac{1}{g_m} \rightarrow$ Equivalent noise impedance $R_n = \frac{\overline{V_n^2}/\Delta f}{4KT}$

$$R_n = \frac{1}{g_m^2} \cdot \frac{\cancel{4KT}\gamma g_{do}}{\cancel{4KT}}$$

$$= \frac{\gamma g_{do}}{g_m^2} = \frac{\gamma}{g_m \alpha}$$

$\xrightarrow{*} G_u = \frac{\delta \omega^2 C_{gs}^2 (1 - |c|^2)}{5 g_{do}}$

$$\overline{i_{ng}^2} = \overline{(i_{ngc} + i_{ngu})^2}$$

$$= 4KT\delta g_g |c|^2 \Delta f + 4KT\delta g_g (1 - |c|^2) \Delta f$$

> since total should be $\overline{i_{ng}^2}$ this is the definition of c - correlation factor.

$\Rightarrow \overline{i_{ngu}^2} = 4KT\delta g_g (1 - |c|^2) \Delta f$

Derivation -

$G_u = \frac{\overline{i_{ngu}^2}}{4KT\Delta f} = \delta g_g (1 - |c|^2)$

We know, $g_g = \frac{\omega^2 C_{gs}^2}{5 g_{do}}$

$\Rightarrow G_u = \frac{\delta \omega^2 C_{gs}^2 (1 - |c|^2)}{5 g_{do}}.$

=> Optimum source admittance Y_s can be calculated.

Minimum NF

We Know, $B_{opt} = -B_c$

$$\Rightarrow B_{opt} = -\omega C_{gs}\left(1-\alpha|c|\sqrt{\frac{\delta}{5\gamma}}\right)$$

$\left\{ c = j\sqrt{\frac{5}{32}} \text{ for (long channel)} \right\}$

B_{opt} is -ve.
=> (inductive)

$$G_{opt} = \sqrt{\frac{G_u}{R_n} + G_c^2} = \alpha\omega C_{gs}\sqrt{\frac{\delta}{5\gamma}(1-|c|^2)}$$

$$F_{min} = 1 + 2R_n(G_{opt} + G_c)$$

$$\boxed{F_{min} = 1 + \frac{2}{\sqrt{5}}\cdot\frac{\omega}{\omega_T}\sqrt{\gamma\delta(1-|c|^2)}}$$

$$\omega_T = \frac{g_m}{C_{gs}+C_{gd}} \simeq \frac{g_m}{C_{gs}}.$$

=> $\omega_T \uparrow \Rightarrow F_{min} \downarrow$

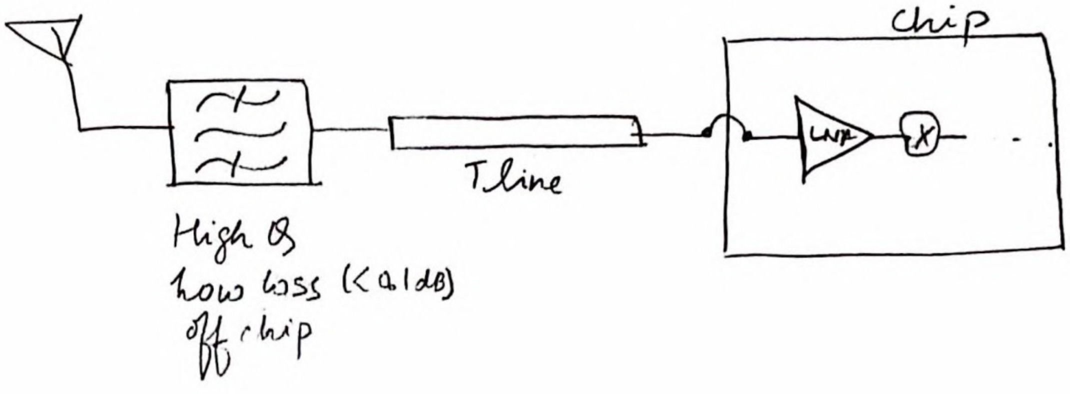
=> $\delta = 0 \Rightarrow F_{min} = 0\,dB$ (If gate noise was zero).

=> If correlation is high b/w gate & drain noise $F_{min} \downarrow\downarrow$.

=> Higher frequency => F_{min} degrades.

LNA Design

Receiver



High Q
Low loss (< 0.1dB)
off chip

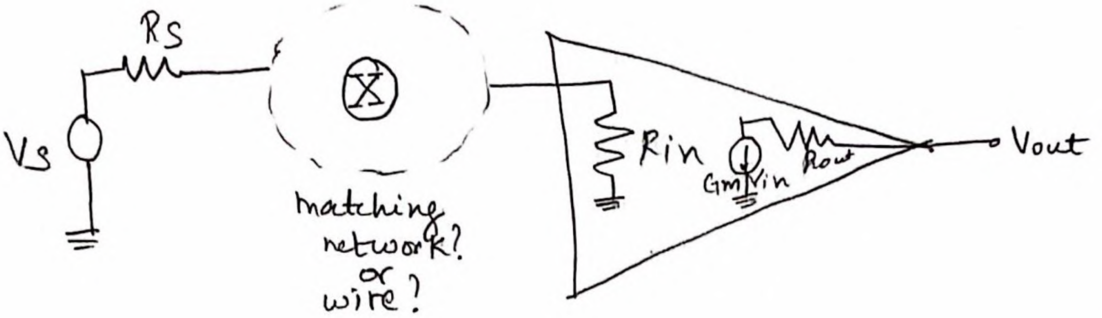
Requirements of LNA

1) Low NF. (Sets the sensitivity)
2) High Power gain (Noise of later stages does not matters)
3) Linearity (Not so important since signal is small)
4) Power consumption
5) Area.
6) Bandwidth (Not larger than necessary - tradeoffs)
7) Dynamic range (Not so important since signal is small)
8) Reverse isolation (Protect antenna & don't transmit power back).
9) Matching (Power match (or) Noise match).

> Obviously we need noise match since it is an LNA (low NF)
But why do we need Power match?

<u>Reason ①</u>:- Input of LNA should match filter (filter is finely tuned).

But there is a more important reason.



Ideally : Rout $= 0$ & Rin $= \infty$ to get maximum voltage gain.

Case ①

→ To get maximum voltage gain : Ⓧ should be short & Rin should be high! ⇒ $V_{in} = V_s$.

Case ②

→ If we use a power match for Ⓧ ⇒ $P_{in} = \frac{1}{2} P_s$.

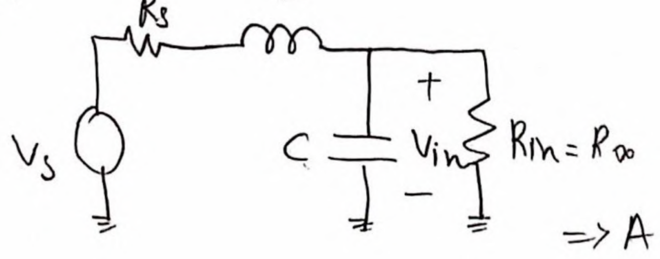
So, it looks like case ① is the best approach for Voltage gain.

<u>But No.</u>

→ Recall that if we use a matching network the voltage across the reactive components is $Q \times V_s$. So using a power match gives higher V_{in} $(V_{in} = Q V_s)\left(Q = \sqrt{\frac{R_{in}}{R_s}}\right)$.
(Proof below) → (Across V_{gs} in MOSFET)

→ Also notice $P_{in} = \frac{V_{in}^2}{R_{in}}$ so for maximum V_{in} we need max P_{in} which also suggest power match is best. Current goes down by a factor of Q (Since power must be conserved) but we care about voltage gain so it's OK.

Proof →



Notice here $V_{in} = Q V_s$.

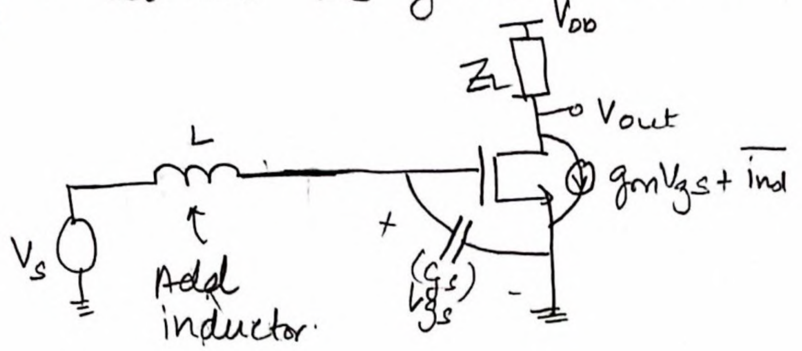
If $R_{in} = \infty$ ⇒ $V_{in} = \infty$!

⇒ A power match also gives maximum voltage gain

Noise match

→ (For a more rigorous treatment notice Bopt needs to be an inductor so we need L to gives best NF) → Can calculate L from Bopt.

> What Z_s gives best NF?



If L_s & C_{gs} resonate,

$V_{gs} \longrightarrow \infty$ $\{R_{gs} = \infty\}$

$\Rightarrow g_mV_{gs} \longrightarrow \infty$

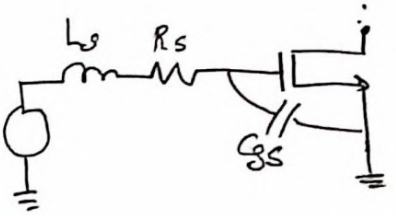
$\Rightarrow \overline{I_{nd}}$ does not matter.

$\Rightarrow NF = 1$, But $BW = 0$.

→ ∴ Need something (—ᴦᴦᴦ—) to resonate with C_{gs} for best NF.

→ In reality we have an R_s.

$\Rightarrow$ Q is finite & $V_{gs} = Q\,V_s$.



$$\Rightarrow \boxed{F = 1 + \frac{g_m R_s \gamma}{\alpha\left(\frac{\omega_T}{\omega}\right)^2}}$$

(Derivation on page 31.1)

$(R_s = 0 \Rightarrow F = 1)$

↳ Same as CS noise with L_s & C_{gs} since they don't change NF.

L_s & C_{gs} resonate

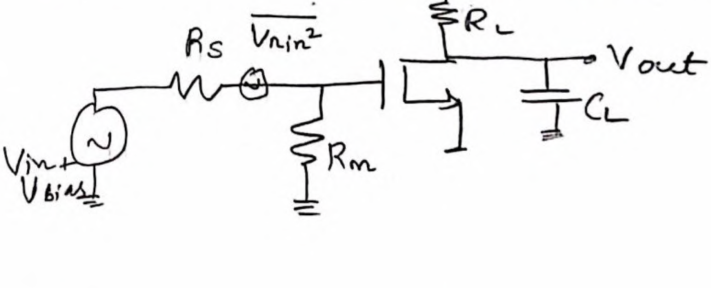
$$V_{gs} = \frac{1}{\omega C_{gs}} \cdot i$$

$$= \frac{V_s}{\omega C_{gs} R_s} = \underline{\underline{Q V_s}}$$

$\Rightarrow V_{gs} = Q\,V_s$.

Power match

> **Simplest approach:**



$R_m = R_s = R$.

$\overline{i_{n,in}^2}$ = O/p noise due to I/p.

$$= g_m^2 V_{gs}^2 = g_m^2\left(\frac{1}{2^2}\overline{V_{in,in}^2}\right) = KT g_m^2 R_s \Delta f$$

$\overline{i_{n,R_m}^2}$ = Same due to R_m $= KT g_m^2 \overline{R_m} \Delta f$

$$\Rightarrow \overline{i_n^2} = 2KT g_m^2 R \Delta f + \underbrace{4KT\gamma g_{d0}\Delta f}$$

=> $\boxed{F = 2 + \frac{4\gamma}{\alpha}\,\frac{1}{g_m R_L}}$ → Pretty high { Problem is R_m }

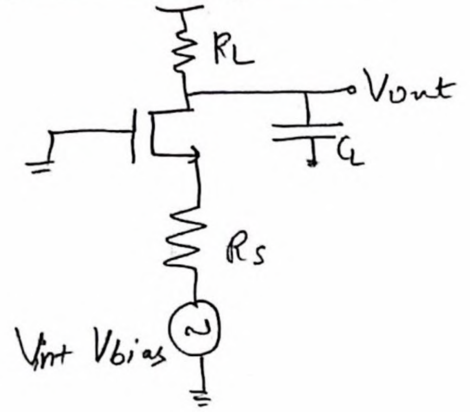
Even ∞ gain gives $F=2$.

=> Power match with just R_m is bad for noise!

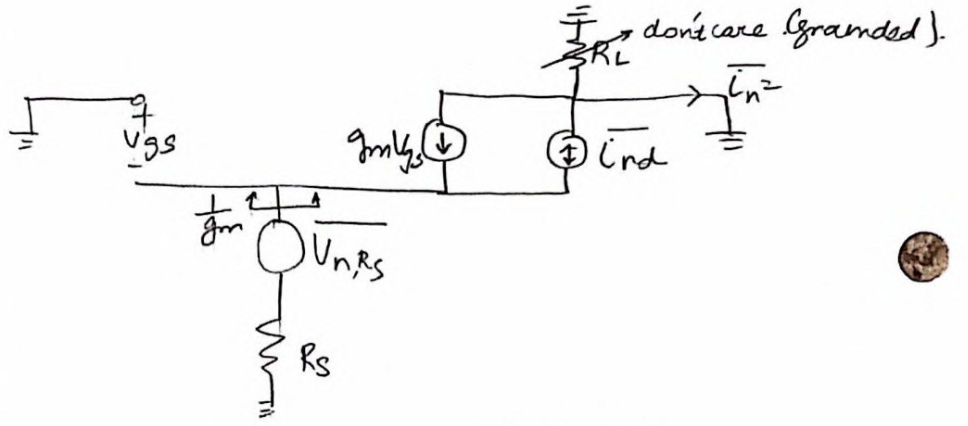
But we need power match!

=> Before we fix this let's see if common gate can help.

Common Gate



$\frac{1}{g_m} = R_s$ for power match => $g_m R_s = 1$



Noise due to input R_s at o/p : $\overline{i^2_{n,op}} = g_m^2 V_{gs}^2 = \frac{g_m^2\left(\frac{1}{g_m}\right)^2}{\left(R_s+\frac{1}{g_m}\right)^2}\cdot\underbrace{4KTR_s\Delta f}_{\overline{V^2_{n,R_s}}}$. — ①

> $\overline{i_{nd}}$ doesnot show up directly at O/p due to R_s (since $V_{gs} \neq 0$).

> What in $\overline{i^2_{n,d}}$? Null $\overline{V_{n,R_s}}$:- $V_{gs} = -R_s(g_m V_{gs} + \overline{i_{nd}})$.

$\overline{i_n} = \overline{i_{nd}} + g_m V_{gs}$. where $\overline{i_{nd}^2} = 4KT\gamma g_{do}\Delta f$

=> $\overline{i^2_{n,d}} = \frac{1}{(1+g_m R_s)^2}\cdot 4KT\gamma g_{do}\Delta f$ — ②

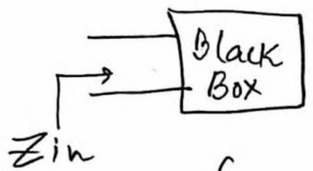
From ①&② => $F = 1 + \frac{\gamma}{\alpha}\cdot\frac{1}{g_m R_s}$ But $g_m R_s = 1$ (power match)

$\boxed{F = 1 + \frac{\gamma}{\alpha}}$ → Pretty high { Problem is that $g_m R_s = 1$ }

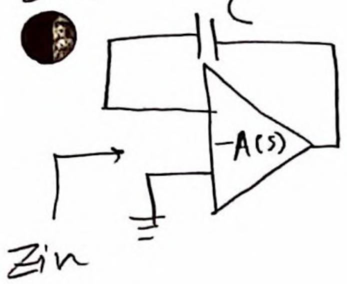
- Lets' go back to CS and see if we can match power with Rm and still have that noise match.

- To get best noise match we needed an L_s to resonate with C_{gs}. So we cannot change the circuit there.
- To get power match we need to change resistance looking into the gate without changing the noise match circuit that is already there. Is it possible?

Lets' look at an example



$Z_{in} = |Z_{in}| e^{j \angle Z_{in}}$ → $=0$ to get a resistor.



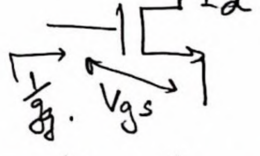
$$Z_{in} = \frac{\frac{1}{sC}}{1 + A(s)} \quad \text{where } A(s) = A_0 e^{-j\phi} \rightarrow \text{-ve phase due to delay.}$$

$$\Rightarrow Y_{in} = j\omega C(1 + A_0 e^{-j\phi})$$

$$= \underbrace{j\omega C(1 + A_0 \cos\phi)}_{\text{capacitor.}} + \underbrace{A_0 \omega C \sin\phi}_{\text{resistor.}}$$

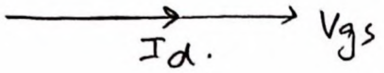
⇒ A delay in the amplifier gives a resistive part to Z_{in}. We can control this resistor by controlling delay.

- Going back to our amplifier.

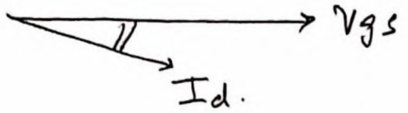


- At the gate we have an RC (g_g & C_{gs}). When V_{gs} changes I_d changes with some delay (g_m is complex). We want to increase this delay in I_d when V_{gs} changes. Use an inductor L_s to

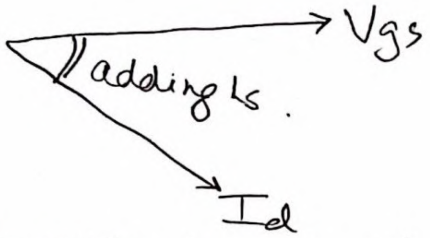
In vectors.

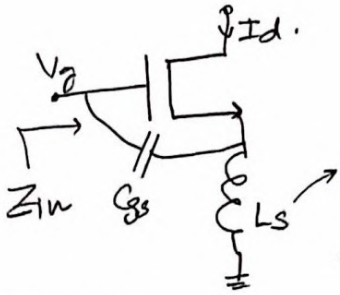


g_m is real. (Only C_{gs} exists) at low frequencies.



g_m is complex. (Both g_g & C_{gs} exist) at high freq. $g_g = \frac{\omega^2 C_{gs}^2}{5\, g_{do}}$.





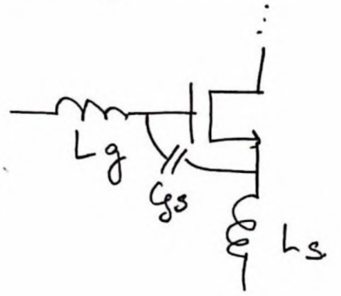
Ls gives feedback if added at source. Like the feedback we saw with . This is probably why we don't use Ld.

Draw small signal & derive Zin

$$\boxed{Z_{in} = \frac{g_m L_s}{C_{gs}} + \frac{1}{sC_{gs}} + sL_s.}$$

↑ resistive! $\boxed{R_{in} = \omega_T L_s}$ if $C_{gs} \gg C_{gd}$

→ Putting it all together.



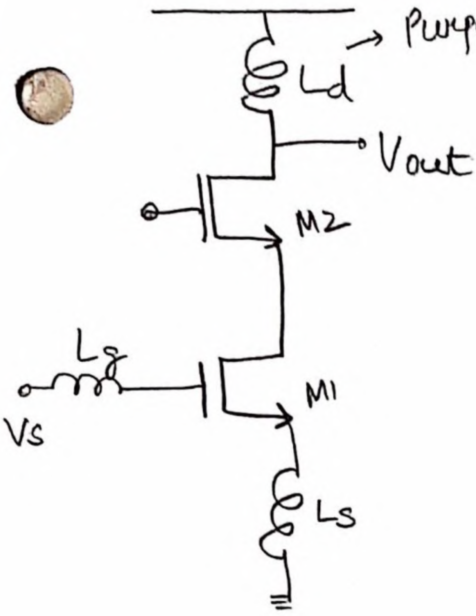
> Ls + Lg resonate with Cgs.

> Ls & Lg are both needed since we need two degrees of freedom to get both noise match and power match.

→ Cgd? 1) Lowers reverse isolation (Large due to miller).
2) Changes phase relationship of input & output.

→ How to get rid of Cgd? Cascode! It also helps with stability and prevents oscillations.

Final LNA



→ Purpose discussed later.

Narrowband LNA

$$F = 1 + \gamma g_{do} R_s' \left(\frac{\omega C_{gs}}{g_m}\right)^2 + \left[1 + \frac{1}{(\omega C_{gs} R_s')^2}\right] \delta g_g R_s'$$
$$+ 2\left(\frac{\omega C_{gs}}{g_m}\right)^2 g_m R_s' |c| \sqrt{\frac{\gamma\delta}{5}}$$

where, $R_s' = \frac{g_m L_s}{C_{gs}}$

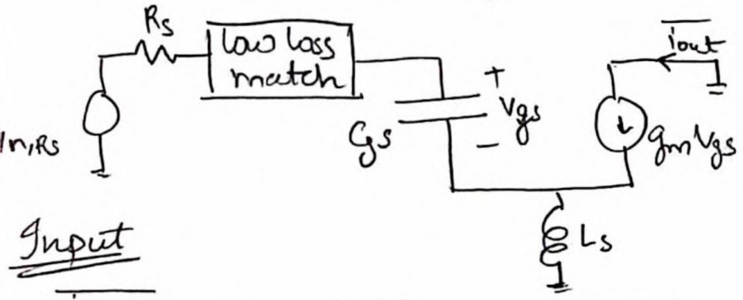
If $\overline{i_{ng}^2} = 0$,

$$\boxed{F = 1 + \gamma g_{do} R_s' \left(\frac{\omega}{\omega_T}\right)^2}$$

$\omega_T = \frac{g_m}{C_{gs}}$

→ We care about ω_T
Higher ⇒ Better
(assuming γ is same)

Derivation (Ignoring C_{gd}, & M_2)



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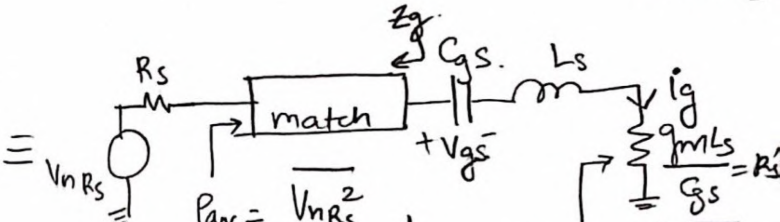


$P_{av} = \frac{\overline{V_{nR_s}^2}}{4R_s}$

$P_{av} = R_s' \cdot \overline{i_g^2}$

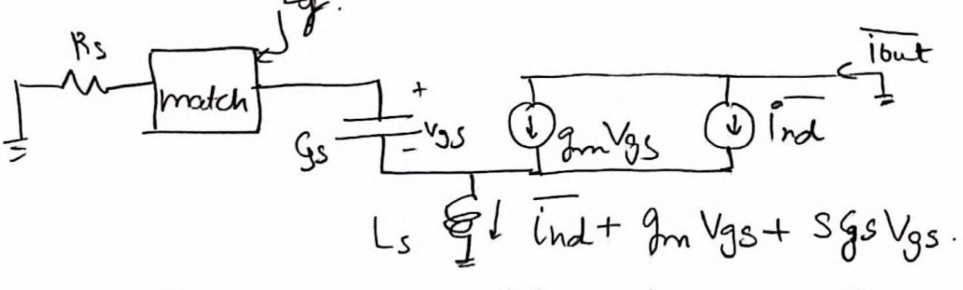
$\Rightarrow \overline{i_g^2} = \frac{\overline{V_{nR_s}^2}}{4R_s R_s'}$

Input

$$\overline{i_{nout}^2} = g_m^2 \overline{V_{gs}^2} = g_m^2 \cdot \frac{\overline{i_g^2}}{(\omega C_{gs})^2} = \left(\frac{g_m}{\omega C_{gs}}\right)^2 \frac{1}{4R_sR_s'} \cdot \overline{V_{nR_s}^2}$$

↳ due to input. — ①

Transistor



$$KVL \Rightarrow sL_s\left[(g_m + sC_{gs})V_{gs} + \overline{i_{nd}}\right] + V_{gs} + Z_g\left[sC_{gs}V_{gs}\right] = 0$$

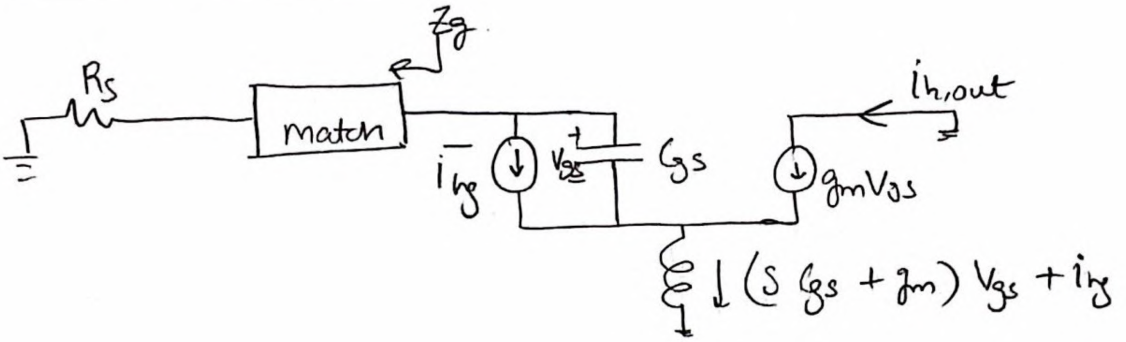
$$\Rightarrow V_{gs} = -\frac{sL_s}{1+sC_{gs}Z_g+sL_s(g_m+sC_{gs})}\,\overline{i_{nd}}$$

$$\overline{i_{nout}} = g_m V_{gs} + \overline{i_{nd}} = \frac{Z_g + \frac{1}{sC_{gs}} + sL_s}{Z_g + \frac{1}{sC_{gs}} + sL_s + \frac{g_m L_s}{C_{gs}}} \cdot \overline{i_{nd}}$$

But for matching we need, $Z_g + \underbrace{\frac{1}{sC_{gs}} + sL_s}_{\text{These two would cancel \& } Z_g = R_s'} = R_s'$

$$\Rightarrow \overline{i_{nout}} = \frac{R_s'}{R_s'+R_s'}\,\overline{i_{nd}} = \frac{\overline{i_{nd}}}{2} \rightarrow \text{noise due to } \overline{i_{nd}}$$

Noise due to $\overline{i_{ng}}$?



KVL: $sL_s\left[(sC_{gs}+g_m)V_{gs} + i_{ng}\right] + V_{gs} + Z_g(i_{ng} + sC_{gs}V_{gs}) = 0.$

$$\Rightarrow V_{gs} = -\frac{sL_s + Z_g}{1+sC_{gs}Z_g + sL_s(sC_{gs}+g_m)} \cdot \overline{i_{ng}}$$

$$i_{out} = g_mV_{gs} = \left[-\frac{g_m}{sC_{gs}} \times \frac{1}{2} \times \left(1 - \frac{1}{sC_{gs}R_s'}\right)\right]\overline{i_{ng}} \quad \text{since } R_s' = Z_g + \frac{1}{sC_{gs}} + sL_s$$

$$\Rightarrow i_{nout,tot} = i_{n,(V_{nRs})} + i_{n,(i_{nd})} + i_{n,(i_{ng})}$$

But What about correlation.

V_{n,R_s} & i_{nd} → Uncorrelated.

V_{nR_s} & i_{ngd} → Uncorrelated.

i_{ng} & i_{nd} → Correlated

$$\Rightarrow \overline{i_{n\,out\,tot}} = \overline{i_{n\,out\,tot} \times i_{n\,out\,tot}^{*}}$$

$$= \overline{i_{nout,(V_{Rs})}^{2}} + \overline{i_{nout\,(ind)}^{2}} + \overline{i_{nout\,(ing)}^{2}} + \overline{i_{nout\,(ind)} \times i_{nout\,(ing)}^{*}}$$

$$+ \overline{i_{nout\,(ing)} \cdot i_{nout\,(ind)}^{*}}$$

$A + A^{*} = 2\,\mathrm{Re}(A)$

$$\overline{i_{nout\,nd} \cdot i_{nout\,ng}^{*}} + \overline{i_{nout\,nd}^{*}\, i_{nout\,ng}} = 2\,\mathrm{Re}\left\{ \overline{i_{nout\,nd}^{*} \cdot i_{nout\,ng}} \right\}$$

$$= 2\,\mathrm{Re}\left\{ \frac{-g_m}{4 s C_{gs}} \left(1 - \frac{1}{s C_{gs} R_s'}\right) \overline{i_{nd}^{*}\, i_{ng}} \right\}$$

$$\boxed{\therefore C = -j|C|}$$

$$= 2\,\mathrm{Re}\left\{ \frac{+\,g_m}{4 \omega C_{gs}} + \frac{j\, g_m}{4 \omega^2 C_{gs}^2 R_s'} \right\} |C| \sqrt{\overline{i_{nd}^2}\, \overline{i_{ng}^2}}$$

$$= \frac{2 g_m}{4 \omega C_{gs}'} \cdot |C| \sqrt{\gamma g_{d0} \delta g_g \cdot 4KT\Delta f}$$

$$g_{d0} g_g = \frac{\omega^2 C_{gs}^2}{5}$$

$$= \frac{g_m}{2} |C| \sqrt{\frac{\gamma \delta}{5} \cdot 4KT\Delta f}.$$

$$\boxed{NF = 1 + \gamma g_{d0} R_s' \left(\frac{\omega C_{gs}}{g_m}\right)^2 + \left[1 + \frac{1}{(\omega C_{gs} R_s')^2}\right] \delta g_g R_s' + 2\left(\frac{\omega C_{gs}}{g_m}\right)^2 g_m R_s' |C| \sqrt{\frac{\delta \gamma}{5}}}$$

$$\omega_T = \frac{g_m}{C_{gs}}$$

$$R_s' = g_m L_s$$

$$\overline{i_{ng}^2} = 0$$

$$\Rightarrow \boxed{NF = 1 + \gamma g_{d0} R_s' (\frac{\omega}{\omega_T}}$$

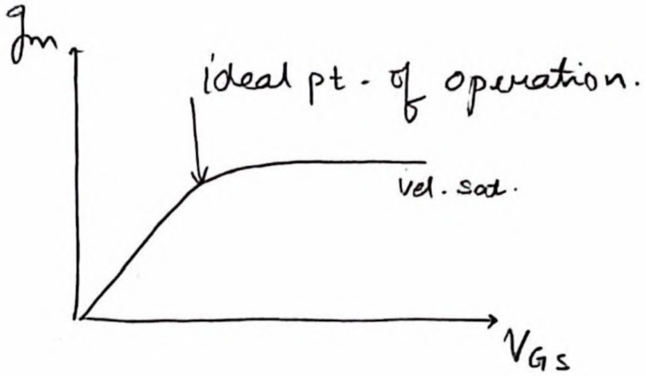
LNA Design example

$f_0 = 1.8 GHz$

$R_s = R_s' = 50 \Omega$

$\gamma = 2$

$BW = 180 MHz$



In velocity saturation.

$$g_m = \frac{\mu_n C_{ox}}{2} . E_c W.$$

$\Rightarrow g_m \propto W$

constant with process.

$$C_{gs} = \frac{2}{3} WL C_{ox} = \frac{2}{3} \varepsilon_{ox} \left(\frac{L}{t_{ox}}\right) W$$

$\Rightarrow C_{gs} \propto W$

Process = 0.25μm CMOS

$C_{gs} = (1.5 - 2.5 fF) W$

$g_m = (0.3 - 0.4 \frac{mA}{V}) W.$

$$\omega_T = \frac{g_m}{C_{gs}} \cong 25 GHz.$$

$$R_s' = \omega_T L_s \Rightarrow L_s = \frac{R_s'}{\omega_T} = \frac{50 \Omega}{25 GHz \times 2\pi} = 0.3 nH.$$

$\Rightarrow L_s = 0.3 nH$

Device size?

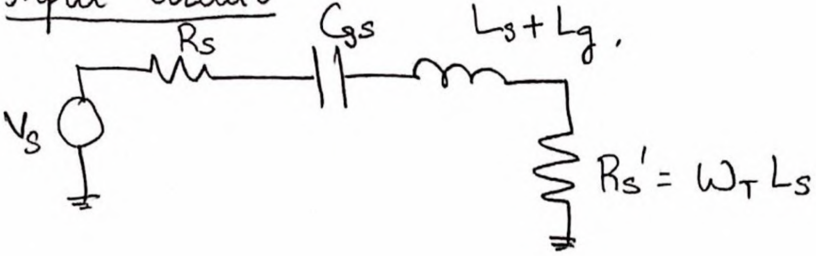
$\hookrightarrow C_{gs} \longrightarrow L_g.$

$\hookrightarrow Q \longrightarrow BW$

Bias?

$\hookrightarrow g_m$ & g_{do}.

Input circuit



$$Q = \frac{1}{(R_s + R_s')\, C_{gs}\, \omega_0} \quad , \quad BW = \frac{\omega_0}{Q}.$$

Tradeoffs on W

$W \downarrow \Rightarrow C_{gs} \downarrow \Rightarrow$
- $Q \uparrow \Rightarrow BW \downarrow$
- $L_g \uparrow \Rightarrow$ Size $\uparrow \Rightarrow \uparrow r_g \Rightarrow \uparrow$ loss $\Rightarrow \uparrow NF$
- $\Rightarrow$ Loss at i/p is bad!

$W \downarrow \Rightarrow g_m \downarrow \Rightarrow g_{do} \downarrow \Rightarrow NF \downarrow$

$\Rightarrow$ If $g_m \uparrow = NF \uparrow$ but input refferred noise $\overline{V_n^2} \downarrow$ since gain $\uparrow$

$\Rightarrow$ Tradeoff $\overline{V_n^2}$ & NF.

$\downarrow NF \rightarrow$ in narrowband $\Rightarrow$ lower g_m.

$\downarrow \overline{V_n^2} \rightarrow$ in broadband $\Rightarrow$ increase g_m.

$W \downarrow \Rightarrow I_d \downarrow \Rightarrow P_{dc} \downarrow$

$BW = 180\,MHz \Rightarrow \overset{Let,}{Q_{i/p}} = 10$

$= \frac{1}{2 R_s C_{gs} \omega_0}$

$\Rightarrow C_{gs} = 88fF$

But $C_{gs} = 2fF \times W$

$\Rightarrow \boxed{W = 44\mu m}$ Huge! But lets see!

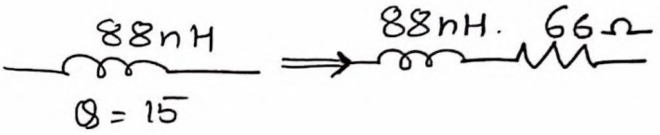
$\Rightarrow L_s + L_g$ should resonate with C_{gs}.

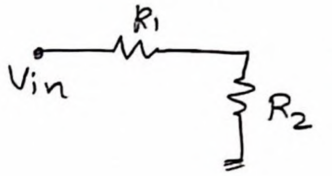
$\Rightarrow (L_s + L_g) C_{gs} \omega_0^2 = 1$

$\Rightarrow \boxed{L_g = 88nH} \rightarrow$ not possible to integrate.

Assume Q of L_g is 15 (Wishful thinking)

$\Rightarrow r_g = \frac{\omega L_g}{Q} = 66\Omega \Rightarrow$





$F = 1 + \frac{R_1}{R_2}$

$NF_{Fet} = 1 + \frac{R_g}{R_s} + \gamma g_{d0} R_s' \left(\frac{\omega}{\omega_T}\right)^2$

$\rightarrow$ i/p is very sensitive to R_g.

$\Rightarrow$ 66Ω at i/p is too high since we don't want to add another matching network to docon convert it. Cannot down convert it with $R_s' = \omega_T L_s$ & current matching network.

Reminder: $W\downarrow \Rightarrow C_{gs}\downarrow \Rightarrow L_g\uparrow \Rightarrow r_g\uparrow \Rightarrow NF\uparrow$

Solution: Lower $^{1}/_{p}\, Q \Rightarrow C_{gs}\uparrow \Rightarrow L_g \downarrow \Rightarrow r_g \downarrow.$

↳ But $W\uparrow$.

$Q_{^1/_p} = 1 \Rightarrow C_{gs} = 880\,fF \Rightarrow W = 440$ [LNA drivers are huge]

$\Rightarrow L_g \simeq 8.5nH.$

If Q of L_g is 15 $\Rightarrow r_g = 6.4\Omega$ (much better).

Current? Remember we bias b/w square law & velocity saturatio

$$\Rightarrow g_m = \frac{\mu_n C_{ox}}{2} E_c W$$

$$\Rightarrow V_{GS} - V_{th} = \frac{E_c L}{2} \Rightarrow I_D = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \cdot \frac{(V_{GS}-V_{th})^2}{1 + \frac{1}{E_c L}(V_{GS}-V_{th})}$$

Comes from $\mu = \frac{\mu_0}{1+\frac{E}{E_c}}$

Where $E = \frac{V_{GS}-V_{t}}{L}$

$$\Rightarrow I_D = \left(\frac{1}{12}\mu_n\left(\frac{\epsilon_{ox}}{t_{ox}}\right)L \cdot E_c^2\right)W$$

$= (0.1mA)\,W$ from simulation.

For $W = 440 \Rightarrow I_D = 44mA$

$\Rightarrow g_m = \omega_T C_{gs} = 140mA/V \Rightarrow$ If $g_{do} = g_m$,

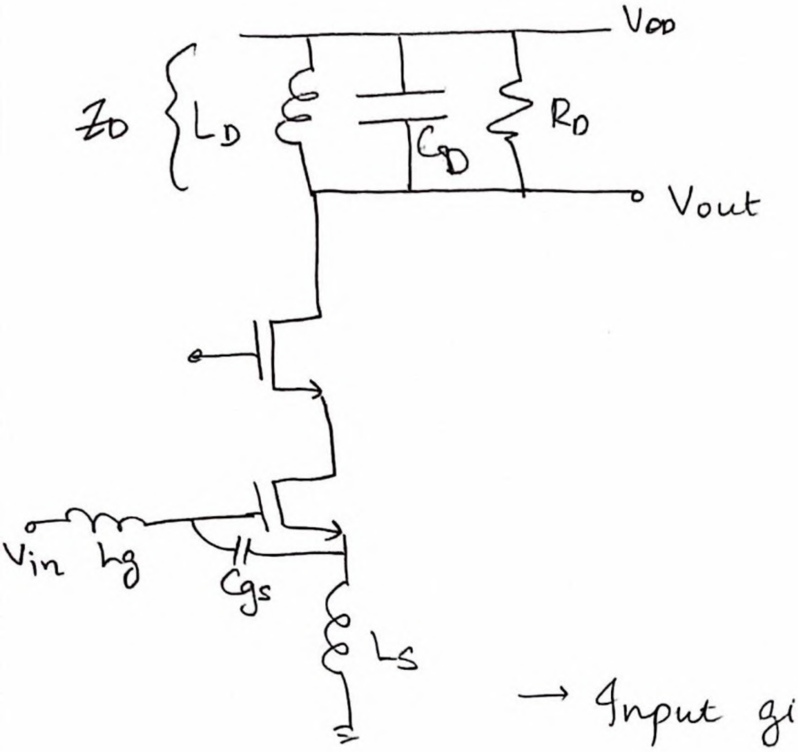
$$F = 1 + \gamma g_{do} R_s' \left(\frac{\omega}{\omega_T}\right)^2 = 1.07$$

$$\Rightarrow \boxed{NF = 0.3dB}$$

With $r_g = 6.4\Omega \Rightarrow F = 1.2$

$$\Rightarrow \boxed{NF = 0.8dB}$$

→ $Q_{i/p}$ is low so it is relatively wideband, ⇒ at the output we use a narrowband (signal is anyway narrowband) & use high $Q_{o/p}$.



Gain?

At resonance.

$$V_{gs} = \frac{V_{in}}{2R_s}\left(\frac{1}{j\omega_0 C_{gs}}\right) = \frac{Q_{in}V_{in}}{j}$$

$$i_D = g_m\left(\frac{Q_{in}}{j}\right)V_{in}$$

$$\text{Gain} = \frac{V_{out}}{V_{in}} = -g_m Z_0\left(\frac{Q_{in}}{j}\right)$$

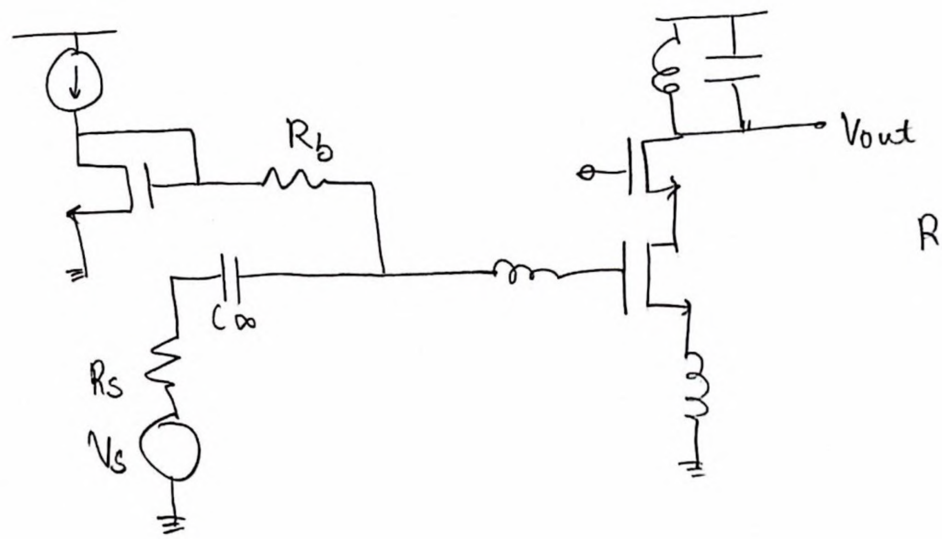
↓

Controls both Gain & BW.

→ Input gives noise & power match.

→ Zo gives Gain & BW.

Biasing.



$$R_b \gg R_s \| R_s' = \frac{R_s}{2}$$

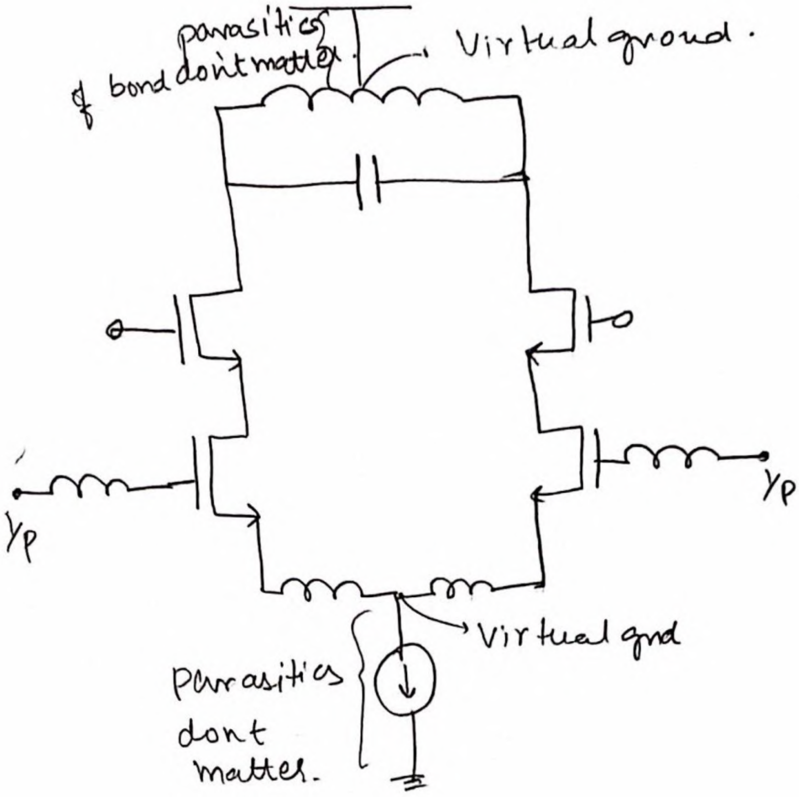
1) Effect of pads & wirebonds.

pads → parasitic cap

wirebonds → parasitic inductance (poor tolerance)

> Could absorb into design at L_0 & C_0.

> Or use differential LNA.



> A balum is implemented at i/p to get rid of parasitics due to bondwire & pad that carry i/p signal to chip!

CGLNA

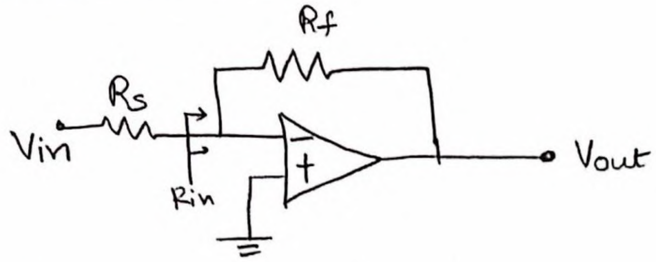
$$NF = 1 + \frac{\gamma}{\alpha}.$$

Could improve to $NF = 1 + \gamma \cdot \frac{g_{do}}{g_{do} + g_m}$ (HW question)

Not so popular.

Broadband LNA

I



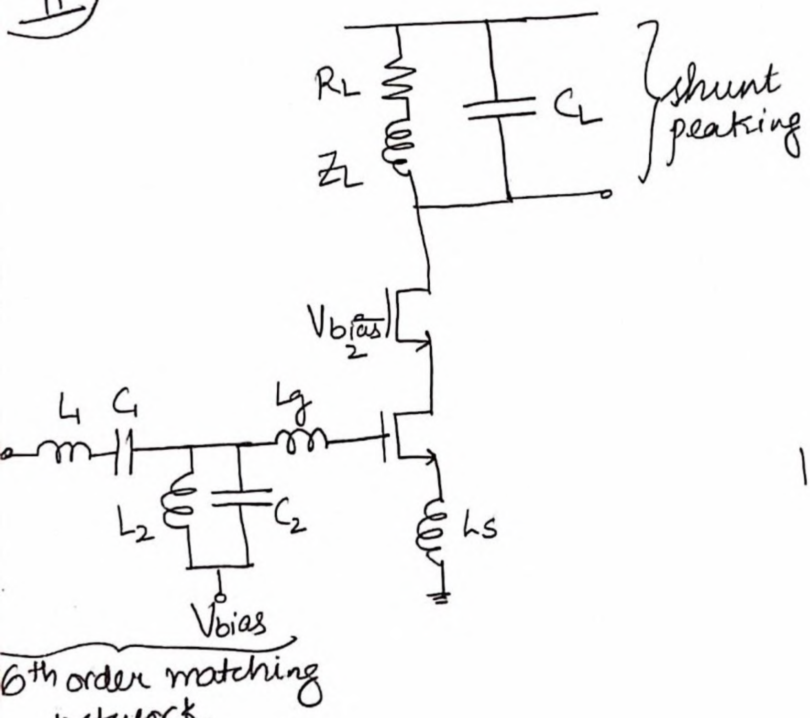
$$\overline{V_{nout}^2} = \left(\frac{-R_F}{R_S}\right)^2 \overline{V_{n_{Rs}}^2} + \overline{V_{n R_F}^2}$$

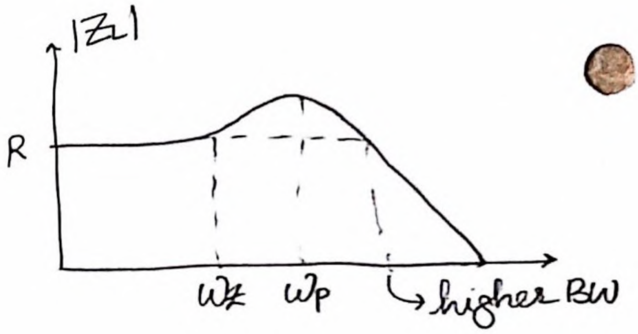
$$F = 1 + \left(\frac{R_s}{R_F}\right)^2 \cdot \frac{4KTR_F\Delta f}{4KTR_s\Delta f} = 1 + \frac{R_s}{R_f}$$

$$R_{in} = \frac{R_f}{1+A} \Rightarrow \text{matched} \Rightarrow \frac{R_f}{1+A} = R_s \Rightarrow R_f = (1+A)R_s$$

$$\Rightarrow \boxed{F = 1 + \frac{1}{1+A}}$$

II





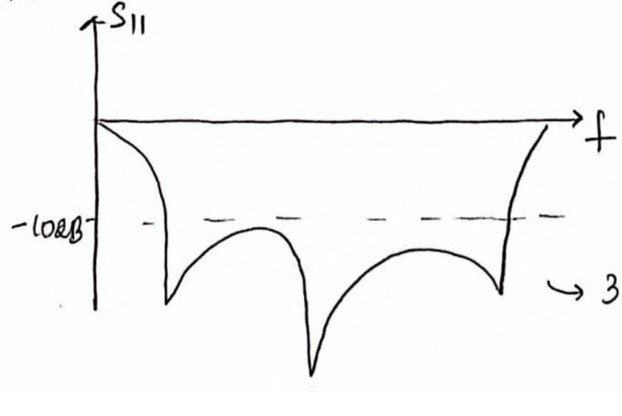
$$\omega_z = \frac{R_L}{L_L}$$

$$|Z_L(j\omega)| = R\sqrt{\frac{(\omega\tau)^2+1}{1-\left(\frac{\omega\tau}{\omega_1}\right)^2+\left(\frac{\omega}{\omega_1}\right)^2}}$$

$$\tau = \frac{L}{R} = \frac{1}{\omega_z}$$

$$\omega_1 = \frac{1}{RC} = -\omega_{-3dB} \text{ without } L.$$

$$m = \frac{RC}{L/R} = \frac{1/\omega_1}{\tau} = \frac{1}{\tau\omega_1}$$



↳ 3 resonant frequencies!

Shunt Peaking.

Maximum BW

$m = \sqrt{2}$ or $\tau = \frac{1}{\sqrt{2}\omega_1}$

$\Rightarrow$ BW = $1.85\omega_1$ but 20% peaking

$\Rightarrow$ Broadband signal is distorted.

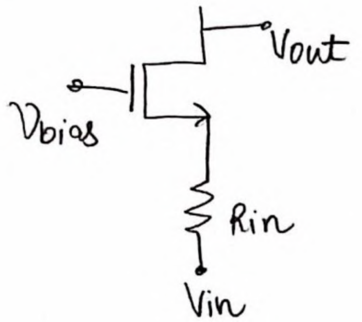
No peaking.

$\tau = \frac{1}{(1+\sqrt{2})\omega_1} \Rightarrow BW = 1.72\omega_1$ ⟶ But phase response is not flat

Flat group delay

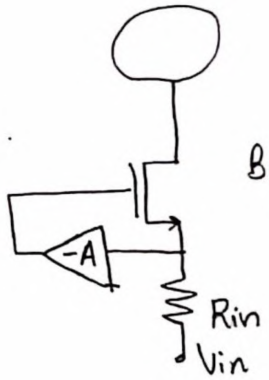
$\tau = \frac{1}{3.1\omega_1} \Rightarrow BW = 1.6\omega_1$

Common Gate LNA



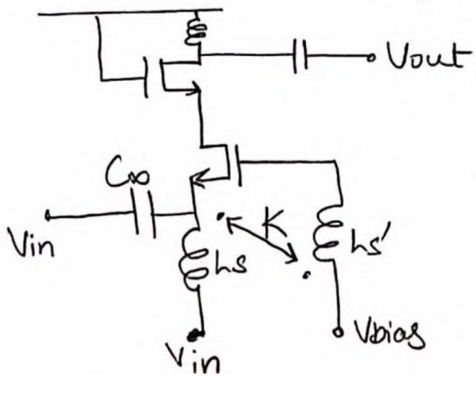
$NF = 1 + \frac{\gamma}{\alpha}$

Increase V_{GS} $\Rightarrow$



Boosts V_{GS} by 1+A

$NF = 1 + \frac{\gamma}{\alpha} \cdot \frac{1}{1+A}$ ⟶ needs to be low noise and needs to be a voltage amplifier $\Rightarrow$ Transformer.

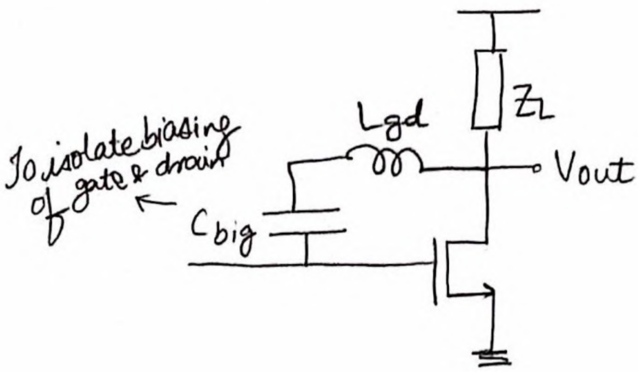


$NF = 1 + \gamma\left(\frac{1}{1+nk}\right)$

(Paper: Xiaoyong Li ISSCC 2005)

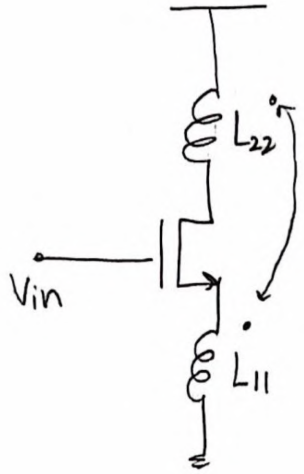
> Reducing the effect of C_{gd}

> Use L_{gd} to resonate with C_{gd} → However this is narrowband



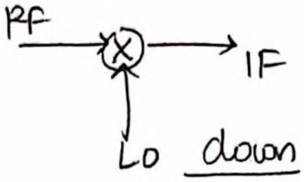
But C_{gd} is small so L_{gd} needs to be large! ⇒ Bad.
Solution: Use a transformer.

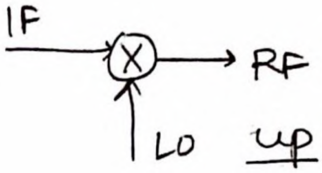
⇓



$$\frac{n}{K} = \frac{C_{gs}}{C_{gd}} \Rightarrow \text{tunes out } C_{gd}!$$

MIXERS

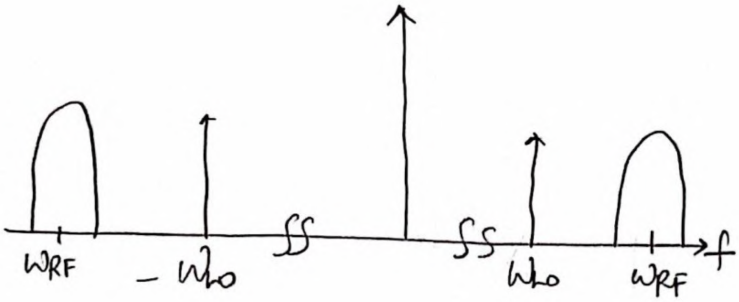




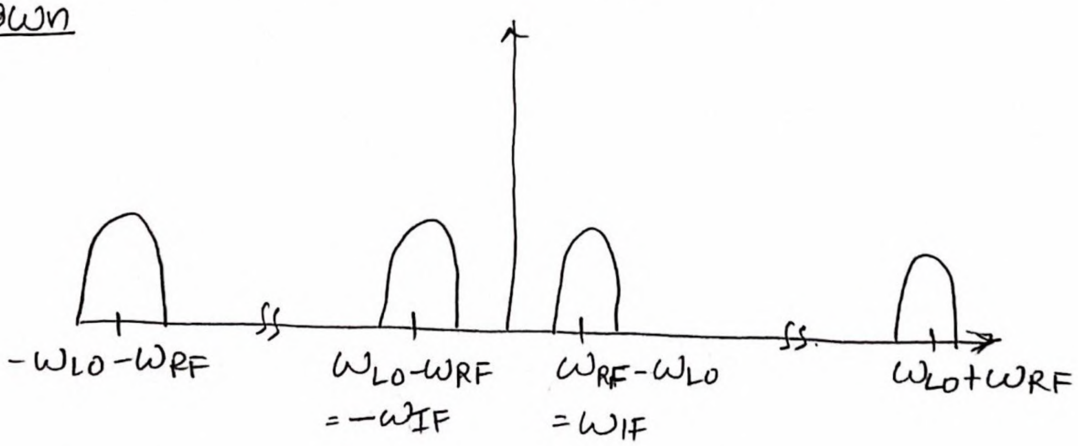
General Considerations.

- NF (not as important as LNA)
- Linearity (quite important)
- Conversion gain (relaxes noise requirement down the chain)
- *** Isolation (B/W LO, RF & IF)
- BW
- Power.

Basic definitions



Down



$W_{IF} = 0 \Rightarrow$ Homodyne, zero IF

$W_{IF} \neq 0 \Rightarrow$ Heterodyne, low IF

Up

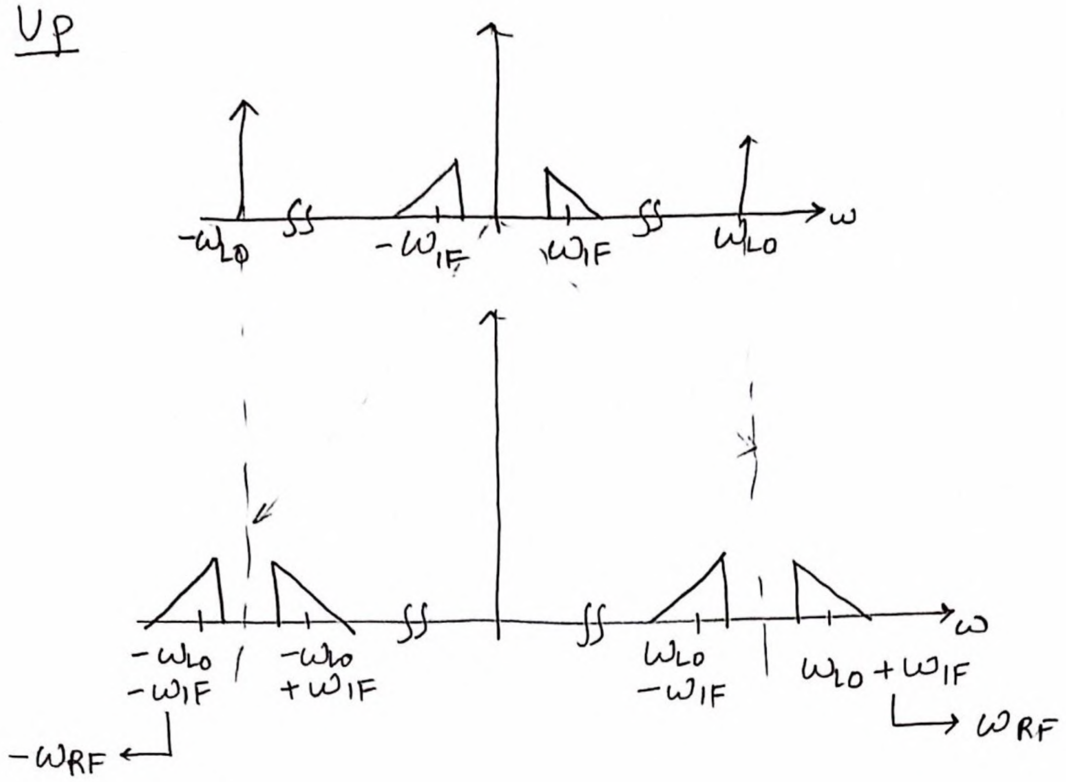
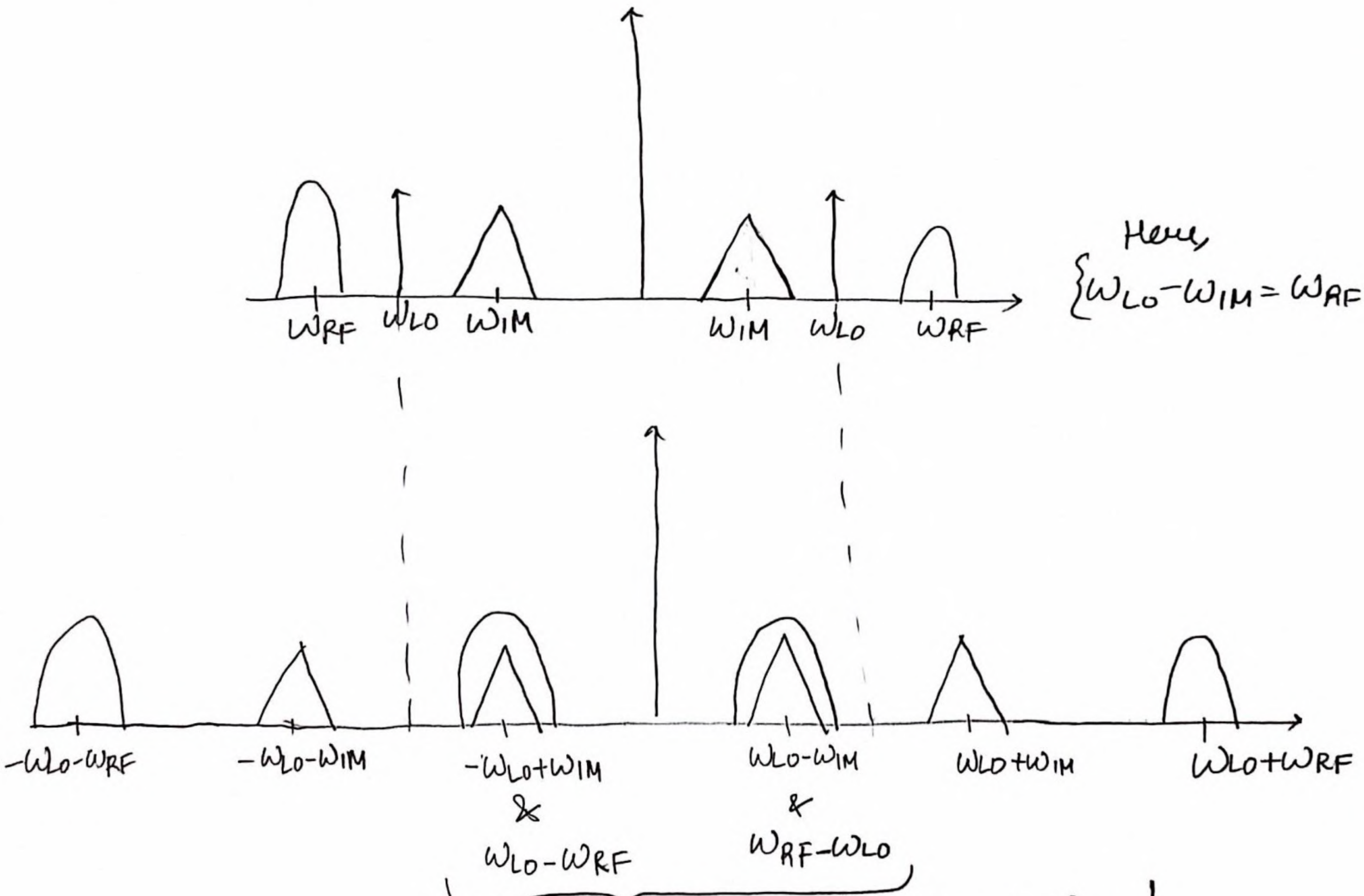


Image Aliasing



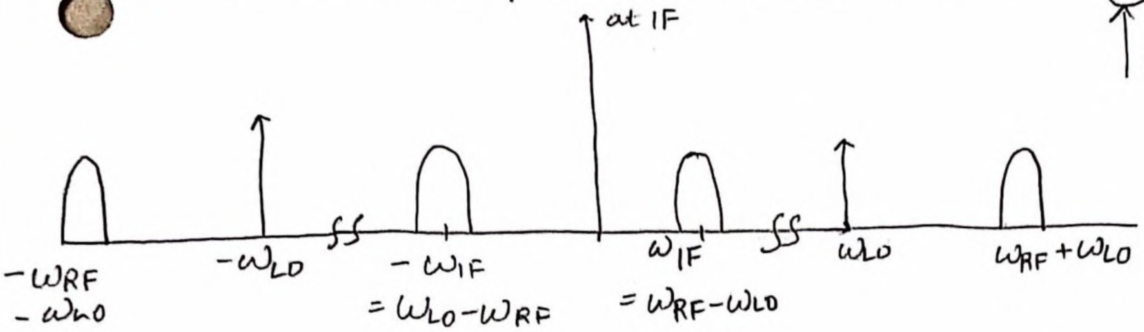
Here,
$\{\omega_{LO} - \omega_{IM} = \omega_{RF} - \omega_{LO}\}$

Image frequency falls at ω_{IF}!

⇒ Need to filter ω_{IM} or use complex mixing (I/Q)

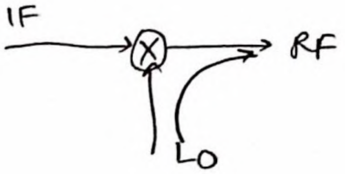
LO Feedthrough.

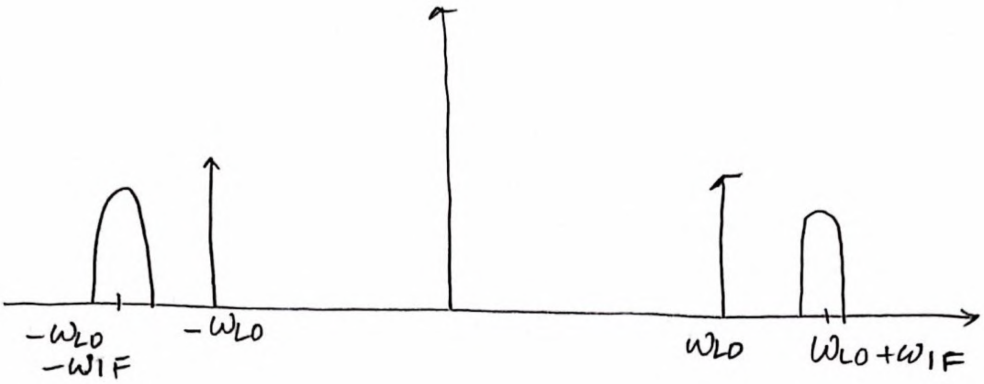
1) LO to IF feedthrough (Down)



If ω_{IF} is chosen low we can easily filter.

2) LO to RF feedthrough (Up)





- Regardless of ω_{IF} it is problematic since we are radiating out of band signals.
- More problematic in zero IF, ω_{LO} is transmitted in band!
- Could cause blocking for following stages (PA).

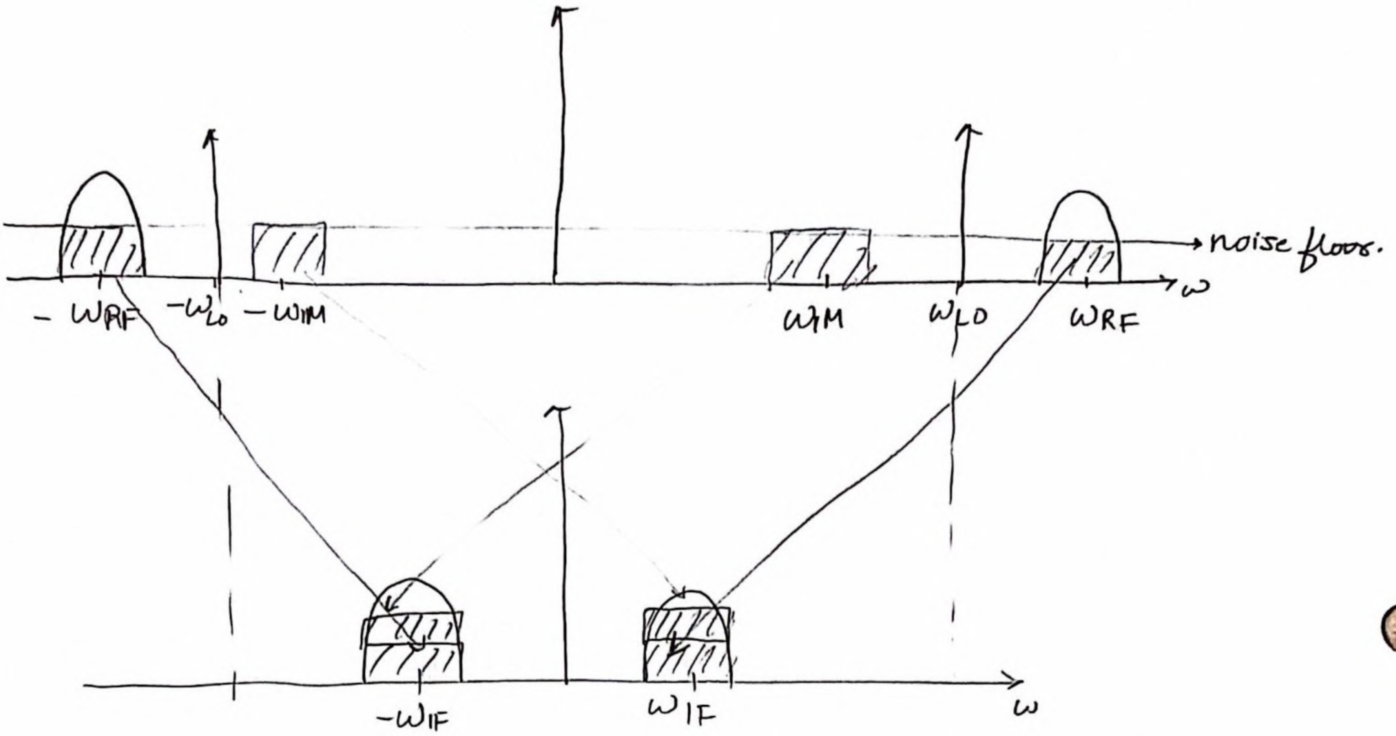
3) LO to RF reverse feedthrough (DOWN)

- Could end up on antenna! Bad.
- Self mixing ⇒ generates a dc signal ⇒ Bigger problem in zero IF ⇒ dc is in band
  - ↳ Could mess up biasing of next stage or saturate it.

→ LO to IF reverse feedthrough in Up. could affect previous stages.

→ RF to LO feedthrough should be low to prevent 'pulling'.

Mixer Noise Figure.



⇒ Even an ideal mixer doubles the noise (at input.).

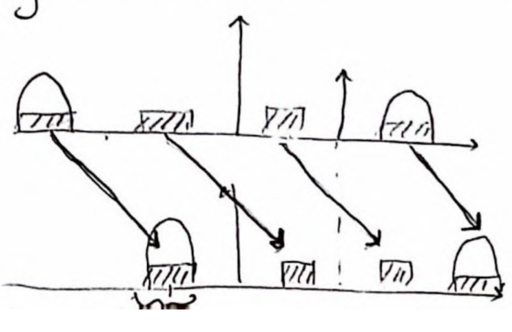
> Here since signal is only SSB, the NF is called SSB.

> Double side band noise figure DSB is the NF when we assume the same signal also exists at ω_{IM}. (eg: in zero IF).

Therefore $\boxed{\text{NF SSB} = \text{NF DSB} + 3\text{dB}}$

Fix: 1) Filter noise at ω_{IM}. (Impractical)

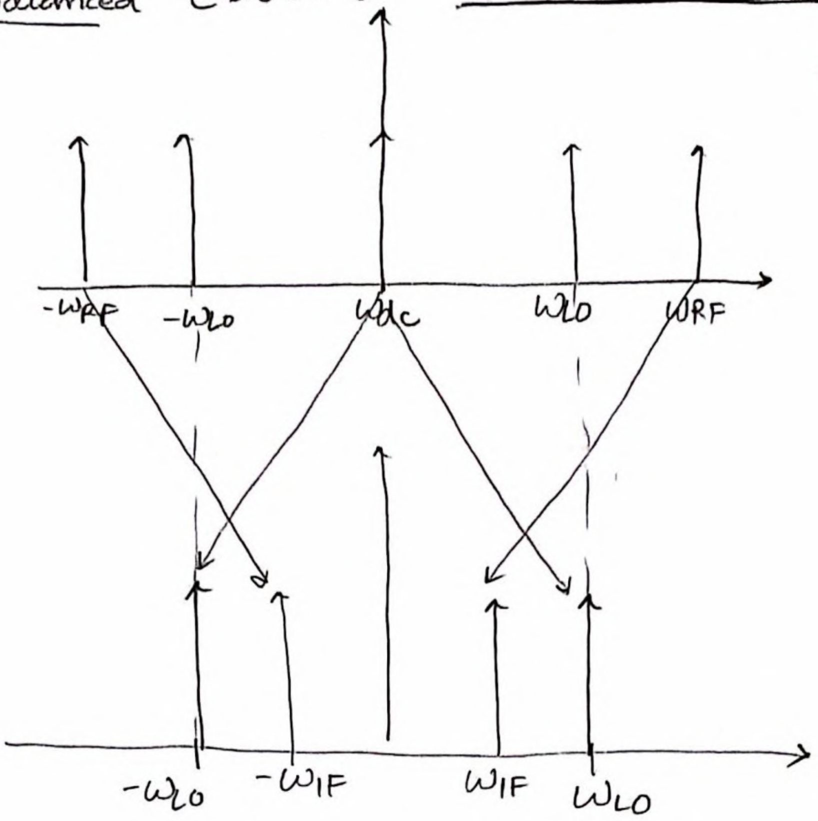
2) Use complex mixing. (Only ω_{LO} exists & no $-\omega_{LO}$ exists).

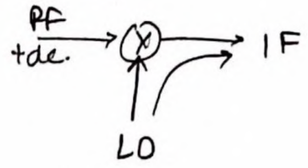


Balance of Mixer

> Balanced ⇒ 0 dc.

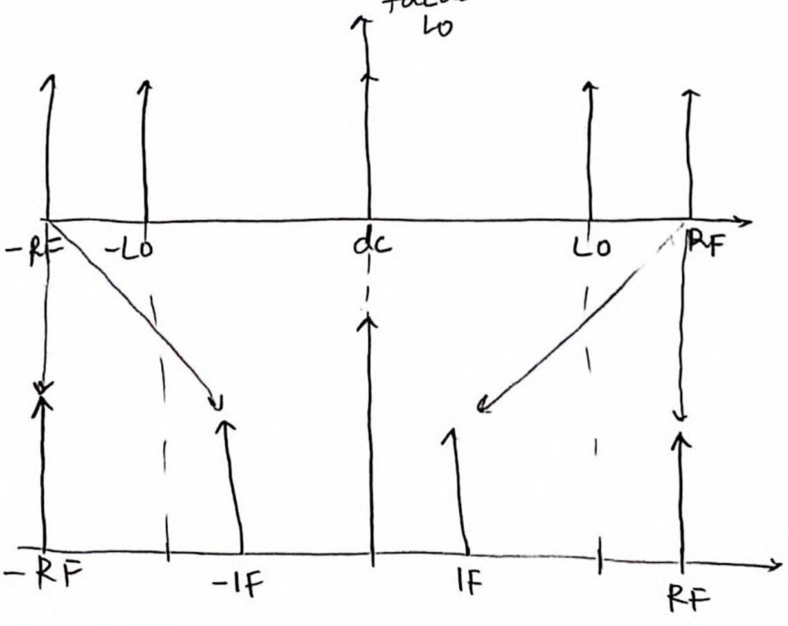
① Eg: Unbalanced (DOWN) — Unbalanced at RF.





→ Like LO feedthrough! ⇒ Problematic in upconversion mixers.

② Eg: Unbalanced at LO (DOWN)

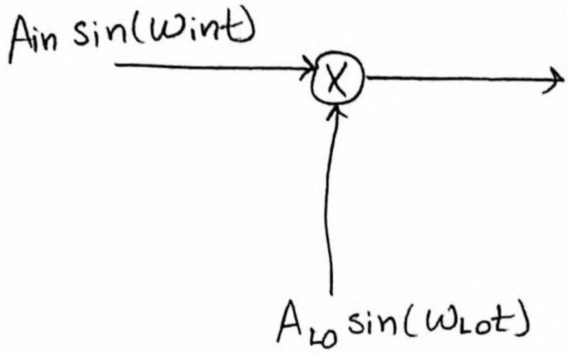


→ Like RF feedthrough

→ Double balanced ⇒ Balanced at RF and LO.

Implementation

> Need nonlinear or Time variant circuits.



$$\rightarrow \frac{A_{in}.A_{LO}}{2}\left[\underbrace{\cos(\omega_{in}-\omega_{LO})t}_{\text{Down}} - \underbrace{\cos(\omega_{in}+\omega_{LO})t}_{\text{Up}}\right]$$

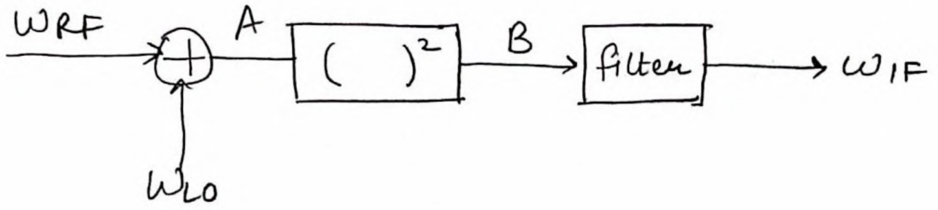
⇒ $\frac{A_{LO}}{2}$ is conversion gain.

⇒ We want large A_{LO}.

Conversion gain (power) (down) = $\dfrac{\text{Power delivered to } R_L \text{ at IF}}{\text{Power available to } R_s \text{ at RF}}$

Conversion gain (power) (up) = $\dfrac{\text{Power delivered to } R_L \text{ at RF}}{\text{Power available to } R_s \text{ at IF}}$

Nonlinearity



A: $A_{RF}\sin(\omega_{RF}t) + A_{LO}\sin(\omega_{LO}t)$

B: $\underbrace{A_{RF}^2\sin^2(\omega_{RF}t)}_{0,\ 2\omega_{RF}\ \times\ \text{filter}} + \underbrace{2A_{RF}A_{LO}\sin\omega_{RF}t\sin\omega_{LO}t}_{\omega_{RF}\pm\omega_{LO}\ \checkmark} + \underbrace{A_{LO}^2\sin^2(\omega_{LO}t)}_{0,\ 2\omega_{LO}\ \times\ \text{filter.}}$

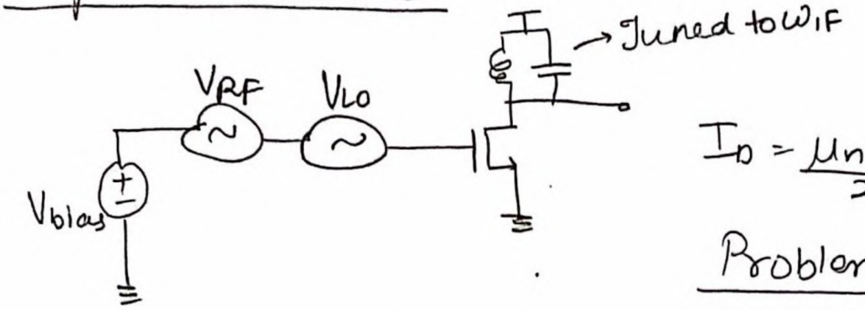
Recall: $\xrightarrow[x(t)]{}\boxed{NL}\rightarrow y(t) = \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$

α_2 → α_2 gives the $(\ )^2$ function

Problems

- α_2 is small => low gain.
- α_3 can affect (we don't want)
- Cannot use differential.

Implementation ①



$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}\left[(V_{bias}+V_{RF}+V_{LO})-V_{tn}\right]^2$$

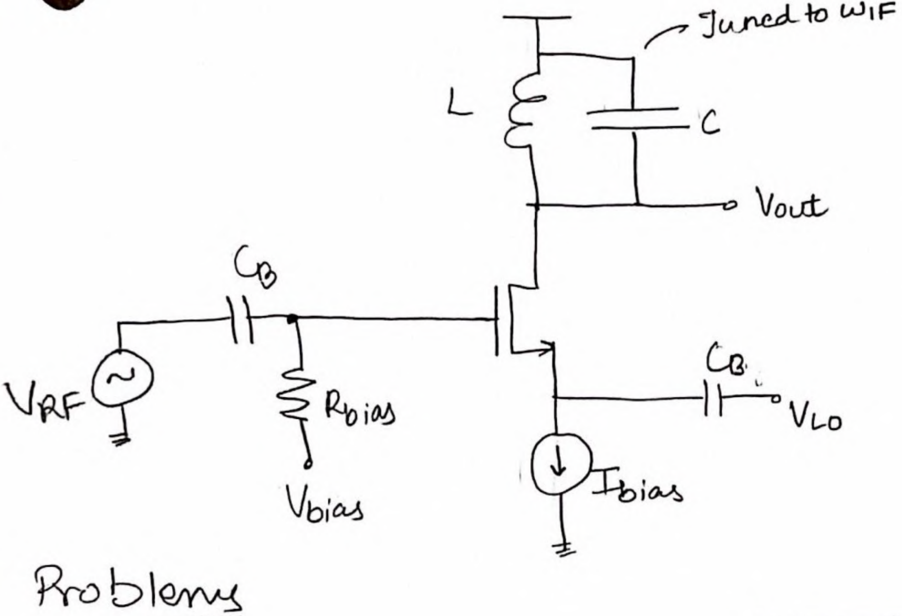
Problems

☺ Simple.

☹ Low gain

×× ☹ No RF, LO isolation.

Implementation ②



$$i_D = \frac{\mu_n C_{ox}}{2}\cdot\frac{W}{L}\left(V_{gs}-V_{tn}\right)^2$$

$$i_D = \frac{\mu_n C_{ox} W}{2L}\left\{(V_{RF}-V_{LO})-V_{tn}\right\}^2$$

$$= \frac{\mu_n C_{ox} W}{2L}\left\{V_{RF}^2+V_{LO}^2+V_{tn}^2 - 2\cdot V_{tn}(V_{RF}-V_{LO}) - 2V_{RF}V_{LO}\right\}$$

Problems

☺ High gain

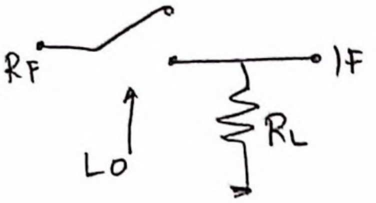
☹ Poor isolation

$$\Rightarrow \boxed{\text{Conversion gain} = \frac{\mu_n C_{ox}}{2}\cdot\frac{W}{L}\cdot V_{LO}}$$

↳ does not depend on V_{bias} as long as device is in saturation. V_{bias} must ensure $V_{RF}-V_{LO} > V_{tn}$!

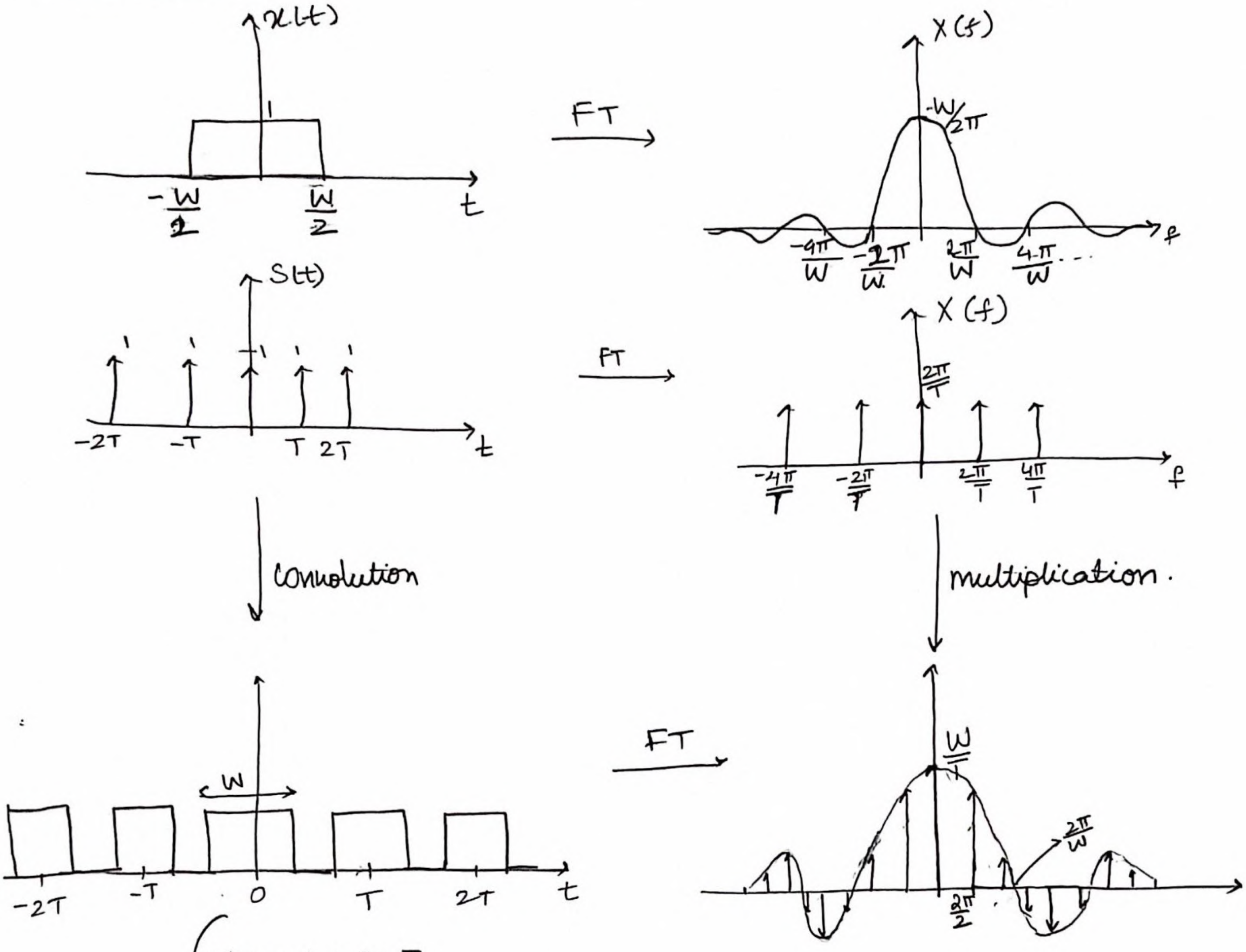
Switching Mixer

LTV



$\sin(\omega_{LO}t) \xrightarrow{\text{switch}} \text{sqr}(\omega_{LO}t).$

Derivation



assuming $w = \frac{T}{2}$.

$$\text{sqr}(\omega_{LO}t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_{LO}t) - \frac{1}{3}\cos(3\omega_{LO}t) + \frac{1}{5}\cos(5\omega_{LO}t) + \ldots\right)$$

V_{LO}

$$V_{IF}(t) = \underset{\overset{\downarrow}{A_{RF}\cos\omega_{RF}t}}{V_{RF}(t)} \cdot V_{LO}(t)$$

$$V_{IF}(t) = \frac{V_{RF}}{2}\cos(\omega_{RF}t) + \frac{2V_{RF}}{\pi}\underbrace{\cos(\omega_{LO}t)\cos(\omega_{RF}t)}_{\text{what we want.}}$$

$$\text{Voltage gain} = \frac{2}{\pi}\cdot\frac{1}{2} = \frac{1}{\pi} \rightarrow \text{low!}$$

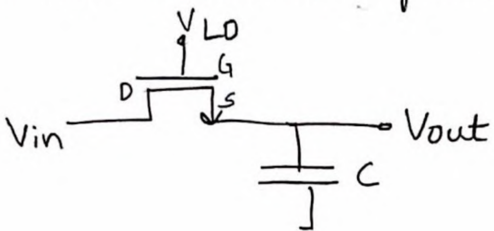
Problems

> LO is unbalanced (due to $\frac{1}{2}$)

> gain is low (it is passive)

Implementation ② → (more widely used)

> Use a swith & capacitor.



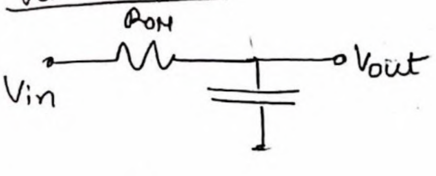
$V_{LO} = 0 \Rightarrow$ OFF

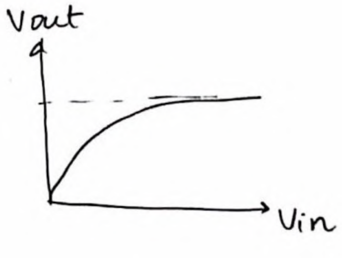
$V_{LO} = V_{DD} \Rightarrow$ ON

For small Vin.

$$R_{ON} = \frac{1}{\frac{\mu n C_{ox}}{2}\frac{W}{L}(V_{DD} - V_{out} - V_{tn})}$$

When ON





As Vout ↑, R ↑

⇒ slower

General rule

I ↓ ⇒ R ↑

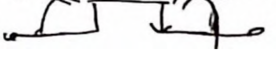
Problems

☹ > slow

😐 > Low pass → cutoff f > RF to work.

> To make switch fast { small C → dominates (limited by parasitics)
small RON ⇒ large $\frac{W}{L}$ ⇒ higher feedthrough

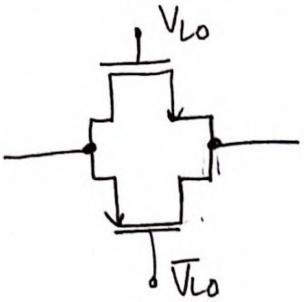
> Tradeoff b/w ON + OFF states.



> Can give gain despite being passive

Fixing problem of variable R.

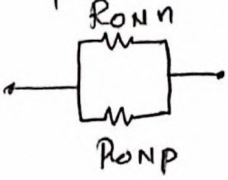
> Use a transmission gate.



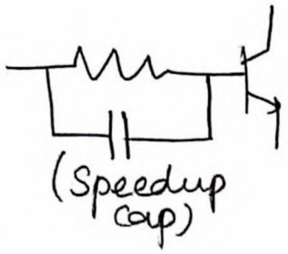
> $R_{ON_n} \parallel R_{ON_p}$ ⇒ dependance on V_{DD} is cancelled

when switch V_{LO} = high. V_{DD}.

V_{DD} term is cancelled.



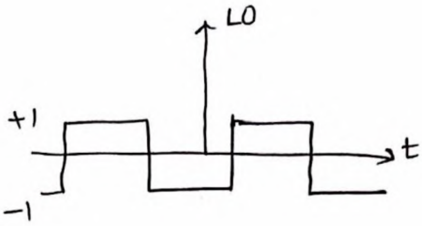
How to make a BJT switch faster.



> To go from OFF to ON charges stored in cap are used to make the transition.

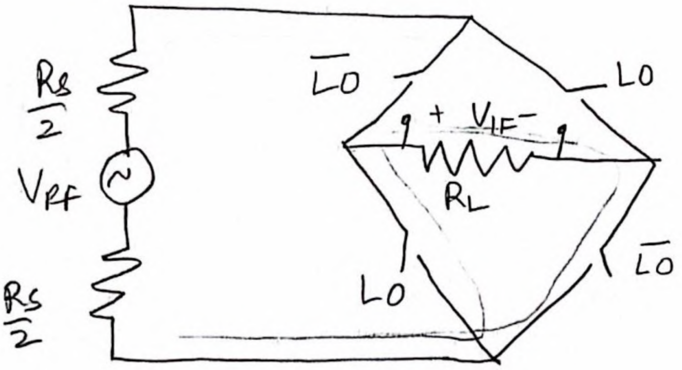
How to make it balanced?

> Use LO signal with 0 dc.



$$\Rightarrow V_{LO} = 0 + \frac{4}{\pi}\left(\cos\omega_{LO}t - \frac{1}{3}\cos(3\omega_{LO}t) + \cdots\right).$$

Double balanced.

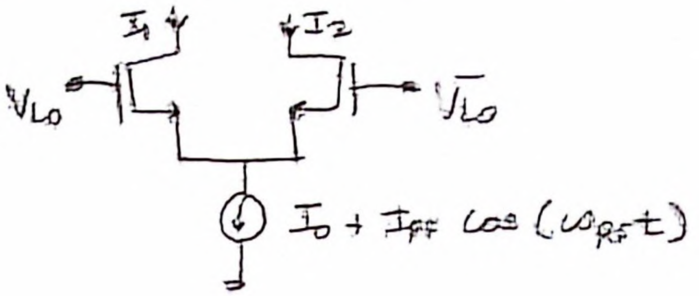


Problems

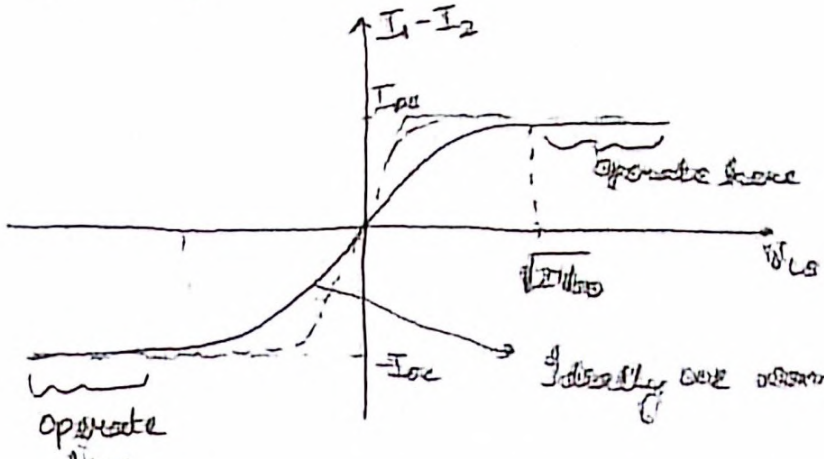
☹ 2 switches ⇒ (R_{ON} & $C_{parasitics}$)

Active Mixers

- Use a diff pair as a switch



- If V_{LO} is very large, current switches between I_1 and I_2



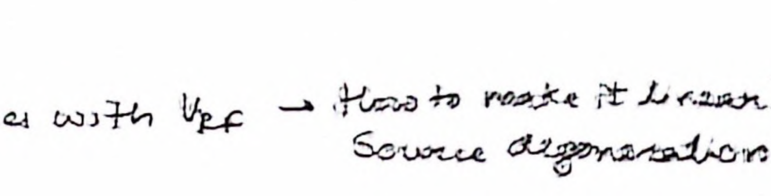
operate here

operate here.

Ideally we want a very steep curve.

→ How to make it steeper ~ Also helps with Flicker noise

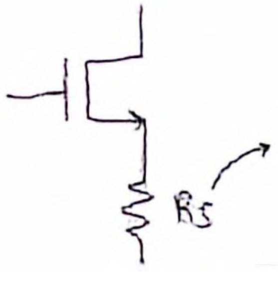
- Increase G_m (or $\frac{W}{L}$)
- How? use a transistor



In saturation $G_m = g_m$.

:) > large g_m.

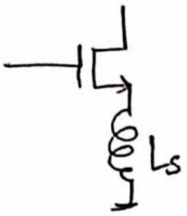
- Nonlinear g_m: G_m varies with V_{RF} → How to make it linear? Source degeneration.



Problems: > Requires headroom

> Noise

>



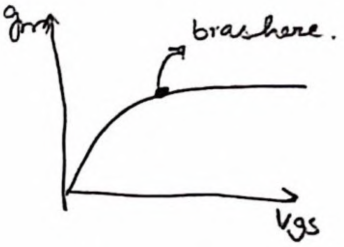
☺ > Linear Gm

☺ > Low noise (no noise if ideal)

☺ > No headroom issues

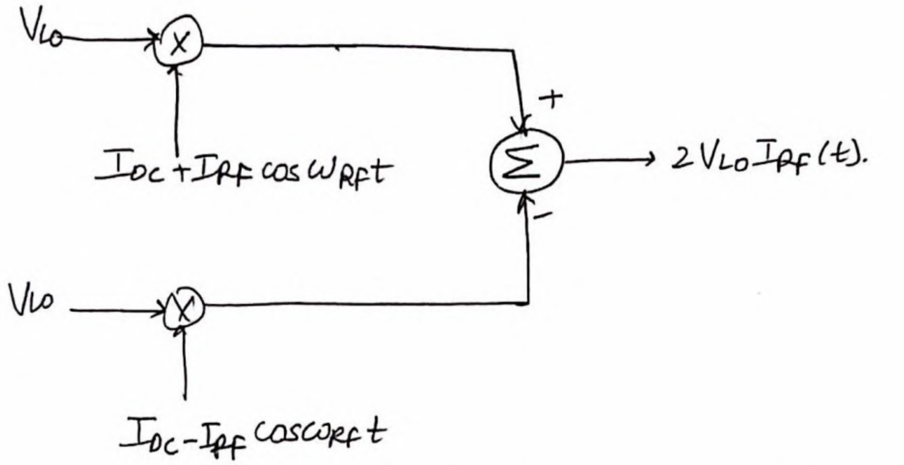
☺ > Could act as a filter for image frequency.
(We want to add filters wherever possible)

> Bias at velocity saturation to make gm (tail) more linear.

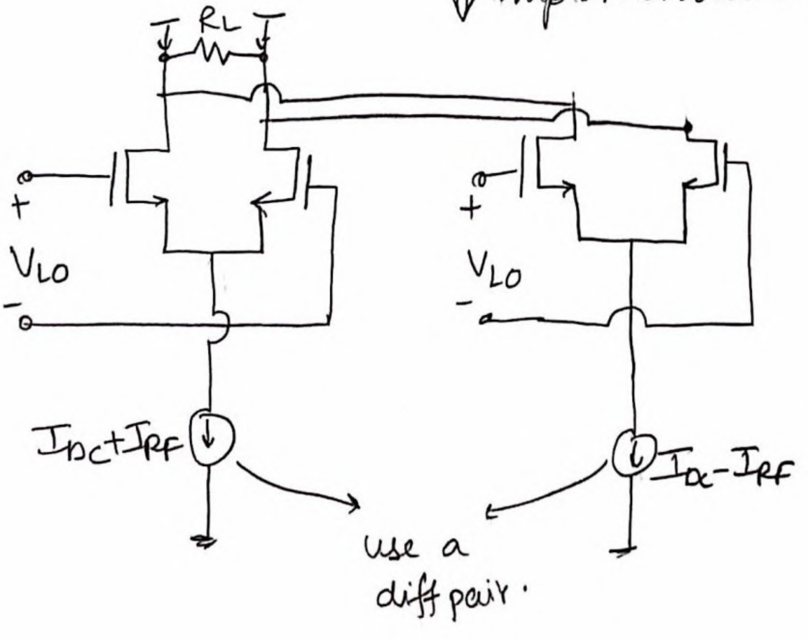


> However this is single balanced.
↳ unbalanced at RF.

Fix:

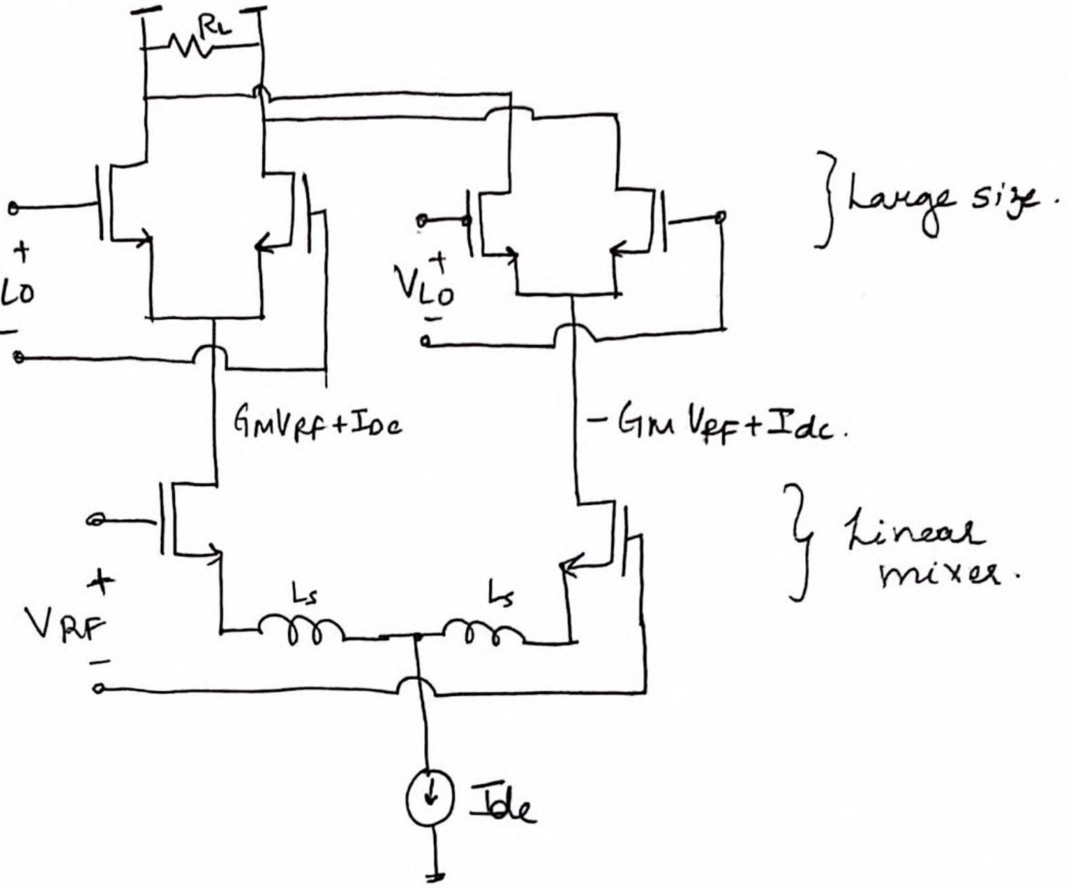


⇓ Implementation



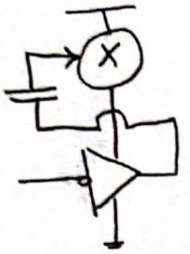
use a
diff pair.

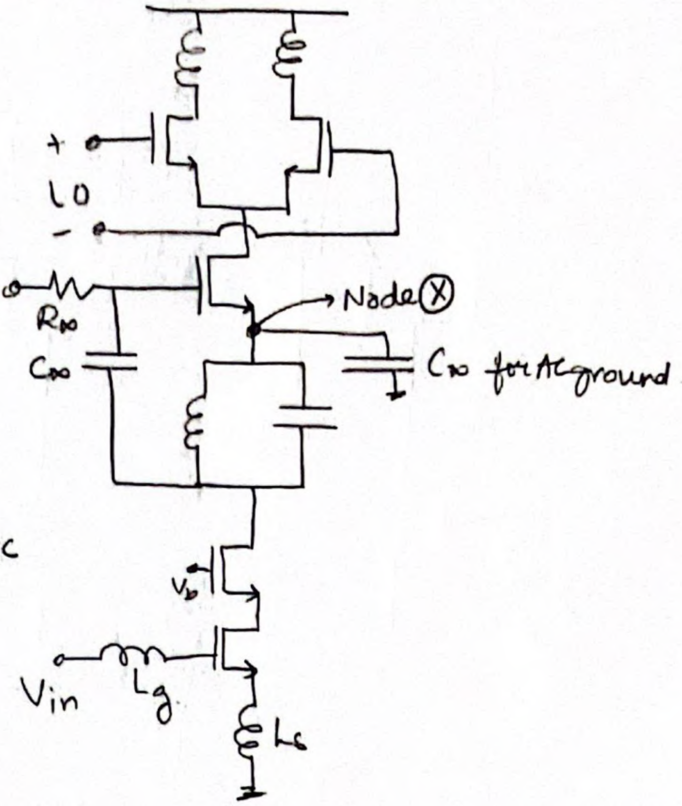
Gilbert Cell Mixer.



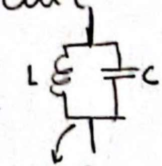
- > ☹ High power.
- > Up conversion is same
- > For noise performance read H. Darabi, ISSCC2005.

Current sharing between LNA and Mixer.



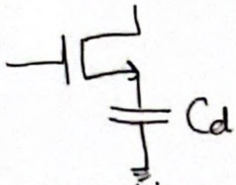


- Could play tricks at node (X)
  - place a resonant circuit to resonate at ω_{IM}



$\Rightarrow$ open at ω_{IM} + mixer is degenerated heavily.

- At ω_{IF} or ω_{RF} > ω_{IM}, it is a Cap!



Gives negative real impedance at Zin.

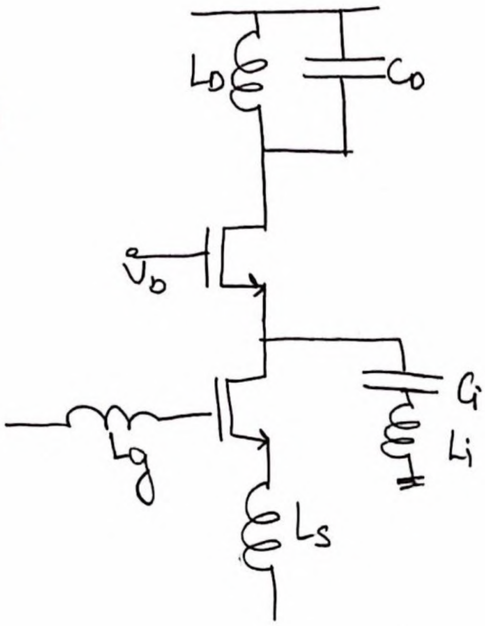
$$Z_{in} = Z_{gs} + Z_s + g_m Z_{gs} Z_s.$$

$$= \frac{1}{sC_{gs}} + \frac{1}{sC} + \underbrace{\frac{g_m}{s^2 C_{gs} C}}_{\text{negative}}.$$

$\Rightarrow$ gain is boosted at ω_{RF} or ω_{IF}

Paper: A. Zolfaghari, ISSC 2003.

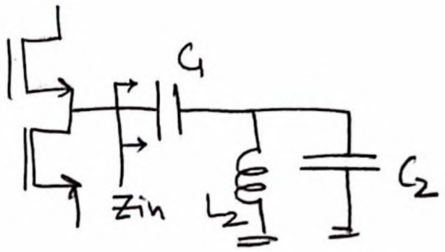
> Could filter IM at LNA



L_i & C_i resonate at ω_{IM} ⇒ short & gain = 0.

$\omega_{RF} > \omega_{IM}$ ⇒ ⌇ at ω_{RF}

In reality

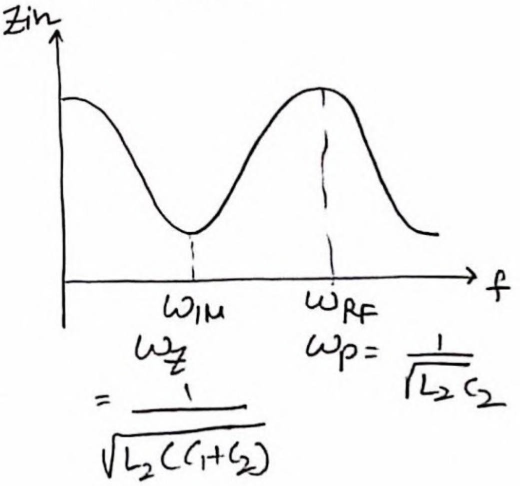


at ω_{IM} :- C_2 & L_2 are an inductor L_{eq}.

L_{eq} & C_1 resonate at ω_{IM}.

at ω_{RF} C_2 & L_2 are an inductor. L_{eq}'

$L_{eq}' < L_{eq}$ ⇒ L_{eq}' & C_1 looks like an inductor.



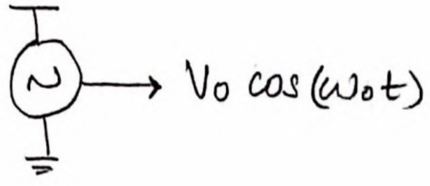
→ This way L_i is small.

Paper : H. Samavati, JSSC.

OSCILLATORS

> Review feedback, root locus, routh hurwitz method to find gain at which poles cross the jω axis

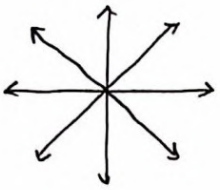
Design

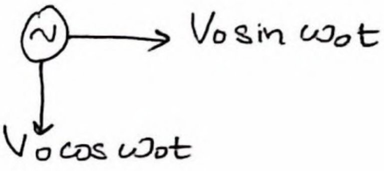


Requirements.

- Phase Noise (jitter) ⟷ related.
- Power consumption (efficiency or o/p power)
- Frequency
- Tuning range of VCO ← tradeoff Phase noise.
- Phase accuracy of multiphase oscillators.

(Tradeoff with phasenoise)





Types

① Resonance based:
- ☺ good phase noise
- ☺ good phase accuracy
- ☹ High power
- ☹ Large size.
- 🤩 Very high frequency.

② Ring oscillator:
- ☺ Low power.
- ☺ Small area.
- ☹ High phase noise
- 😐 Not as accurate.

③ Relaxation oscillators:
- ☺ Very low power.

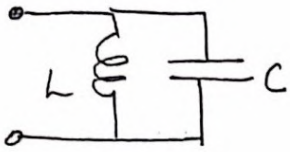
Basics

$$G = \frac{V_{out}}{V_{in}} = \frac{H}{1+H}.$$

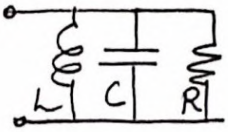
> Phase around loop should be 0° $\Rightarrow$ $$\left.\begin{array}{l} |H| = 1 \\ \measuredangle H = (2n+1)\pi \end{array}\right\}$$ Barkhausen Criteria.

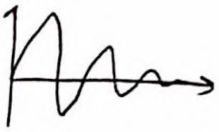
Negative Resistance Viewpoint

>

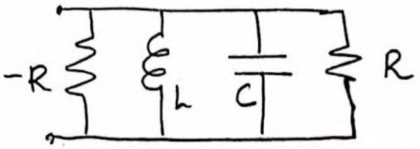


$\omega_0 = \frac{1}{\sqrt{LC}}$ $\xrightarrow{\text{reality}}$

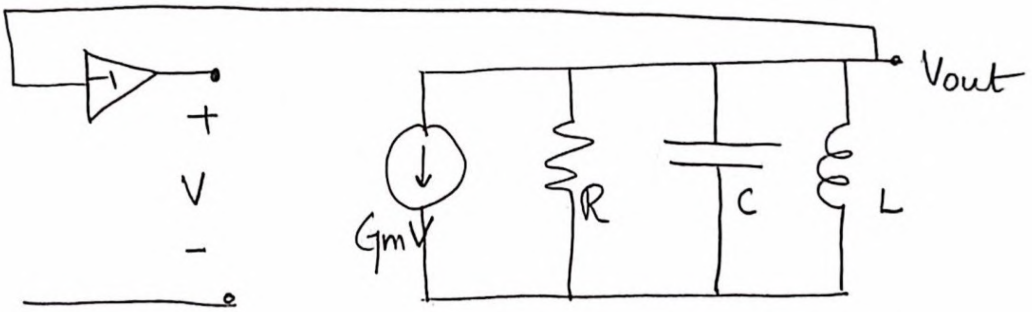




↓ add '-R'



↓ Implementation

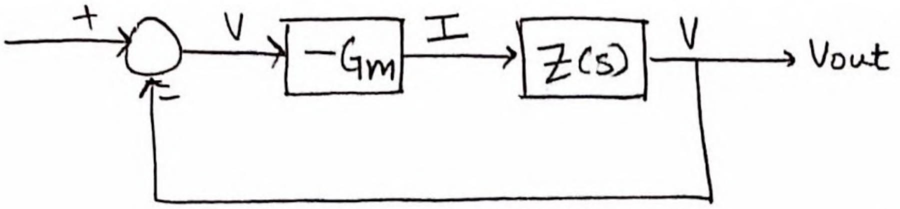


$V = -V_{out}$ $\Rightarrow$ $G_mV = -G_mV_{out}$.

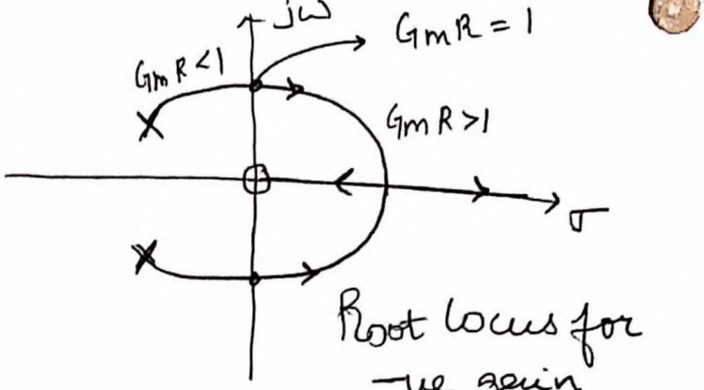
$\Rightarrow$ $I = -G_mV_{out}$ $\longrightarrow$ Current source whose value ∝ voltage across it ⇒ resistor.

$$\Rightarrow \boxed{R = -\frac{1}{G_m}}$$

$$\Rightarrow \frac{-1}{G_m} = R \text{ gives oscillation!}$$



$$Z(s) = \frac{Ls}{LCs^2 + \frac{L}{R}s + 1} \Rightarrow$$



Root locus for −ve gain (aka positive feedback RL)

$$\Rightarrow \boxed{G_{m\,\text{desired}} = \frac{1}{R}}$$

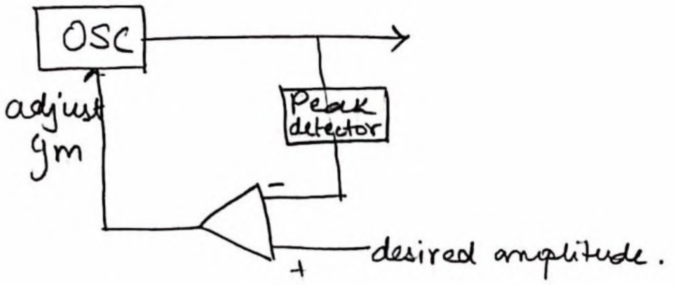
At resonance $Z(s) = R$.

$$\Rightarrow \left.\begin{aligned} |H| &= G_m R = 1 \\ \measuredangle H &= 180^\circ \end{aligned}\right\}$$ Satisfies Barkhausen!

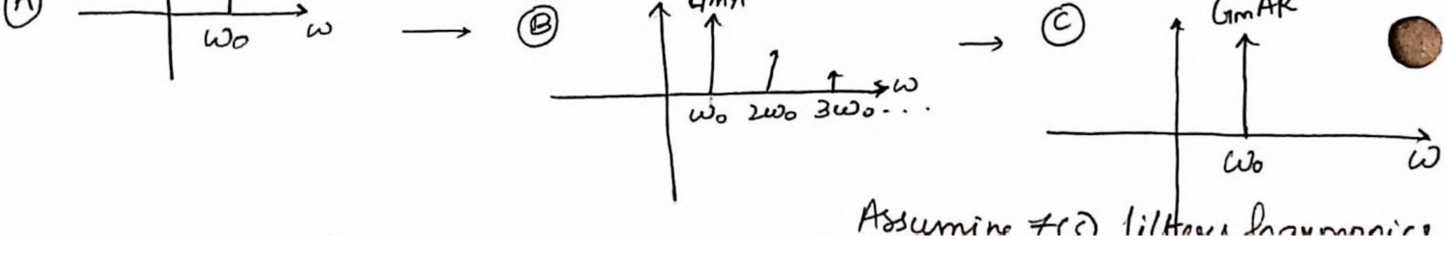
> How can we make $g_m = R$ exactly?

① Impractical solution (Feedback)

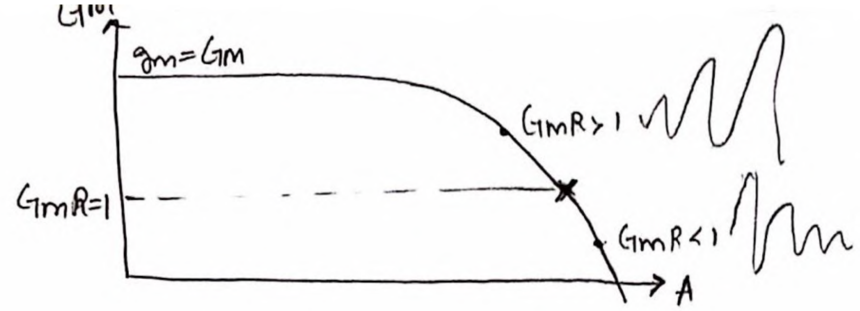
- ☹ Complex.
- ☹ Power ↑
- ☹ Noise ↑
- ☹ Stability
- ☹ Slow



② Nonlinearity.



Assuming Z(s) filters harmonics



G_M = ratio of (fundamental) current / I/p voltage.

> Startup condition : $G_m R > 1$ for small A.

> Circuit stabilizes itself.

BJT example

$$i_C = I_s e^{\frac{V_{BE}}{V_T}}$$

$$V_{BE} = V_{DC} + V_1 \cos \omega t .$$

$$\Rightarrow i_C = I_S \exp\left(\frac{V_{DC}}{T} + \frac{V_1 \cos\omega t}{V_T} \right)$$

$$= I_s \exp\left(\frac{V_{DC}}{V_T}\right) \left[I_0 + 2I_1 \cos\omega t + 2I_2 \cos 2\omega t + \dots \right]$$

Where,

$$I_0(x) = \frac{1}{2\pi} \int_{2\pi} e^{x\cos\theta} d\theta .$$

$$I_n(x) = \frac{1}{2\pi} \int_{2\pi} e^{x\cos\theta} \cos n\theta \, d\theta$$

$$x = \frac{V_1}{V_T} \qquad V_T \rightarrow \frac{KT}{q} = 25 mV$$

$$\Rightarrow i_c = I_s e^{\frac{V_{DC}}{V_T}} . I_0(x) \left[1 + 2 \sum_n \frac{I_n(x)}{I_0(x)} \cos n\omega t \right]$$

Average.

$$\bar{i}_c = I_{DC} = I_s e^{\frac{V_{DC}}{V_T}} . I_0(x)$$ ∵ avg of cos=0

$\underline{x=0 \Rightarrow}$ $$\bar{i}_c = I_{DC} = I_s e^{\frac{V_{DC}}{V_T}}$$

$\underline{x \text{ is small} \Rightarrow}$ $$\begin{cases} I_0(x)=1 \\ I_1(x)=\frac{x}{2} \end{cases} \Rightarrow i_c = I_{DC} + \frac{2\cdot\frac{x}{2}}{1}\cdot\cos\omega t + \underset{0}{\cancel{\cdots}}$$

$$\Rightarrow \quad i_c = I_{DC} + x I_{DC} \cos\omega t$$

$$= I_{DC} + \frac{V_1}{V_T} . I_{DC} \cos\omega t$$

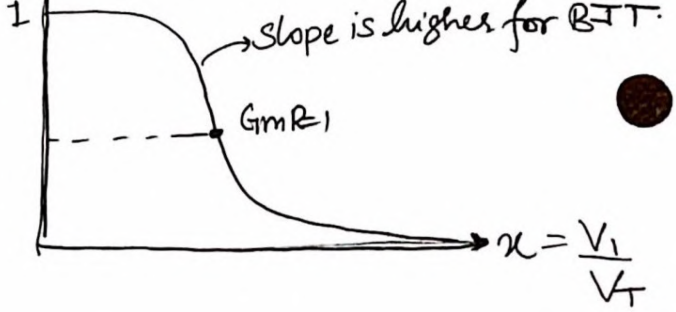
$$i_c = I_{DC} + V_1 \underbrace{g_m}_{\text{small signal } g_m} \cos\omega t$$

$\underline{x \text{ is large}}$ $$\Rightarrow \quad G_m = \frac{\text{fund. of } I}{V} = \frac{2 I_{DC} I_1(x) \cos\omega t / I_0(x)}{V_1 \cos\omega t}$$

$$= \frac{2 I_{DC}}{V_1} \cdot \frac{I_1(x)}{I_0(x)}$$

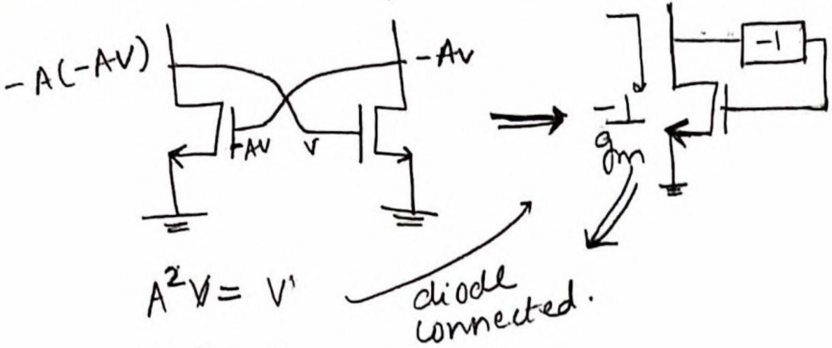
$$= \frac{2}{x} \cdot \frac{I_{DC}}{V_T} \cdot \frac{I_1(x)}{I_0(x)}$$

$$\boxed{\frac{G_m}{g_m} = \frac{2}{x} \cdot \frac{I_1(x)}{I_0(x)}}$$



Circuit

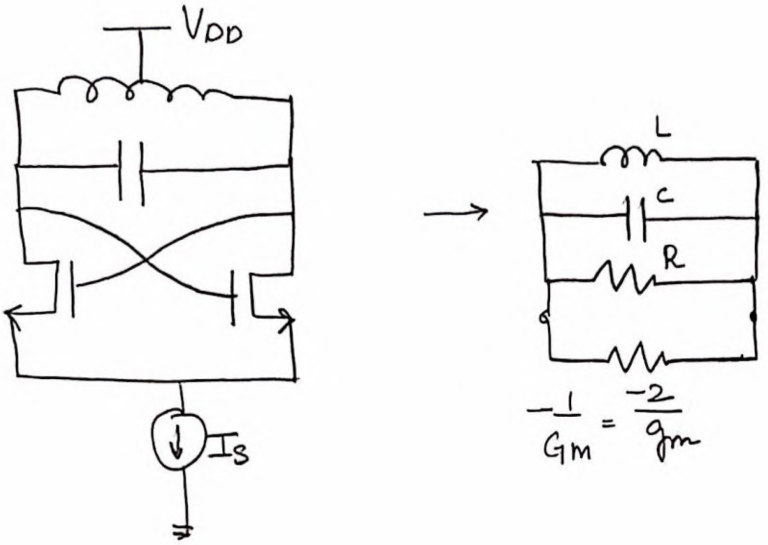
(I) Cross coupled oscillator



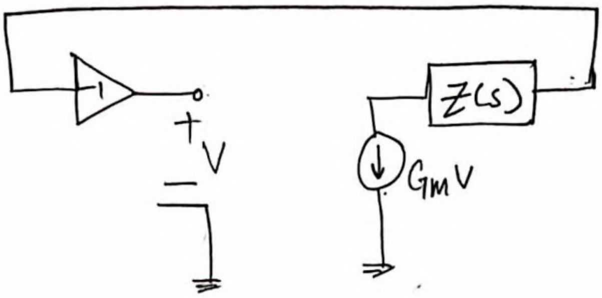
$\xrightarrow[\text{series}]{\text{2 in}} \Rightarrow -\frac{2}{g_m}$

$A^2V = V'$

$\Rightarrow A = 1$



At small signal
$G_M = \frac{g_m}{2}$



Poles at $1 + Z(s)G_m = 0$

$$Z(s) = \frac{sL}{s^2LC + s\frac{L}{R} + 1}$$

$$\Rightarrow 1 + \frac{sL}{s^2LC + s\frac{L}{R} + 1} \cdot -\frac{g_m}{2} = 0 \xrightarrow[\text{eq.}]{\text{ch.}} (LC)s^2 + \left(\frac{1}{R} - \frac{g_m}{2}\right)Ls + 1 = 0$$

Startup ⇒ the poles ⇒ $g_m > \frac{2}{R}$

$$\boxed{g_m R > 2}$$

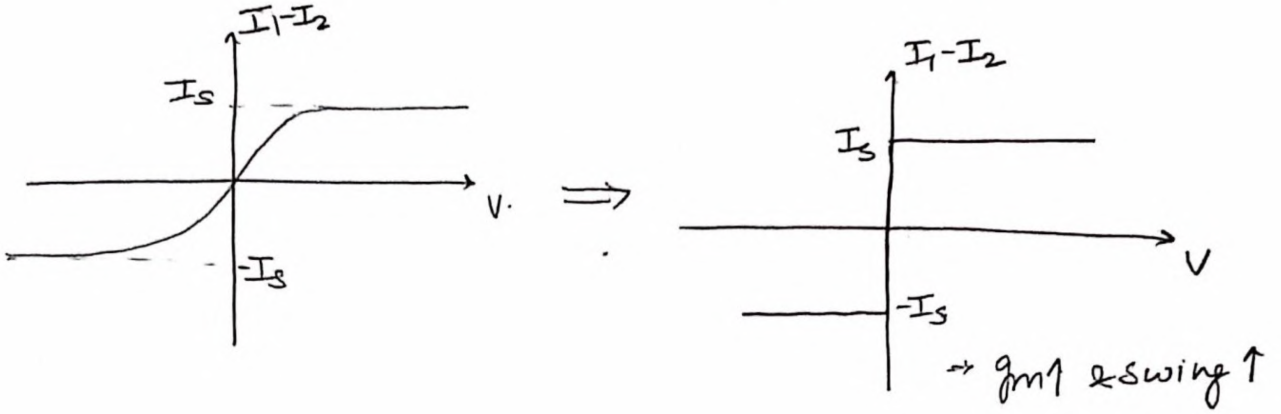
$\{ R = R_p$ of tank at resonance.

Amplitude of oscillations.

Assumptions

① Q of LC tank is large enough to filter harmonics ($Q \gg 2$)

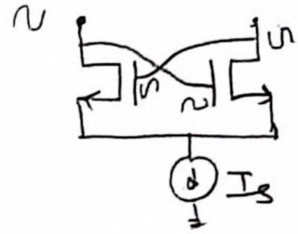
② Assume the diff pair switches "ideally" ⇒ step function.

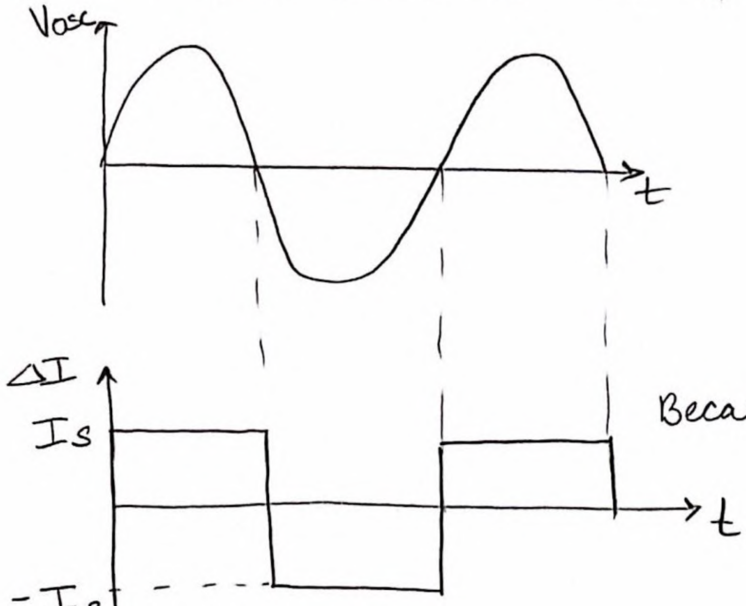


→ gm↑ & swing↑

Derivation.

① Assume some oscillation at o/p.





Because of Assumption #2.

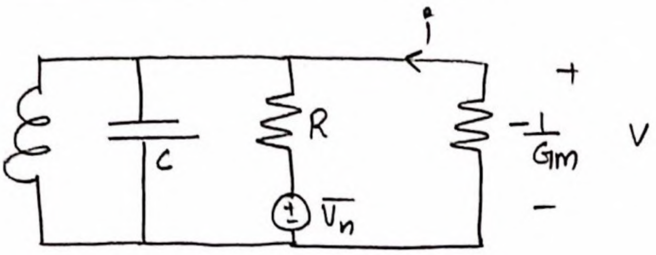
$$\Delta I = \frac{4 I_s}{\pi}\left(\sin \omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t \ldots\right)$$

↳ Fourier series of square wave

> Assuming harmonics are filtered

$$\boxed{V_0 = \frac{4 I_s}{\pi} R.} \Rightarrow \boxed{G_m = \frac{4 I_s}{\pi V_0}} \rightarrow \text{Need to ensure } \boxed{g_m R_p \gg 2}$$

Oscillator derivation.



$$\frac{V}{-i} = -\frac{1}{Gm} \Rightarrow \frac{V}{i} = \frac{1}{Gm}.$$

$$KCL \Rightarrow \quad V Gm = \frac{V}{SL} + VSC + \frac{V - V_n}{R}.$$

$$\Rightarrow \quad V\left[Gm\right] = V\left[\frac{1}{SL} + SC + \frac{1}{R}\right] - \frac{V_n}{R}$$

$$\Rightarrow \quad V\left[Gm - \frac{1}{Z(s)}\right] = -\frac{V_n}{R}.$$

$$\Rightarrow \quad \frac{V}{V_n} = \frac{-Z(s)/R.}{Gm Z(s) - 1}$$

$$\Rightarrow \quad \frac{V}{V_n} = \frac{Z(s)/R.}{1 - Gm Z(s).}$$

Characteristic equation: $1 - Gm Z(s). = 0.$

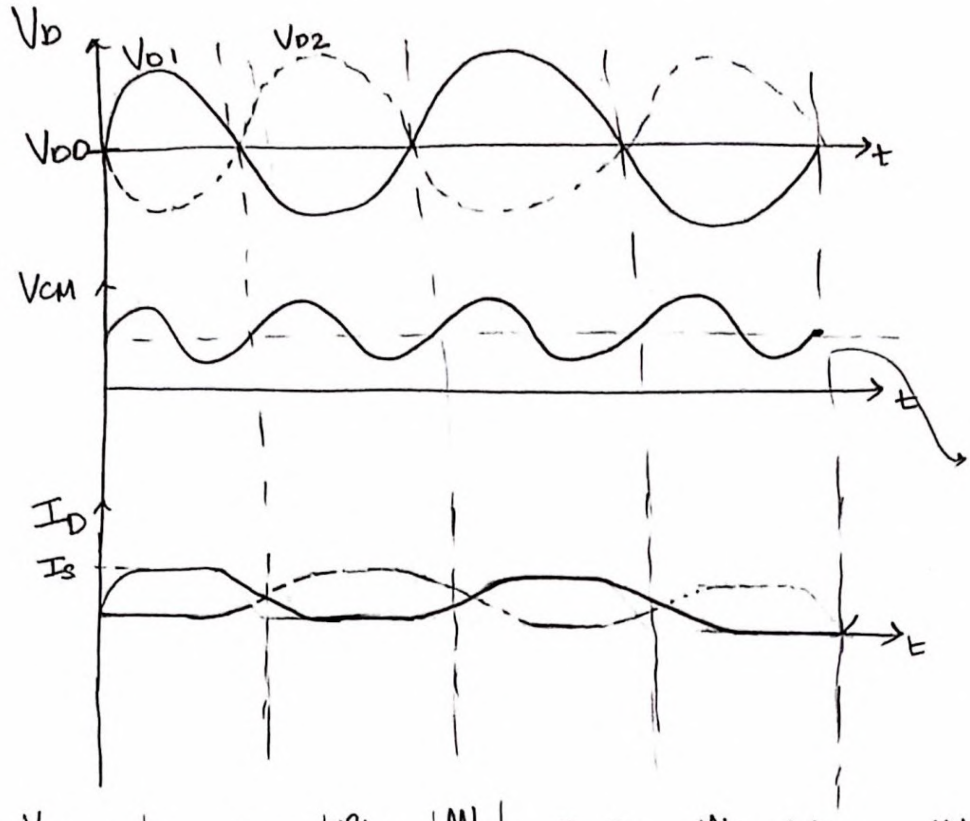
$$\Rightarrow \quad 1 - Gm\left[\frac{SL}{S^2LC + S\frac{L}{R} + 1}\right] = 0$$

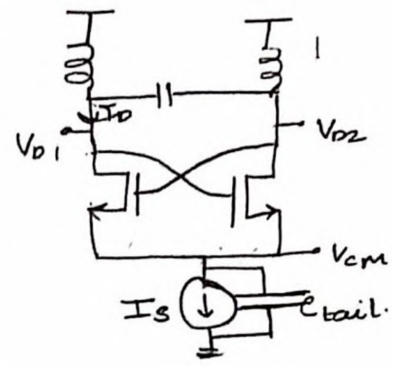
$$\Rightarrow \quad S^2LC + S\left[\frac{L}{R} - GmL\right] + 1 = 0$$

To get oscillations we need damping factor < 0

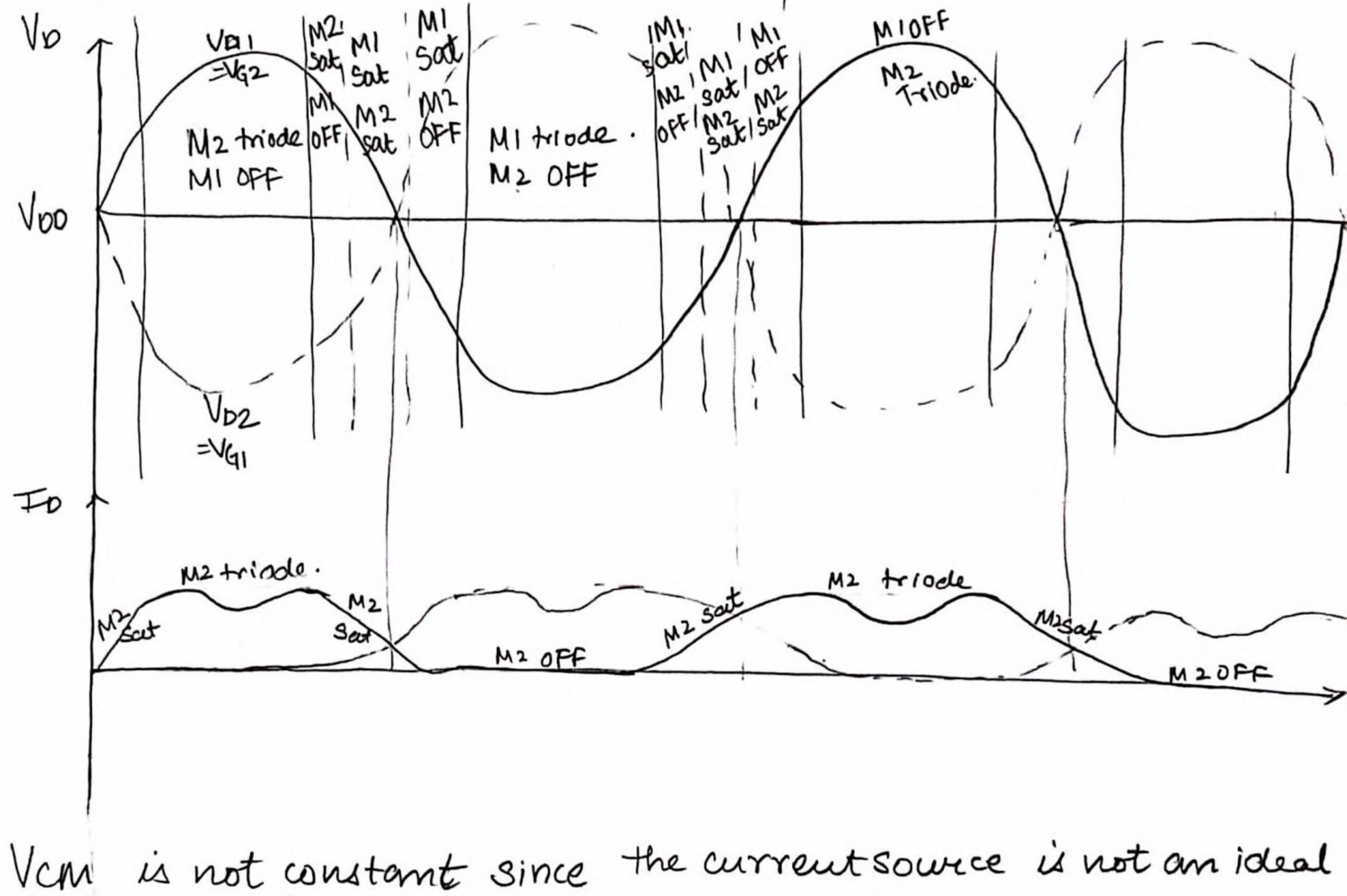
$$\Rightarrow \quad \frac{L}{R} - GmL < 0$$

$$\Rightarrow \quad \boxed{Gm > \frac{1}{R}}$$



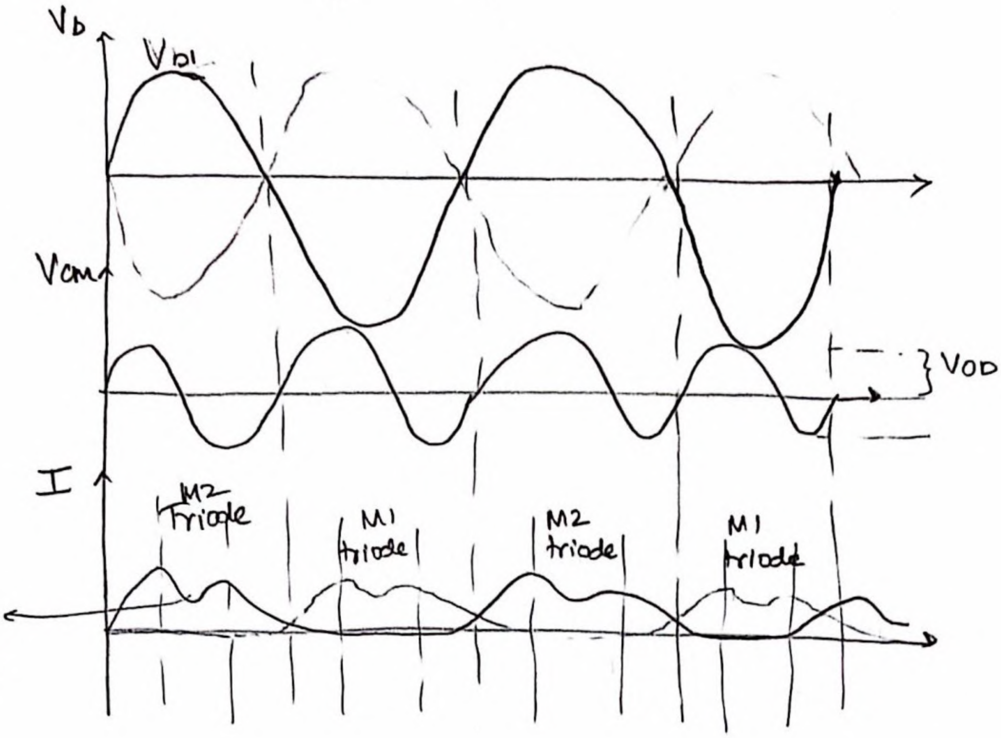


Twice the frequency.
Why?



→ Vcm is not constant since the current source is not an ideal AC open. The capacitence Ctail "steals" current every time M1 or M2 turns ON [since $I_{D1}+I_{D2}$ is not exactly I_S]. Therefore, its voltage goes up twice every cycle.

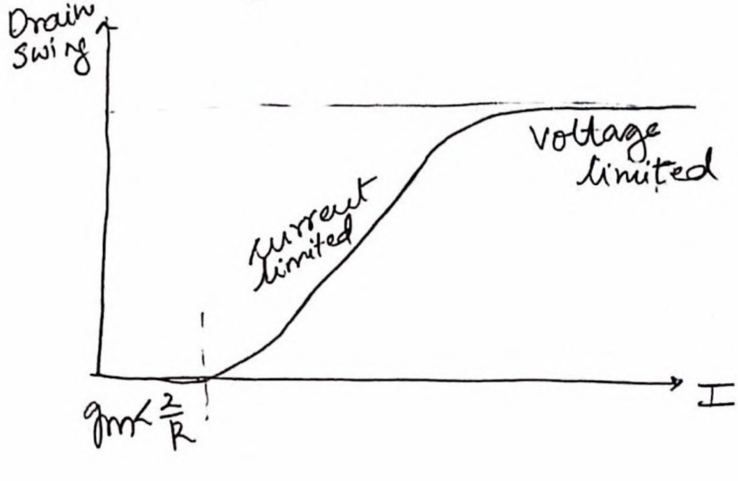
- Large swing ⇒ I_s ↑ ⇒ Current limited Region.
- Drain voltage swing ↑ with I_s as long as it is above V_{cm} (by $1 V_{OD}$).
- V_{cm} should be large enough for I_{tail} to be in saturation.



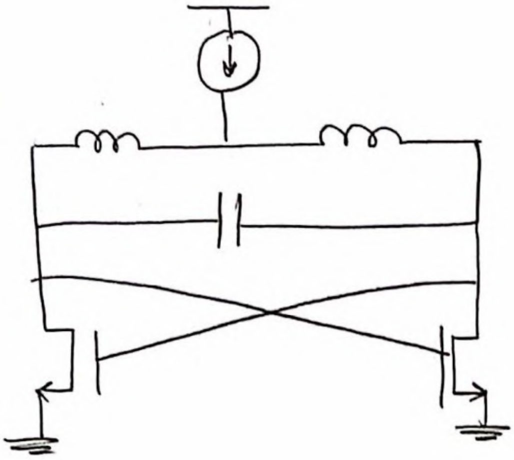
Refer to previous graph.

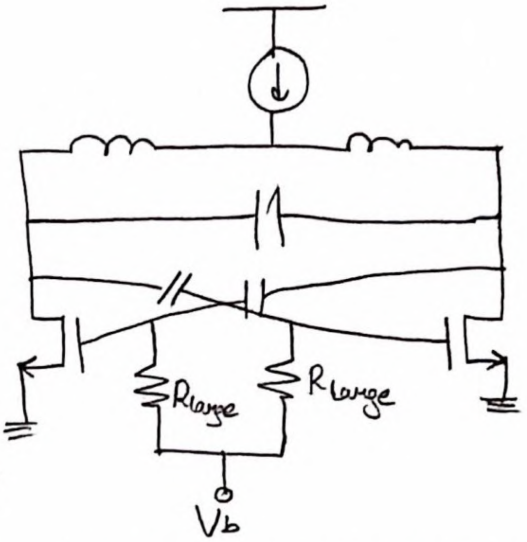
Max oscillation voltage = $\pm(V_{DD} - V_{CM})$ → (ignoring triode)

- Voltage limited region: M_1 & M_2 go into triode during small period.
- Current through branches would be Q times larger.

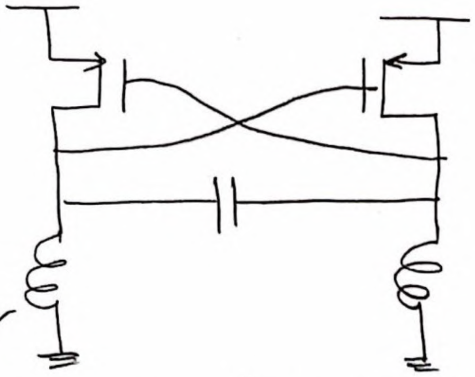


Variations.



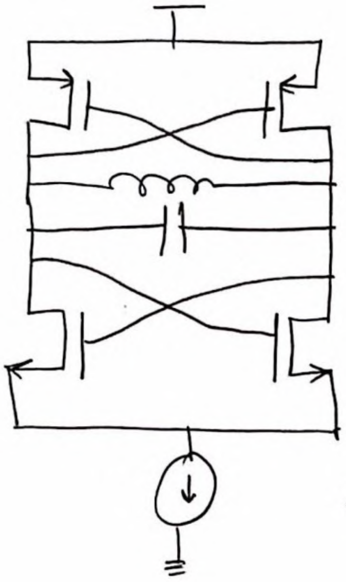


→ Bias gates seperately to improve phase noise as M1&M2 do not stay in triode region for long.
⇒ lower CM of gate to less than V_{DD}.



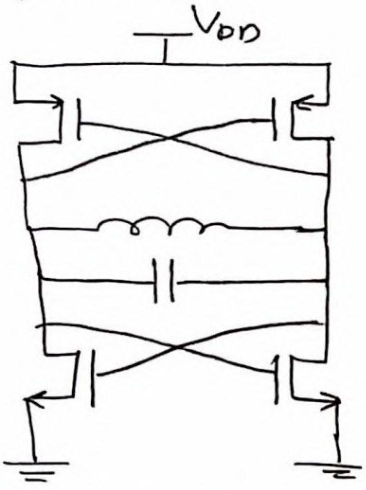
→ Lower flicker noise of PMOS due to burried channel.

*→



- > Low phase noise since ISF is symmetric
- > Higher g_m for same bias current
- ⇒ Higher swing for same bias current
- > Lower swing in voltage limited region.
- > Difficult to bias & needs higher V_{DD}.

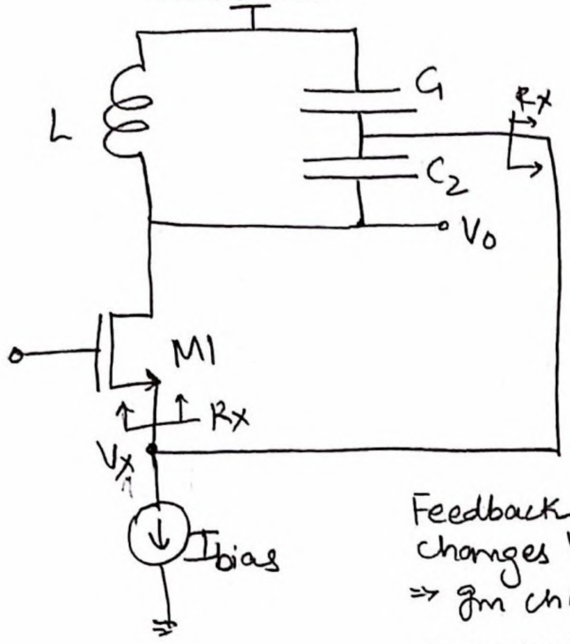
> To remove tail noise and reduce V_{DD} use voltage biased configuration



> Sizing needs to be right to limit current.

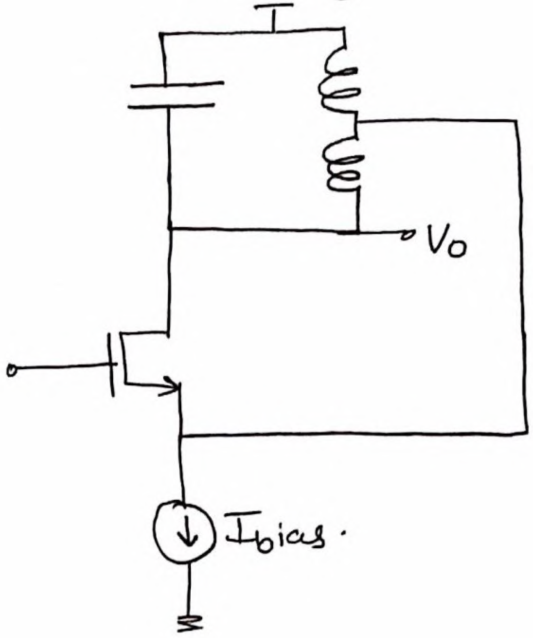
Three point oscillators

Colpitts

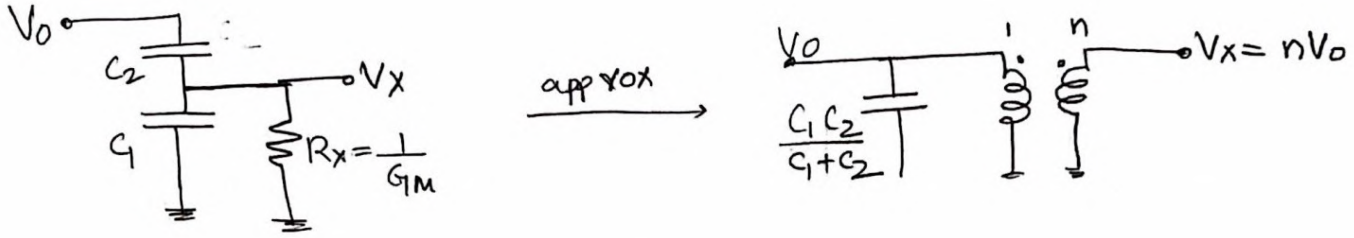


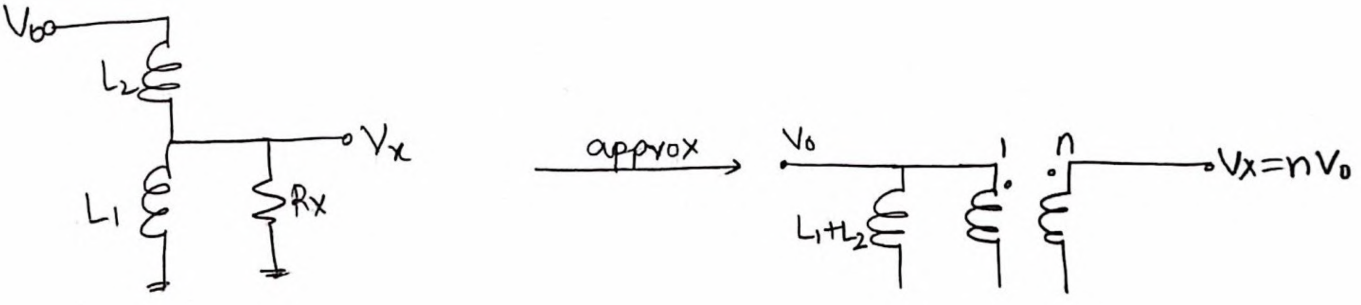
Feedback changes Vgs.
=> gm changes
=> -ve resistance.

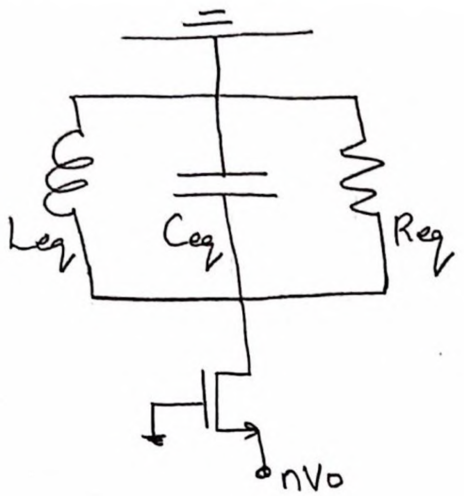
Hartley



Analysis







Colpitts

$L_{eq} = L$

$C_{eq} = \frac{C_1 C_2}{C_1+C_2}$

$n = \frac{C_2}{C_1+C_2}$

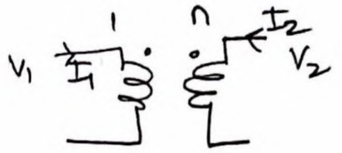
Hartley

$L_{eq} = L_1+L_2$

$C_{eq} = C$

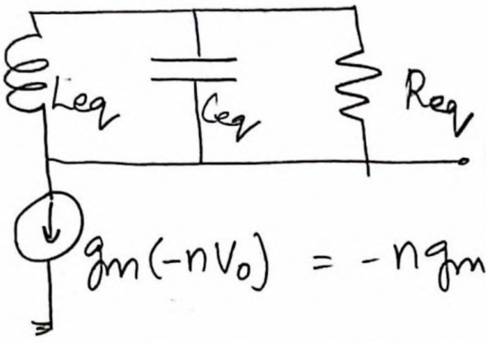
$n = \frac{L_1}{L_1+L_2}$

$R_{eq} = \frac{R_X}{n^2}$ since $\frac{V_1}{I_1} = \frac{R_X}{n^2}$



$R_{eq} \overset{\text{small}}{\underset{\text{signal}}{=}} \frac{\frac{1}{g_m}}{n^2}$

→



$g_m(-nV_o) = -n g_m V_o$

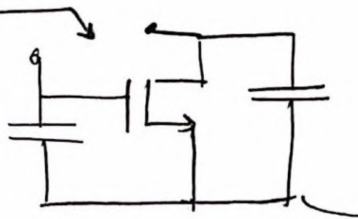
$\Rightarrow \boxed{R = \frac{-1}{n g_m}}$

$$\boxed{\omega_{osc} = \frac{1}{\sqrt{L_{eq} C_{eq}}}}$$

Startup $-n g_m + \frac{1}{R_{eq}} < 0 \Rightarrow \boxed{g_m > \frac{1}{n R_{eq}}}$

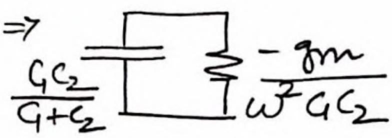
Easier derivation

Zin



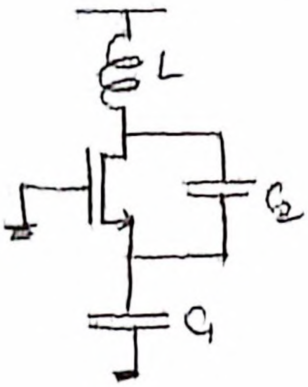
$\left.Z_{in} = \frac{1}{sC_1} + \frac{1}{sC_2} + \frac{g_m}{s^2 C_1 C_2}\right\} \rightarrow$ negative resistance.

$\Rightarrow$

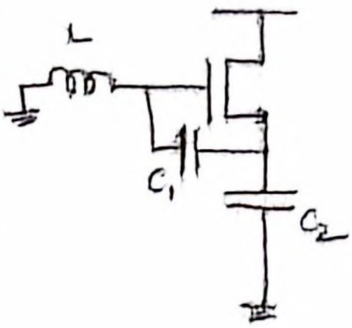


→ Three point oscillator.

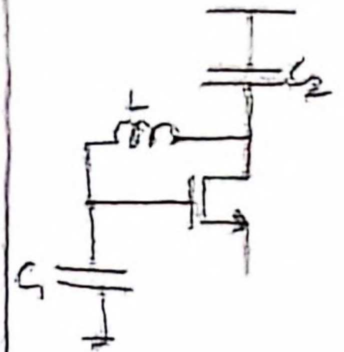
Ground the gate



Ground the drain

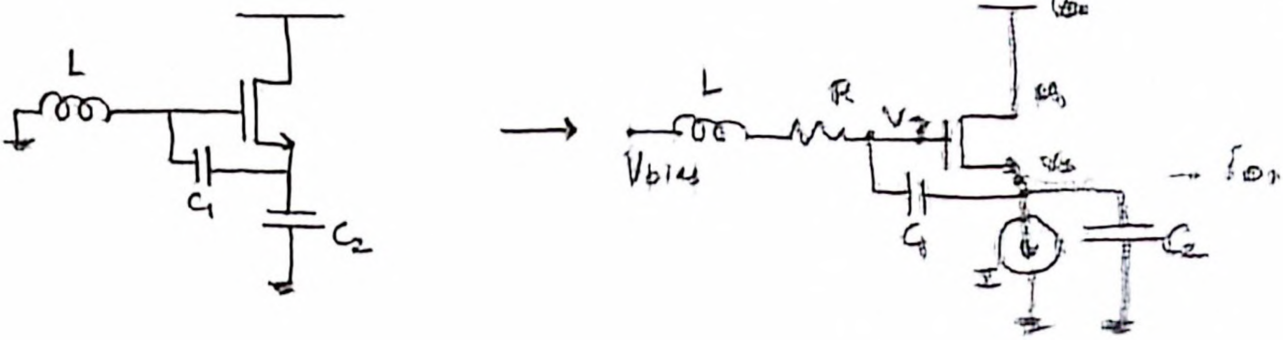


Ground the source



> Noise performance differs but ω_{osc} is same!

Analysis



Find voltages and current waveforms

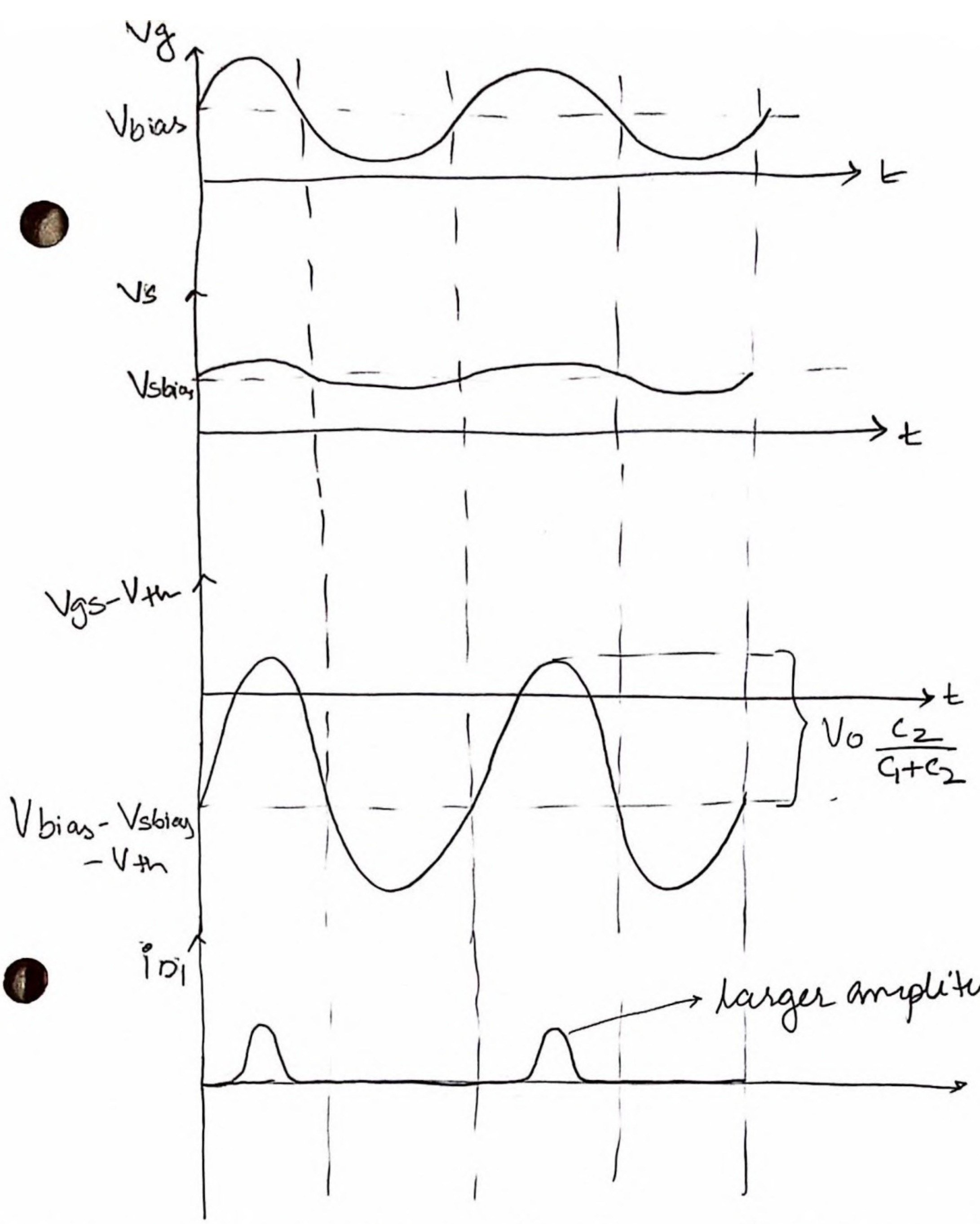
> Assume high Q, ⇒ Voltages are single tone

> $$\omega_0 = \frac{1}{\sqrt{L \cdot \frac{C_1 C_2}{C_1 + C_2}}}$$

$$V_g = V_{bias} + V_0 \sin \omega_0 t$$

$$V_s = V_{sbias} + \frac{C_1}{C_1 + C_2} V_0 \sin \omega_0 t$$

$$V_{gs} = V_{bias} - V_{sbias} + \frac{C_2}{C_1 + C_2} V_0 \sin \omega_0 t$$



Energy is pumped into tank in short impulses.

Avg $\langle i_{D_1} \rangle = I_{bias}$

$$i_{D_1}(t) = I_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

We care about fundamental. for amplitude of voltage.

$$\Rightarrow a_n = \frac{2}{T}\int_0^T i_{D_1}(t)\cos(n\omega_0 t) = 0 \rightarrow \text{assuming single tone sinusoid.}$$

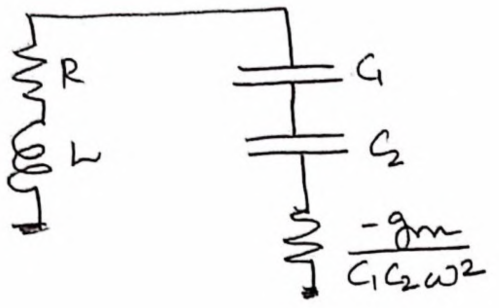
$$b_1 = \frac{2}{T}\int_0^T i_{D_1}(t)\sin(\omega_0 t) \simeq \frac{2}{T}\int_0^T i_{D_1}(t) \rightarrow$$

when we have current flowing the sine is close to the pea

$\Rightarrow \quad b_1 = 2 I_{bias}.$

Oscillation amplitude

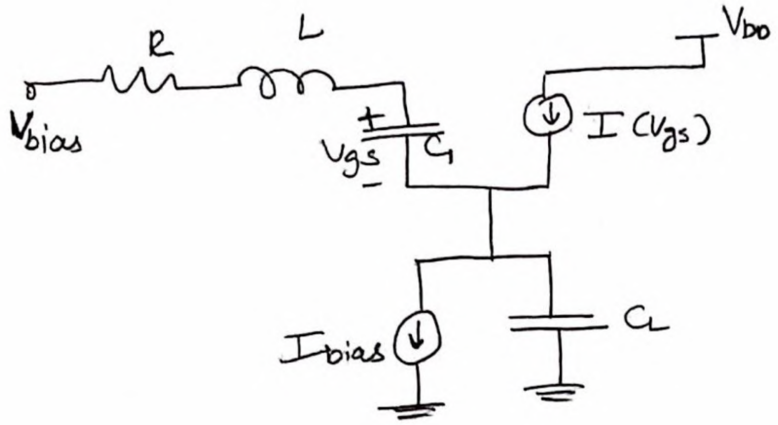
Use small signal.



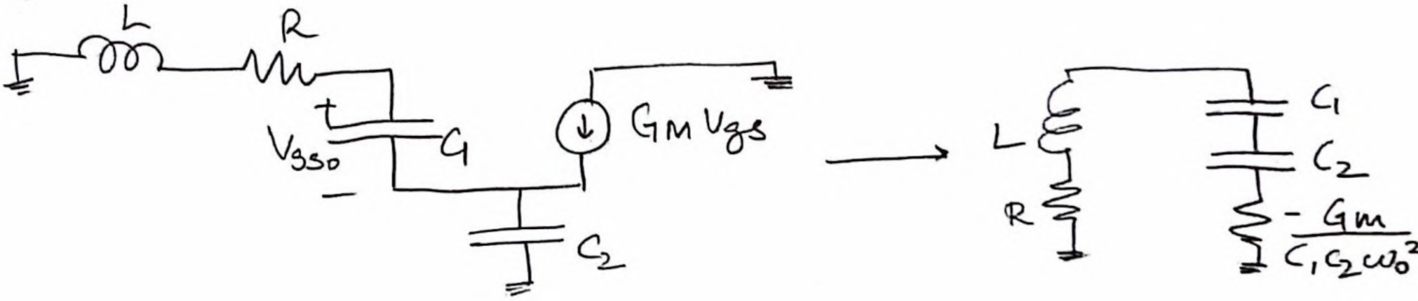
Startup $\Rightarrow$ $g_m > C_1C_2\omega^2 R$.

In steady state, small signal model cannot be used.

$\Rightarrow$



At fundamental,



At steady state:
$$R - \frac{G_M}{C_1C_2\omega_0^2} = 0$$

$$\Rightarrow G_m = C_1C_2\omega_0^2 R$$
$$= \frac{R(C_1+C_2)}{L}$$

But, $G_m = \frac{I_{fund}}{V_{gs0}}$

$$\Rightarrow G_M = \frac{2 I_{bias}}{V_{gs0}} = \frac{C_1+C_2}{L} \cdot R$$

$$\Rightarrow \boxed{V_{gs0} = \frac{L}{C_1+C_2} \cdot \frac{1}{R} \cdot 2 I_{bias}}$$

$$V_{gs0} = \frac{C_2}{C_1+C_2} V_0$$

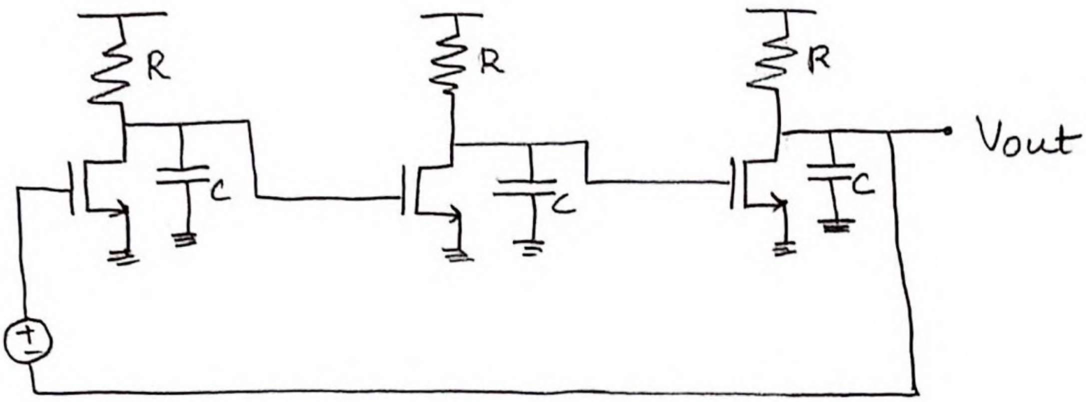
V_0 → Voltage swing at fundamental

$$\Rightarrow \boxed{V_0 = \frac{L}{C_2} \cdot \frac{1}{R} \cdot 2 I_{bias}}$$

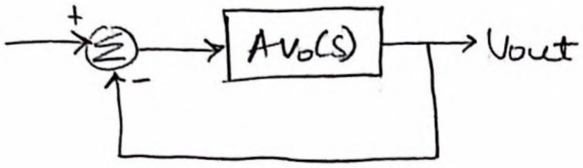
Current limited region.

→ Independant of transistor.
↓
just acts as a switch to "pump" energy

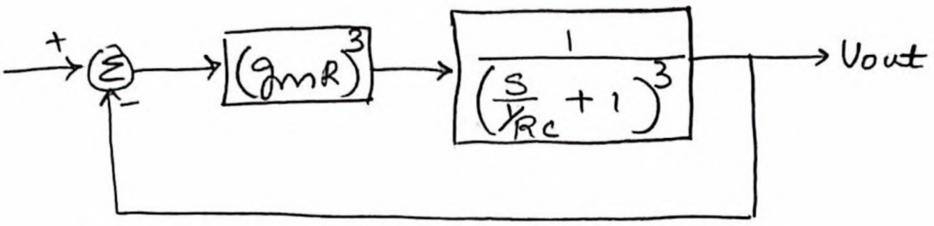
RING OSCILLATOR

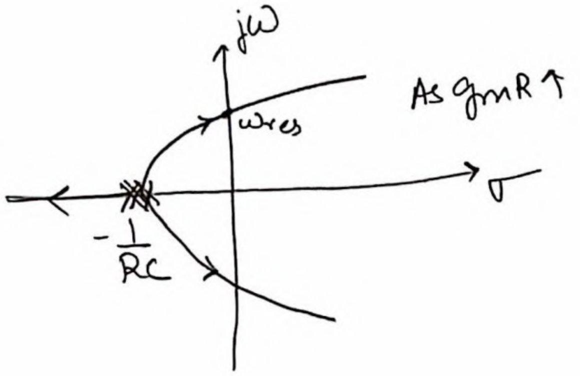


Model



$$A_{Vo}(s) = \left(\frac{g_m R}{RCs+1}\right)^3$$ ignoring 2nd order parasictics.





$$\frac{(2n+1)180^\circ}{3} \rightarrow 60^\circ, 180^\circ, 300^\circ$$

$$\sigma_a = -\frac{1}{RC}$$

$$\omega_0 \simeq \frac{1}{RC} \cdot \tan 60^\circ$$

$$\boxed{\omega_0 = \frac{\sqrt{3}}{RC}}$$

Using RH table to find gmR.

$$(RCs+1)^3 + (g_m R)^3 = 0$$

$$\Rightarrow (RC)^3 s^3 + 3(RC)^2 s^2 + 3RCs + 1 + (g_m R)^3 = 0$$

$$\begin{array}{c|cc} s^3 & (RC)^3 & 3RC \\ s^2 & 3(RC)^2 & 1+(g_mR)^3 \\ s^1 & \dfrac{3(RC)^2\cdot 3RC-(RC)^3[1+(g_mR)^3]}{3(RC)^2} & 0 \\ s^0 & 1+(g_mR)^3 & 0 \end{array}$$

$$\Rightarrow 9(RC)^3-(RC)^3[1+(g_mR)^3] < 0$$

$$\Rightarrow \boxed{g_mR > 2} \longrightarrow \text{why 2?}$$

T.F of each stage is $\frac{g_mR}{RCs+1}$ ⟶ We need $|TF|=1$

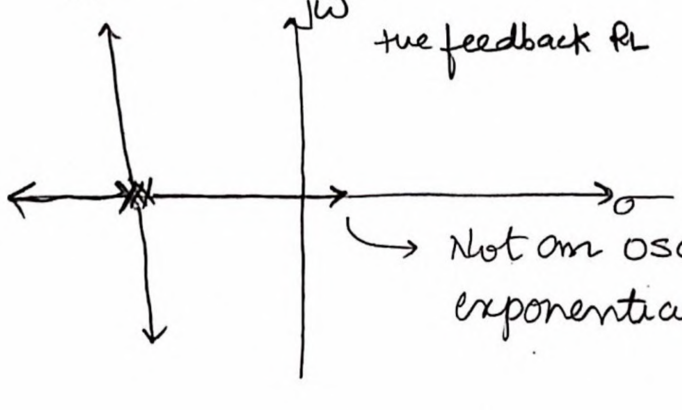
$\measuredangle TF = 120°$

$\Rightarrow s = \frac{j\sqrt{3}}{RC}$. Solving, we get $g_mR = 2$.

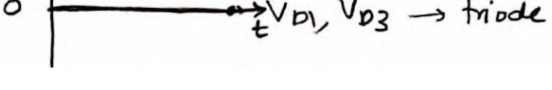
> ω_{res} could be much lower. Why?

> Inputs to transistors are large ⇒ The capacitor could be slewing. ⇒ Slew rate limited. ⇒ Dont use large current sources.

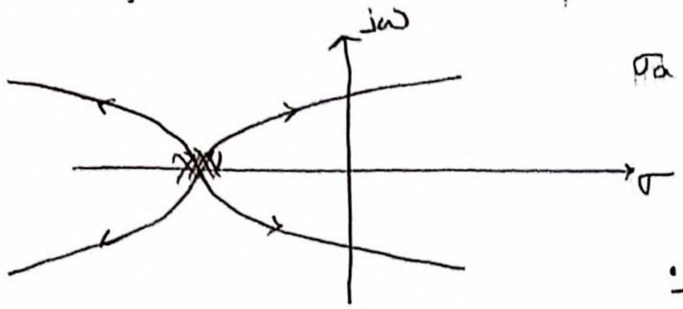
> What happens if we use 4 stages.



↳ Not an oscillator, amplitude grows exponentially and 'latches'.



> However if we had -ve feedback.



$\sigma_a = -\frac{1}{RC}$

$\omega_0 = \frac{1}{RC} \tan 45°$

☹ > $\omega_0 = \frac{1}{RC}$ → lower.

☺ > More phases.

$1 + A_v(s) = 0$

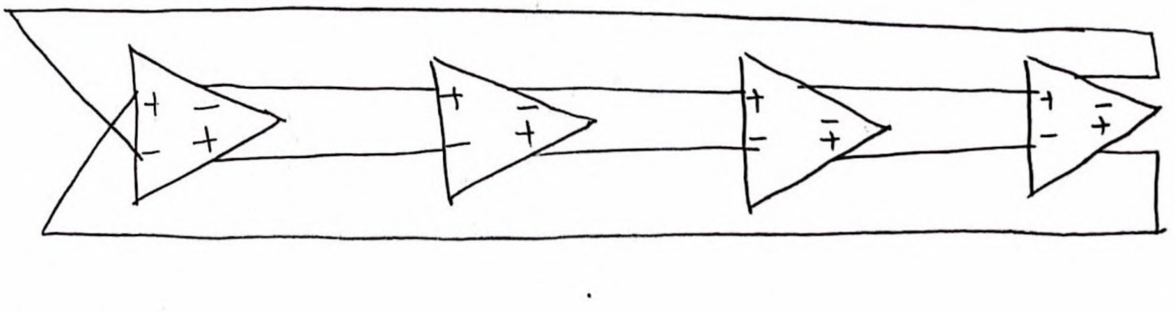
$$\hookrightarrow \left(\frac{g_mR}{RCs+1}\right)^4 + 1 = 0$$

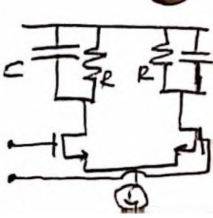
$$(RCs)^4 s^4 + 4(RC)^3 s^3 + 6(RC)^2 s^2 + 4RCs + 1 + g_mR = 0$$

s^4	$(RC)^4$	$6(RC)^2$	$1+(g_mR)^4$
s^3	$4(RC)^3$	$4RC$	0
s^2	$5(RC)^2$	$1+(g_mR)^4$	0
s^1	$\frac{20(RC)^3 - 4(RC)^3(1+(g_mR)^4)}{5(RC)^2}$	0	0
s^0	$1+(g_mR)^4$	0	0

> $g_mR > \sqrt{2}$

⇒ Each stage must have 90° of phase. However for an RC stage ω → ∞ for 90° of phase. Fix? Differential.





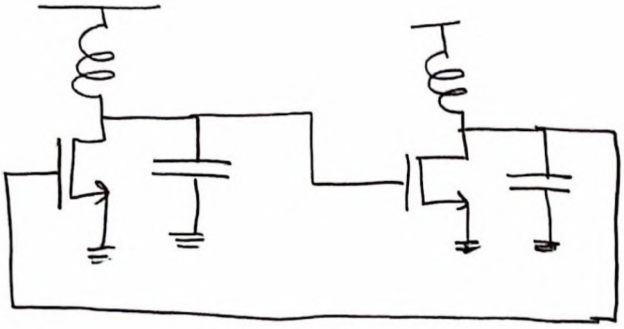
☹ > Tolerance of gm & R is poor ⇒ ωres is not well defined.

☺ > Low power.

☺ > Small

☺ > Multiphase

→ Resonance based Ring oscillator.



→ Cross coupled!

→ Only C cannot give 180° of phase for each stage since that happens at ω=0

→ R, L, C can give −90 → −270° of phase per stage.

→ ω is set when each cell has gain=1 & phase =180°.

→ Read paper by Afshari & Momeni on designing oscillators close to fmax

→ Phase Noise : [Hajimiri Lee paper.]

Leeson : (Empirical) $L\{\Delta f\} = 10 \log \left\{ \frac{2KFT}{P_{sig}} \left[1 + \left(\frac{1}{2Q} \frac{f_0}{\Delta f} \right)^2 \right] \left[\frac{1 + \Delta f_{1/f^3}}{|\Delta f|} \right] \right\}$

F → Correcting coefficient

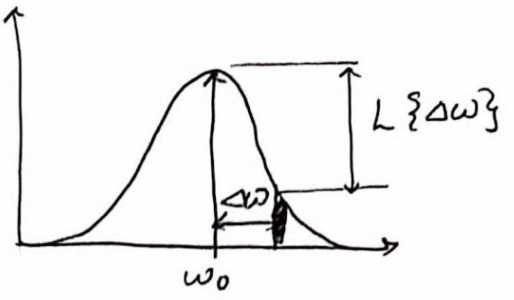
> Phase noise $\propto \frac{1}{P_{sig}} \propto \frac{1}{Q}$

> Active boosting of Q doesn't work since F changes.

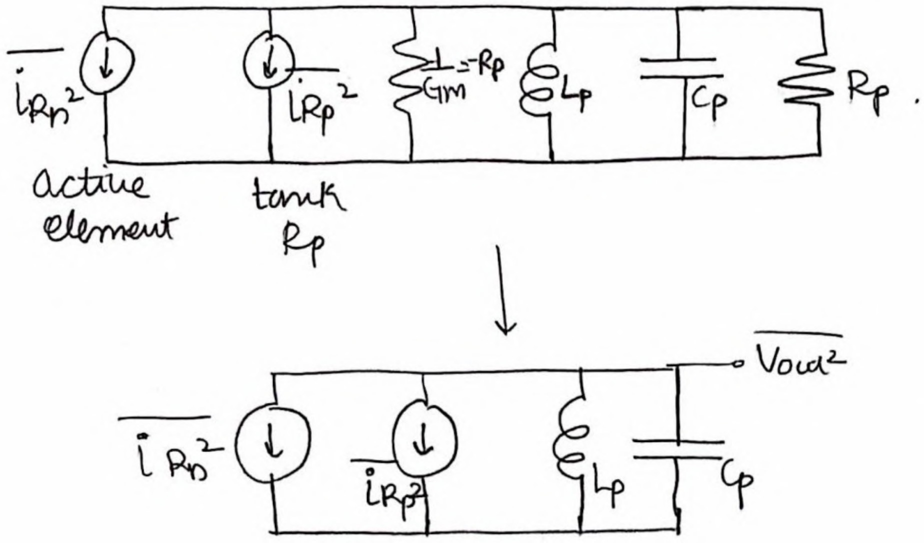
Phase noise basics

$$\underset{\frac{dBc}{Hz}}{\mathcal{L}\{\Delta\omega\}} = 10\log \frac{\text{Power in 1Hz BW } \Delta\omega \text{ away from } \omega_0}{\text{Total carrier power.}}$$

$$A(t)\cos(\omega_0 t + \phi(t))$$



Phase noise model



$$Z_{tank}(\omega) = \frac{1}{jC_p\omega} \,\|\, jL_p\omega = \frac{jL_p\omega}{1-\omega^2 L_p C_p}$$

↳ $\omega = \omega_0$
$\Rightarrow Z_{tank} = \infty$
$\Rightarrow$ amplifies current to ∞.

$$Z_{tank}(\omega_0+\Delta\omega) = \frac{jL_p(\omega_0+\Delta\omega)}{1-(\omega_0+\Delta\omega)^2 L_p C_p}$$

$$= \frac{jL_p\,\omega_0}{\underbrace{\cancel{(1-L_pC_p\omega_0^2)}}_{0} - 2\omega_0\Delta\omega L_pC_p + \underbrace{\cancel{\Delta\omega^2 L_pC_p}}_{0}} \simeq \frac{-j}{2}\,\frac{1}{\omega_0 C_p}\,\frac{\omega_0}{\Delta\omega}$$

> Even when $\Delta\omega \neq 0$, Z_{tank} is high and it amplifies the noise.

$$\boxed{Z_{tank}(\omega_0+\Delta\omega) = \frac{-j}{2}\cdot\frac{R_p}{Q_s}\cdot\frac{\omega_0}{\Delta\omega}}$$

$$\frac{\overline{V_{out}^2}}{\Delta f} = \left(\frac{\overline{i_n R_p^2}}{\Delta f} + \frac{\overline{i_n R_n^2}}{\Delta f}\right)\left|Z_{tank}(f)\right|^2$$

$$= \left(\frac{4KT}{R_p} + \frac{4KT}{R_n}\right)\left(\frac{R_p}{2Q}\left(\frac{f_0}{\Delta f}\right)\right)^2$$

> from Equipartition theorem, half of noise power goes to amplitude and half to phase.

$$\frac{\overline{V_{out}^2}}{\Delta f} = \left(\frac{2KT}{R_p} + \frac{2KT}{R_n}\right)\left(\frac{R_p}{2Q}\cdot\frac{f_0}{\Delta f}\right)^2$$

$$L\{\Delta f\} = 10 \log \frac{S_{noise}}{P_{signal}}$$

Assume $R_p = R_n$

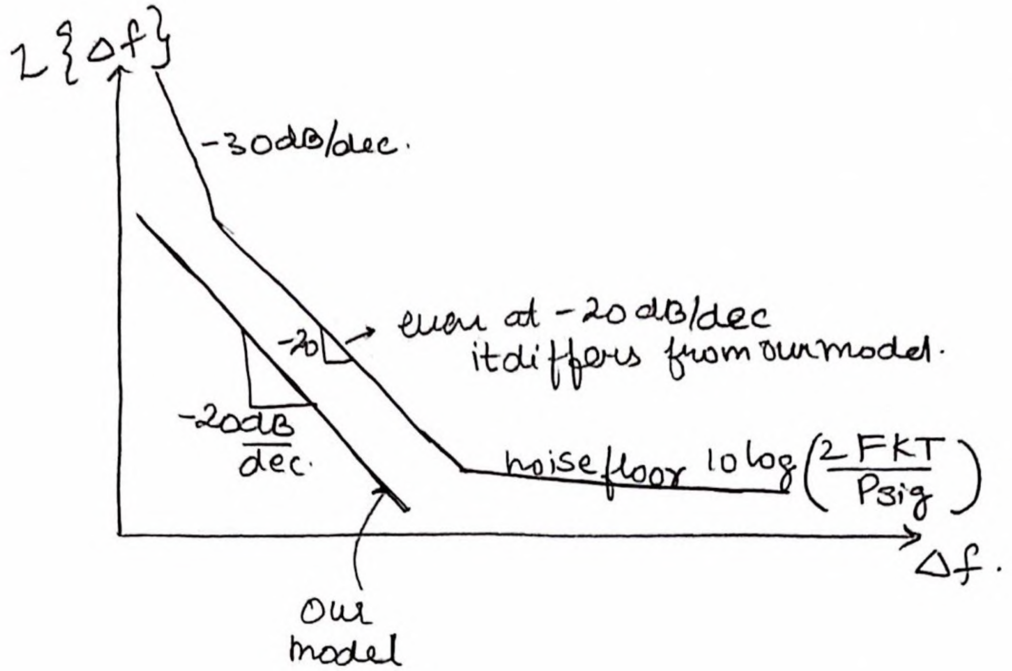
$$\Rightarrow L\{\Delta f\} = 10 \log \left\{ \frac{4KT}{P_{sig}}\left(\frac{1}{2Q}\cdot\frac{f_0}{2\Delta f}\right)^2 \right\}$$

Check derivation! Doesn't seems right

> Define a parameter $F = \dfrac{\text{total noise in tank @}\Delta f}{\text{noise in tank due to tank loss @}\Delta f}$

$$= 1 + \frac{\overline{i_{Rn}^2}/\Delta f}{\overline{i_{Rp}^2}/\Delta f}$$

> If we include flicker noise, we would see th -30dB/dec part



> Noise floor could be from measurement setup or output buffer or from more complex models of L & C for high frequency.

> Improved model → Leeson model.

> More improved → ISF model

> Most recent / accurate (used in Cadence) → Demir model.

Transceiver Considerations

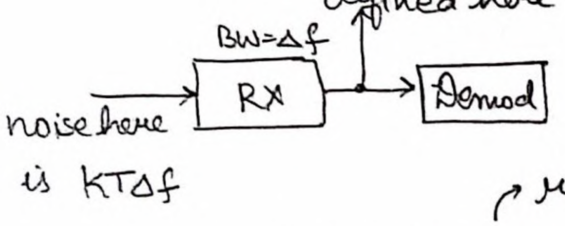
(I) Receiver

General Considerations

1) Frequency
2) Dynamic range
3) Gain
4) P_{dc}
5) Image rejection.

Noise Floor

Noise power at the output of receiver, before the demodulator



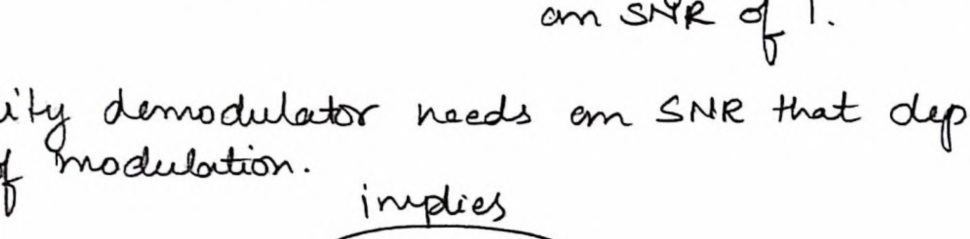
$$\Rightarrow \text{Noise floor} = KT\Delta f \cdot \overset{\nearrow \text{receiver gain}}{G} \cdot \underset{\hookrightarrow \text{noise of receiver.}}{F}$$

Sensitivity: Function of noise floor and SNR required.

"Min. signal that can be detected".

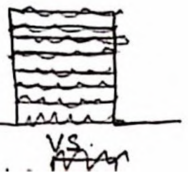
$S = KT\Delta f \cdot F$ → Assuming demodulator only needs an SNR of 1.

> In reality demodulator needs an SNR that depends on type of modulation.



> Higher order modulation ⇒ SNR needed is higher

$\boxed{S = KT\Delta f \cdot F \cdot SNR_{RX}}$ → depends on modulation and BER



> SNR_{Rx} would be less than 1, but BW required would be high.

→ $$S(\text{dBm}) = \overset{KT @ 300K}{-174\,\text{dBm}} + 10\log\frac{\Delta f}{1\,\text{Hz}} + NF(\text{dB}) + SNR(\text{dB}).$$

Eg: BPSK → BER is 10^{-6} ⇒ SNR required = 12.5 dB

QAM is 10^{-6} ⇒ SNR required = 27 dB

to retrieve information.

Eg: BER = 10^{-6} ⇒ SNR = 14 dB QPSK.

$\Delta f = 1\,\text{GHz}$ assuming NF = 9 dB.

$$\Rightarrow S = -174 + 10\log\frac{1\,\text{GHz}}{1\,\text{Hz}} + 9 + 14.$$

$$\boxed{S = -61\,\text{dBm}.}$$

⇒ In Watt

Eg: 5 GHz LAN ⇒ $\Delta f = 20\,\text{MHz}$, SNR = 14 dB, NF = 6 dB,

$$S = -174 + 10\log\frac{2M}{1\,\text{Hz}} + 6 + 14$$

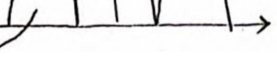
$$\boxed{S = -81\,\text{dBm}}$$

Phase Noise

$$\underbrace{L\{\Delta f_{in}\}}_{\text{channel BW}} = \text{max PN that you can tolerate}$$

$$= \underbrace{P}_{\substack{\text{desired signal level}\\ \text{fn of S}}} - \underbrace{C}_{\substack{\text{adjacent channel}\\ \text{rejection}}} - \underbrace{I}_{\substack{\text{Interference}\\ \text{power level}}} - 10\log\underbrace{\Delta f_{in}}_{\text{channel BW}}$$

→ Approximating that interference is constant in each band.



Eg:

- 60 GHz signal
- $P = -60\,dBm$
- $C = 14\,dB$

$I = -38\,dBm$

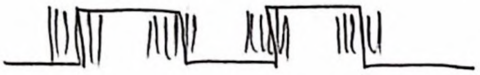
$\Delta f = 1.8\,GHz \rightarrow$ Huge offset

$$\mathcal{L}\{1.8\,GHz\} = -60 - 14 + 38 - 92.55$$

$$= -128.55\ dBc/Hz$$ at 1.8 GHz offset.

Transmitter Specifications

- freq. of operation
- Pout → Based on standard. (Japan 60GHz → 10dBm
  USA 60GHz → 27dBm.
- EVM → Error vector magnitude.
- Noise



jitter is more important.

- Rough estimate: $t_{j_{rms}} \cong \frac{1}{f_0}\sqrt{2 \cdot L \cdot \Delta f}$

Link budget analysis

$$P_{Rx}(d) = \overbrace{P_{TX}\ \ G_{TX}}^{EIRP} \cdot\ G_{Rx} \cdot \left(\frac{\lambda}{4\pi d}\right)^2$$

EIRP → like an omnidirectional transmitter.

Antenna gain

prop. loss in free space

G = directivity × Efficiency

EIRP: $Power_{TX} \cdot G_{TX}$

$\boxed{P_{Rx}(d) > S} \Rightarrow$ Link works.

Usually $P_{Rx}(d) > S$ by 10dB.

Eg: $f = 60\,GHz$.

$d = 2m$

$P_{TX} = 0\,dBm$

QPSK modulation: $BER = 10^{-6} \Rightarrow SNR_{RX} = 14\,dB$.

$\Delta f = BW = 2\,GHz. \Rightarrow \underset{\text{bit rate}}{R_b} = 4\,Gbps$ from Nyquist.

$NF_{RX} = 7\,dB$

$G_{RX} = G_{TX} = 25\,dB$ → very good.

$$P_{RX} = P_{TX} + G_{TX} + G_{RX} + 20\log\left(\frac{\overset{\curvearrowright 5mm}{\lambda}}{4\pi \underset{\hookrightarrow 2m}{d}}\right)$$

$$= 0+25+25-74 = \underline{\underline{-24\,dBm}}$$

$$S = -174\,dBm + 10\log\underbrace{(2\times10^9)}_{\Delta f} + 7\,dB + 14\,dB$$

$$\boxed{S = -60\,dBm.} \Rightarrow \text{Link works.}$$

Modulations

ASK

$$S(t) = m(t)\cos(\omega_0 t) \qquad m = 0, 1.$$

Prob of error $P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{4n_0}}$ $\qquad \underset{\text{Bit energy in one period}}{E_b} = \int_0^T s^2(t)\,dt$

n_0 = noise level [W/Hz]

S = Signal power = $E_b \times R_b$ ↳ bit rate [bits per second]

N = noise power = $n_0 \cdot \Delta f$

$$\frac{E_b}{n_0} = SNR \cdot \frac{\Delta f}{R_b}$$

> BER → ratio definition : no. of error bits per no. of bits.
→ rate definition : no. of errors in one time period.

Prob. of error $P_e = \overline{BER}$ (using ratio def.)
↳ avg BER.

Usually $P_e \longleftrightarrow$ BER are considered same.

FSK Easy to design PA since amplitude is constant.

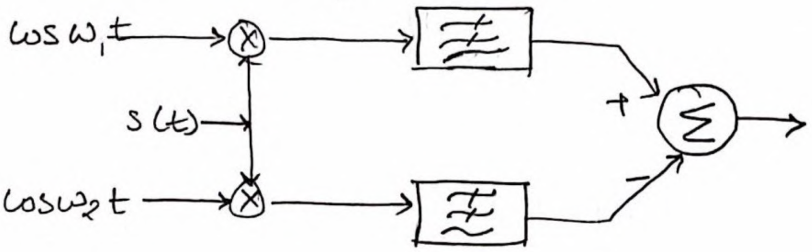
$$s(t) = \cos[(\omega_2 + m(t)\Delta\omega)t] \qquad \Delta\omega = \omega_1 - \omega_2 = 2\pi[f_1 - f_2]$$

$$BW = 2(\Delta f + \frac{2}{T})$$

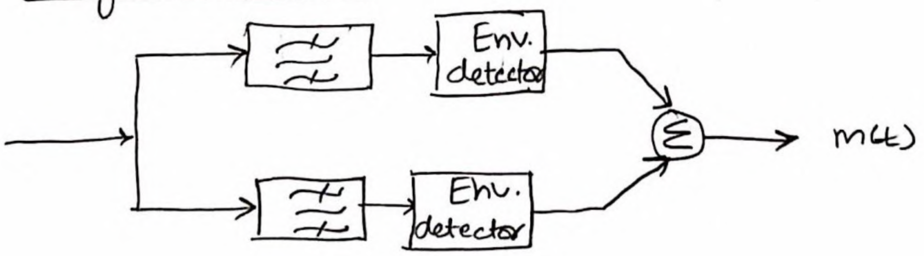
→ depends on demodulation.
T → period of the signal / binary data rate. or m(t).

Demodulation

Coherent / synchronous



Asynchronous



$$P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{2n_0}} \longrightarrow$$ Better?
If we normalize to power (in E_b) it is not.

PSK

$$s(t) = m(t) \cdot \cos(\omega_0 t)$$

$\hookrightarrow m(t) = +1, -1$ polar NRZ

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{n_0}}$$ → After normalizing it is same.

MPSK

$$S_i(t) = A \cos(\omega_0 t + \phi_i)$$

$\phi_i = 2\pi \cdot \frac{i}{M}$ $M = 2^n$; $i = 0, \ldots, M-1$; n = no. of bits per symbo

BPSK → m = 2 ; QPSK → m = 4.

› Could use I & Q to represent M phase states.

QAM

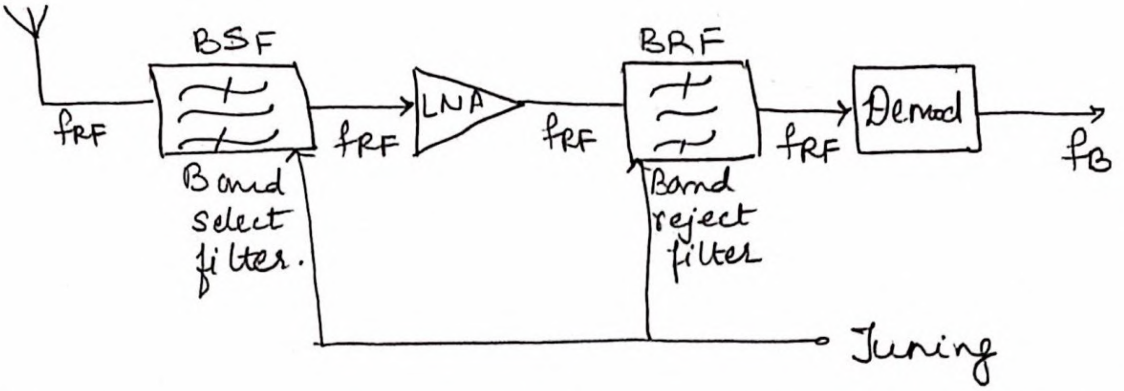
$$S_K(t) = a_K \cos(\omega_0 t) + b_K \sin(\omega_0 t).$$

For BER = 10^{-6}

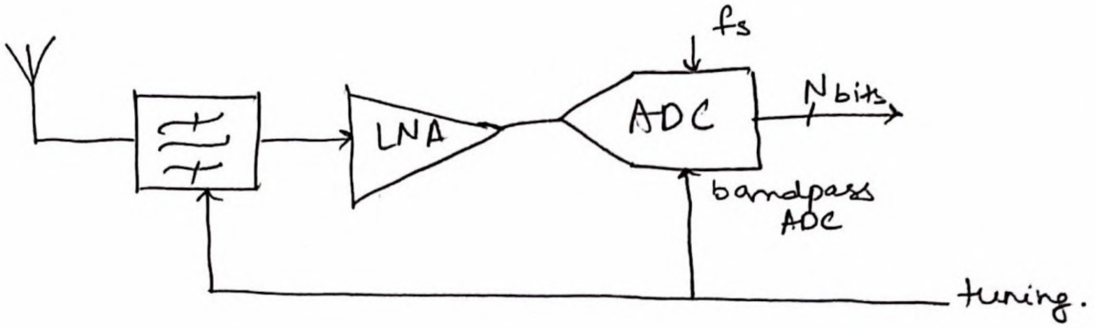
modulation	BW efficiency η	SNR required (dB)
BPSK	1	12.5
4FSK	2 (bits per symbol)	17
QPSK	2	14
8PSK	3	19
16QAM	4	21
64QAM	6	27

Receiver Architechtures

1) Tuned radio receiver. (aka tuned homodyne)



Square law detector ∴ Only detects amplitude ⇒ AM, ASK.



> Poor sensitivity ⇒ LNA & filter need large gain

$s(t) = \text{RF input} = \underbrace{A(t)}_{\text{slow changing \& contains information.}} \cos(\omega_{RF} t)$

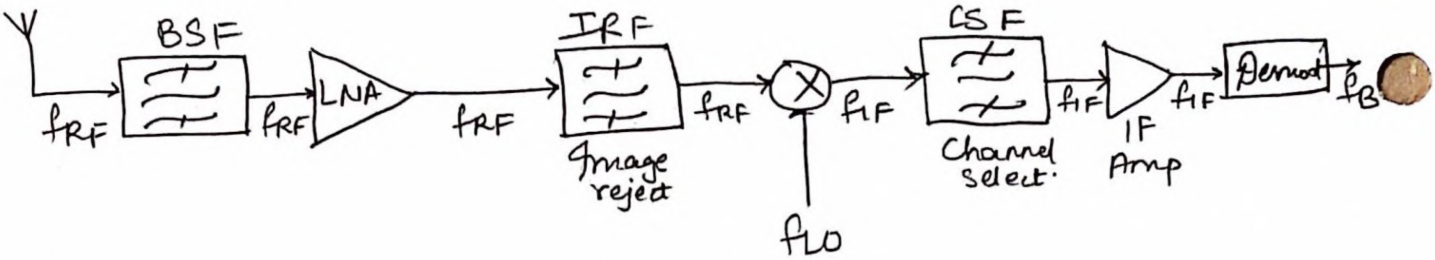
$\text{Output} = [\underbrace{A_v}_{\text{gain of receiver.}} s(t)]^2$

$$\text{Output} = \frac{A_v^2 A(t)^2}{2}\left[1 + \cos(2\omega_{RF} t)\right]$$

$$\text{Output} \underset{\text{LPF \& sq.root}}{=} \frac{A_v A(t)}{\sqrt{2}}$$

Used in AM radio, automatic cruise control.

2.) Heterodyne or Superheterodyne (1906)



$f_{RF} = f_{LO} \pm f_{IF}$

Choosing f_{IF}

High f_{IF} → Relaxes. IRF design. ☺

High f_{IF} → CSF & IF amp are at high freq. ☹

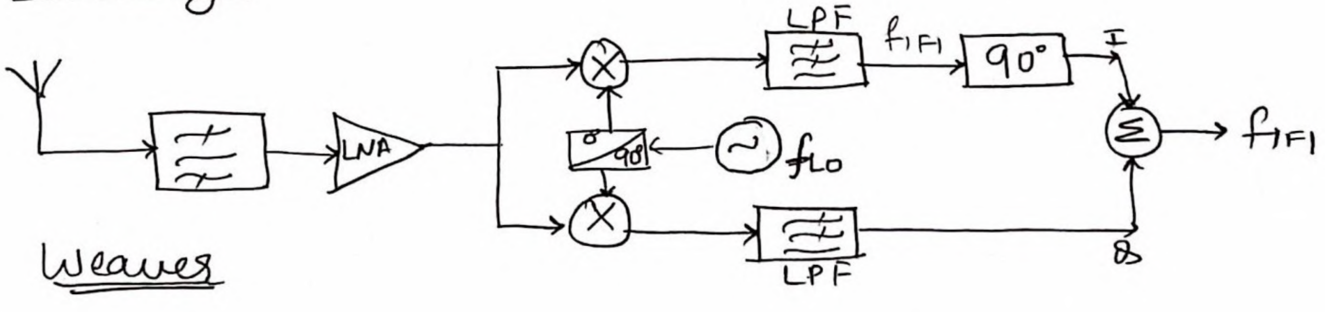
$S(t) = A(t) \cos[\omega_{RF}t + \alpha(t)] \cos(\omega_{LO}t)$

$\underline{\text{After IRF}} \rightarrow \underbrace{\frac{A(t)}{2}}\cos[\omega_{IF}t - \alpha(t)]$ → Both amp & phase information.

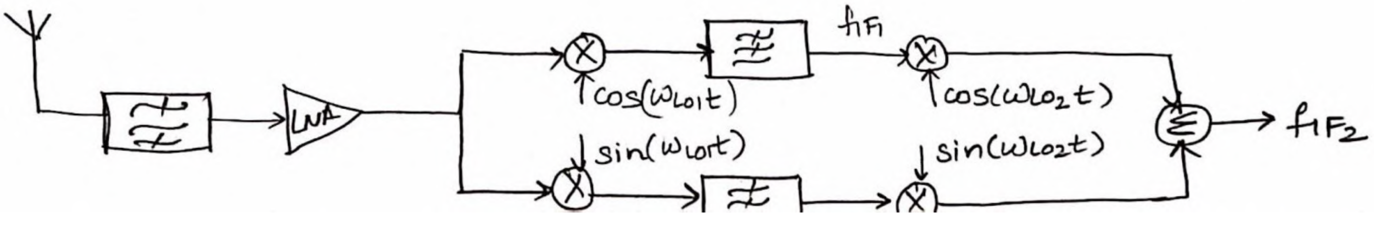
› Problem is the image. rejection filter.

› To eliminate IRF use Hartley (1928) and Weaver (1956) topologies.

Hartley



Weaver



Hartley

$S_{RF}(t) = A_{RF}\cos(\omega_{RF}t + \alpha_{RF})$

$S_{IM}(t) = A_{IM}\cos(\omega_{IM}t + \alpha_{IM})$

$f_{RF} = f_{LO} - f_{IF}$; $f_{IM} = f_{LO} + f_{IF}$

I path

$$S_I(t) = A_{RF}\cos(\omega_{RF}t + \alpha_{RF})\cos(\omega_{LO}t) + A_{IM}\cos(\omega_{IM}t + \alpha_{IM})\cos(\omega_{LO}t)$$

$$\xrightarrow{LPF} S_I(t) = \frac{A_{RF}}{2}\cos(\omega_{IF}t - \alpha_{RF}) + \frac{A_{IM}}{2}\cos(\omega_{IF}t + \alpha_{IM})$$

$$\xrightarrow{90^\circ} S_I(t) = \frac{A_{RF}}{2}\sin(\omega_{IF}t - \alpha_{RF}) + \frac{A_{IM}}{2}\sin(\omega_{IF}t + \alpha_{IM}). \quad ①$$

$$S_Q(t) = \frac{A_{RF}}{2}\sin(\omega_{IF}t - \alpha_{RF}) - \frac{A_{IM}}{2}\sin(\omega_{IF}t + \alpha_{IM}) \quad ②$$

① + ② ⇒ $\boxed{S_{IF}(t) = A_{RF}\sin(\omega_{IF}t - \alpha_{RF})}$

> Having 90° phase shift over wideband is difficult.

⇒ use Weaver

Weaver

$$S_{I_1}(t) = \frac{A_{RF}}{2}\cos(\omega_{IF_1}t - \alpha_{RF}) + \frac{A_{IM}}{2}\cos(\omega_{IF}t + \alpha_{IM})$$

$$S_{Q_1}(t) = \frac{A_{RF}}{2}\sin(\omega_{IF_1}t - \alpha_{RF}) - \frac{A_{IM}}{2}\sin(\omega_{IF}t + \alpha_{IM})$$

→ X $\cos(\omega_{LO2}t)$

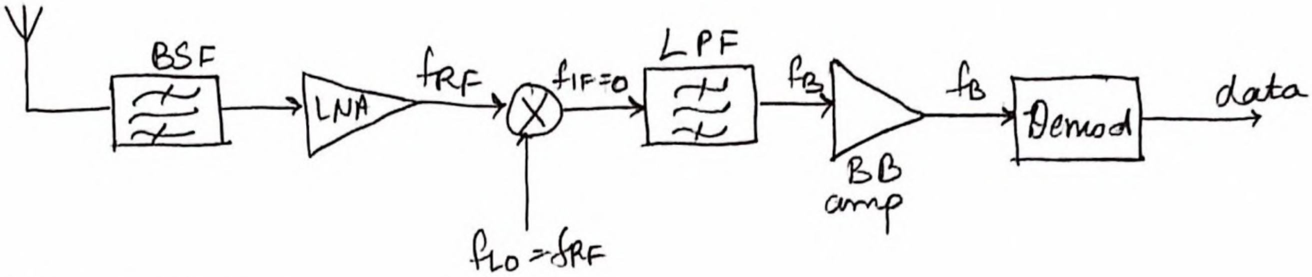
→ X $\sin(\omega_{LO2}t)$ & add them.

$$\Rightarrow S_{IF_2}(t) = \frac{A_{RF}}{2}\cos(\omega_{IF}t - \alpha_{RF}) - \frac{A_{IM}}{2}\cos[\underbrace{(\omega_{IF_1} + \omega_{LO2})t}_{\text{Image pushed to high frequency.}} - \alpha_{IM}]$$

Both amp & phase info.

3) Direct conversion receivers → (Zero IF, Homodyne)

1924



- ☺ No image.
- ☺ CSF is just a LPF
- 😐 Need to track f_{RF} to adjust f_{LO}
- ☹ f_{RF} 'pulls' f_{LO}.

Heterodyne: $S_{IF}(t) = \frac{A(t)}{2} \cos(\omega_{IF} t - \alpha(t))$

Homodyne: $S_{IF}(t) = \frac{A(t)}{2} \cos(\alpha(t))$

↳ ideal for amplitude modulation since $\cos\alpha(t)$ is constant

> To get FSK & PSK need I & Q.

- ☹ DC offset.
- ☹ Flicker noise of mixer is in band.