EECS 411 - Microwave Circuits

Transmission Line Theory.

Lossless Transmission Lines.

> Telegrapher's Equations

$$\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial^2 i}{\partial z^2} = LC\frac{\partial^2 i}{\partial t^2}$$

> general solution is of the form $v(z,t) = F(z \pm ut)$
$i(z,t) = F(z \pm ut)$

> where u is the velocity of propagation of the signal. Substituting them into the Telegrapher's Equations gives $\boxed{u = \frac{1}{\sqrt{LC}}}$

Therefore,

$$\left.\begin{aligned} v &= v^+\left(z - \frac{1}{\sqrt{LC}}t\right) + v^-\left(z + \frac{1}{\sqrt{LC}}t\right) \\ i &= i^+\left(z - \frac{1}{\sqrt{LC}}t\right) + i^-\left(z + \frac{1}{\sqrt{LC}}t\right) \end{aligned}\right\} \text{General solutions of 2 waves.}$$

> Apply $\frac{\partial v}{\partial z} = -L\frac{\partial i}{\partial t} \rightarrow v^+ = \sqrt{\frac{L}{C}}\, i^+$ and $v^- = -\sqrt{\frac{L}{C}}\, i^-$

Therefore, we define $\boxed{Z_0 = \sqrt{\frac{L}{C}}}$ $\quad V^+ = Z_0 i^+ \; ; \; V^- = -Z_0 i^-$

> Using phasor notation and assuming sinusoidal solutions.

$$\left.\begin{aligned} \frac{d^2 V}{dz^2} &= -\omega^2 LCV \\ \frac{d^2 I}{dz^2} &= -\omega^2 LCI \end{aligned}\right\} \text{Telegrapher's Equations.}$$

Let $\boxed{\beta = \omega\sqrt{LC}}$ Phase constant.

Relative permittivity $\epsilon_r = \left(\frac{c}{u_g}\right)^2$

The general solutions now become,

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{-j\beta z} = V^+(z) + V^-(z)$$

$$I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z} = I^+(z) + I^-(z).$$

Converting back to time domain ($\times$ with $e^{j\omega t}$ & take Re part)

$$v(z,t) = V_0^+ \cos(\omega t - \beta z) + V_0^- \cos(\omega t + \beta z)$$

$$i(z,t) = i_0^+ \cos(\omega t - \beta z) + i_0^- \cos(\omega t + \beta z)$$

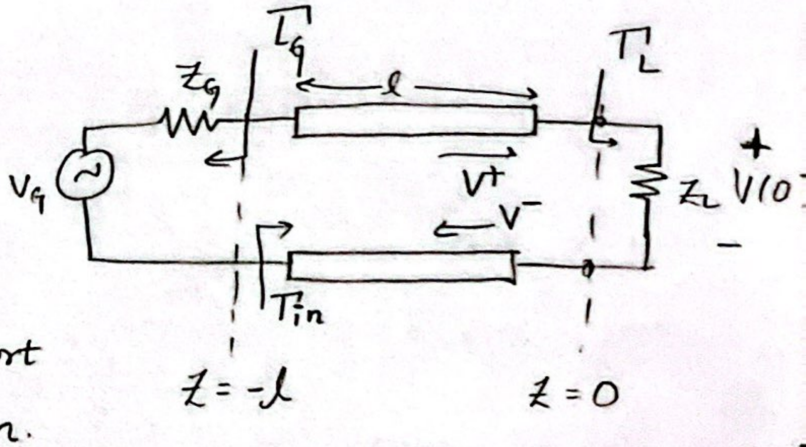
> $\boxed{u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}}$ Phase velocity. $\boxed{\beta = \frac{2\pi}{\lambda} = \omega\sqrt{LC}}$ phase constant

$V^+ = (V + i Z_0)/2$ $\qquad i^+ = (i + Y_0 V)/2$ $\qquad Y_0 = \frac{1}{Z_0}$

$V^- = (V - i Z_0)/2$ $\qquad i^- = (i - Y_0 v)/2$

<u>Reflection Coefficient</u>

$$\Gamma = \frac{V^-}{V^+} = \frac{Z(z) - Z_0}{Z(z) + Z_0}$$



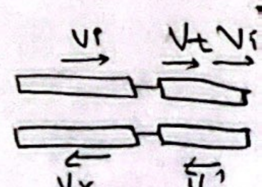
$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$\Gamma_L = -1 \Rightarrow$ short

$\Gamma_L = 1 \Rightarrow$ open.

$\Gamma_L = 0 \Rightarrow$ matched.

<u>Transmission Coefficient</u>



$$\tau_L = \frac{V(0)}{V^+(0)} = \frac{V^+(0) + V^-(0)}{V^+(0)} = 1 + \Gamma_L = \frac{2Z_L}{Z_L + Z_0}$$

$$\tau_L = \frac{V_t}{V_i} = \frac{V_i + V_r}{V_i} = \frac{V_i' + V_r'}{V_i}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \qquad \Gamma_G = \frac{Z_G - Z_0}{Z_G + Z_0}$$

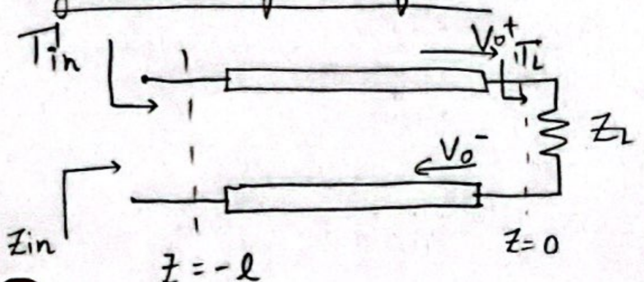
Power

Average power $\langle P \rangle = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt$

In phasor $\langle P \rangle = \frac{1}{2} \mathrm{Re}[V I^*]$

$P_i = \frac{1}{2}\frac{|V^+|^2}{|Z_0|}$ $\quad P_r = \frac{1}{2}\frac{|V^-|^2}{|Z_0|} = \frac{1}{2}\frac{|V^+|^2}{Z_0}\cdot|\Gamma|^2$ $\Rightarrow$ $\boxed{\frac{P_r}{P_i} = |\Gamma|^2 \quad \frac{P_t}{P_i} = 1 - |\Gamma|^2}$

General form of Γ



$\boxed{\Gamma_{in} = \Gamma_L e^{-2j\beta l}}$ $\Rightarrow$ only phase changes along lossless line

$$Z_{in} = Z_0 \cdot \frac{1+\Gamma_L e^{-2j\beta l}}{1-\Gamma_L e^{-2j\beta l}} = Z_0 \frac{1+\Gamma_{in}}{1-\Gamma_{in}}$$

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}}$$

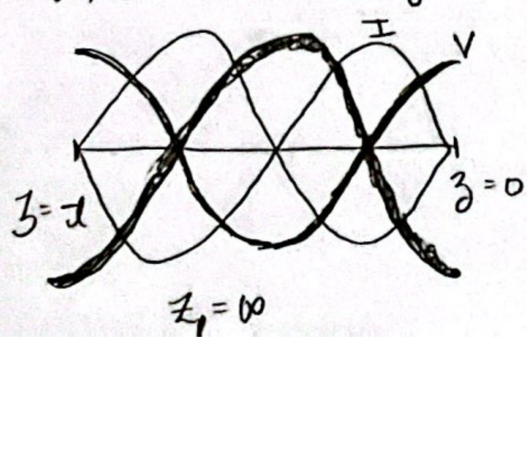
βl = electrical length in degrees.

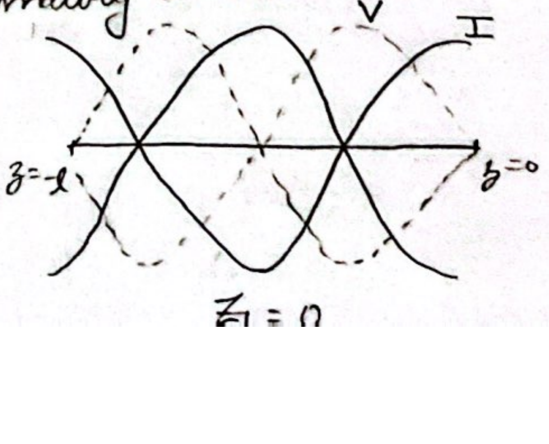
$\theta = \beta l = 2\pi \frac{l}{\lambda}$

> βl is a measure of how many wavelengths can fit inside the line. More wavelengths can fit $\Rightarrow$ high βl.

> Γ and Z are repeated every $\lambda/2$ along the line.

> $Z_L = \infty, 0 \Rightarrow$ Standing waves on the line & V, I are $90°$ out of phase due to boundary condition.





Voltage Standing Wave Ratio. (VSWR)

$$\boxed{VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}} \quad ; \quad |\Gamma| = \frac{VSWR-1}{VSWR+1}$$

VSWR = ∞ ⇒ pure standing wave ⇒ $Z_L = 0, \infty$

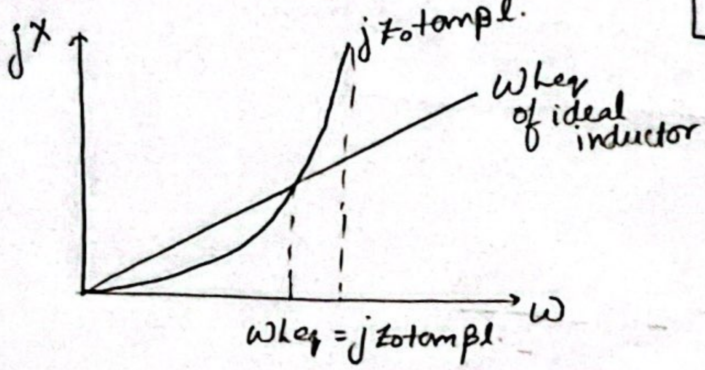
VSWR = 1 → only travelling wave ⇒ $Z_L = Z_0$.

Short circuited line.

$$\boxed{Z_{short} = j Z_0 \tan \beta l.}$$

$l < \lambda/4$ ⇒ inductor. with inductance

$$\boxed{L_{eq} = \frac{Z_0}{\omega} \tan \beta l.}$$



Open circuited line.

$$\boxed{Z_{open} = -j Z_0 \cot \beta l.}$$

$l < \lambda/4$ ⇒ Capacitor with capacitance

$$\boxed{C_{eq} = \frac{Y_0}{\omega} \tan \beta l.}$$

Quarter wave Transformer.

$$\boxed{Z_{in} = \frac{Z_0^2}{R_L}}$$

⇒ To deliver maximum power $Z_0 = \sqrt{R_G R_L}$

must be real.

Lossy Transmission Lines.

- Let $Z = R + j\omega L \quad Y = G + j\omega C \quad , \quad \gamma^2 = ZY$

$$\frac{d^2 V(z)}{dz^2} = \gamma^2 V(z) \quad ; \quad \frac{d^2 I(z)}{dz^2} = \gamma^2 I(z) \Big\}$$ Freq. domain telegraphers equations with sinusoidal excitation

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

γ: complex prop. constant
α: attenuation constant
β : phase constant

α : Nepers/meter.
β : Radians/meter.

$$\boxed{dB/m = 8.686 \text{ Nepers/m}}$$

Solutions to Telegraphers

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

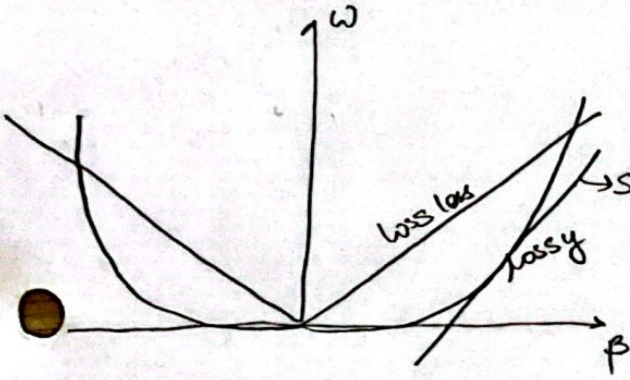
Timedomain solutions

$$v(z,t) = V_0^+ e^{-\alpha z}\cos(\omega t - \beta z) + V_0^- e^{-\alpha z}\cos(\omega t + \beta z)$$
$$i(z,t) = \frac{1}{Z_0}\left[V_0^+ e^{-\alpha z}\cos(\omega t - \beta z) - V_0^- e^{-\alpha z}\cos(\omega t + \beta z)\right]$$

Phase velocity: $\boxed{u_p = \frac{\omega}{\beta}}$ and $\boxed{\beta = \frac{2\pi}{\lambda} = \frac{\omega}{u_p}}$

Group velocity: $\boxed{u_g = \frac{d\omega}{d\beta}}$ Velocity of propagation is now a fn of f.
$\Rightarrow$ Dispersion.

Lossless line : $u_g = \frac{d\omega}{d\beta} = \frac{d\omega}{d\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = u_p$



> u_p can be greater than u_g
$\Rightarrow$

> Attenuation after length l
$\Rightarrow \alpha l$ in Nepers

> $\boxed{\text{Loss in } dB = 8.686 \times \alpha l}$

Low loss approximation

If $R << \omega L$ and $G << \omega C$

$\beta \simeq \omega\sqrt{LC}$, $\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$, $\gamma = \alpha + j\beta$

$Z_0 \simeq \sqrt{\frac{L}{C}}$

Integrated Circuit T lines

Note $\sqrt{j} = \frac{1+j}{\sqrt{2}}$

If $G \simeq 0$ and $R >> \omega L$

$\alpha = \frac{1}{\sqrt{2}}\sqrt{\omega RC}$, $\beta = \frac{1}{\sqrt{2}}\sqrt{\omega RC}$, $\gamma = \alpha + j\beta$

$Z_0 = \frac{1-j}{\sqrt{2}}\sqrt{\frac{R}{\omega C}}$

$u_p = \sqrt{\frac{2\omega}{RC}}$, $u_g = 2\sqrt{\frac{2\omega}{RC}}$ $\Rightarrow u_g \neq u_p$ $\Rightarrow$ dispersion.

Distortionless lossy Tline.

If $\frac{R}{L} = \frac{G}{C}$

$\alpha = R\sqrt{\frac{C}{L}}$, $\beta = \omega\sqrt{LC}$, $\gamma = \alpha + j\beta$

$u_p = u_g = \frac{1}{\sqrt{LC}}$.

Terminated lossy Tline.

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}.$$

$$\Gamma_{in} = \Gamma_L e^{-2\alpha l} e^{-2j\beta l}.$$

The Smith Chart

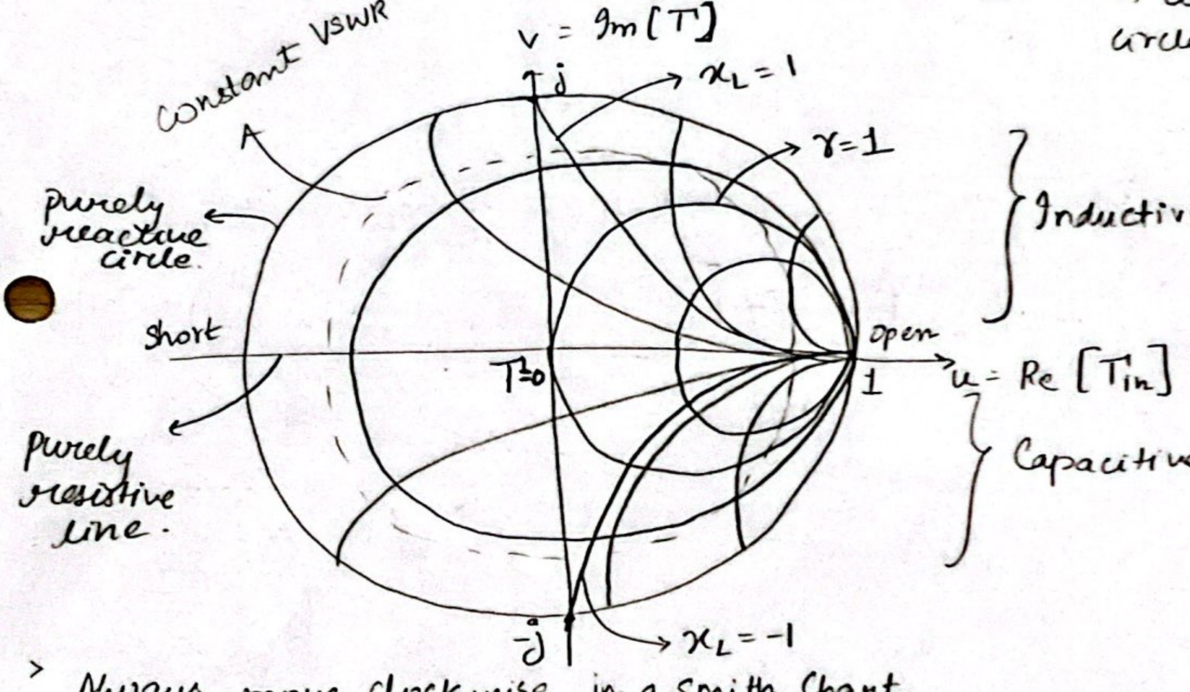
$$Z_{in} = Z_0 \cdot \frac{1 + \Gamma_L e^{-2j\beta l}}{1 - \Gamma_L e^{-2j\beta l}}$$

$$r + jx = \frac{1 + \Gamma_L e^{-2j\beta l}}{1 - \Gamma_L e^{-2j\beta l}} \quad \text{where} \quad \Gamma_{in} = \Gamma_L e^{-2j\beta l} = u + jv$$

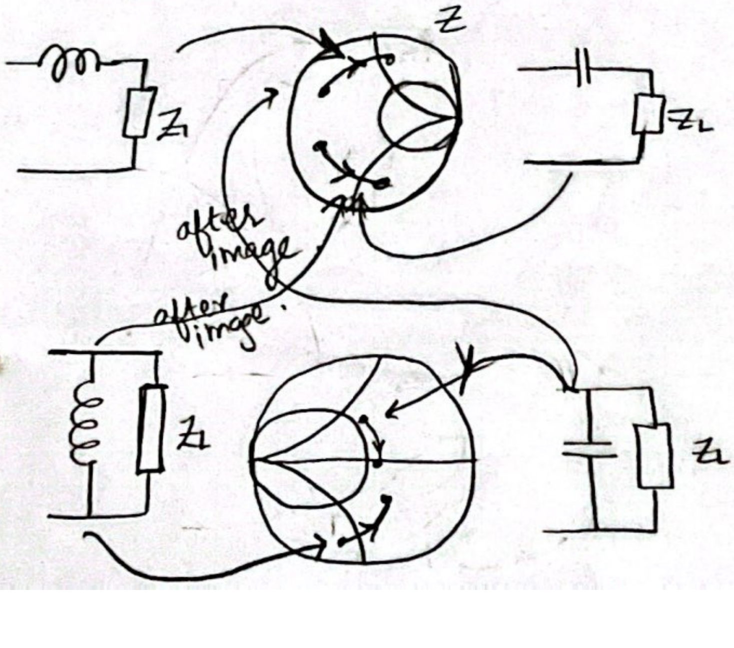
$$\Rightarrow \bar{Z}_{in} = \frac{1 + u + jv}{1 - (u+jv)} = r + jx \Rightarrow \begin{cases} \left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2} \\ (u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \frac{1}{x^2} \end{cases}$$

constant r ⇒ circles centred at $\left(\frac{r}{1+r}, 0\right)$, radius $\frac{1}{1+r}$

↳ constant x ⇒ circles with $c = \left(1, \frac{1}{x}\right)$, $r = \frac{1}{x}$



> Always move clockwise in a Smith Chart.



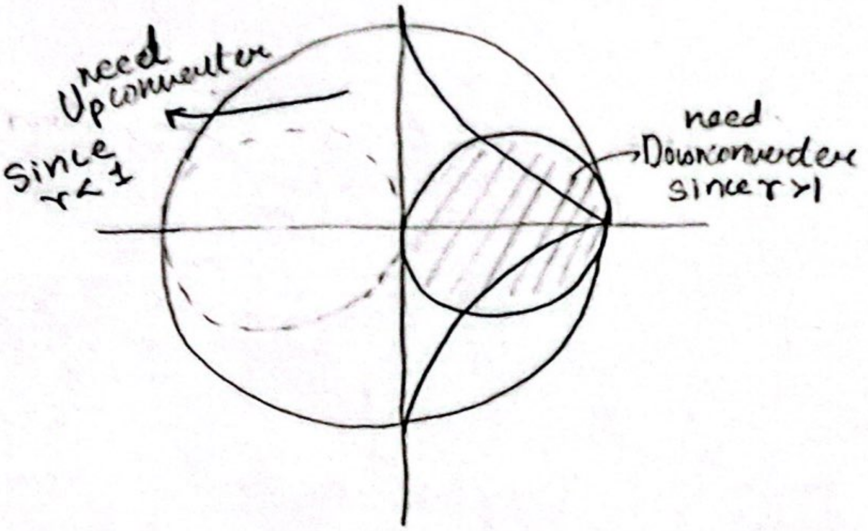
Move along constant VSWR circle in clockwise direction when extending the line length. (electrical length).

L series or shunt ⇒ ↑ move up in Z or Y

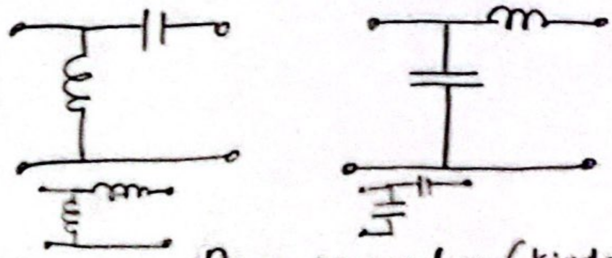
C series or shunt ⇒ ↓ move down in Z or Y

Impedance Matching

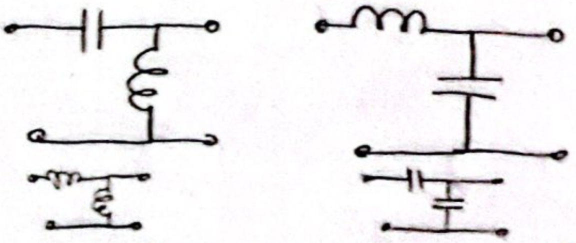
- $Z_L = Z_g^*$
- $\Rightarrow \Gamma_L = \Gamma_g^*$



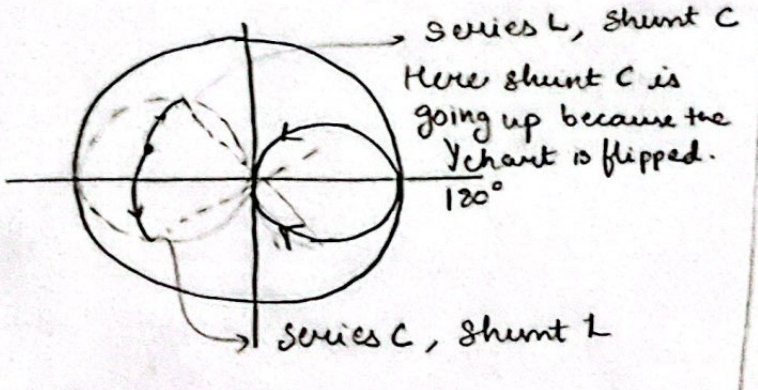
Up converter



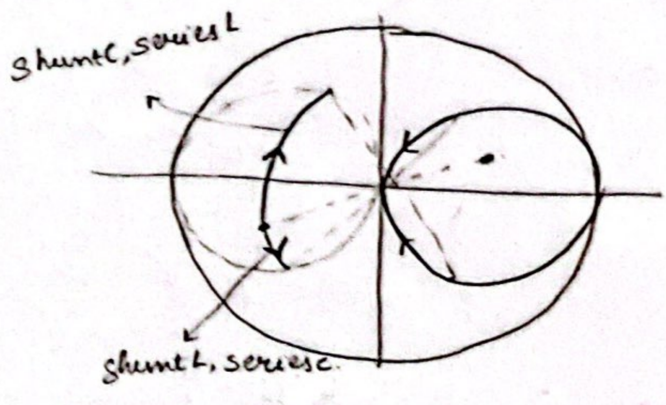
Down converter (kind of)



Upconversion.



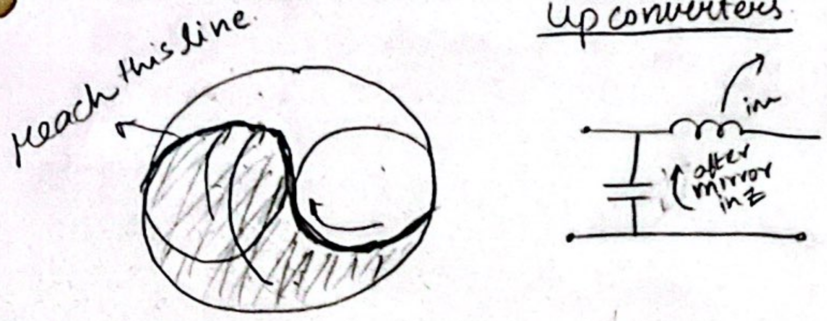
Down conversion.

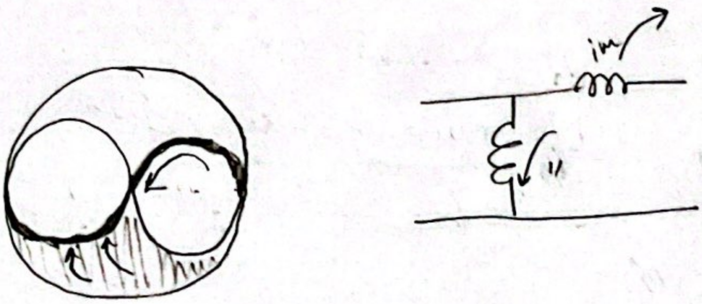


- When dealing with shunts take mirror image w.r.t origin
- Remember to renormalize correctly in the end.
- The "Downconverter" Circuits can also upconvert some circuits as shown on Page 9.

Allowed regions → Shaded

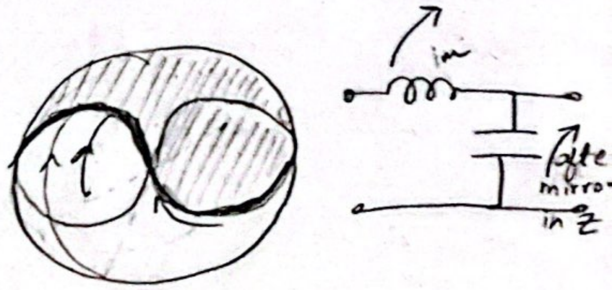
Up converters

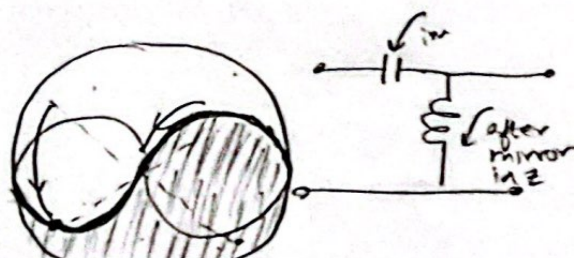


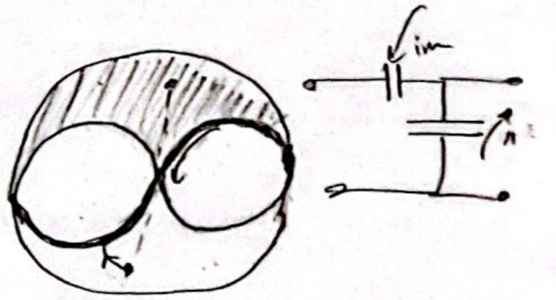


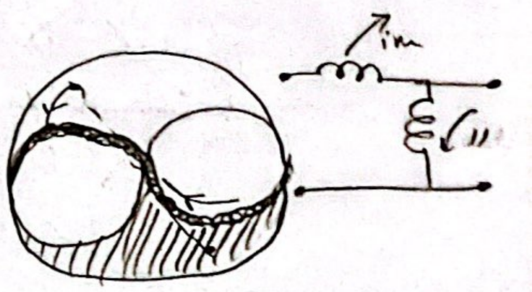
(Kind of)

Downconverters







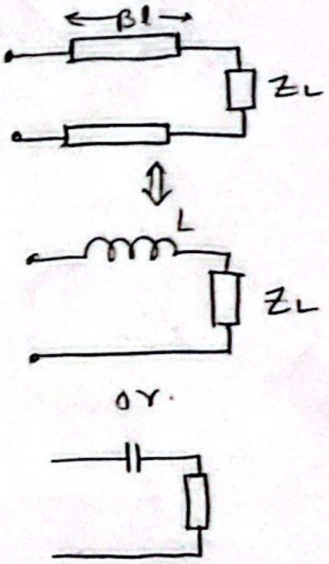


Matching using transmission lines

→ Move along constant VSWR circle clockwise.

→ Stub moves exactly like shunt capacitor (or shunt inductor after mirror) after mirror

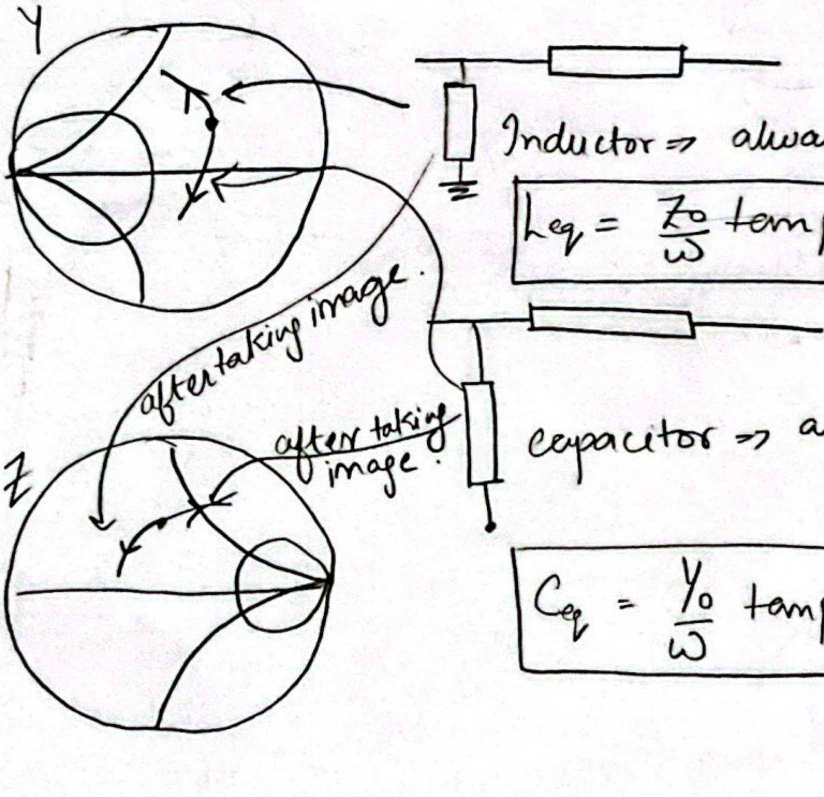
→ Z_0 is high and length is short ⇒ T.L acts as an inductor with $\boxed{L = \frac{Z_1 l}{u_p}}$. Regardless of load Z_L as long as $Z_0 = Z_1 > 3|Z_L|$



→ Z_0 is low & length is short

⇒ $\boxed{C = \frac{l}{Z_1 u_p}}$ $\quad Z_1 = Z_0 < 3|Z_L|$

→ For Tline stub matching use ZY chart (preferred)



Inductor ⇒ always goes up.

$\boxed{L_{eq} = \frac{Z_0}{\omega} \tan\beta l}$

capacitor ⇒ always goes down.

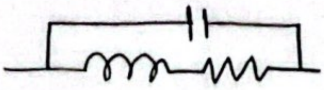
$\boxed{C_{eq} = \frac{Y_0}{\omega} \tan\beta l}$

Lumped Elements and Lines

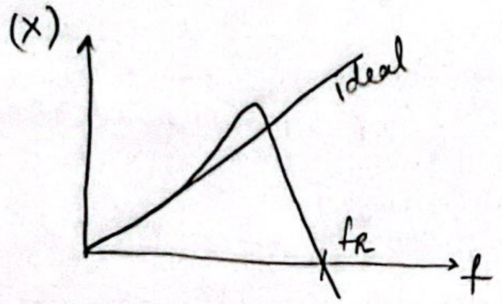
→ **Inductors** are the issue

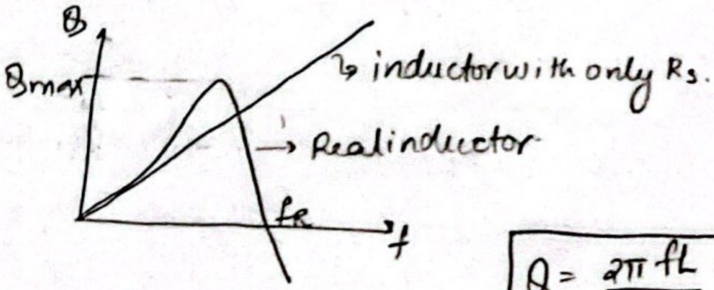
> $Q = \dfrac{X}{R} = \dfrac{Im(Z)}{Re(Z)} = \dfrac{\text{Energy stored}}{\text{Power loss/cycle}}$.

> Equivalent model.



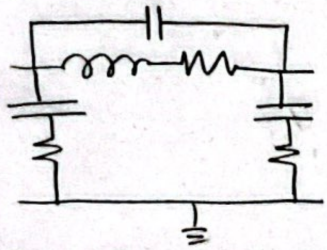
$f_R = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad f < f_R \Rightarrow$ inductor. $\quad f > f_R \Rightarrow$ capacitor.

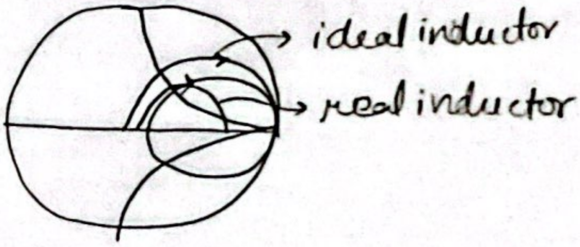




$$\boxed{Q = \frac{2\pi f L}{R}}$$

Integrated inductor model.



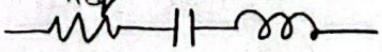


> At high frequency we can use a line of wire of length l & diameter d. Inductance is empirically given by.

> $L = 0.002l\left[\log\left(\dfrac{4l}{d}\right) - 1\right]\ \mu H$

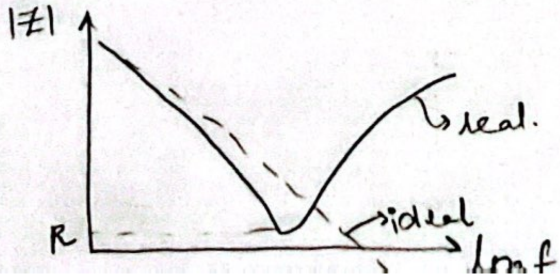
→ Capacitors.

model:



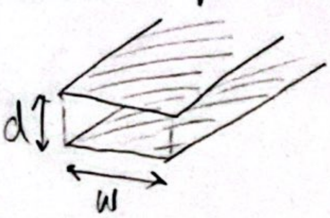
R is a function of frequency.

$Q = \dfrac{1}{2\pi f RC}$ $\quad f_R = \dfrac{1}{2\pi\sqrt{LC}}$



Transmission Lines and Waveguides

Parallel plate TRL



Assume $W >> d$.

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{\mu\epsilon}}{C} = \frac{1}{v_p C}$$

$$\boxed{C = \epsilon \frac{W}{d}} \Rightarrow \boxed{Z_0 = \eta \frac{d}{W}}$$

Characteristic wave impedance of the medium b/w plates.

From Pozar

Transverse Electro Magnetic Waves.

> For TEM, atleast 2 conductors must be used ($\Rightarrow$ no waveguides)

> $K_c = K^2 - \beta^2$ is the cut off wavenumber, $\boxed{K = \beta} \Rightarrow \underline{K_c = 0}$

$$\therefore \beta = K = \omega\sqrt{\mu\epsilon}$$

> The E field, H field & scalar potential ϕ satisfy Laplace's Equation. Therefore the transverse fields are like static fields.

> TEM modes have a well defined voltage, current & Z_0 for the structure.

> $$Z_{TEM} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} = \eta \quad \text{or} \quad Z_{TEM} = \frac{-E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$

$$\therefore \vec{h}(x,y) = \frac{1}{Z_{TEM}} \hat{z} \times \vec{e}(x,y).$$

> Z_0 relates V, I travelling on the line & is a fn of both geometry and material.

> η or Z_{TEM} relates field components & is only a fn of material.

TE Mode

- $E_z = 0 \; ; \; H_z \neq 0$

$\beta = \sqrt{k^2 - k_c^2}$ is a fn of frequency and geometry.

$$\boxed{Z_{TE} = \frac{k\eta}{\beta}} = \frac{\omega\mu}{\beta}$$

fn of freq.

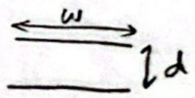
H field now satisfies HelmHoltz & is a travelling wave in z direction.

TM mode.

$E_z \neq 0 \quad H_z = 0 \quad , \quad \beta = \sqrt{k^2 - k_c^2}$

$$Z_{TM} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k}$$

(I) Parallel Plate Waveguide.



- **TEM modes**

$$\bar{E} = -\hat{y}\,\frac{V_0}{d}\,e^{-jkz} \qquad \bar{H} = \hat{x}\,\frac{V_0}{\eta d}\,e^{-jkz} \qquad k = \omega\sqrt{\mu\epsilon}$$

potential on top plate

$$V = V_0 e^{-jkz} \; ; \; I = \frac{W V_0}{\eta d} e^{-jkz} \; ; \; Z_0 = \frac{\eta d}{W} \; ; \; v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\alpha_d = \frac{k \tan\delta}{2} \text{ Np/m} \; ; \; \alpha_c = \frac{R_s}{\eta d}.$$

dielectric attenuation

TM modes

$$k_c = \frac{n\pi}{d} \; , \; n = 0, 1, 2, 3 \ldots$$

$$\beta = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2}$$

$$E_z = A_n \sin\frac{n\pi y}{d}\, e^{-j\beta z} \qquad H_x = \frac{j\omega\epsilon}{k_c} A_n \cos\frac{n\pi y}{d}\, e^{-j\beta z}$$

- $$E_y = \frac{-j\beta}{k_c} A_n \cos\frac{n\pi y}{d}\, e^{-j\beta z} \qquad E_x = H_y = 0.$$

n=0

$\Rightarrow TM_0 \Rightarrow E_z = 0 \rightarrow$ Identical to TEM mode.

→ $k > k_c \Rightarrow$ <u>β is real.</u> $k = \omega\sqrt{\mu\epsilon} \Rightarrow f_c$ corresponding to k_c is the cut off frequency below which $k < k_c$ & β is imaginary $\Rightarrow$ evanescent modes & no power propagates.

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{n}{2d\sqrt{\mu\epsilon}}$$

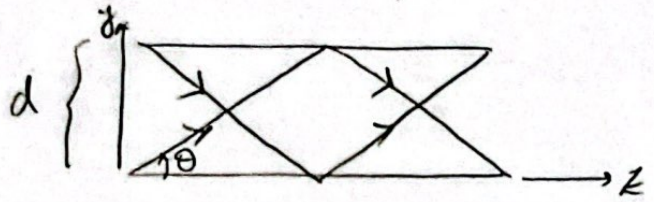
Dominant mode = TM1 @ $f_c = \dfrac{1}{2d\sqrt{\mu\epsilon}}$

$$Z_{TM} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k}$$ → real when $k > k_c$.

$$v_p = \frac{\omega}{\beta} > \frac{\omega}{k} \Rightarrow v_p > c$$

$\lambda_g = \dfrac{2\pi}{\beta}$ → guide wavelength distance b/w equiphase planes.

> Could be interpreted as a pair of bouncing planewaves.



At $f = f_c$ $\theta = 90° \rightarrow$ no propagation.

Phase velocity of each planewave along $z = \dfrac{1}{\sqrt{\mu\epsilon}\cos\theta} > c$.

$$\alpha_c = \frac{2\omega\epsilon R_s}{\beta d} = \frac{2kR_s}{\beta\eta d}.$$

<u>TE modes</u>

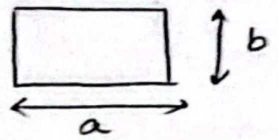
> β, f_c are same.

> $Z_{TE} = \dfrac{E_x}{H_y} = \dfrac{k\eta}{\beta}$

There is no TE_0 mode.

TE_1 & TM_1 occur at the same frequency.

(II) Rectangular Waveguide



$a > b$

> No TEM.

TE Modes.

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$k > \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \Rightarrow \text{propagation!}$$

$$f_{c_{mn}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

> Since $a > b$, TE_{10} occurs at the lowest frequency & is dominant

> There is no TE_{00} mode.

> $Z_{TE} = \dfrac{k\eta}{\beta}$; $\lambda_g = \dfrac{2\pi}{\beta} > \dfrac{2\pi}{k} = \lambda$ wavelength of plane wave in the same medium.

TE_{10} : $k_c = \frac{\pi}{a}$; $\beta = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$

TM modes

> β & $f_{c_{mn}}$ are identical.

> However there are no TM_{00}, TM_{10}, TM_{01} modes (due to the E, H formulae)
so lowest TM mode is TM_{11} & therefore TE_{10} is the dominant mode.

> $Z_{TM} = \dfrac{\beta\eta}{k}$.

First five modes :- TE_{10}, TE_{20}, TE_{01}, TE_{11}, TM_{11}.

> $\alpha_d = \dfrac{k^2 \tan\delta}{2\beta}$ Np/m > $\alpha_c = \dfrac{R_s}{a^3 b\beta k\eta}(2b\pi^2 + a^3k^2)$

(III) Circular Waveguide.

Solutions to HelmHoltz' or Laplace's equations in cylindrical coordinates gives Bessel's functions of integer order.

Bessel Equation: $\rho^2 \frac{d^2R}{d\rho^2} + \rho \frac{dR}{d\rho} + (\rho^2 K_c^2 - K_\phi^2) R = 0$.

TE modes.

$K_{c_{nm}} = \frac{P'_{nm}}{a}$ → roots of $J_n'(x)$ s.t P'_{nm} is the m^{th} root of J_n'

$J_n'(x)$ is the derivative of $J_n(x)$ → Bessel fn of first kind.

$\beta_{nm} = \sqrt{K^2 - K_{c_{nm}}^2}$ $\qquad f_{c_{nm}} = \frac{K_{c_{nm}}}{2\pi\sqrt{\mu\epsilon}}$

> TE_{11} is the dominant mode, (it has the smallest P'_{nm})

> There is no TE_{10} mode, but there is a TE_{01} mode.

> $Z_{TE} = \frac{\eta K}{\beta}$

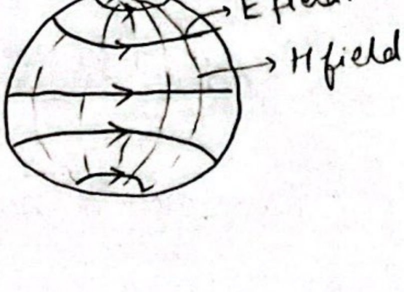
TM modes.

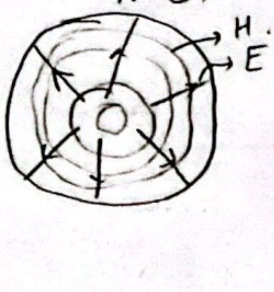
$K_c = \frac{P_{nm}}{a}$ → m^{th} root of $J_n(x)$; $\beta_{nm} = \sqrt{K^2 - K_c^2}$

$Z_{TM} = \frac{\eta \beta}{K}$ $\qquad f_{c_{nm}} = \frac{K_c}{2\pi\sqrt{\mu\epsilon}}$

> First TM mode is TM_{01} & comes after TE_{11}.

> Attenuation of TE_{01} is very low @ high freq, but it excites other modes.





(IV) Coaxial line.

- $\phi(\rho, \phi) = \dfrac{V_0 \ln b/\rho}{\ln b/a}$ for TEM mode.

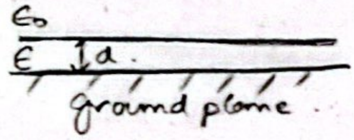
Higher order modes.

> Hard to analyze.

> TE11 is the dominant waveguide mode.

> $k_c = \dfrac{2}{a+b}$ - empirical equation.

(V) Surface waves on a Grounded Dielectric Sheet



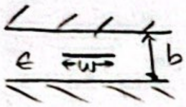
TM: $f_c = \dfrac{nc}{2d\sqrt{\epsilon_r - 1}}$ → cut off for TM_n where $n > 0$

- > TM0 is dominant & has 0 cut off freq. & therefore always exists.

TE: > No TE0 exists. For $n > 0$ $f_c = \dfrac{(2n-1)c}{4d\sqrt{\epsilon_r - 1}}$

Order of modes: TM0, TE1, TM1, TE2, TM2 . . .

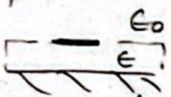
(VI) Stripline.



TEM: $v_p = c/\sqrt{\epsilon_r}$; $\beta = \dfrac{\omega}{v_p} = \sqrt{\epsilon_r}\, k_0$ $Z_0 = \dfrac{1}{v_p C} = \sqrt{\dfrac{L}{C}}$

There are empirical formula for L, C. Check Pozar Pg. 142.

(VII) Microstripline.



Partial dielectric ⇒ v_p is different in

- dielectric & air ⇒ cannot use phase matching ⇒ No TEM, only quasiTEM

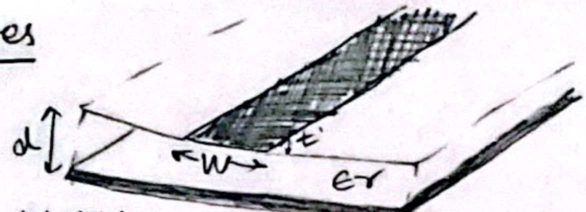
→ $1 < \epsilon_e < \epsilon_r$, where ϵ_e is the effective permittivity.

→ Empirical formulae for ϵ_e, Z_0, $\frac{W}{d}$, α_c are in Pozar. Pg 148

$v_p = c/\sqrt{\epsilon_e}$; $\beta = k_0\sqrt{\epsilon_e}$.

Back to Lecture Notes

Microstrip Lines



Assume $W \gg d$; $W \ll \lambda$

$$u_p = \frac{u_c}{\sqrt{\epsilon_e}}$$ $u_c \rightarrow$ speed of light 3×10^8 m/s , $1 \le \epsilon_e \le \epsilon_r$

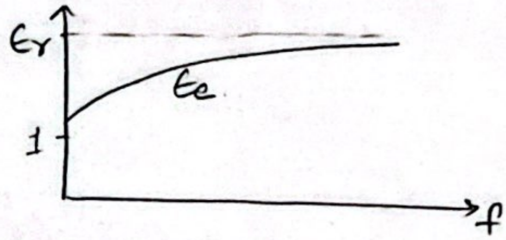
$\epsilon_e = \epsilon_r$ if $W \gg d$.

$$Z_0 = \frac{Z_{0air}}{\sqrt{\epsilon_e}}$$ $Z_{0air} \rightarrow Z_0$ when dielectric is air

$\epsilon_e \simeq \frac{1}{2}(\epsilon_r + 1)$ if $W \ll d$.

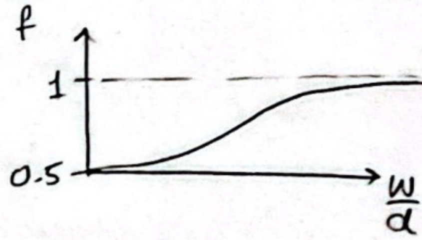
Guide wavelength

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_e}}}$$



Filling Factor (portion of EM energy inside the substrate)

$$f = \frac{\epsilon_e - 1}{\epsilon_r - 1} \qquad \underset{(W \ll d)}{\frac{1}{2}} < f < \underset{(W \gg d)}{1}$$



Dielectric Loss.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= \underbrace{j\omega\sqrt{\mu\epsilon}}_{j\beta}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$$

$$\gamma \simeq \underbrace{\frac{1}{2}\sigma\sqrt{\frac{\mu}{\epsilon}}}_{\alpha} + \underbrace{j\omega\sqrt{\mu\epsilon}}_{\beta}$$

Loss tangent

$$\boxed{\tan\delta = \frac{\sigma}{\omega\epsilon}}$$

Conductance $G = \omega C \tan\delta$.

For Low loss line

$$\alpha = \alpha_c + \alpha_d = \frac{1}{2}R\sqrt{\frac{C}{L}} + \frac{1}{2}G\sqrt{\frac{L}{C}} \quad ; \quad \alpha_d = \frac{1}{2}G\sqrt{\frac{L}{C}}$$

$$\boxed{\alpha_d = \frac{Z_0}{2}\omega C \tan\delta}$$

$$f_d = \frac{\epsilon_r}{\epsilon_e} f = \frac{\epsilon_r}{\epsilon_e}\left(\frac{\epsilon_e - 1}{\epsilon_r - 1}\right)$$

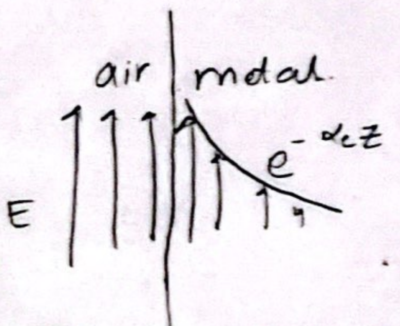
$$\boxed{\alpha_d = \frac{\omega}{2}\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_e}\tan\delta \ \ \text{Np/m}} \Rightarrow \boxed{\alpha_d = \frac{\omega}{2}\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_e}\,\frac{\epsilon_r}{\epsilon_e}\left(\frac{\epsilon_e - 1}{\epsilon_r - 1}\right)\tan\delta}$$

Np/m

T Lines continued

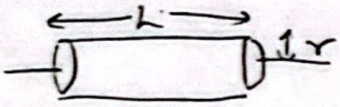
Skin depth.

$$\delta_s = \frac{1}{\sqrt{\pi f \mu \sigma}} \qquad \alpha_c = \sqrt{\pi f \mu \sigma}$$



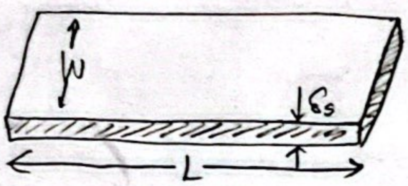
DC resistance $R_{dc} = \frac{1}{\sigma} \frac{L}{\pi r^2}$ of a cylindrical conductor

→ length (L) ; ↳ radius (r)



RF resistance $R_{RF} = \frac{1}{\sigma} \frac{L}{2\pi r \delta_s}$

For a sheet



$R = \frac{1}{\sigma} \frac{L}{W \delta_s}$ since $R = \frac{\rho L}{A}$

Let $R_s = \frac{1}{\sigma \delta_s}$ is the sheet resistance

We know $\delta_s = \frac{1}{\sqrt{\pi f \mu \sigma}}$ since current only flows in this depth & not in the real depth.

$$\boxed{R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}}}$$

& $R = R_s \frac{L}{W}$

Conductor loss α_c in Microstrip line.

$$\boxed{\alpha_c = \frac{R_s}{2 Z_0 W}}$$

If skin is rough.

$$\alpha_c' = \alpha_c \left[1 + \frac{2}{\pi} \tan^{-1}\left(1.4 \frac{\Delta}{\delta_s}\right)\right]$$

(Δ → rms surface roughness)

Power.

Power at some point $\boxed{P = P_0 e^{-2\alpha z}}$ → Attenuates fast as a fn of α.

→ Wide line ⇒ low Z_0 & low loss since $Z_0 = \frac{1}{u_p C}$

→ Choose $h < \frac{\lambda_g}{4}$ & $W < \frac{\lambda_g}{2}$ to prevent excitation of higher order modes.

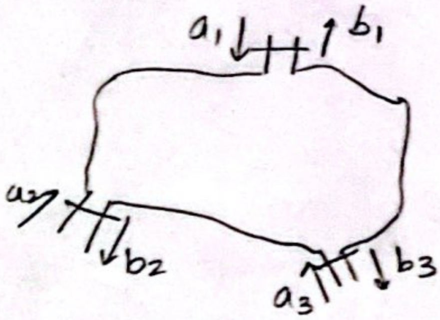
Network Analysis

$[V]_{n\times 1} = [Z]_{n\times n}[I]_{n\times 1}$ $\quad [I] = [Y][V]$ $\quad$ and $[Z] = [Y]^{-1}$

Reciprocal $\Rightarrow$ $Z_{ij} = Z_{ji}$ or $Y_{ij} = Y_{ji}$

Lossless $\Rightarrow$ Z, Y are purely imaginary

Scattering Matrix



$$a_j = \frac{V_j^+}{\sqrt{Z_0}} \qquad b_j = \frac{V_j^-}{\sqrt{Z_0}}$$

$$[b] = [S][a]$$

Reciprocal $\Rightarrow$ $S_{ij} = S_{ji}$

Lossless $\Rightarrow$ $[S]^{\dagger}[S] = I$ $\Rightarrow$ Sum of squares of columns = 0.

$P_{loss} = 1 - |S_{11}|^2 - |S_{21}|^2 - |S_{31}|^2$

↳ Fraction of power lost to network.

Return loss $\quad RL = -20\log|S_{11}|$ dB

Insertion loss $\quad IL = -20\log|S_{21}|$ dB

Insertion gain $\quad IG = 20\log|S_{21}|$ dB.

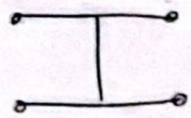
Mismatch loss $\quad ML = -10\log[1 - |S_{11}|^2]$

Power in dB_m $= +10\log(\text{Power in mWatt})$

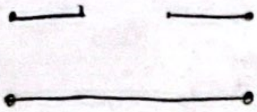
high = good match

Γ	VSWR	Reflected power(%)	RL (dB)	ML(dB)
0.89	17.4	79.4	1dB	6.87 dB
~0.708	~5.85	50%	3dB	3dB
~0.32	~1.92	10%.	10dB	~0.45dB

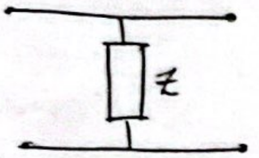
S parameters contd.



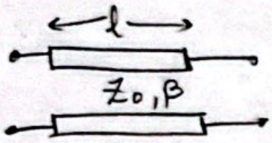
$$\Rightarrow [S] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



$$[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



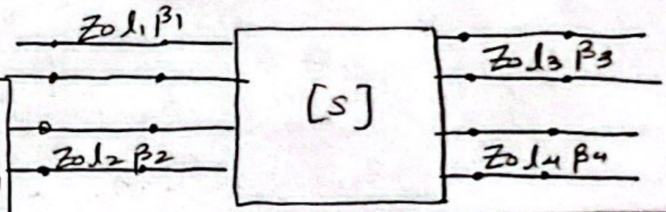
$$[S] = \begin{bmatrix} \frac{-Z_0}{2Z+Z_0} & \frac{2Z}{2Z+Z_0} \\ \frac{2Z}{2Z+Z_0} & \frac{-Z_0}{2Z+Z_0} \end{bmatrix}$$



$$[S] = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

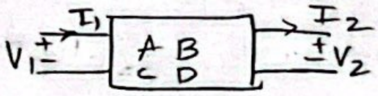
Shift in reference plane.

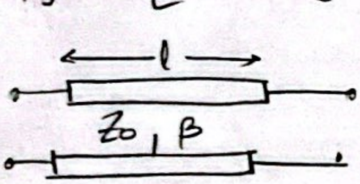
$$[S'] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ 0 & & \ddots\ e^{-j\theta_4} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ 0 & & \ddots\ e^{-j\theta_4} \end{bmatrix}$$



ABCD parameters

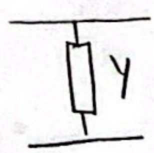
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



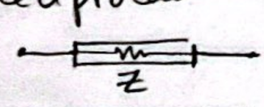


$$\Rightarrow [A] = \begin{bmatrix} \cos\beta l & jZ_0 \sin\beta l \\ jY_0 \sin\beta l & \cos\beta l \end{bmatrix}$$

Reciprocal $\Rightarrow$ $\boxed{AD - BC = 1}$

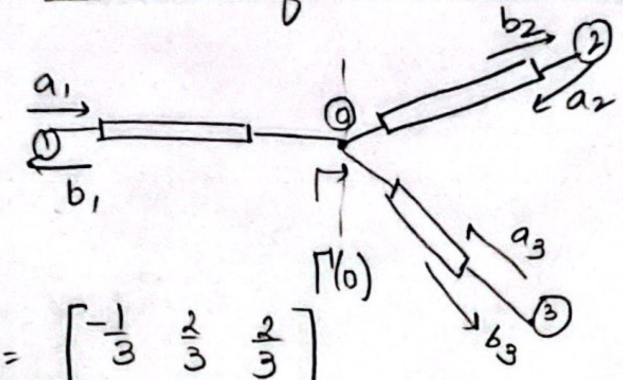


$$A = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$



$$A = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

S matrix of lossless T junction.



Assuming lengths are equal.

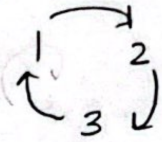
$\Gamma(0) = -\frac{1}{3}$

$a_1 + b_1 = a_2 + b_2 = a_3 + b_3$ since the lengths are same & the waves must be equal at 0.

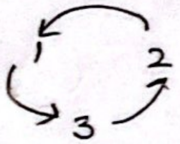
$$[S] = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

Circulator ⇒ nonreciprocal.

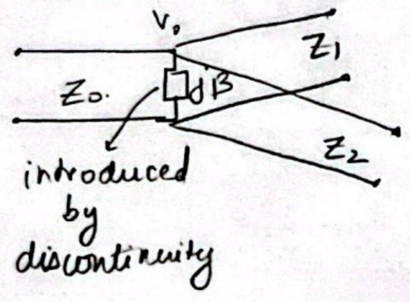
$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



T jn power divider. (not matched). Lossless & reciprocal.

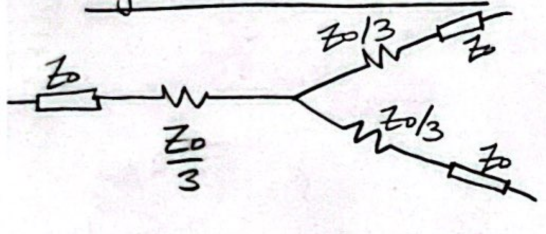


$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0^2}$ $\quad P_1 = \frac{1}{2}\frac{V_0^2}{Z_1}$ $\quad P_2 = \frac{1}{2}\frac{V_0^2}{Z_2}$

3dB ⇒ $P_1 = P_2 = P_{in}$ ⇒ $Z_1 = Z_2 = 2Z_0$.

⇒ Input alone is matched but no isolation.

T jn loss divider.



$$S = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

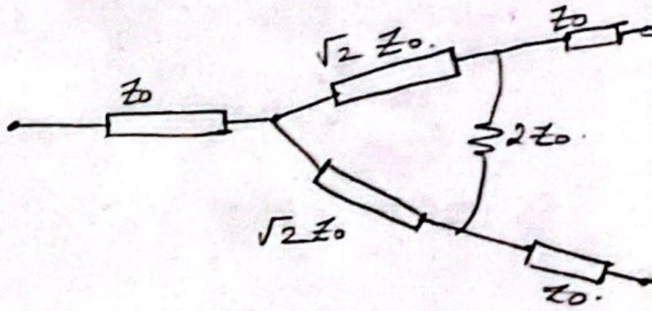
→ 6dB insertion loss.
→ $\frac{1}{4}$ power split b/w o/ps & $\frac{1}{2}$ power dissipated.
⇒ High bandwidth.
→ Measurement devices.

Wilkinson Power Divider

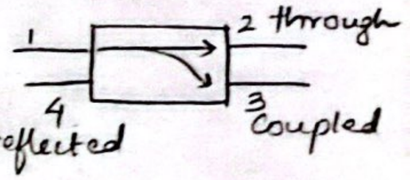
$$[S] = \begin{bmatrix} 0 & \frac{-j}{\sqrt{2}} & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 \\ \frac{-j}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

→ All ports are matched.
→ lossless only for power injected to port ① or to both ② & ③ equally.
→ Reciprocal.

→ Insertion loss = 3dB.



Four Port Networks



Coupling factor $C = -20 \log |S_{31}|$ dB

Directivity $D = 20 \log \frac{|S_{31}|}{|S_{41}|}$ dB

↳ determines how well coupled & reflected waves are isolated.

Ideal coupler ⇒ $S_{41} = 0 \rightarrow D = \infty$

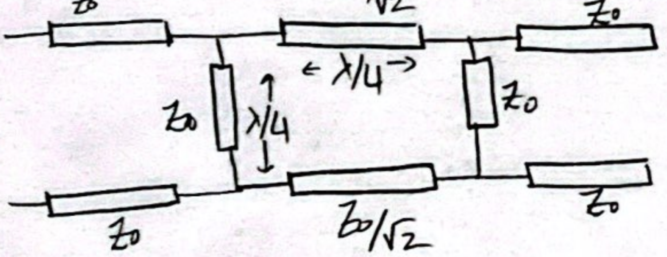
Isolation $I = -20 \log |S_{41}|$ dB

$RL = -20 \log |S_{11}|$ dB

$\boxed{I = D + C}$ in dB

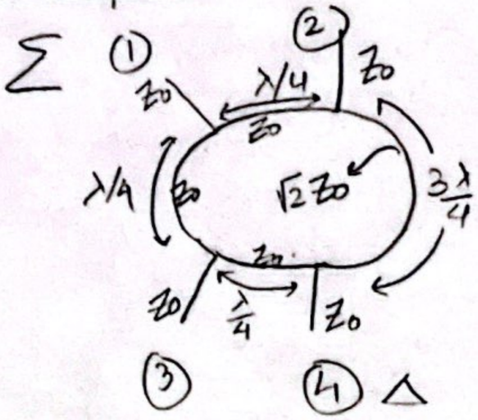
$IL = -20 \log |S_{12}|$ dB or $-20 \log |S_{21}|$ dB.

Branched line Coupler. [3dB Hybrid coupler]



$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Rat race coupler

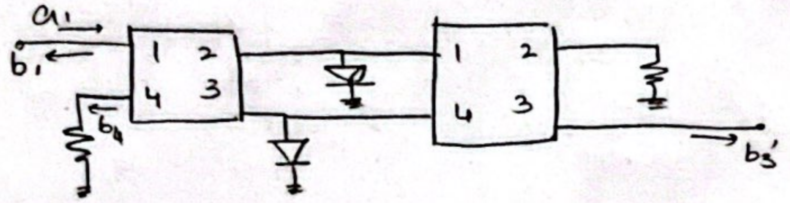


$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

→ Inputs to ② & ③ → Σ at ① & Δ at ④

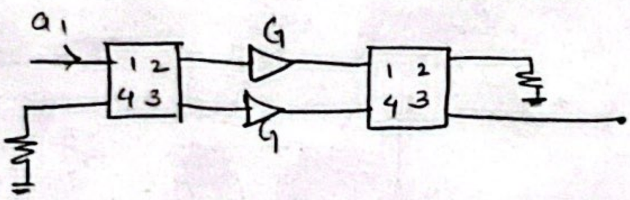
Applications of BLC

Matched switches

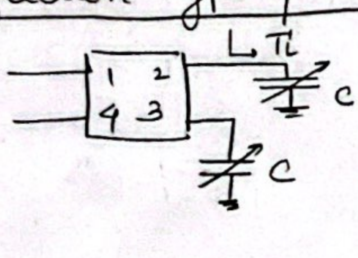


→ b_1 is always 0 regardless of open or closed diodes.

Wideband amplifier



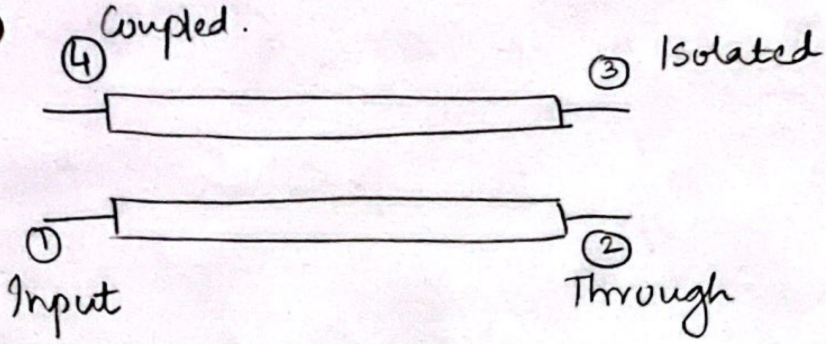
Reflection type phase shifter



$\angle S_{41} = -90° + \angle\theta$

$\Gamma_L = 1\angle\theta$

Coupled Line Coupler



$Z_{oe} \rightarrow$ even mode Z_o. $Z_{oo} \rightarrow$ odd mode impedance.

$$Z_o = \sqrt{Z_{oe} Z_{oo}}$$

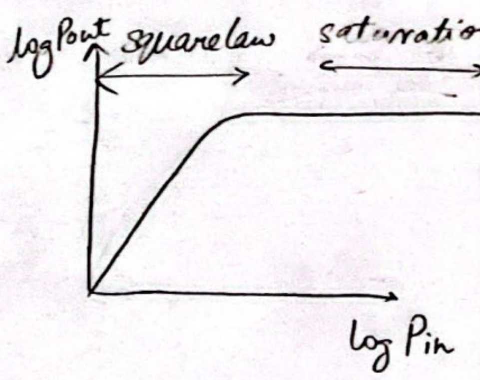
Coupling factor $C = \dfrac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \Rightarrow Z_{oe} = Z_o\sqrt{\dfrac{1+C}{1-C}} \quad Z_{oo} = Z_o\sqrt{\dfrac{1-C}{1+C}}$

$$\left.\begin{aligned} S_{11} &= S_{22} = S_{33} = S_{44} = 0 \\ S_{21} &= -j\sqrt{1-C^2} \\ S_{31} &= C \\ S_{41} &= 0 \end{aligned}\right\} \text{At the design frequency} \Rightarrow \theta = \frac{\pi}{2}$$

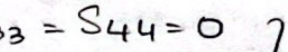
Diodes.

> Schottky diodes are commonly used.

$I(V) = I_s(e^{\alpha V} - 1)$ where $\alpha = \dfrac{q}{nkT}$



> PIN diodes (Ptype - Intrinsic - Ntype).

Forward bias $\Rightarrow$ 

Reverse bias $\Rightarrow$ 

- Single pole reflective switch
- Single pole double throw
- Switched line phase shifter.

} Common applications.

Noise

Thermal noise

RMS value of thermal noise across resistor:

$$V_n = \sqrt{\frac{4hfBR}{e^{hf/KT}-1}}$$

$h = 6.626 \times 10^{-34}$ Js → planck's const.

$K = 1.38 \times 10^{-23}$ J/K → Boltzmann's const.

f → center freq

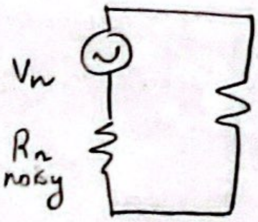
B → BW

At microwave freq. $hf << KT$

$$\Rightarrow V_n = \sqrt{4KTBR}.$$

Available thermal noise power $= P_n = \frac{V_L^2}{R} = \frac{(V_n/2)^2}{R} = \frac{V_n^2}{4R}.$

$$\Rightarrow P_n = KTB$$



At room temp. (290 K)

$$P_n = -174 \text{ dBm} + 10 \log B.$$

Equivalent noise temperature → (sensitive scale used in astronomy)

A resistor at T_e produces the same noise as our noise source with a gain of G & BW → B.

$$N_o = (K T_e B) G$$

$$\Rightarrow T_e = \frac{N_o}{KGB}$$

Noise figure.

$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1$$

$$\boxed{T_e = (F-1) T_o}$$

not in dB

$$NF = 10 \log F$$

Cascaded noise figure.

$$F_{casc} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots$$

nothing is in dB.

→ Noise figure of a matched lossy network is equal to its loss in dB at room temperature.

→ N.F of attenuator = Insertion loss in dB.

→ If the reference temperature of attenuator ≠ T_o

$$NF = 1 + \frac{(\overset{\to \text{loss}}{L}-1)T}{T_o}$$

LNA Design.

Noise figure of a two port network.

$$F = F_{min} + \frac{R_N}{G_s}|Y_s - Y_{opt}|^2$$

$$Y_s = G_s + jB_s$$

$R_N \to$ equivalent noise resistance.

$Y_s \to$ source admittance.

$Y_{opt} \to$ optimum source resistance to minimize $F \to F_{min}$.

$$Y_{opt} = \frac{1}{Z_o} \cdot \frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

$$\Rightarrow F = F_{min} + \frac{4R_N}{Z_o} \frac{|\Gamma_s - \Gamma_{opt}|^2}{(1-|\Gamma_s|^2)(|1+\Gamma_{opt}|^2)}$$

For a constant F, the equation describes a circle in Γ plane.

Center: $C_F = \dfrac{\Gamma_{opt}}{N+1}$

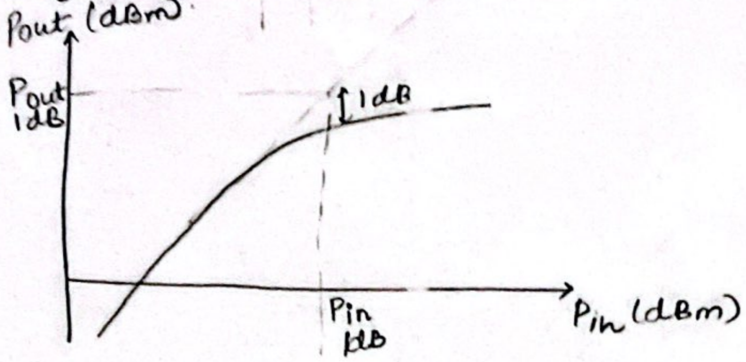
Radius: $R_F = \dfrac{\sqrt{N(N+1-|\Gamma_{opt}|^2)}}{N+1}$

where,

$$N = \frac{F-F_{min}}{4R_N/Z_o}|1+\Gamma_{opt}|^2$$

Nonlinearity and Distortion - Dynamic Range

Gain compression



$$y(x) = K_1 f(x) + K_2 f^2(x) + K_3 f^3(x) + \cdots$$

$$f(x) = A_1 \cos\omega_1 t$$

$$\Rightarrow y(x) = K_1 A_1 \cos\omega_1 t + K_2 A_1^2 \left(\frac{1+\cos 2\omega_1 t}{2}\right) + K_3 \left[A_1^3 \left(\frac{3\cos\omega_1 t}{4}\right)\right] + \cdots$$

$$\Rightarrow \text{gain at } \omega_1 = \frac{y(x) \text{ at } \omega_1}{f(x) \text{ at } \omega_1} = \frac{K_1 A_1 + K_3 A_1^3 (3/4)}{A_1}$$

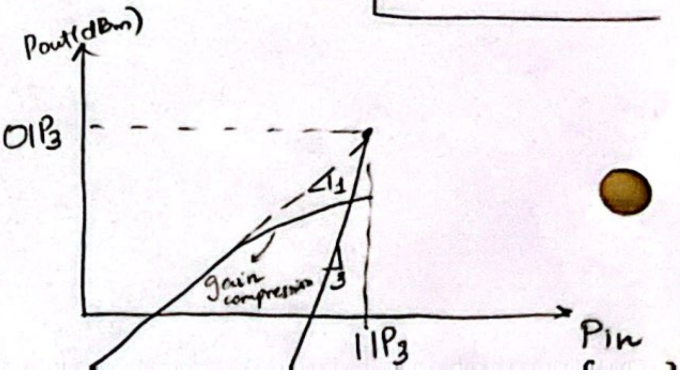
$$\boxed{\text{Gain} = K_1 + K_3\left(\frac{3}{4} A_1^2\right)}$$

K_3 → usually negative.

Intermodulation & Distortion.

2 tone test: $f(x) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$ generates $2\omega_1 - \omega_2$ & $2\omega_2 - \omega_1$ that are in band.

If $A_1 \approx A_2$, At the output amplitude of IM terms is $\frac{3}{4} K_3 A_1^2 A_2$ & amplitude of the signal is $K_1 A_1$. Therefore IM terms grow very fast.



SFDR Spurious Free Dynamic Range.

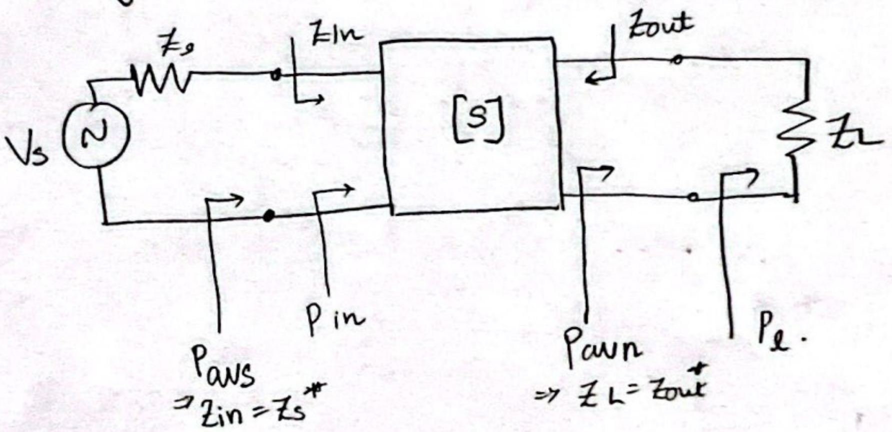
$$DR \text{ (in dB)} = \frac{2}{3}\left[IIP_3 \text{ (dBm)} - S_{i\,min} \text{ (dBm)} \right]$$

$$= \frac{2}{3}\left[IIP_3 \text{ (dBm)} - \underset{\text{output noise}}{N_o} \text{ (dBm)} \right]$$

Cascaded OIP3

$$\frac{1}{OIP3} = \frac{1}{OIP3_1\, G_2 G_3} + \frac{1}{OIP3_2\, G_3} + \frac{1}{OIP3_3}$$

Power Gain Definitions



$$P_{avs} = \frac{V_s^2}{8 R_s}$$

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11}\Gamma_s}$$

$$P_{in} = \frac{|V_s|^2}{8 Z_0} \frac{|1 - \Gamma_s|^2}{|1 - \Gamma_{in}\Gamma_s|^2} (1 - |\Gamma_{in}|^2)$$

$$P_l = \frac{|V_s|^2}{8 Z_0} \frac{|S_{21}|^2 (1 - |\Gamma_L|)^2 |1 - \Gamma_s|^2}{|1 - S_{22}\Gamma_L|^2 |1 - \Gamma_{in}\Gamma_s|^2}$$

$$P_{avs} = \frac{|V_s|^2}{8 Z_0} \frac{|1 - \Gamma_s|^2}{1 - |\Gamma_s|^2}$$

$$P_{avn} = \frac{|V_s|^2}{8 Z_0} \frac{|S_{21}|^2 |1 - \Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2 (1 - |\Gamma_{out}|^2)}$$

Operating power gain (independant of Z_s)

$$G_P = \frac{P_L}{P_{in}} = \frac{1}{1-|\Gamma_{in}|^2} |S_{21}|^2 \frac{(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}$$

Available Power gain (independant of Z_L)

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{(1-|\Gamma_{out}|^2)}$$

Transducer Power Gain (Depends on both Z_s & Z_L)

$$G_T = \frac{(1-|\Gamma_s|^2)}{|1-\Gamma_s\Gamma_{in}|^2} |S_{21}|^2 \frac{(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}$$

G_{P_L}

P_{AVS}

$\rightarrow G_{T\max} = G_T = G_A = G_P = G_{A\max} = G_{P\max}$ if simultaneous conjugate matching.

$\rightarrow$ Unilateral $\Rightarrow$ $S_{12} = 0$

$$\Rightarrow G_{TU} = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

RF Amplifier Design

- $G_T = G_S G_0 G_L$

$G_S = \dfrac{1 - |\Gamma_s|^2}{|1 - \Gamma_{in}\Gamma_s|^2}$ gain contribution due to input matching

$G_0 = |S_{21}|^2$ device gain

$G_L = \dfrac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$ gain contribution due to output matching.

$\Gamma_{out} = S_{22}$, $\Gamma_{in} = S_{11}$ if $S_{12} = 0 \Rightarrow$ unilateral.

→ $|\Gamma_{in}|$ or $|\Gamma_{out}| > 1$ at some frequency $\Rightarrow$ potentially unstable!

Stability

a) Unconditional stability

- $|\Gamma_{in}| < 1$ & $|\Gamma_{out}| < 1$ for all possible source & load impedances

b) Conditional stability

$|\Gamma_{in}| < 1$ & $|\Gamma_{out}| < 1$ for some set of source & load impedances.

→ Unilateral device $\Rightarrow$ $|S_{11}| < 1$ & $|S_{22}| < 1$

→ For bilateral device: Use stability circles.

Loci in the Γ plane for which $|\Gamma_{in}| = 1$ or $|\Gamma_{out}| = 1$.

→ Locus of Γ_Ls to make $|\Gamma_{in}| = 1$ give output stability circles

$$\left| S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1$$

- Let $\Delta = S_{11}S_{22} - S_{12}S_{21}$

Output Stability circles

$C_L = \dfrac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$

$R_L = \left| \dfrac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$

Input Stability circles

$C_S = \dfrac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}$

$R_S = \left| \dfrac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$

> We know that the circle defines boundary of stability, but how do we know if inside or outside is stable?

Use K Δ Test

An amplifier is unconditionally stable iff $K > 1$ & $|\Delta| < 1$

Where,

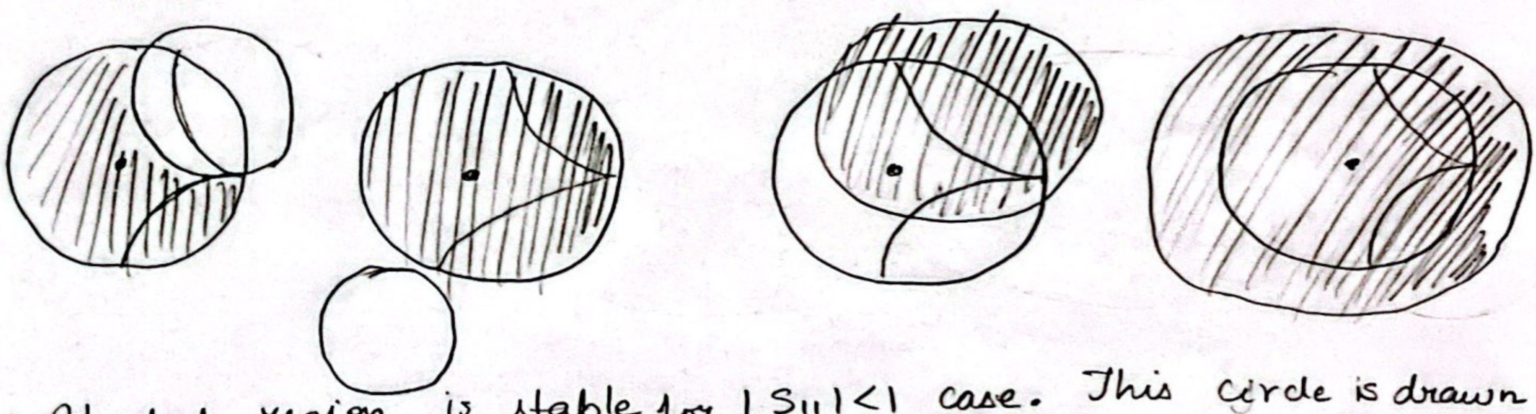
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12} S_{21}|} > 1$$

$$|\Delta| = |S_{11} S_{22} - S_{12} S_{21}| < 1$$

Test for the center of smith chart

If $Z_L = Z_0 \Rightarrow |\Gamma_{in}| = |S_{11}|$

> If $|S_{11}| < 1 \rightarrow |\Gamma_{in}| < 1$ for $Z_L = Z_0 \Rightarrow$ Center of smith chart is stable. Now we can identify if inside or outside the stability circles is stable.



> Shaded region is stable for $|S_{11}| < 1$ case. This circle is drawn at one frequency. Different frequencies have different circles.

> Pick any load in the shaded region and the circuit would be stable.

> Unconditional stability → entire smith chart is stable since it represents all passive impedances.

Amplifier Design

Unilateral

> Match input and output seperately to get $G_{TU\max} = \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2}$

$\Gamma_{in} = S_{11}$, $\Gamma_{out} = S_{22}$

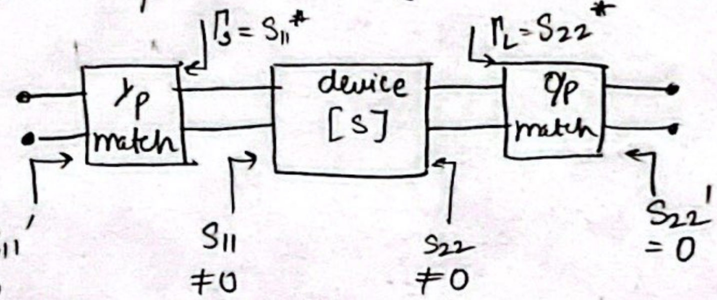
$Z_S = Z_{in}^*$, $Z_L = Z_{out}^*$

$\Gamma_S = S_{11}^*$, $\Gamma_L = S_{22}^*$ → use this to get Z_S & Z_L that give max gain.

> This maximizes G_S & G_L & we in fact get gain > 1 from G_S & G_L

> If input & output are matched does it not imply $S_{11} = 0$ & $S_{22} = 0$?

=> Γ_S & $\Gamma_L = 0$ => $G_{TU\max} = |S_{21}|^2$? No, here S_{11} & S_{22} are of the device. S_{11}' & S_{22}' are in fact 0 but these are for the entire 2 port including the device.



> ## Unilateral approximation.

Error in transducer gain $= \frac{G_T}{G_{TU}}$ by making the unilateral approximation

This error is bounded by $\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$

Where $U = \frac{|S_{21}||S_{12}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$ → unilateral figure of merit

In general $U \leq 0.1$ => make the approximation.

Bilateral amplifier design.

Simultaneous conjugate matching.

$$\Rightarrow \Gamma_s^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = \Gamma_{in}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} = \Gamma_{out}$$

$$B_1 = 1+|S_{11}|^2-|S_{22}|^2-|\Delta|^2$$

$$B_2 = 1+|S_{22}|^2-|S_{11}|^2-|\Delta|^2$$

$$C_1 = S_{11}-\Delta S_{22}^*$$

$$C_2 = S_{22}-\Delta S_{11}^*$$

$$\left.\begin{aligned} \Rightarrow \Gamma_{ms} &= \frac{B_1 \pm \sqrt{B_1^2-4|C_1|^2}}{2C_1} \\ \Gamma_{ml} &= \frac{B_2 \pm \sqrt{B_2^2-4|C_2|^2}}{2C_2} \end{aligned}\right\}$$

→ Required source & load reflection coefficient for S.C.M → use to find Z_{ms} & Z_{ml}.

∴ Maximum available gain

$$G_{Tmax} = \frac{1}{1-|\Gamma_{ms}|^2}|S_{21}|^2\frac{1}{1-|\Gamma_{ml}|^2}$$

$$G_{Tmax} = \frac{|S_{21}|}{|S_{12}|}\left(K+\sqrt{K^2-1}\right)$$

→ K is only defined if $|S_{12}|>0$ since $K \propto \frac{1}{|S_{12}|}$

Simultaneous conjugate match is only possible if $K>1$ → unconditional stability!

<u>Note</u>

$B_1 > 0 \Rightarrow$ use −ve sign

$B_1 < 0 \Rightarrow$ use +ve sign

$B_2 > 0 \Rightarrow$ use −ve sign.

$B_2 < 0 \Rightarrow$ use +ve sign.

When $K=1$ we get max Stable gain MSG

$$MSG = \frac{|S_{21}|}{|S_{12}|}$$

→ Cannot go to ∞ if $|S_{12}|=0$ Since $K\rightarrow\infty$ & G_{Tmax} defn doesn't make sense.

Constant gain circles.

> Locus of Γ_s or Γ_L which provides constant gain (G_s or G_L)

Unilateral

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \quad \& \quad G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$\Gamma_S = S_{11}^*$ & $\Gamma_L = S_{22}^* \Rightarrow$ max gain $G_{S\,max} = \dfrac{1}{1 - |S_{11}|^2}$ & $G_{L\,max} = \dfrac{1}{1 - |S_{22}|^2}$

Normalized gains: $g_s = \dfrac{G_S}{G_{S\,max}}$ & $g_L = \dfrac{G_L}{G_{L\,max}}$

$$g_s = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2}(1 - |S_{11}|^2) \qquad 0 \le g_s \le 1$$

$$g_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}(1 - |S_{22}|^2) \qquad 0 \le g_L \le 1$$

g_s, g_L = constant $\Rightarrow$ circles on Γ plane.

$$C_S = \frac{g_s S_{11}^*}{1 - (1 - g_s)|S_{11}|^2} \qquad C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L)|S_{22}|^2}$$

$$R_S = \frac{\sqrt{1 - g_s}\,(1 - |S_{11}|^2)}{1 - (1 - g_s)|S_{11}|^2} \qquad R_L = \frac{\sqrt{1 - g_L}\,(1 - |S_{22}|^2)}{1 - (1 - g_L)|S_{22}|^2}$$

Filters.

2 methods: i) Insertion loss method.
ii) Image parameter method.

Insertion loss method.

$P_{LR} = \frac{1}{|S_{12}|^2}$ if load & source are matched.

$$P_{LR} = \underset{\text{loss}}{\overset{10\log P_{LR}}{\text{Insertion}}} = \frac{\text{Power available from source}}{\text{Power delivered to load}} = \frac{P_{avs}}{P_{Load}} = \frac{1}{1-|\Gamma(\omega)|^2}$$

In general,

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$ where M & N are polynomials.

If matched $IL = 10\log P_{LR} = -20\log S12$

Maximally flat aka Butterworth aka Binomial. response.

$$P_{LR} = 1 + k^2\left(\frac{\omega}{\omega_c}\right)^{2N}$$

$\omega_c \rightarrow$ cut off freq,
$N \rightarrow$ number of lumped elements aka order.

→ Flattest possible passband, slow roll off.

Equiripple aka Chebyshev response.

$$P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

$T_N(X)$ is the Chebyshev polynomial of the first kind of order N.

→ Fast roll off & ripple in pass band.

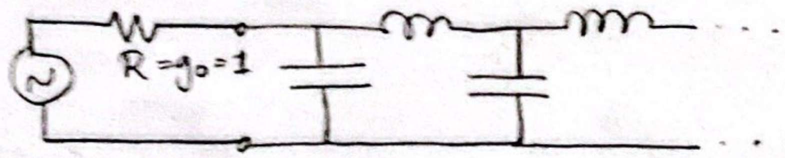
Elliptic response.

> Look up table
> Fastest roll off with ripple in pass & stop band.

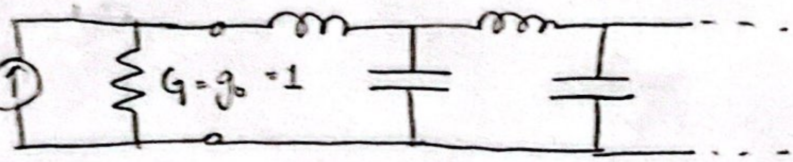
Steps in Insertion Loss method

a) Based on specs, design a prototype.
b) Using scaling & conversion rules, obtain the desired filter.
c) Implement filter using lumped elements or transmission lines.

Low pass prototypes



Reflects as a short as $f \uparrow$



Reflects as an open as $f \uparrow$

$\Rightarrow$ phase response is different

$g_0 = \begin{cases} \text{generator resistance for } \Pi \\ \text{generator conductance for } T \end{cases}$

$\underset{K=1\ldots N}{g_K} = \begin{cases} \text{inductance for series element} \\ \text{Capacitance for shunt element} \end{cases}$

$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is shunt cap.} \\ \text{load conductance if } g_N \text{ is series ind.} \end{cases}$

> For a type of filter g values are got from a Table & design proceeds.

> This is designed for $\omega_c = 1$ & $g_0 = \underbrace{g_{N+1}}_{\rightarrow \text{not always } 1} = 1$

Filter Transformations

→ **Impedance scaling**

$L' = Z_0 L$ $\quad R_s' = Z_0$ → where Z_0 is the source resistance.

$C' = \dfrac{C}{Z_0}$ $\quad R_L' = Z_0 R_L$

→ **Frequency scaling**

Scale the frequency response by $\frac{1}{\omega_c}$ ⇒ replace $\omega \longrightarrow \frac{\omega}{\omega_c}$

$\therefore L_K' = \dfrac{L_K}{\omega_c}$, $C_K' = \dfrac{C_K}{\omega_c}$.

⇒ Combining the 2 scaling laws:

$$L_K' = \frac{Z_0 L_K}{\omega_c} \qquad C_K' = \frac{C_K}{Z_0 \omega_c}.$$

→ **Low pass to High pass transformation**

$$\omega \longrightarrow -\frac{\omega_c}{\omega}$$

$C_K' = \dfrac{1}{Z_0 \omega_c L_K}$ inductor → cap

$L_K' = \dfrac{Z_0}{\omega_c C_K}$. cap → ind.

Band Pass/Stop transformation.

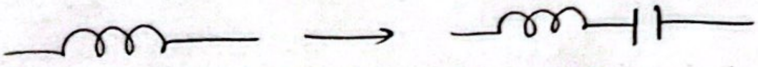
$\Delta = \dfrac{\omega_2 - \omega_1}{\omega_0}$ → Fractional Bandwidth. , $\omega_0 = \sqrt{\omega_1 \omega_2}$.

$$\omega \longrightarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

ω_0 = center frequency = $\sqrt{\omega_1 \omega_2}$

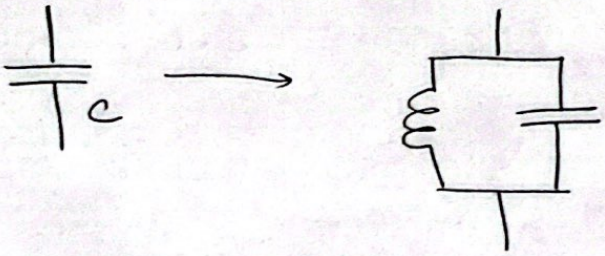
use $\left|\dfrac{\omega}{\omega_c = 1}\right| - 1$ to find N on the table

Band pass.



L

$$L_k' = \frac{L_k Z_0}{\Delta \omega_0} \quad (L_k \rightarrow g_k) \qquad C_k' = \frac{\Delta}{\omega_0 L_k Z_0} \quad (L_k \rightarrow g_k)$$

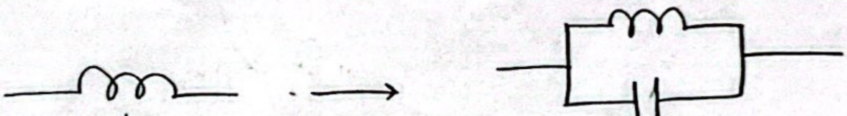


C

$$L_k' = \frac{\Delta Z_0}{\omega_0 C_k} \quad (C_k \rightarrow g_k)$$

$$C_k' = \frac{C_k}{\Delta \omega_0 Z_0} \quad (C_k \rightarrow g_k)$$

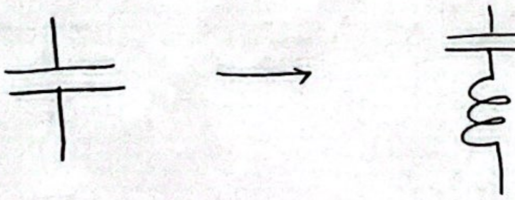
Band stop.



L

$$L_k' = \frac{\Delta L_k Z_0}{\omega_0}$$

$$C_k' = \frac{1}{\omega_0 \Delta L_k Z_0}$$



C

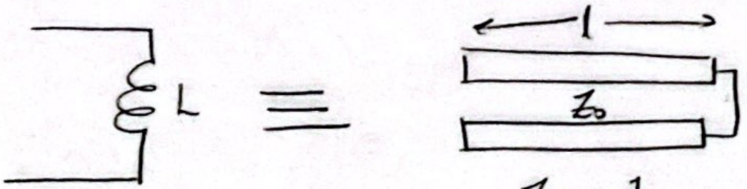
$$L_k' = \frac{Z_0}{\omega_0 \Delta C_k}$$

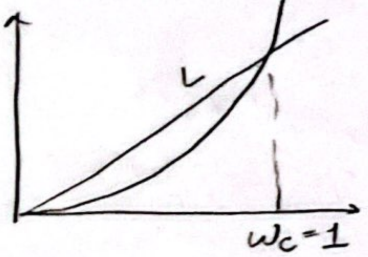
$$C_k' = \frac{\Delta C_k}{\omega_0 Z_0}$$

Filter Implementation

→ Implementing lumped elements as T Lines.

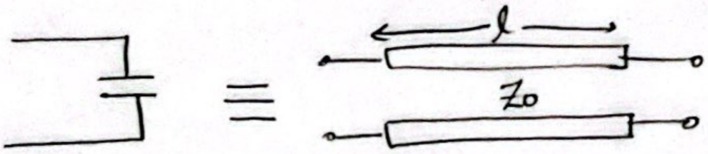
Richard's Transformation





$Z_0 = L$

$l = \frac{\lambda}{8}$ at ω_c



$Y_0 = C \Rightarrow Z_0 = \frac{1}{C}$

$l = \frac{\lambda}{8}$ at ω_c.

→ Frequency response becomes periodic → problem!!

→ Impedance scaling must be done by multiplying Z_0 with desired filter impedance.

→ **Problem:** Series stubs cannot be fabricated in planar process.

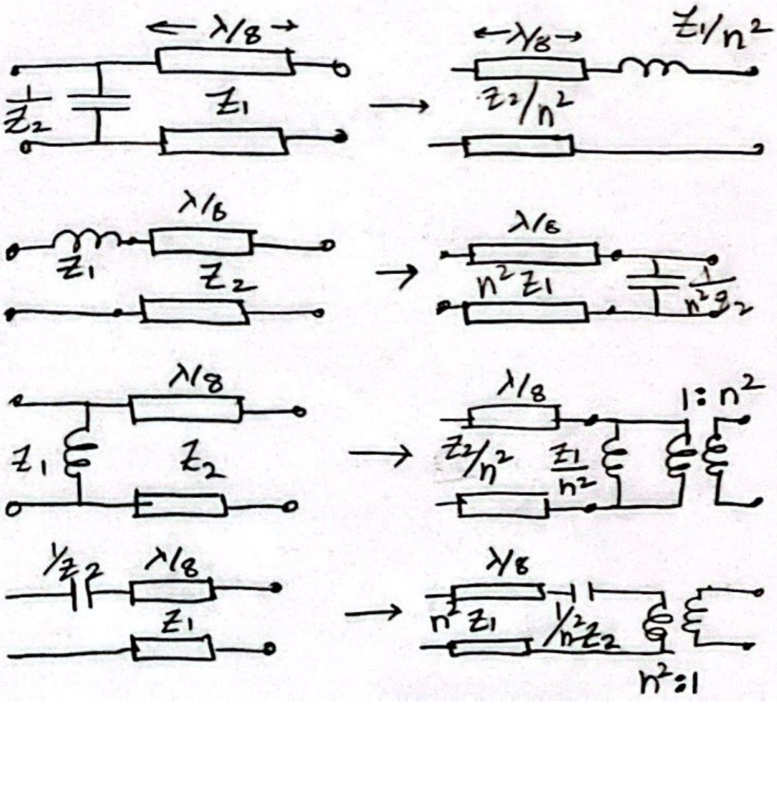
Kuroda's Identity.

→ To seperate stubs.

→ To transform series stubs to shunt stubs

→ To get realisable characteristic impedances.

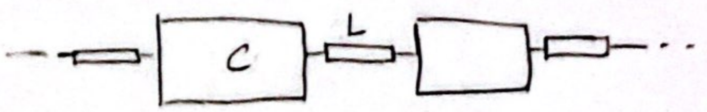
→ $n^2 = 1 + \frac{Z_2}{Z_1}$



Stepped Impedance Low Pass Filters.

→ Series inductor → high impedance line ($\beta l < \lambda/4$)
Shunt capacitor → low impedance line ($\beta l < \lambda/4$)

→ $L = \dfrac{Z_h l}{U_p} = \dfrac{Z_h \beta l}{\omega}$



$C = \dfrac{\beta l}{\omega Z_l}$

→ $\dfrac{Z_h}{Z_l}$ should be as large as possible.

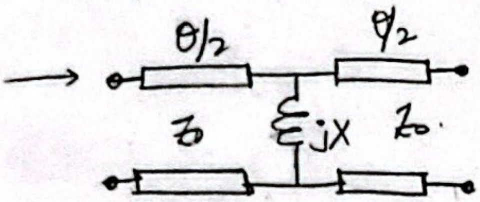
→ $\beta l = \dfrac{Z_0 L_k}{Z_h}$ for series inductor.

$\beta l = \dfrac{C_k Z_l}{Z_0}$ for shunt capacitor.

Inverters.

Transform load impedance to it's inverse.

→ $\lambda/4$ line.

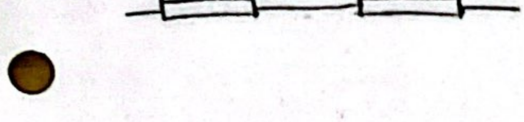


$K = Z_0 \tan|\theta/2|$

$X = \dfrac{K}{1 - |K/Z_0|^2}$

$\theta = -\tan^{-1}\dfrac{2X}{Z_0}$

θ_0 is $-ve$ ⇒ it can be absorbed into existing lines.



$J = Y_0 \tan|\theta/2|$

$B = J/1-(J/Y_0)^2$

$\theta = -\tan^{-1}\dfrac{2B}{Y_0}$

→ Band pass & Bandstop filters using $\frac{\lambda}{4}$ resonators.

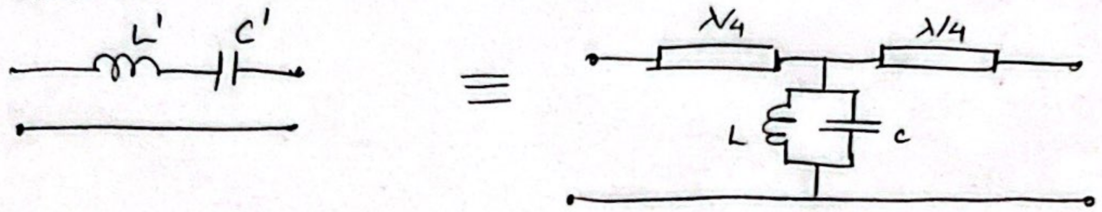
> An open circuited $\lambda/4$ Tline with characteristic impedance Z_{on} acts like a series resonant circuit.

$$\rightarrow \omega_o L_n = \frac{\pi}{4} Z_{on} \quad \text{where } \omega_o = \frac{1}{\sqrt{L_n C_n}}.$$

> Short circuit $\lambda/4$ → parallel resonant circuit.

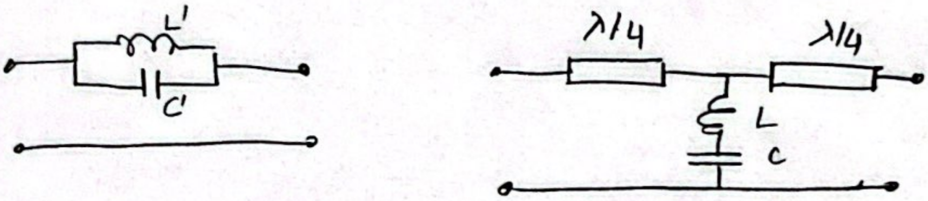
$$\rightarrow \omega_o C_n = \frac{\pi}{4} \cdot \frac{1}{Z_{on}} \quad \text{where } \omega_o = \frac{1}{\sqrt{L_n C_n}}.$$

→ Convert series resonant circuit → parallel using inverters.



K inverter

$$L = \frac{Z_o^2}{W_o^2 L'} \qquad C = \frac{1}{\omega_o^2 C' Z_o^2}$$

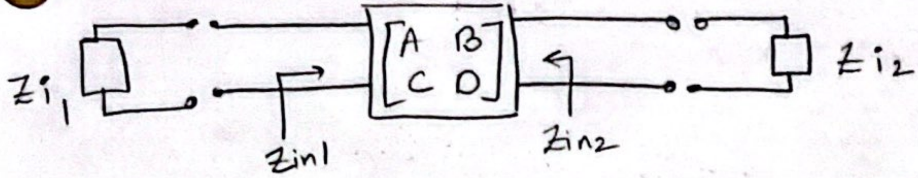


J inverter.

Steps.

1) Choose filter type (# of sections, g values)
2) Use filter transformations
3) Determine L_Ks & C_Ks for resonators.
4) Transform series resonators to shunt resonators as shown above.
5) Implement all shunts with $\lambda/4$ S-C Tlines (or open circuit in case of band stop - refer to top of this page).

Image impedance of 2 port networks



→ If connecting Z_{i_1} & Z_{i_2} ensures $Z_{in1} = Z_{i_1}$ & $Z_{in_2} = Z_{i_2}$
we call Z_{i_1} & Z_{i_2} image impedances of the 2 port.

→ $Z_{i_1} = \sqrt{\dfrac{AB}{CD}}$ $\qquad Z_{i_2} = \sqrt{\dfrac{BD}{AC}}$

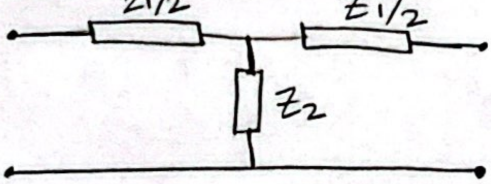
→ Symmetric network ⇒ $Z_{i1} = Z_{i2}$ since $A = D$

- $\dfrac{V_2}{V_1} = \sqrt{\dfrac{D}{A}}\left(\sqrt{AD} - \sqrt{BC}\right)$

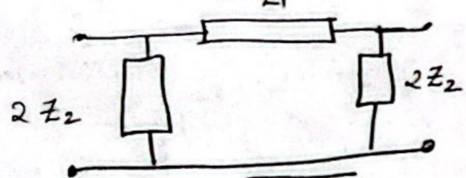
  $\dfrac{I_2}{I_1} = \sqrt{\dfrac{A}{D}}\left(\sqrt{AD} - \sqrt{BC}\right)$

  $e^{-\gamma} = \sqrt{AD} - \sqrt{BC}$ where $\gamma = \alpha + j\beta$

→



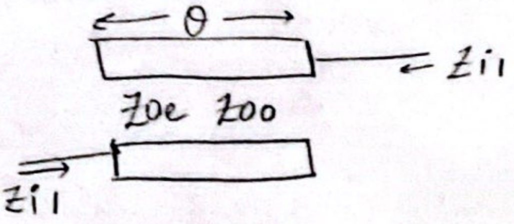
$Z_i = \sqrt{Z_1 Z_2}\sqrt{1 + \dfrac{Z_1}{4Z_2}}$



$Z_i = \dfrac{\sqrt{Z_1 Z_2}}{\sqrt{1 + Z_1/4Z_2}}$

- $e^{\gamma} = 1 + \dfrac{Z_1}{2Z_2} + \sqrt{\dfrac{Z_1}{Z_2} + \dfrac{Z_1^2}{4Z_2^2}}$

Coupled Tline



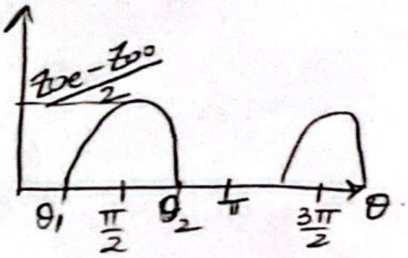
$$Z_i = \frac{1}{2}\sqrt{(Z_{oe} - Z_{oo})^2 \operatorname{cosec}^2\theta - (Z_{oe} + Z_{oo}) \cot^2\theta}$$

when $\theta = \frac{\pi}{4}$

$$Z_i = \frac{1}{2}(Z_{oe} - Z_{oo})$$

$$\cos\beta = \frac{Z_{oe} + Z_{oo}}{Z_{oe} - Z_{oo}} \cos\theta$$

} Band pass response.



→ Cascading them gives desired response.

Recipe.

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} \qquad \frac{\omega}{\omega_c} \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$\left|\frac{\omega}{\omega_c}\right| - 1$ gives N

$$J_1 Z_0 = \sqrt{\frac{\pi\Delta}{2g_1}} \qquad J_n Z_0 = \frac{\pi\Delta}{2\sqrt{g_{n-1}\, g_n}} \qquad J_{n+1} Z_0 = \sqrt{\frac{\pi\Delta}{2 g_n\, g_{n+1}}}$$

$$Z_{oe} = Z_0\left[1 + JZ_0 + (JZ_0)^2\right]$$

$$Z_{oo} = Z_0\left[1 - JZ_0 + (JZ_0)^2\right]$$

$$\theta = \frac{\pi}{2}$$

Mixers

→ Conversion loss in dB = $10 \log \dfrac{\text{IF output power}}{\text{RF input power}}$

Typical diode mixers → Loss = 6dB.

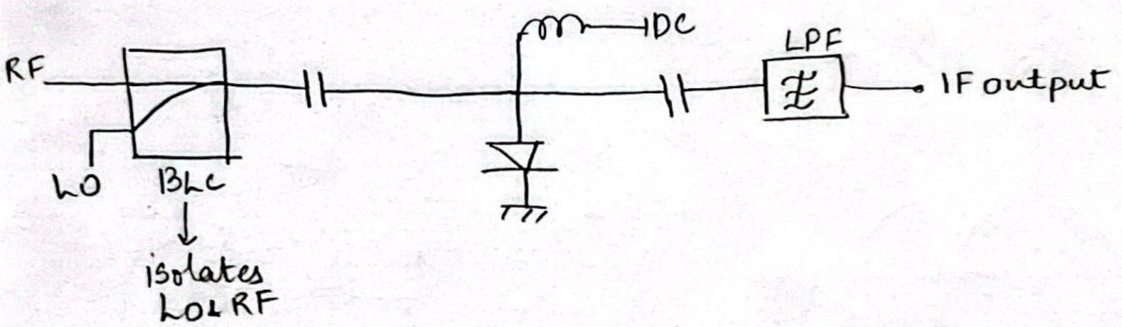
→ In case of passive mixers, the noise figure is atleast equal to the loss

→ LO - RF isolation in dB = $10 \log \dfrac{\text{LO power (leak) at RF port}}{\text{LO Input power}}$

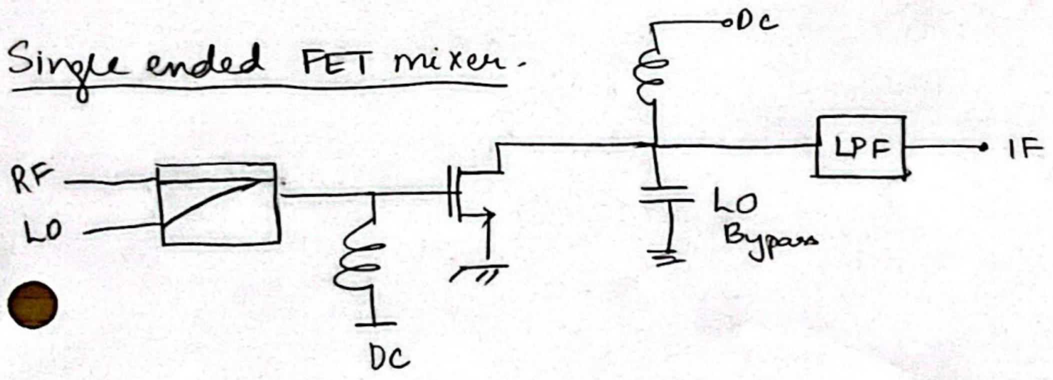
→ RF - IF isolation in dB = $10 \log \dfrac{\text{RF power at IF port}}{\text{Input RF power}}$

→ LO - IF isolation in dB = $10 \log \dfrac{\text{LO power at IF port}}{\text{LO input power}}$

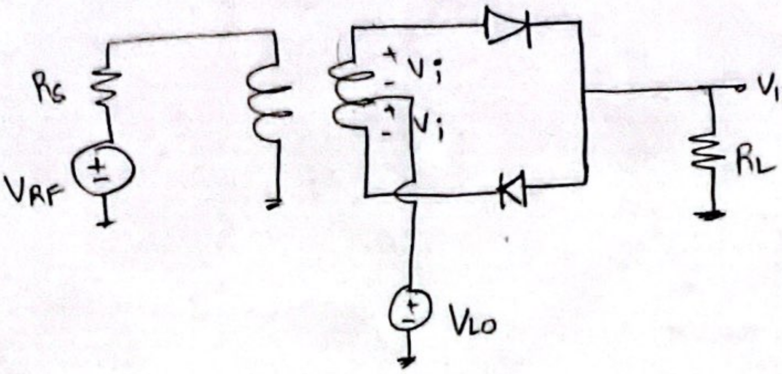
Single ended diode mixer



Single ended FET mixer.



Switching Type Mixers



$V_{LO} \gg V_i$

$V_{LO} > 0 \Rightarrow u(t) = 1$

$V_{LO} < 0 \Rightarrow u(t) = -1$

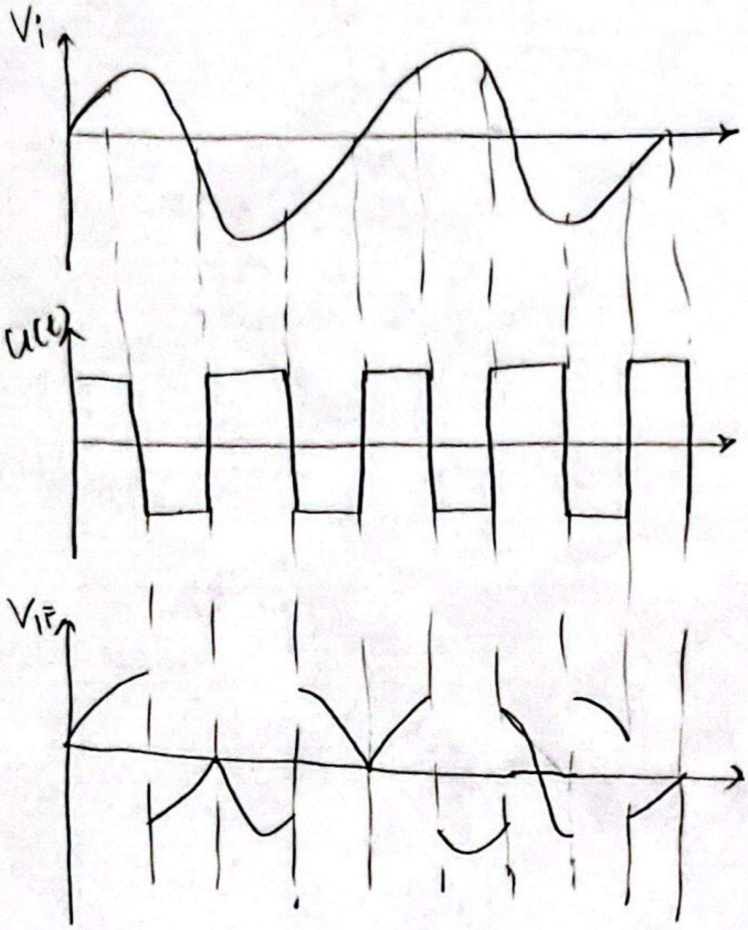
$V_i = V_i \sin \omega_i t$

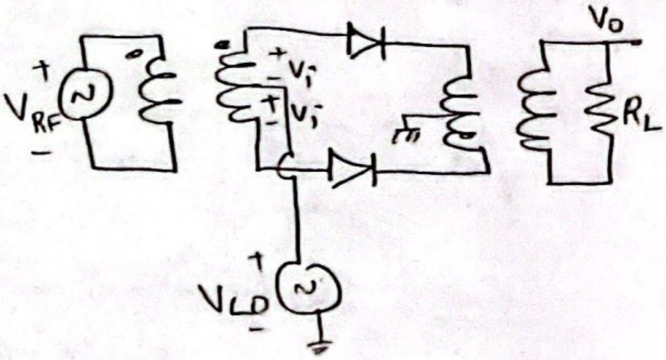
$u(t) = \frac{4}{\pi}\left[\sin \omega_{LO} t + \sin 3\omega_{LO} t + \dots\right]$

$V_{IF} = V_{LO} + u(t) V_i$

$\Rightarrow$ LO leaks to IF port

$\rightarrow$ To prevent this use a balanced mixer.



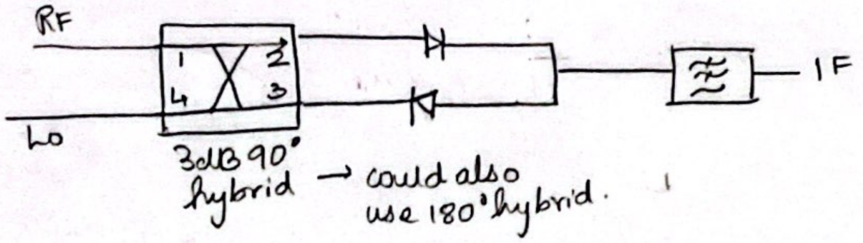


$V_{LO} > 0 \Rightarrow$ Both diodes are ON $\Rightarrow V_o = V_i$

$V_{LO} < 0 \Rightarrow$ Both are OFF $\Rightarrow V_o = 0$.

$\Rightarrow$ Sampling mixer!

Balanced mixer at microwave.



Double Balanced Mixer.

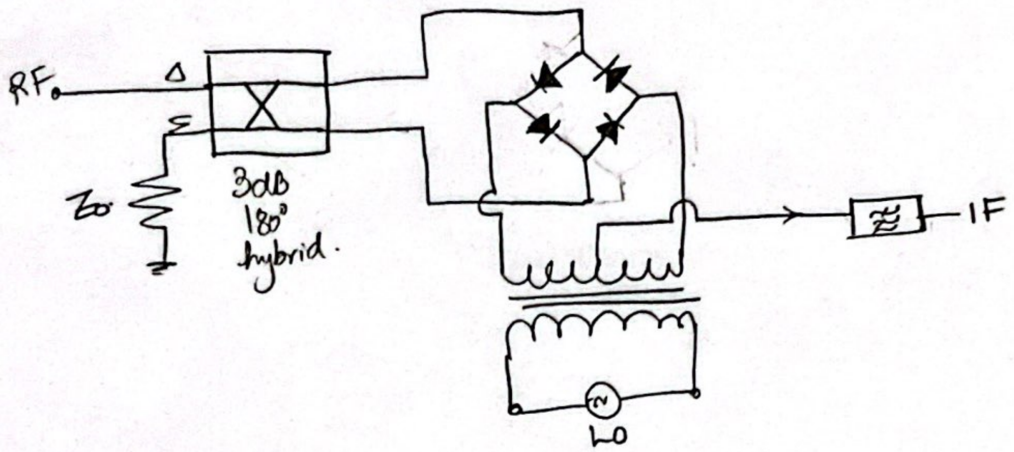
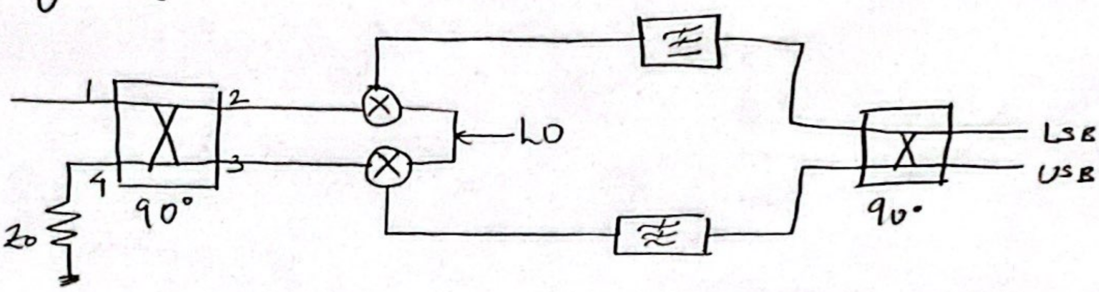


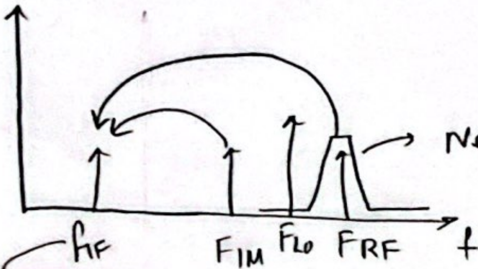
Image Reject Mixers



Receiver Basics.

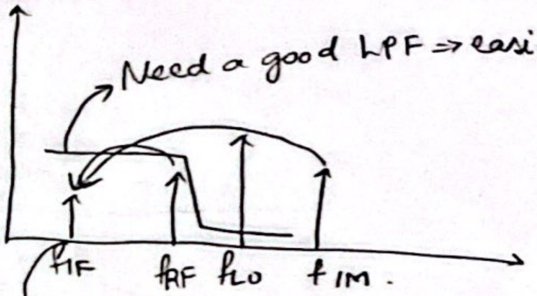
Image frequency → Downconverter

$f_{RF} > f_{LO}$



$f_{RF} - f_{LO}$
& $f_{LO} - f_{IM}$

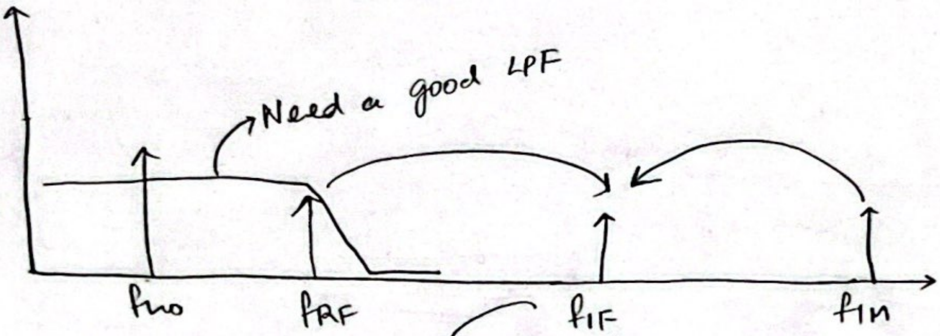
$f_{RF} < f_{LO}$



$f_{LO} - f_{RF}$
$f_{IM} - f_{LO}$

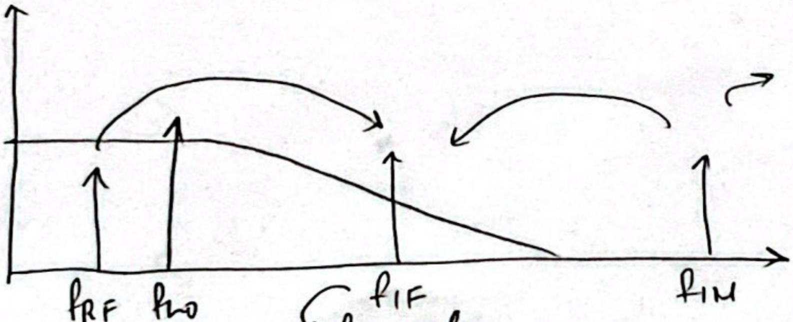
Upconverter

$f_{RF} > f_{LO}$



$f_{RF} + f_{LO}$
$f_{IM} - f_{LO}$

$f_{RF} < f_{LO}$

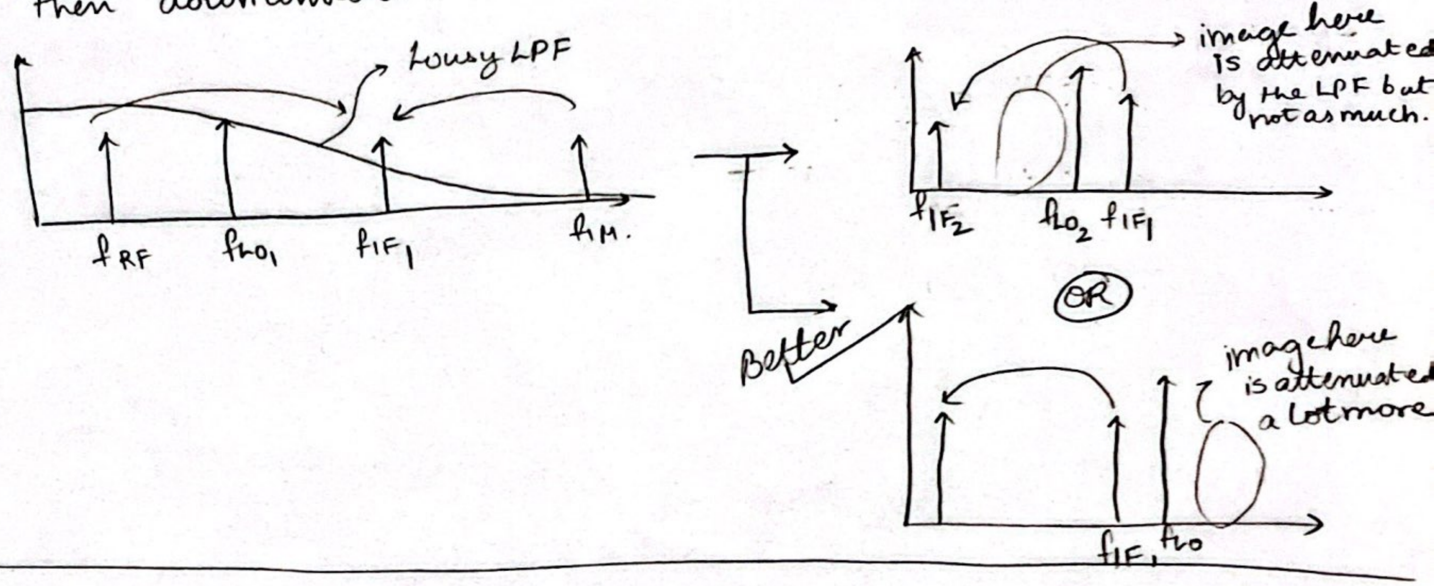


Since f_{LO} is higher than earlier, f_{IM} must be much higher to fall on f_{IF} therefore a shallow roll off LPF does the job.

$f_{RF} + f_{LO}$
$f_{IM} - f_{LO}$

→ In both up & down converters, in case of $f_{RF} < f_{LO}$ notice that as $f_{LO} \uparrow$, $f_{IM} \uparrow\uparrow$, therefore LPF design specs become easier. High f_{LO} ⇒ good for image rejection.

→ Dual conversion mixers. → First up convert → LPF for image & then downconvert.



Receiver Sensitivity

$$P_{in,min}\ (dBm) = -174 + 10\log(\text{BW of channel}) + NF\ (dB) + SNR_{out,min}\ (dB)$$

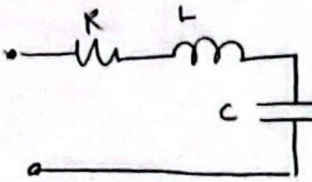
$SNR_{out,min}$ ↓ min SNR at out for a specified BER or modulation

Microwave Resonators

$$Q = \omega \cdot \frac{\text{average energy stored}}{\text{energy loss/second}} \qquad Q = \omega \cdot \frac{W_m + W_e}{P_{loss}} = \omega \frac{2W_m}{P_{loss}}$$

at resonance

a) Input impedance of series resonator $\qquad Q = \frac{1}{\omega_0 RC} = \frac{\omega_0 L}{R}$



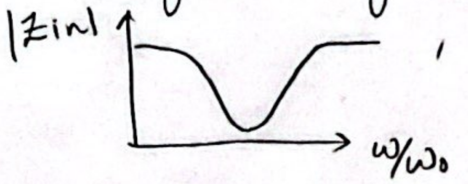
$$Z_{in} = R + 2j \frac{RQ}{\omega_0} \Delta\omega$$

$$Z_{in} = R\left(1 + 2Qj\frac{\Delta\omega}{\omega_0}\right)$$

$$Z_{in} = \frac{\omega_0 L}{Q} + j\, 2L(\omega - \omega_0)$$

$\Rightarrow \quad R = \frac{\omega_0 L}{Q}$. Therefore a lossless resonator can be made lossy in analysis by $\omega_0 \longrightarrow \omega_0\left(1 + \frac{j}{2Q}\right)$

$BW = \frac{\omega_0}{Q}$

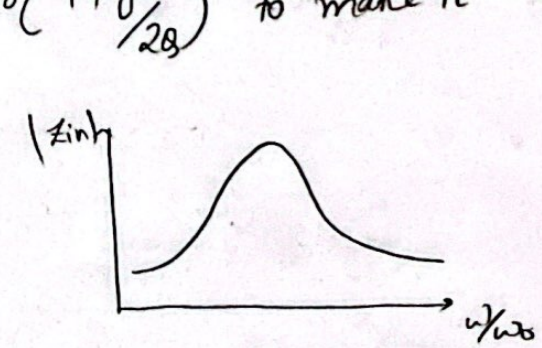


b) Input impedance of parallel resonator

$$Z_{in} = \left(\frac{1}{R} + j\frac{\Delta\omega}{\omega_0^2 L} + j\Delta\omega C\right)^{-1} = \frac{R}{1 + j\frac{2Q\Delta\omega}{\omega_0}}$$

Again replace $\omega_0 \longrightarrow \omega_0\left(1 + j/_{2Q}\right)$ to make it lossy.

$BW = \frac{\omega_0}{Q}$



Loaded Q_L

Series resonant circuit $\Rightarrow$ $Q_L = \dfrac{\omega_0 L}{R+R_L}$

Parallel resonant circuit $\Rightarrow$ $Q_L = \dfrac{RR_L/R+R_L}{\omega_0 L}$

$$\boxed{\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q}}$$

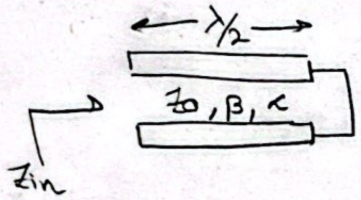
Q_L: loaded Q with lossy resonator.

Q $\rightarrow$ unloaded Q with lossy resonator.

Q_e $\rightarrow$ loaded Q with lossless resonator.

Transmission line resonator

> Short circuit $\lambda/2$ line acts as a series resonant circuit.



lossless $\Rightarrow$ $Z_{in} = jZ_0 \tan\beta l$.

lossy $\Rightarrow$ $Z_{in} = Z_0 \dfrac{\tanh\alpha l + j\tan\beta l}{1 + j\tan\beta l \tanh\alpha l}$.

Near resonance, $\omega = \omega_0 + \Delta\omega$

$$\beta l = \frac{\omega l}{u_p} = \frac{\omega_0 l}{u_p} + \frac{\Delta\omega l}{u_p}$$

at ω_0: $l = \dfrac{\lambda}{2} = \dfrac{u_p}{2f_0} = \dfrac{\pi u_p}{\omega_0}$ $\Rightarrow$ $\beta l = \pi + \dfrac{\Delta\omega\pi}{\omega_0}$

$\tan\beta l \cong \dfrac{\Delta\omega\pi}{\omega_0}$ if α is small $\tanh\alpha l \cong \alpha l$

$$\Rightarrow Z_{in} = Z_0\left(\alpha l + j\frac{\Delta\omega\pi}{\omega_0}\right)$$

$$\omega_0^2 = \frac{1}{LC}$$

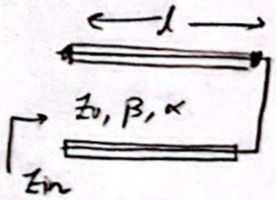
Let $R = Z_0 \alpha l$

$$L = \frac{Z_0 \pi}{2\omega_0}$$

$$C = \frac{2}{Z_0 \pi \omega_0}$$

$$Q = \frac{\pi}{2\alpha l} = \boxed{\frac{\beta}{2\alpha}}$$ → conductor loss + dielectric loss.

→ Short circuit $\frac{\lambda}{4}$ line behaves like a parallel resonant circuit.



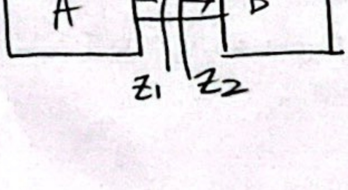
$$Z_{in} = \frac{1}{\frac{\alpha l}{Z_0} + j\frac{\pi}{2}\frac{\Delta\omega}{\omega_0 Z_0}}$$

$\Rightarrow$ $R = \frac{Z_0}{\alpha l}$, $C = \frac{\pi}{4\omega_0 Z_0}$, $Q = \omega_0 RC \Rightarrow \boxed{Q = \frac{\beta}{2\alpha}}$

→ open circuit $\frac{\lambda}{2}$ line ≡ Parallel resonant circuit

$R = \frac{Z_0}{\alpha l}$; $C = \frac{\pi}{2\omega_0 Z_0}$; $L = \frac{1}{\omega_0^2 C}$; $Q = \frac{\beta}{2\alpha}$

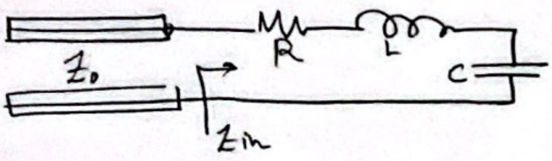
→



If AB constitute a resonant circuit, at resonance $Z_1 = Z_2^*$

Excitation of Resonators

Critical Coupling.



At resonance $Z_{in} = R \Rightarrow$ great!
$\Rightarrow$ Critical coupling.

Coupling coefficient $g = \dfrac{Q}{Q_e}$

$Q \longrightarrow$ loss R + load loss

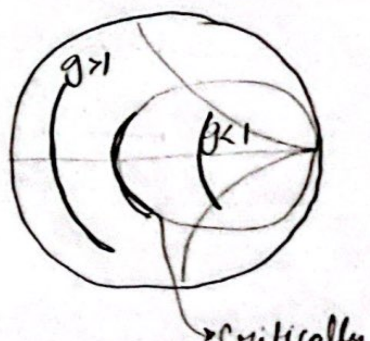
$Q_e \longrightarrow$ no loss + load loss. (Q external)

Series resonance $\Rightarrow g = \dfrac{Z_0}{R}$

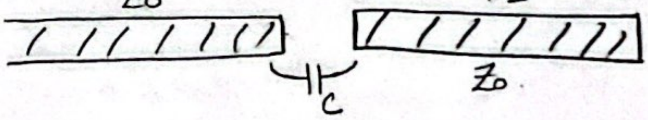
parallel resonance $\Rightarrow g = \dfrac{R}{Z_0}$

$g = 1 \Rightarrow$ The resonator is critically coupled.

$g < 1 \Rightarrow$ undercoupled
$g > 1 \Rightarrow$ over coupled
} Bad.

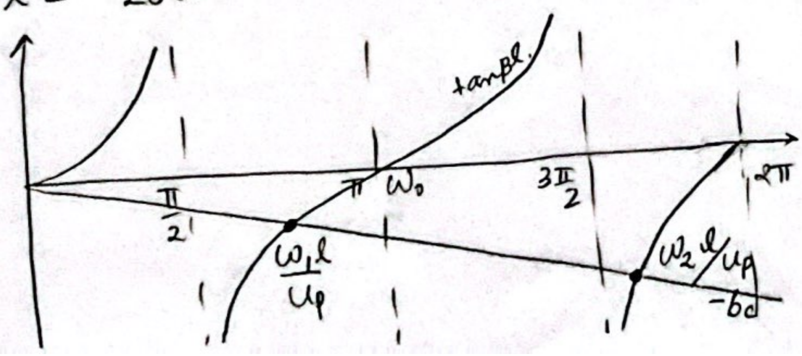


Gap Coupling.



$\rightarrow$ OC $\lambda/2$ $\Rightarrow$ parallel.

$\tan \beta l = -Z_0 \omega C$.



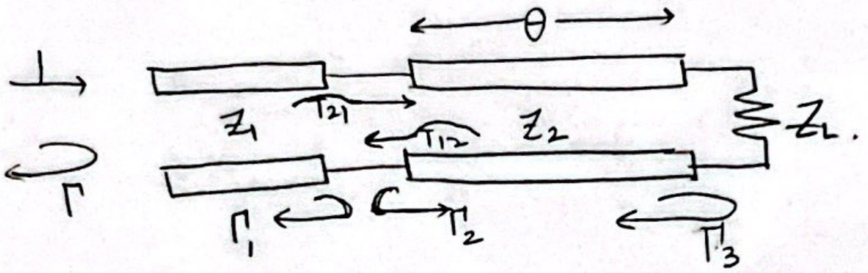
Gap coupling reduces the resonant frequency from $\omega_0 \longrightarrow \omega_1$. If $b_c \ll Y \Rightarrow \omega_1$ is close to ω_0.

At resonance $R = \frac{Z_0 \pi}{2Q b_c^2}$ $\Rightarrow$ $\boxed{b_c = \sqrt{\frac{\pi}{2Q}}}$ For critical coupling

$$\boxed{g = \frac{2Q b_c^2}{\pi}}$$

→ Approximate theory of multisection quarterwave transformers for wideband impedance matching.

Theory of small reflections.



$$\boxed{\Gamma \cong \Gamma_1 + \Gamma_3 e^{-2j\theta}}$$

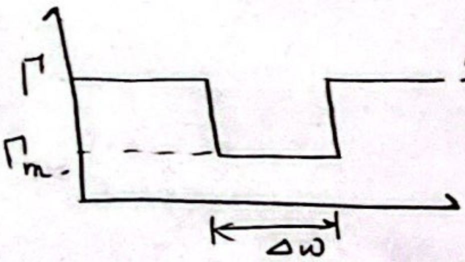
Book has the full proof.

→ This idea can be extended to multisection $\frac{\lambda}{4}$ line matching & tapered wideband matching.

Bode-Fano Criteria.

Parallel RC network as load.

$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|} d\omega \leq \frac{\pi}{RC}.$$



$\Gamma(\omega) = 0$ at some point $\Rightarrow$ the width of the match must be 0.

$\Rightarrow$ 0 BW.

→ Similar formula exist for series RC & RL loads.