Antennas - Lectures

Books: Balanis, or Kraus. (easier), Stutzmann

Lec 1

→ MW Radio :- 530 – 1620 KHz use $\lambda/4$ monopole antenna. → 75m tall.

→ Cellphones :- CDMA, GSM 900, GSM 1800, 3G, 4G, WiFi/Bluetooth (2.4 GHz) same as microwave oven.

(monopole → normal mode helical + monopole → microstrip + printed monopole.)

→ Cell towers: (monopole, dipole, arrays)

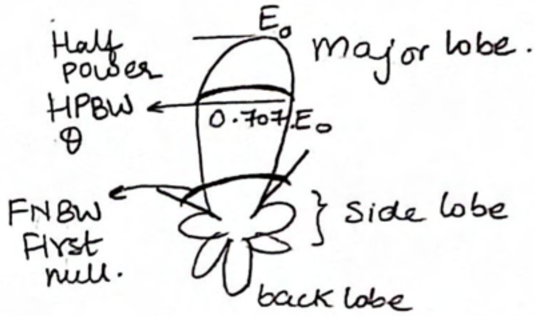
→ Satellite and defense (Use all types).

Radiation pattern

> Isotropic. (Does not really exist)

> Omnidirectional. [Equal power in all directions of one plane].

Directional



Polarization

> Linear (Horizontal or Vertical) → defined by E field (by convention)

> Elliptical

> Circular (LH & RH polarization).

Gain and Directivity

$$D = \frac{41253}{\theta_E \; \theta_H} = \frac{4\pi A}{\lambda^2}$$

HPBW → $\theta_E \; \theta_H$; A → Aperture area.

Gain = ηD (η → efficiency, D → directivity).

→ At one frequency if $A\uparrow \Rightarrow D\uparrow$ but HPBW$\downarrow$.

Reflection Coefficient

$$\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0}$$

Γ (0 to 1)

Z_A → impedance of antenna. → usually complex!

Z_0 → line impedance

VSWR

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

VSWR (1 to ∞)

Bandwidth: Could be defined in many ways.

> Frequency range over which VSWR $\leq 2 \Rightarrow |\Gamma| = 1/3$; $P_r = 1/9 = 11.1\%$

Lec 2

Link Budget

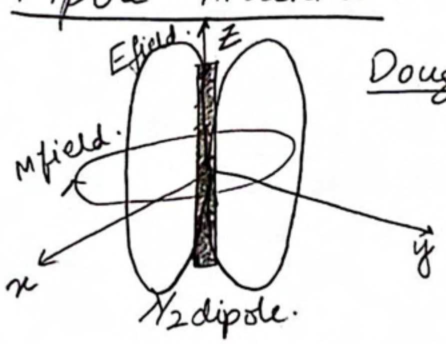
Friis Equation of Transmission

$$P_r = P_t\, G_t\, G_r \left[\frac{\lambda}{4\pi r}\right]^2 \text{(Watt)}$$

$\uparrow f \Rightarrow P_r \downarrow\downarrow$
$\Rightarrow$ High frequency $\Rightarrow$ Short range.

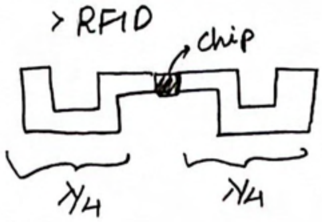
> Always design link budget with 10dB margin atleast.

Dipole Antenna



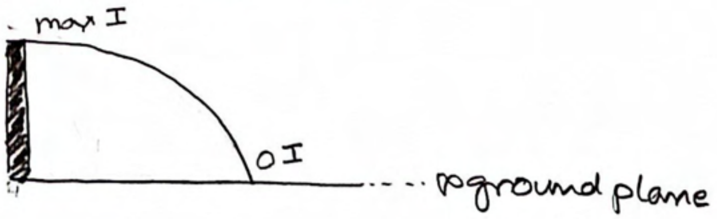
Doughnut

Applications
> RFID

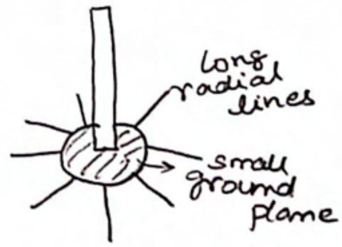


> Requires differential feed.
> Balanced feed!
> Use BALUN!

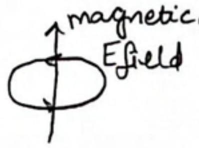
Monopole Antenna



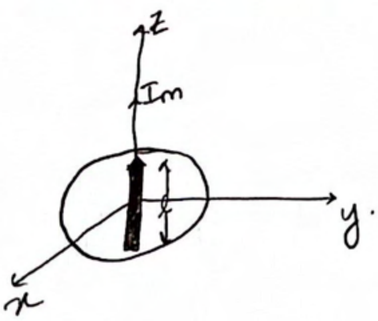
in reality ground plane is finite



Loop Antenna



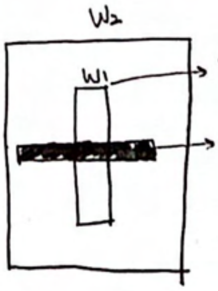
Think of it as a magnetic dipole.



> Efield & H field are reversed b/w dipole & loop.
> Could use square or triangular loop if size is small.
> Could use multiple turns for better impedance matching.

> Slot Antenna

> Compliment of printed dipole antenna.
> Slot is cut in ∞ ground plane.

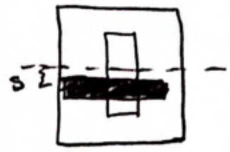


→ V = 0 & I is maximum → compliment of dipole.

→ microstrip line feed.

> If length of line is $\frac{\lambda}{2}$, current is max at the slot centre.

→ Impedance is ∞ at the centre of Slot ⇒ Use an offset feed.



> Linear and Planar Arrays.

> For very large gain.
> For directionality.
> Phased arrays :- Change phase of feed.

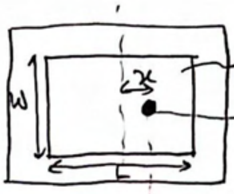
> Type of element
> Amplitude
> Phase
> Spacing
> Feed network

} Determine performance.

> Spacing must be less than λ.
> spacing $\xrightarrow{\lim} 0$ ⇒ could be used to model continuous antennas.
↳ Very powerful technique!

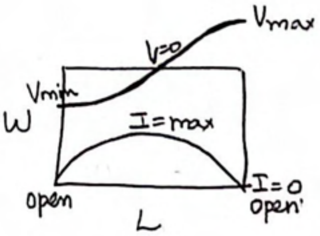
> Microstrip Antenna.

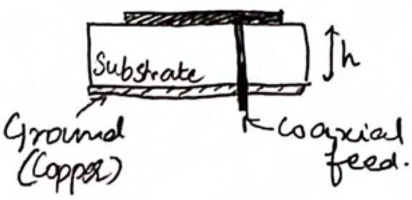
> Different shapes. > Broadband > Compact > Multiband
> Dual polarization > Circular polarization > Linear and Planar array.

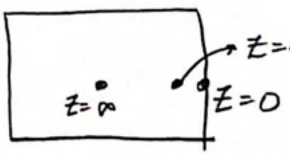


→ patch!
→ offset feed since $Z = \infty$

L should be $\simeq \frac{\lambda}{2}$







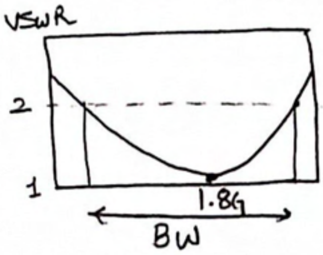
→ $Z = 50\,\Omega$ ⇒ feed Here!!

> Does not need matching network or BALUN.
> Could be flexible (PCB).

Lec 3

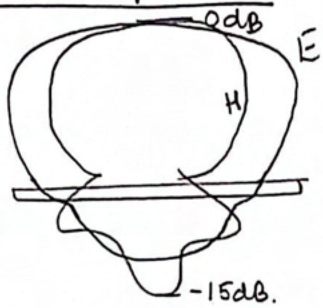
Microstrip continued. (MSA)

> Designed at 1.8 GHz.



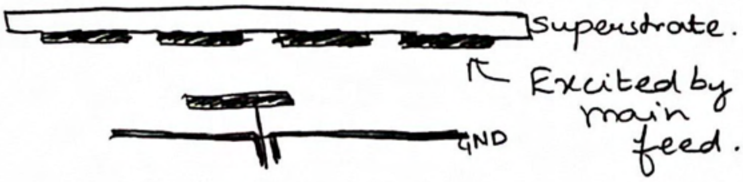
BW for VSWR ≤ 2
$1.76 \rightarrow 1.855$

Radiation pattern.



F/B ratio. = Front/Back ratio.

→ Space fed MSA array.



Helical antenna.

3 modes : (Based on frequency)

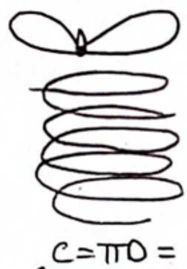
Normal mode

$C = \pi D << \lambda$

Axial mode

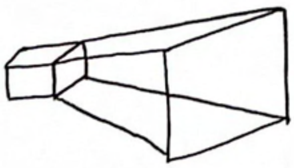
$C = \pi D = \lambda$

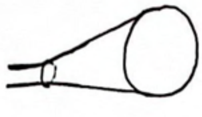
Conical mode.



$C = \pi D = n\lambda$
(rarely used)

Horn antenna (Pyramidal and Conical)



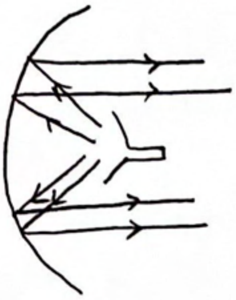


Reflector Antenna. (Plane reflector, conical reflector, corner reflector, parabolic reflector)

parabolic reflector → Very high gain.

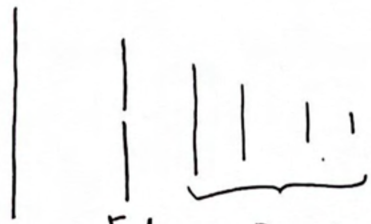
> Omnidirectional → Directional.

- Increases gain!



> Very high gain (Satellite & interstellar).
> Large space
> Can use multiple types of feed.

Yagi Uda antenna.

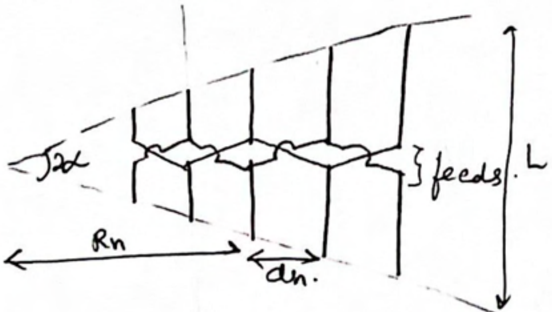


→ max field direction. more directors ⇒ more gain. } upto 7dB with 3-5 elements.

> Not high BW.

Log periodic antenna.

Many dipoles feed alternatively out of phase.



$$\frac{l_{n+1}}{l_n} = \frac{1}{\tau} \; ; \; \frac{R_{n+1}}{R_n} = \frac{1}{\tau} \; ; \; \frac{d_{n+1}}{d_n} = \frac{1}{\tau}$$

lengths vary logarithmically. } $\frac{l_2}{l_1} = \frac{1}{\tau}$ → log periodic ratio.
spacing vary "

> High BW. [1:10] ⇒ 10%
> Could have high gain [8-10dB]

> Conclusion

→ • Need : 1) Broad band.
2) Multi band.
3) Multi polarization.
4) MIMO
5) Compact.
6) High efficiency.

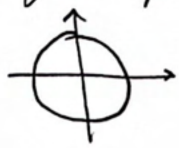
→ High frequency ⇒ small size ⇒ precision manufacturing.
(Leave 5-10% margin)

Lec 4 Antenna Fundamentals.

Radiation pattern

Isotropic antenna : "Sphere with equal power in all directions"

Directivity $D = 1 = 0dB$.



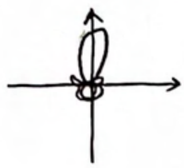
> Don't exist in reality.

Omnidirectional antenna : "Doughnut" → Max field in one plane.

$\frac{\lambda}{2}$ dipole antenna ; $D = 1.64 = 2.1dB$.

Directional antenna : "Pole" → max field in one axis.

Microstrip array: $D = 500 = 27dB$



HPBW :- Angle θ.

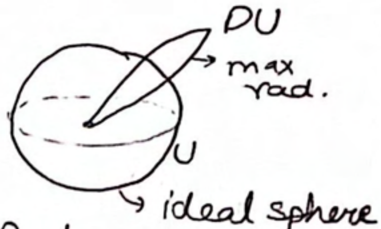
$$\boxed{FNBW \simeq 2.25 \times HPBW}$$

Sidelobe level :- Should be low for directional. ≤ 30dB
power in any of the side lobe.

F/B ratio :- < 20dB.

Directivity

"Ratio of radiation density in the direction of maximum radiation to the radiation density averaged over all directions".



$$D = \frac{\text{max radiation intensity}}{\text{avg. radiation intensity}}$$

avg. radiation intensity → (for isotropic case) → $\frac{P_{rad}}{4\pi r^2}$ (where $r=1$)

$$= \frac{U_{max}}{U_0} = \frac{U_{max}}{\frac{P_{rad}}{4\pi}} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi U_{max}}{U_{max}\Omega_A} = \frac{4\pi}{\Omega_A}$$

Ω_A → Beam solid angle.

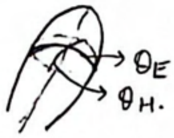
$$\Omega_A = \frac{1}{F(\theta,\phi)\big|_{max}} \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta \, d\theta \, d\phi$$

$F(\theta,\phi) \simeq \left[|E_\theta^\circ(\theta,\phi)|^2 + |E_\phi^\circ(\theta,\phi)|^2\right]$

(Too avoid math

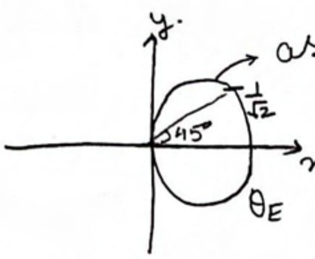
approximation $D \simeq \frac{4\pi}{\theta_E \theta_H}$ → not always used.

$\theta_E \theta_H$ → in radians.



→ For infinitesimal dipole: $\theta_E = \pi/2$; $\theta_H = 2\pi$ → $D = \frac{4\pi}{\frac{\pi}{2}.2\pi} \simeq 1.3$

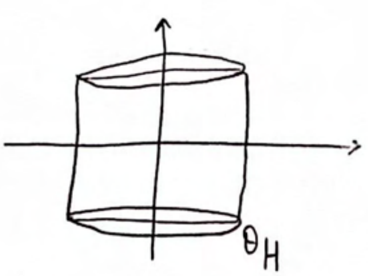
{In reality it is 1.5 so, close but not accurate}



→ assume sinusoid.

$\Rightarrow$ at 45° it is $\frac{1}{\sqrt{2}} \Rightarrow 0.707\ E_{max}$

$\Rightarrow 2 \times 45° = \frac{\pi}{2}$ → θ_E



> H plane beam width is 360°

For small antennas.

$$D = \frac{4\pi}{\theta_E \theta_H} = \frac{41253}{\theta_E \theta_H}$$

(radian) (degrees)

For large antenna.

$$D \simeq \frac{32400}{\theta_E \theta_H}$$ → accounts for power lost to side lobes.

within ±1dB.

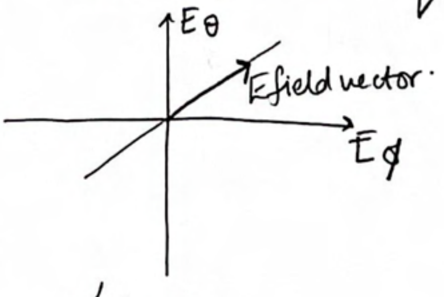
→ $D = \frac{4\pi A_{eff}}{\lambda^2}$ → Effective aperture antenna (= L × W for array)

Q:- Find gain in dB of a parabolic reflector antenna at 15 GHz having diameter of 1m. Assume $\eta = 0.6$.. Gain at 36 GHz?

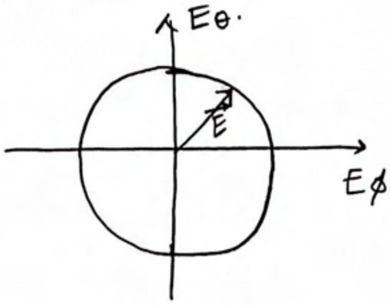
$$G = \eta \cdot D \;; \quad D = \frac{4\pi \cdot \pi D^2}{4 \cdot c^2} \cdot f^2 = \left(\frac{\pi D f}{c}\right)^2 = 44\,dB\ @\ 15\,GHz.$$

$$51.5\,dB\ @\ 36\,GHz$$

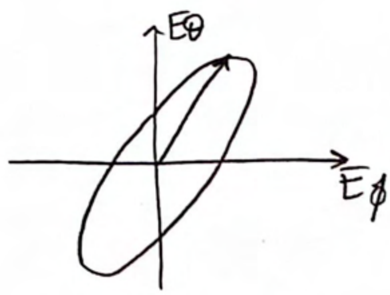
Polarization of Antenna.



Linear



Circular



Elliptical.

$$E = a_\theta E_\theta \cos \omega t + a_\phi E_\phi \cos(\omega t + \alpha)$$

> $\alpha = 0$ or π → Linear

> $\alpha = \pm\frac{\pi}{2}$ and $E_\theta = E_\phi$ → Circular

> $\alpha = \pm\frac{\pi}{2}$ and $E_\theta \neq E_\phi$ → Elliptical

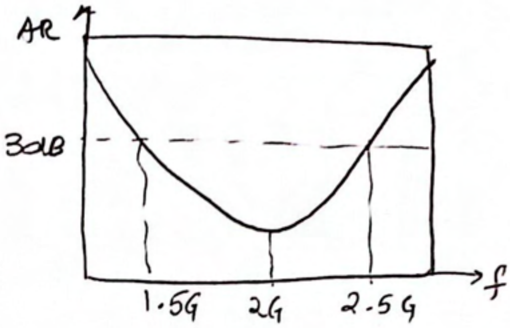
Axial Ratio of antenna.

$$AR = \frac{\text{Major axis of polarization}}{\text{minor axis of polarization}}$$

$AR = \infty \Rightarrow$ linear.
$AR = 1 \Rightarrow$ circular
$1 < AR < \infty \Rightarrow$ elliptical.

> Axial ratio bandwidth : Frequency range for which $AR \leq 3dB$.

> Circular $\Rightarrow$ usually defined as when $AR \leq 3dB$



$\Rightarrow$ AR BW = 1 GHz.

Input impedance and VSWR.

$$Z_A = R_A + j X_A$$

$R_A \rightarrow$ power lost in the antenna.
$X_A \rightarrow$ power stored in the near field of the antenna.

$$R_A = R_r + R_L$$

$R_r \rightarrow$ radiation resistance $\rightarrow$ mathematical model.
$R_L \rightarrow$ loss in antenna.

$$e_r = \frac{R_r}{R_A} = \frac{R_r}{R_r + R_L}$$

$e_r \rightarrow$ radiation efficiency.

$$\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0} \qquad VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

> R_r is a math model for power radiated. $R_r = \frac{P_{radiated}}{i^2}$

Example :- $Z_A = 20 + j30$.

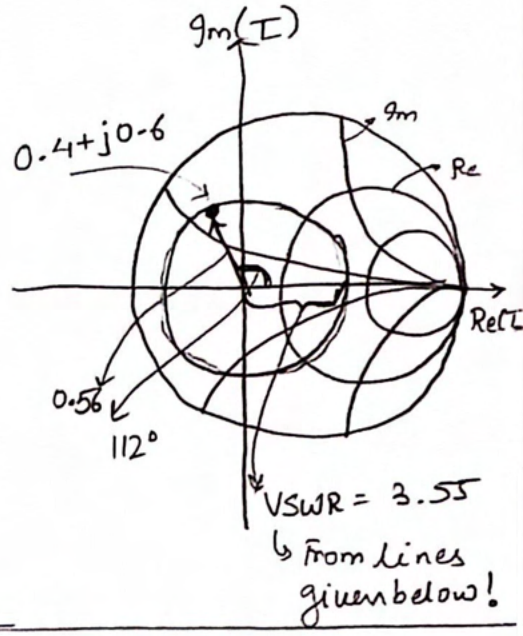
$$\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0} = -0.2 + 0.52j = 0.56\angle 112^\circ$$

$$VSWR = \frac{1+0.56}{1-0.56}$$

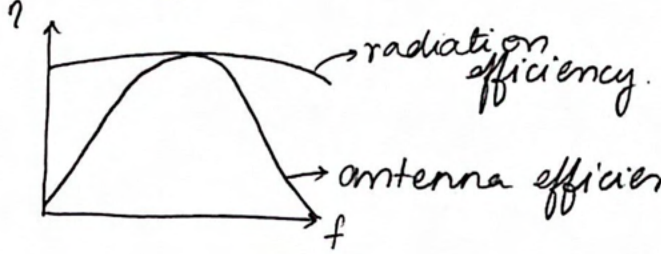
$$= 3.55$$

(OR) Use smith chart!

$$Z_{A\,norm} = \frac{Z_A}{Z_0} = \frac{20+j30}{50} = 0.4 + j0.6.$$

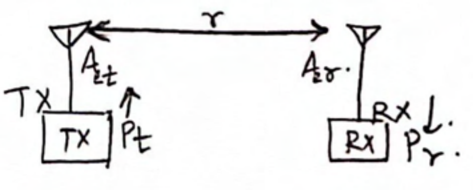


Lec 5 Efficiency vs. Frequency.



→ due to reflection coefficient.

Link Budget



$$P_d = \frac{P_t \cdot G_t}{4\pi r^2} \; \frac{\text{Watt}}{m^2}$$ → power density.

$$P_r = P_d \cdot A_{er} \text{ Watt}$$ where $G = \frac{4\pi A_e}{\lambda^2} \Rightarrow A_{er} = \frac{G_r}{4\pi\lambda^2}$

A_{er} → Effective aperture.

$$\Rightarrow P_r = \frac{P_t G_t A_{er}}{4\pi r^2} = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 r^2}$$

$$\Rightarrow \boxed{P_r = P_t G_t G_r \left[\frac{\lambda}{4\pi r}\right]^2 \text{ Watt}}$$

Friis Transmission Equation.

Eg: A GSM 1800 cell tower is transmitting 20W of power in the frequency range of 1840 to 1845MHz. The antenna gain is 17dB. Find the power density at a distance of (a) 50m (b) 300m. in the direction of max. radiation.

50

Sol:- $P_d = \frac{P_t G_t}{4\pi r^2} = \frac{20 \times 50}{4\pi\, 50^2} = 31.8\ mW/m^2$

$= \frac{20 \times 50}{4\pi \times 300^2} = 0.88\ mW/m^2$

> Phone radiation is linked to brain tumor? WHO classifies it as a class 2B carcinogen. Keep your phone atleast 1foot away (use earphones!)

Le 6: Radiation Hazards

> Radiation pollution: Microwave heating principle.
  > Microwave penetrates skin and is trapped inside creating multiple hot "spots" since its wavelength is ≈ cm.
  > Nonthermal:- DNA damage.
  > Thermal :- heating.
> 20 min usage ⇒ ear lobe temperature ↑ 1°c.
  ↳ hearing loss & tumors!

> SAR (Specific absorption rate):- SAR limit of human body is close to 18-24 mins.

> Interphone study conclusion: 0.5 hrs/day over 8-10yrs ⇒ doubled or quadrupled brain tumor risk.

> Govt. raises money from spectrum auction :⇒ they lobby against awareness. (WHO is funded by government). It should be carcinogen 2A or maybe even carcinogen class 1

Lec 7

> Cell tower radiation.

CDMA - 869 - 890 M

GSM900 - 935-960 M

GSM1800 - 1805-1880M

3G - 2110-2170 M.

4G - 2300-2400M.

WiFi, Bluetooth - 2400 - 2500M

> Microwave oven power ≃ 500W @ 2.4GHz. A single antenna tower transmits around 20W in India.

> Should not live within 50-300m of tower!

> Radiation pattern is like a flat doughnut cut in half.

Horizontal.

Vertical.

> In USA: In urban areas only 0.5 - 1W/m^2 is allowed.

In India: Close to 20W/m^2 is allowed & not strictly enforced.

> From research we have seen it should be < 0.5mW.

> Yield from crops is also affected.

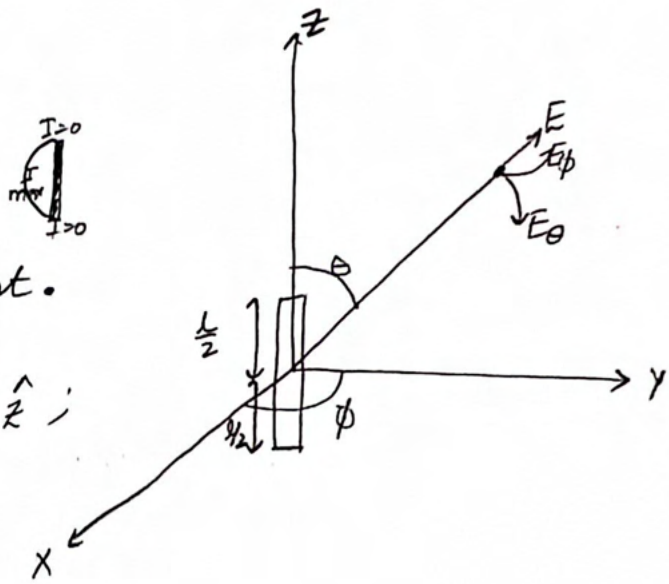
DIPOLE ANTENNA

Infinitesimal dipole aka Hertzian Dipole. $[L < \frac{\lambda}{50}]$

> Assume an infinitesimal current element of length dl carrying an AC current I_0.

$$i(t) = I_0 e^{j\omega t} \hat{i}_z$$

→ Instantaneous Current



→ In reality

Assuming current distribution is constant.

$$\vec{A} = A_z . \hat{z} = \frac{\mu_0}{4\pi} I_0 dl . \frac{e^{-jkr}}{r} e^{j\omega t} \hat{z} ;$$

where $K = \frac{2\pi}{\lambda}$

$\vec{A} \rightarrow$ Vector magnetic potential.

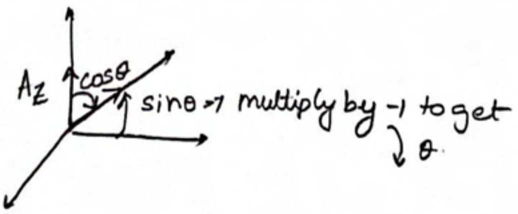
$\vec{A} = A_z . \hat{z}$ since current is only in $\hat{z}$ and is uniform
$\hookrightarrow \hat{a}_z$

$$= \frac{\mu I_0 l}{4\pi r} . e^{-jkr} . \hat{a}_z$$

$A_r = A_z \cos\theta .$

$A_\theta = - A_z \sin\theta .$

$A_\phi = 0$



From Maxwell

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A} = \hat{a}_\phi \frac{1}{\mu r} \left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

$$\vec{E} = -j\omega \vec{A} - j \frac{1}{\omega \mu \epsilon} \nabla (\nabla . A).$$

$$P = \frac{1}{2} \iint_S \vec{E} \times \vec{H}^* . d\vec{s}$$

$$E_r = \eta \frac{I_0 l}{2\pi r^2} \cos\theta \left[1 + \frac{1}{jkr}\right] e^{-jkr}$$

$\eta \rightarrow$ Free space impedance $= 120\pi = 377\,\Omega$

$$E_\theta = j\eta \frac{k I_0 l}{4\pi r} \sin\theta \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right] e^{-jkr}$$

$$E_\phi = H_r = H_\theta = 0$$

$$H_\phi = j \frac{k I_0 l}{4\pi r} \left[1 + \frac{1}{jkr}\right] \sin\theta\, e^{-jkr}$$

(Derivation in Balanis) [For far field we can ignore all $\frac{1}{r^2}, \frac{1}{r^3}$ terms]

Near Field

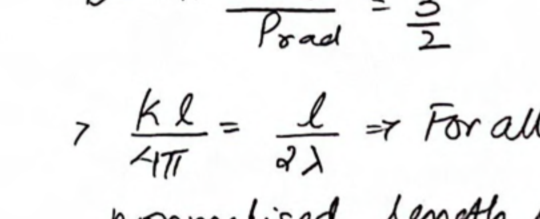
$r \ll \frac{\lambda}{2\pi} \rightarrow r < \frac{\lambda}{6}$ Near reactive field region.

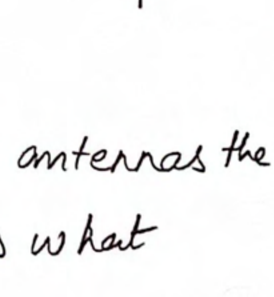
$\frac{\lambda}{6} < r < \frac{2d^2}{\lambda}$ Near radiative field region.

Far Field

$r \gg \frac{\lambda}{2\pi}$ $\quad r > \frac{2d^2}{\lambda}$ where d is the max. dimension of antenna.

(need to satisfy both)





Far field ($kr \gg 1$).

$$E_\theta = \frac{j\eta k I_0 l}{4\pi r} \sin\theta$$

$$D = 4\pi \frac{U_{max}}{P_{rad}} = \frac{3}{2}$$

$$H_\phi = j \frac{k I_0 l}{4\pi r} \sin\theta$$

> $\frac{kl}{4\pi} = \frac{l}{2\lambda} \Rightarrow$ For all antennas the normalised length is what matters!

$$E_r = E_\phi = H_r = H_\theta = 0$$

$\frac{E_\theta}{H_\phi} = \eta = 120\pi \longrightarrow$ E & H field are perpendicular!

$\downarrow$

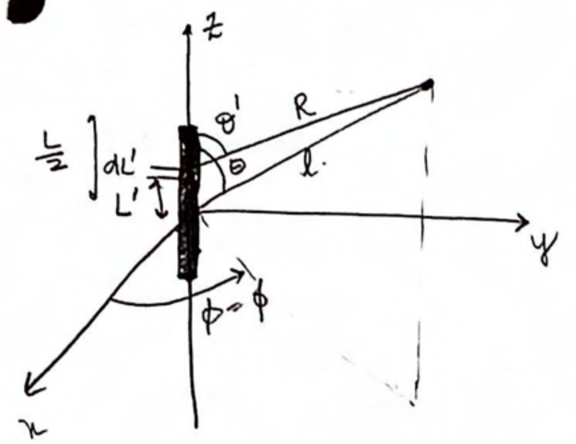
Impedance of free space.

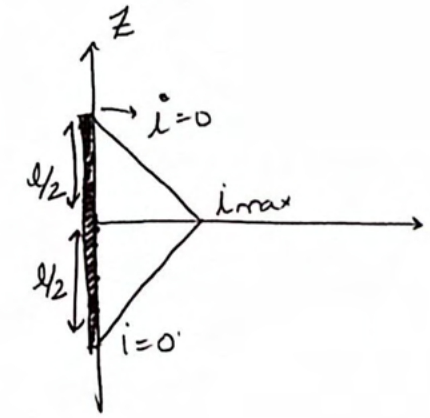
> Very low efficiency

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$$

Small dipole antenna. $\lambda/50 < l < \lambda/10$.

> Assume triangular current distribution.





$$I_0(x', y', z') = \hat{a}_z I_0 \left[1 - \frac{2}{l} z'\right], \quad 0 \le z' \le l/2$$

$$\hat{a}_z I_0 \left[1 - \frac{2}{l} z'\right] \quad -l/2 \le z' \le 0$$

Vector Potential

$$A(x, y, z) = \frac{\mu}{4\pi}\left[\hat{a}_z \int_{-l/2}^{0} I_0\left(1 + \frac{2}{l} z'\right) \frac{e^{-jkR}}{R}\, dz' + \hat{a}_z \int_{0}^{l/2} I_0\left(1 - \frac{2}{l} z'\right) \frac{e^{-jkR}}{R}\, dz'\right]$$

Assuming average value of current on solving integral. (due to triangular distr.).

Far field ($kr \gg l$)

$$E_\theta = j\eta \frac{k I_0 l e^{-jkr}}{8\pi r} \sin\theta$$

$$H_\phi = j \frac{k I_0 l e^{-jkr}}{8\pi r} \sin\theta.$$

$$E_r = E_\phi = H_r = H_\theta = 0$$

$$R_r = \frac{2 \overbrace{P_{rad}}^{\left(\frac{1}{2} \iint \vec{E} \times \vec{H}^* \cdot d\vec{s}\right)}}{|I_0|^2} = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$$

$\eta = 120\pi = 377\,\Omega$

$l = \lambda/10 \Rightarrow R_r = 2\,\Omega$.

$l = \lambda/4 \Rightarrow R_r = 12.3\,\Omega \rightarrow$ small dipole? NO.

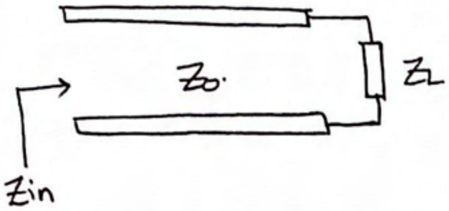
> Remember here we are ignoring the loss & reactive components.

> $\boxed{R_r = \frac{2 P_{rad}}{|I_0|^2}} \rightarrow$ The real deal! Use this equation.

> Needs a balanced feed.

> What about the reactive component

• To find reactive part we use T. lines.



$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

<u>Case ①</u> :- $Z_L = 0 \Rightarrow Z_{in} = jZ_0 \tan \beta l.$

<u>Case ②</u> :- $Z_L = \infty \Rightarrow Z_{in} = \dfrac{Z_0}{j \tan \beta l}$ → dipole case!

<u>Case ③</u> :- $Z_L = Z_0 \Rightarrow Z_{in} = Z_0$

$\beta = \dfrac{2\pi}{\lambda}$ if $l < \dfrac{\lambda}{4} \rightarrow \tan \beta l = +ve. \Rightarrow Z_{in} = \dfrac{Z_0}{j(+ve)} \Rightarrow$ Capacitive!

$\dfrac{\lambda}{4} < l < \dfrac{\lambda}{2} \rightarrow \tan \beta l = -ve.$

⇒ Small dipole antennas have a capacitive component!

Lec 9

For larger dipole antennas: $\frac{\lambda}{4} < l < \frac{\lambda}{2} \Rightarrow \tan\beta l = -ve.$

$\Rightarrow$ For open circuit $Z_{in} = Z_0 \left[\frac{1+0}{0 + j\tan\beta l} \right]$

$$= \frac{Z_0}{j\tan\beta l}.$$

$\Rightarrow Z_{in} = (+ve)\, j Z_0 \longrightarrow$ Inductive reactance.

Half Wavelength Dipole ($l = \frac{\lambda}{2}$)

$$E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \cdot \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \right]$$

$$H_\phi = j \frac{I_0 e^{jkr}}{2\pi r} \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \right]$$

$D_0 = 4\pi \frac{U_{max}}{P_{rad}} \simeq 1.643 = 2.1\,dB$ [higher than small dipole]

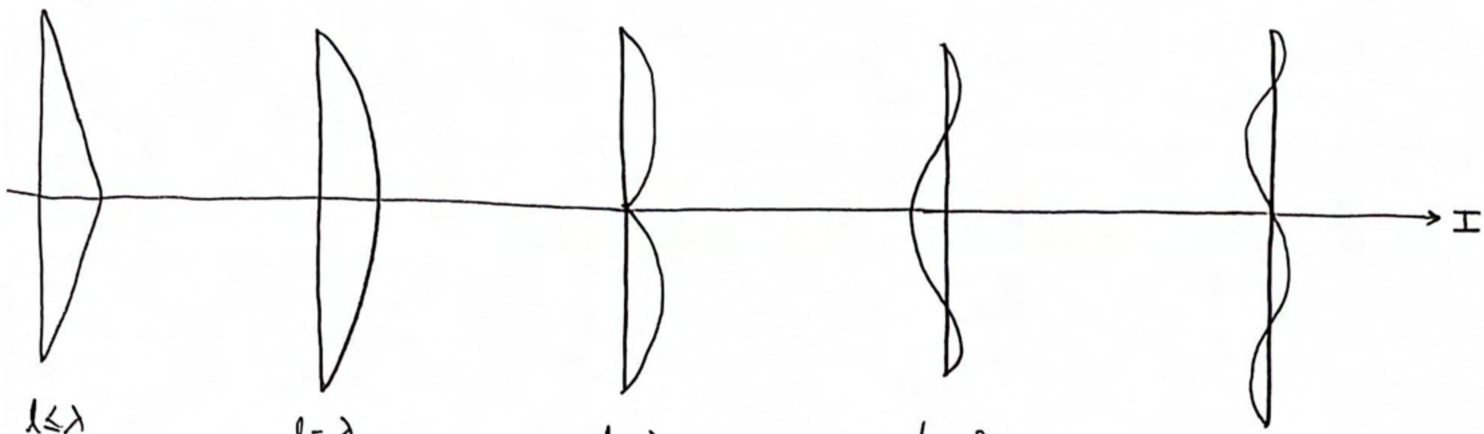
$R_r = \frac{2 P_{rad}}{|I_0|^2} = 73$

Z_{in} for $\lambda/2$ $= 73 + j42.5\,\Omega \rightarrow$ To make it real we reduce the length until it is purely real.

? Intuitively $\lambda/2$ should have purely real impedance, but due to fringing fields at the edges the effective length is higher. Therefore we reduce the length a little until resonance point is hit. Effective length should be $= \frac{\lambda}{2}$

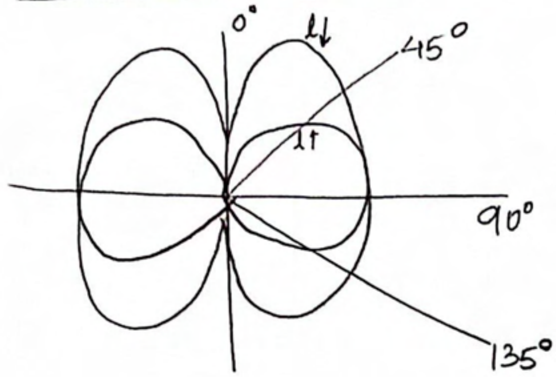
- Fringing fields are due to a finite diameter. We want $l+d = 0.48\lambda$ to get an effective length $\simeq 0.5\lambda$. The real impedance then is $\simeq 68\,\Omega$. $\boxed{f_{res} = \frac{0.48c}{l+d}}$

Current distributions



$l \leq \frac{\lambda}{4}$ $\quad l = \frac{\lambda}{2}$ $\quad l = \lambda$ $\quad l = \frac{3\lambda}{2}$ $\quad l = 2\lambda$

Radiation Pattern



As $l \uparrow \Rightarrow$ HPBW $\downarrow \Rightarrow$ gain $\uparrow$.

$\frac{\lambda}{50}$ small dipole $\Rightarrow$ 3dB Beamwidth = 90°

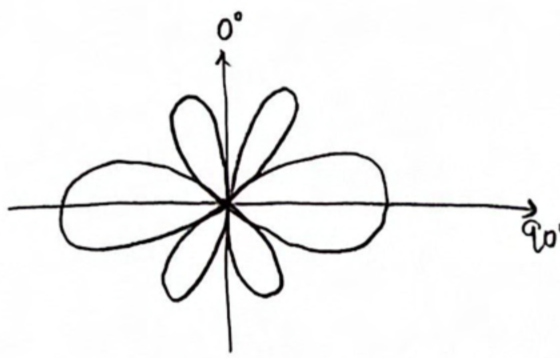
$\frac{\lambda}{4}$ — — — = 87°

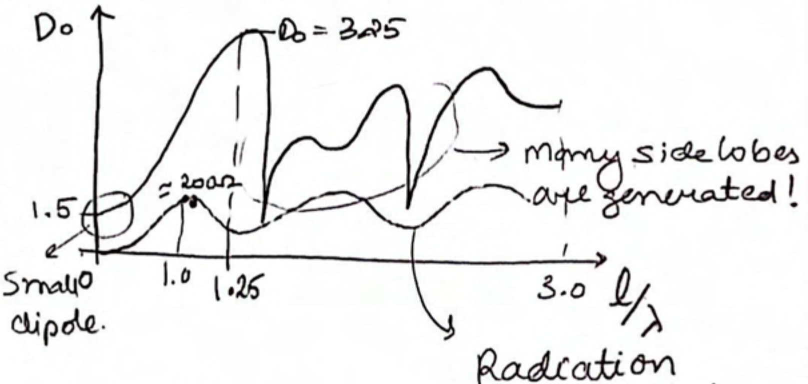
$\frac{\lambda}{2}$ — — — = 78°

$\frac{3\lambda}{4}$ — — — = 64°

λ — — — = 47.8°

When $l = 1.25\lambda$ there is another minor lobe that is added.





> At 1.25 we get maximum directivity. $D_o = 3.25$.

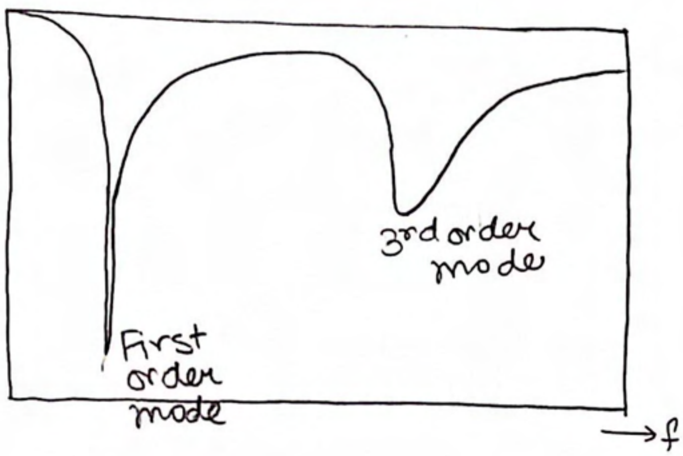
> At around 1.0, $R_r \simeq 200\,\Omega$. [R_{in} does not follow R_r due to current waveforms corresponding to different lengths]

> For more info look at Balanis!

> As $l\uparrow$, imaginary part of impedance becomes inductive to capacitive to inductive and so on.

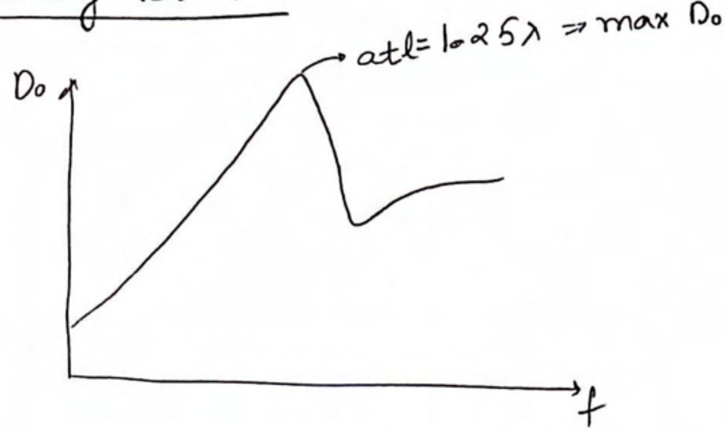
> BW for $|S_{11}| \leq -10dB \Rightarrow$ reflected power = 10% $\Rightarrow$ VSWR $\simeq 2$.

> S_{11} ↑

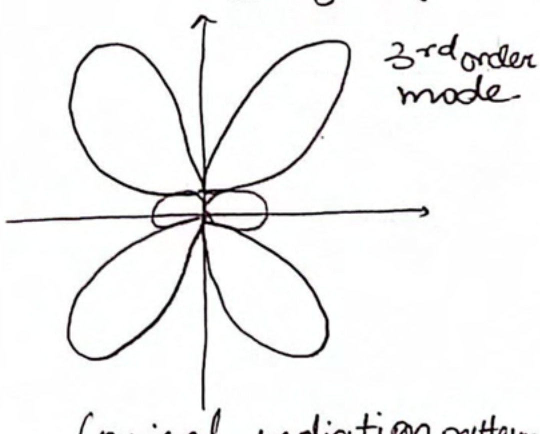


> 2nd order mode i/p Z is $\rightarrow \infty$ & therefore doesn't show up.

> Directivity vs. f



At high f!



Conical radiation pattern.

Lec 10

> If dipole is vertical, the polarization (defined w.r.t E) is vertical.
> High gain ⇒ use array instead of increasing length.
> Bandwidth of dipole antenna can be increased by increasing the diameter. [BW ∝ diameter]. But we cannot keep increasing it because current distribution inside the antenna will not remain uniform. Rule of thumb. $[d \leq \frac{\lambda}{10}]$. Otherwise resonance modes occur across the diameter. Can use a hollow pipe, performance doesn't change much.

> Biconical dipole antenna. [Extremely large Bandwidth]

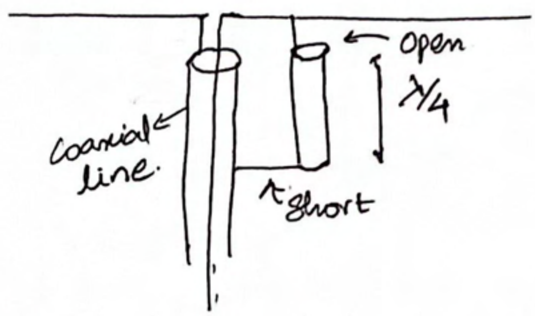
> Could use a hollow grid otherwise.

> Need balanced feed

BALUN

? $\lambda/4$ Coaxial BALUN?:

Narrowband.



Bazooka/Sleeve BALUN?
Ferrite Core Balun?
} Complex so use a monopole antenna.

Microstrip Balun Dipole antenna.??

Top loaded microstrip Balun

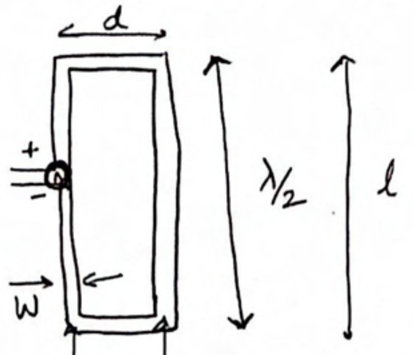
> Triangular dipole

→ Higher bandwidth.

Folded Dipole

→ $Z_{in} = N^2 \cdot Z_{11} = N^2 \cdot Z_r$.. Impedance is N^2 times higher.

⇒ 2 folds ⇒ $Z_{in} = 4 Z_r$.



→ L + D = 0.48 D needs to be used.

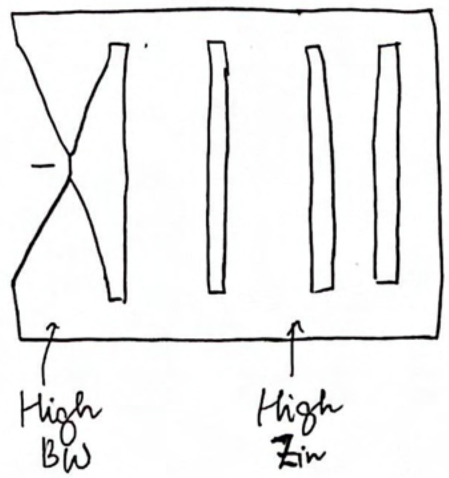
→ Could approximate D = W.

→ Assuming d is ≃ 0.

→ This is like a loop antenna.

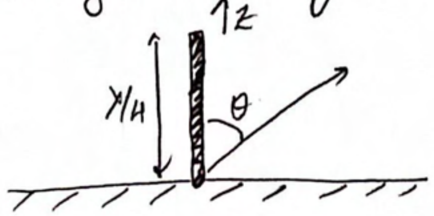
If this diameter < this diameter ⇒ Z_{in} is slightly more than $4 Z_r$.

→ Folded Broadband Dipole



Lec 11 Monopole antenna on Infinite GND plane.

> λ/4 length is only valid when ground is ∞.



> Replicates half of dipole when gnd is ∞.

$$E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$H_\phi = j \cdot \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

} Same as dipole on upper half.

$Z_{in} = \frac{Z_{in}(\text{dipole})}{2} = \frac{73 + j42.5}{2} = 36.5 + j21.25$.

$D_0 = 2 \times D(\text{Dipole}) = 2 \times 1.643 = 3.286 = 5\text{dB}$

↳ radiation is only on one side ⇒ 2×.

In reality. consider radius

$h + r = 0.24\lambda$ where $r \leq \lambda/20$

> As radius of monopole ↑ ⇒ inductance ↓ and BW ↑ & ↑ weight.
> Resonance $f = \dfrac{0.24\,c}{h+r}$

directivity doesn't change & gain also remains same ⇒ Gain BW ↑↑
Radiation pattern remains same.

> In reality gain changes slightly due to reflection coefficient Γ.

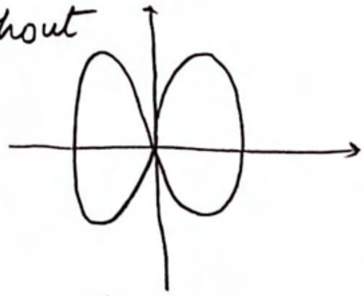
> At constant 'l' and 'r' what happens if ground plane changes? Assuming circular ground plane.

> As R_{gnd} ↑, input impedance becomes more inductive.
As R_{gnd} ↓, Z_{in} is capactive. ⇒ We can choose R_{gnd} to get perfect input match

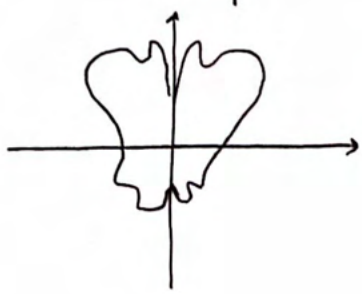
> f_{res} changes : $R \uparrow \Rightarrow f_{res} \downarrow \Rightarrow Re\{Z_{in}\}$ goes down & up.

> $R \to 0 \Rightarrow$ End fed dipole antenna.

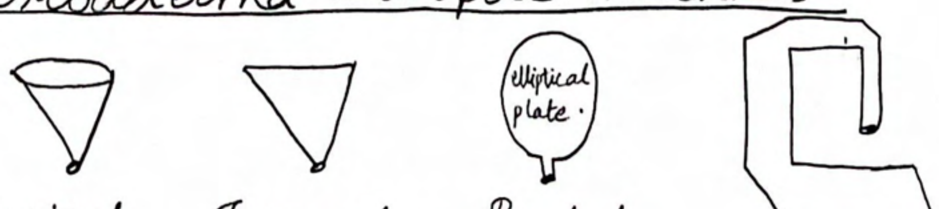
> $R_{gnd} < \lambda \Rightarrow$ radiation pattern is almost same as dipole. Therefore we can get same rad. pattern without balanced feed. Gain is $\simeq 2dB$



> $R_{gnd} > \lambda \Rightarrow$ radiation pattern is screwed up! But gain ↑. It looks like it is slowly changing into monopole on ∞ R_{gnd}.

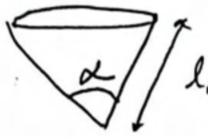


> Broadband Monopole antennas



Conical — could be hollow.
Triangular
Printed Elliptical (Very high BW↑)
Bent. → Both vertical & horizontal polarization.

Conical Monopole antenna.



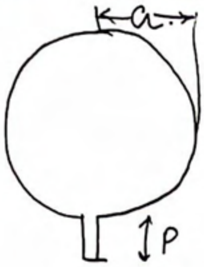
BW for VSWR ≤ 2 is from 175 to 1615 MHz!!

) $l_{slant} = \lambda/4$ at lowest frequency of operation.

$Z_{in} = 60 \ln\left[\cot\left[\frac{\alpha}{4}\right]\right]$ where α is cone angle.

$\alpha = 90° \Rightarrow Z_{in} = 52.9\,\Omega$

Lec 12 Broadband Circular monopole.



> BW is from $1.17 \rightarrow 13$ GHz $\rightarrow$ could be higher.
> But radiation pattern changes over the BW.

) > If $a = 2.5$ cm $\Rightarrow 2a = 5$ cm. Recall $h + r = 0.24\lambda$. Use $l = 2a$ and equate $r = \frac{\text{Area}}{2\pi h}$ [$\because$ Equating areas. Area $= (2\pi r)h$] $\rightarrow r = \frac{\pi a^2}{2\pi(2a)}$

$\Rightarrow r = \frac{\pi a^2}{4\pi a} = \frac{a}{4}$ - Using this approximation gives 5-8% error.

$\Rightarrow$ $\boxed{2a + \frac{a}{4} = 0.24\lambda}$ To calculate fref at lower end.

→ Circular shape has many higher order modes. They all come together and give a very broadband antenna. This also is the reason for radiation pattern changing.

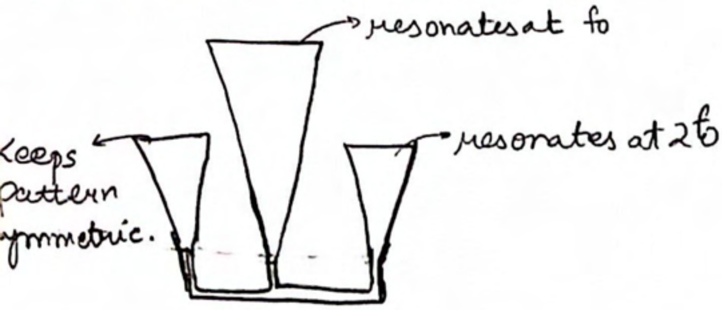
> Could use different shapes for both the pole and the ground plane.

)

> To get stable radiation pattern, cut out parts of the circular plate that removes some modes. Called Dual ring antenna. Most of the current flows in the out regions.

Another example is called Dual band Trident monopole.

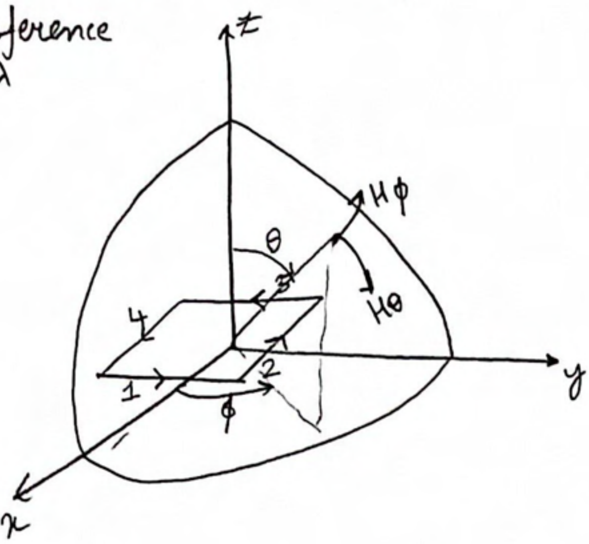
Broadband Trident

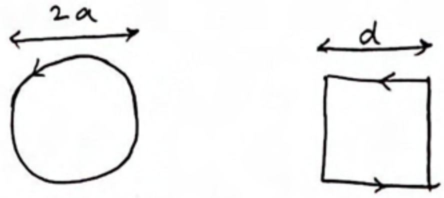


> Radiation pattern is fairly constant.
> Vertically polarized.
> High frequency.

Lec 13 Loop Antenna.

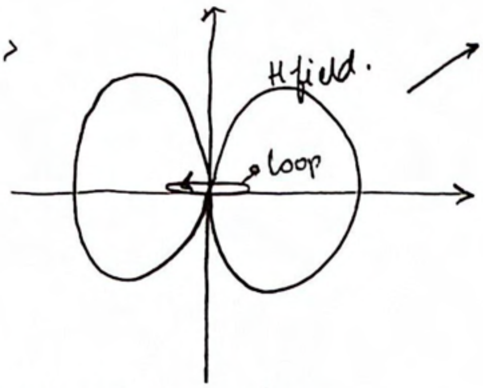
> Small loop → magnetic dipole : E & H field are interchanged.
circumference $\ll \lambda$



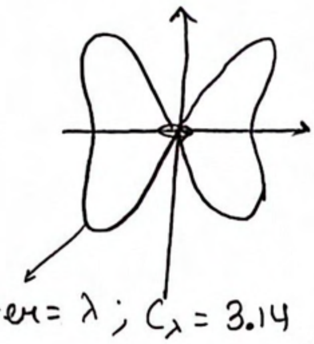


> Assume magnetic dipole with constant current I_m

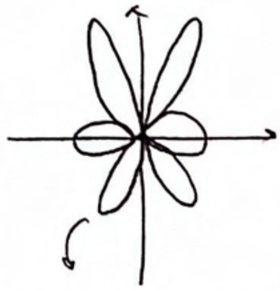
Diameter = $\lambda/10$; $C_\lambda = 0.314$ (circumference/λ)



> E field is uniform around loop.



Diameter = λ ; $C_\lambda = 3.14$



Diameter = $\frac{3\lambda}{2}$
$C_\lambda = 4.71$

> In reality as diameter ↑ the shapes of radiation are quite different since current distribution is not uniform. They are rarely used.
> We usually use the small loop case.

Single turn loop antenna. (Small loop)

> $R_r = 20\pi^2 \left(\frac{C}{\lambda}\right)^4$. where $C = 2\pi a$

> N turns $\Rightarrow$ $R_r = 20\pi^2 N^2 \left(\frac{C}{\lambda}\right)^4$

> Large loop $\Rightarrow$ $(C \geq 3.14\lambda)$ $\Rightarrow$ $R_r = 60\pi^2 \left(\frac{C}{\lambda}\right)$

If $\frac{C}{\lambda} = 0.1$ Need 50 turns to get $R_r = 20\pi^2 N^2 \left(\frac{C}{\lambda}\right)^4 = 50\Omega$

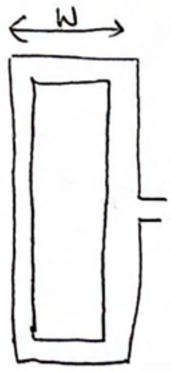
→ Ferrite core loop antenna.

$R_f = \mu_{cer}^2 R_r$
↳ effective permeability (of core + air) $= \left(\frac{\mu_{ce}}{\mu_0}\right)^2$

> Reduces no. of turns required.

> Directivity (small loop) → same as dipole.

> Can model folded dipole as a single rectangular loop.



> As $W\uparrow \Rightarrow Z_{in}\uparrow$ and $f_{res}\downarrow$
Since $W\uparrow \Rightarrow C\uparrow$

> Various modes are at $C = n\lambda$

Lec 14 · Slot Antenna.

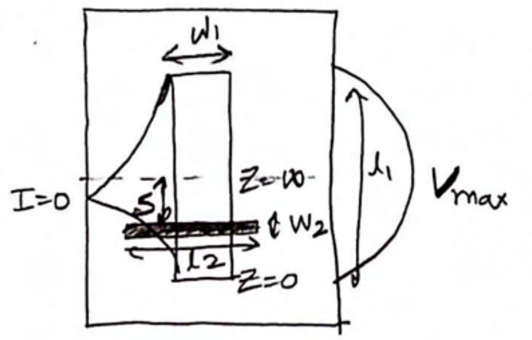
> Practically slot is cut in a finite ground plane.

$$\underset{\text{slot}}{Z_s} \cdot \underset{\text{dipole}}{Z_c} = \frac{\eta^2}{4}$$

> E & H field are exchanged with dipole case. It is a compliment to the dipole.

> $Z_s \cdot Z_c =$ constant $\Rightarrow$ In theory BW is ∞.

> Slot $\Rightarrow \frac{H}{E}$ Dipole $= \frac{E}{H} \Rightarrow \frac{E}{H} \cdot \left[\frac{H}{E}\right]^{-1} \rightarrow \eta^2$ [Intuitively]

> 

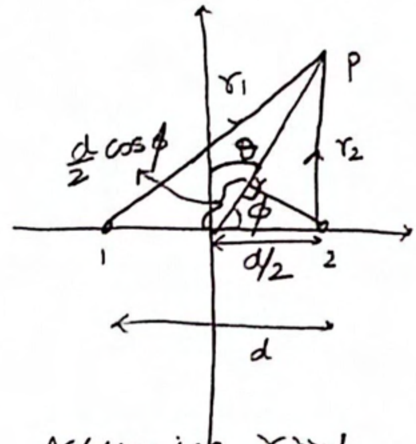
$\frac{V}{I} = \infty$ at centre so feed offcentre at a pt. where impedance is 50Ω.

> Effective length is smaller than physical length since fringing fields are on the inside. $\Rightarrow$ Physical length $> \frac{\lambda}{2}$

> Slot length $\uparrow \Rightarrow$ fres $\downarrow$ & i/p impedance rotates on Smith chart
> Slot width $\uparrow \Rightarrow$ fres same & i/p Zin $\downarrow$ & BW $\uparrow$
> Feed length $\uparrow \Rightarrow$ fres $\downarrow$ & Zin $\downarrow$.
> Feed width $\uparrow \Rightarrow$ fres same & Zin $\downarrow$.
> offset $\uparrow \Rightarrow$ fres $\downarrow$ since loading changes to different points.
& Zin rotates

Lec 15 Arrays

> Arrays of two isotropic point Sources.



$$E = E_0 e^{-j\beta r_1} + E_0 e^{-j\beta r_2} \qquad \beta = K = \frac{2\pi}{\lambda}$$

$$\left.\begin{aligned} r_1 &= r + \frac{d}{2}\cos\phi \\ r_2 &= r - \frac{d}{2}\cos\phi \end{aligned}\right\} \text{Assuming, } r \gg d,\ \phi = 90 - \theta.$$

Assuming $r \gg d$
$\Rightarrow$ Point P receives equal amplitude but different phases.

$$E = E_0 e^{-j\beta r}\left[e^{-j\beta\frac{d}{2}\cos\phi} + e^{j\beta\frac{d}{2}\cos\phi}\right]$$

$$= E_0 e^{-j\beta r}\left[e^{-j\frac{\psi}{2}} + e^{+j\frac{\psi}{2}}\right]$$

$$= 2E_0 \cos\frac{\psi}{2}$$

$$\boxed{E = 2E_0 \cos\left[\frac{\pi d}{\lambda}\cos\phi\right]}$$

$$\psi = \beta d\cos\phi = \frac{2\pi d}{\lambda}\cos\phi$$
$$= \beta d\sin\theta = \frac{2\pi d}{\lambda}\sin\theta$$

Normalised

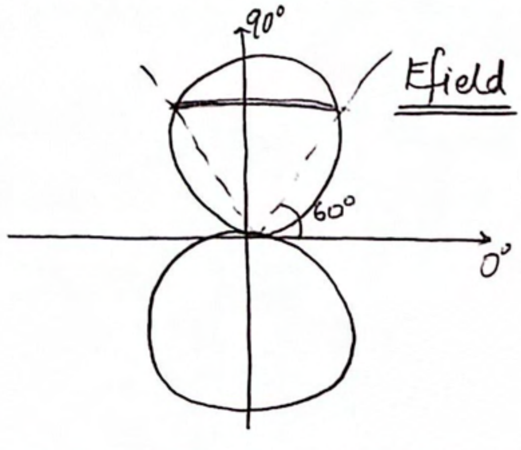
$$E = \cos\left(\frac{d_r}{2}\cos\phi\right) \qquad \text{where } d_r = \frac{2\pi d}{\lambda} = \beta d.$$

For $d = \frac{\lambda}{2}$; $E = \cos\left(\frac{\pi}{2}\cos\phi\right)$

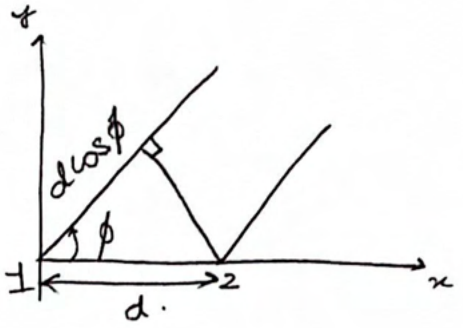
ϕ	$0°$	$90°$	$60°$
E	0	1	$1/\sqrt{2}$

$\Rightarrow$ HPBW = $60°$ in one plane and $360°$ in another.

For $\frac{\lambda}{2}$



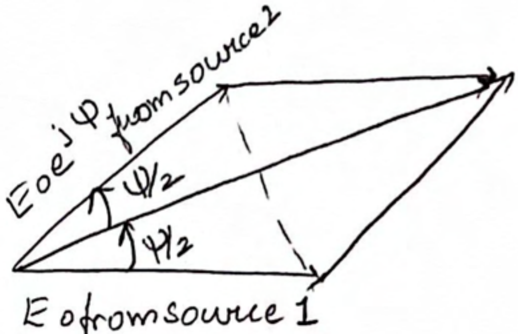
Origin at Element 1



$$E = E_0(1 + e^{j\psi})$$

$$= 2E_0 e^{j\psi/2}\left[\frac{e^{j\psi/2} + e^{-j\psi/2}}{2}\right]$$

$$= 2E_0 e^{j\psi/2} \cos\frac{\psi}{2}$$



Normalising

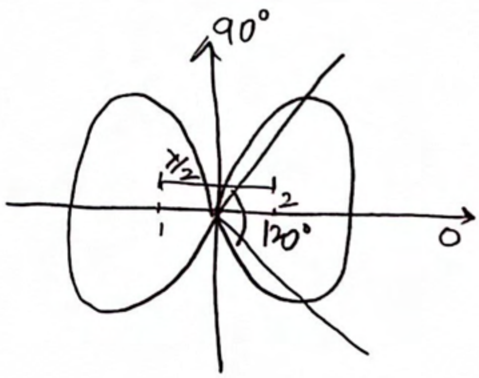
$$E = e^{j\psi/2} \cos\frac{\psi}{2}$$

$$= \cos\psi/2 \angle \psi/2$$

> Two Isotropic point sources of same amplitude and opposite phase.

$$E = E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2}$$

$$E = 2jE_0 \sin\frac{\psi}{2} = 2j\, E_0 \sin\left(\frac{d_r}{2}\cos\phi\right)$$



> For $d = \frac{\lambda}{2}$; $E = \sin\left(\frac{\pi}{2}\cos\phi\right)$

ϕ	0°	90°	60°
E	1	0	$1/\sqrt{2}$

HPBW = 120° in both orthogonal planes

> When $d = \frac{\lambda}{2}$, each antenna does not load the next one, so they can be individually optimized.

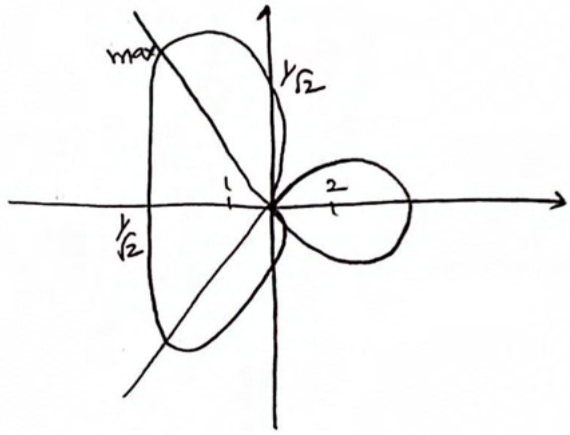
> Two isotropic point sources of same amplitud but 90° phase diff at $\frac{\lambda}{2}$

$$E = E_0 \exp\left[+j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right] + E_0 \exp\left[-j\left(\frac{d_r \cos\phi}{2} + \frac{\pi}{4}\right)\right]$$

$$E = 2E_0 \cos\left[\frac{\pi}{4} + \frac{d_r}{2}\cos\phi\right]$$

Letting $2E_0 = 1$ and $d = \frac{\lambda}{2}$; $E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)$

ϕ	0°	60°	90°	120°	180°
E	$1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$

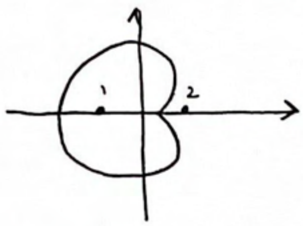


> HPBW = 180° on one side.

> If $d = \lambda/4$ in this case.

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right)$$

ϕ	0°	90°	120°	150°	180°
E	0	$1/\sqrt{2}$	0.924	0.994	1



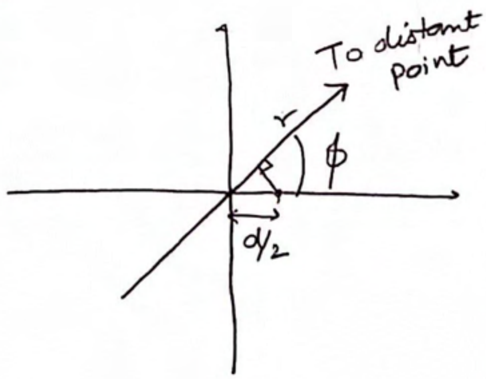
> Negligible back radiation.

> Two isotropic point sources of same amplitude and any phase difference

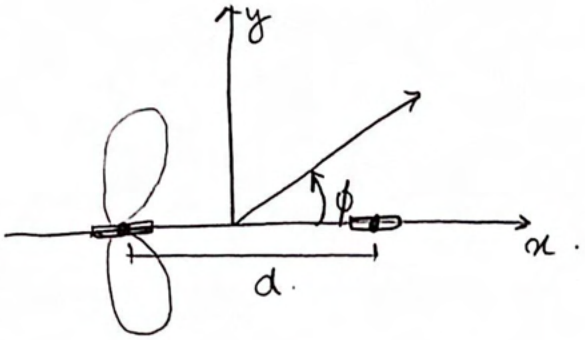
> $\psi = d_r \cos\phi + \delta$; $E = E_0\left(e^{j\psi/2} + e^{-j\psi/2}\right) = 2E_0 \cos\frac{\psi}{2}$

Normalising by setting $2E_0 = 1$; $E = \cos\frac{\psi}{2}$.

> Based on the value of δ we can change the value of $\psi \Rightarrow$ can control beam maxima direction.



Two same dipoles and pattern Multiplication.



$E_0 = E_0' \sin\phi$ since dipole is horizontal

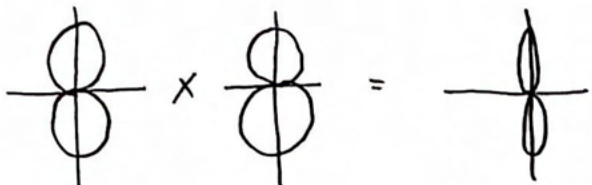
$AF = \cos(\psi/2)$

$E = \sin\phi \cos\frac{\psi}{2}$ where, $\psi = d_r \cos\phi + \delta$

For $\delta = 0$, AF will give max radiation in Broadside direction.

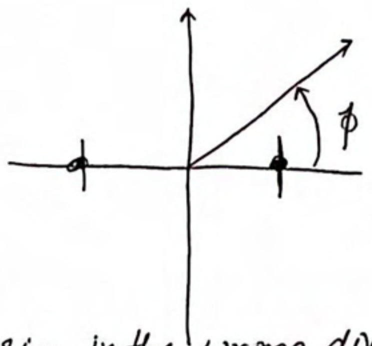
* Where ϕ is measured from matters

Dipole × AF = Final pattern



Fat × Fat → thin.

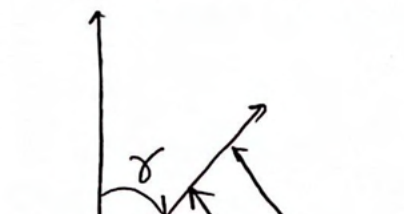
$E_0' \cos\phi$ $\cos\phi \cos\left(\frac{\pi}{2}\cos\phi\right)$



Array of two vertical dipoles.

↳ measuring in the wrong direction gives wrong result.

N Isotropic Point Sources of Equal Amplitude and Spacing.



① $E = 1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \dots + e^{(n-1)j\psi}$

$\times e^{j\psi}$

where, $\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta = d_r \cos\phi + \delta$.

② $- Ee^{j\psi} = e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \dots + e^{jn\psi}$

①−② $\Rightarrow E - Ee^{j\psi} = 1 - e^{jn\psi} \Rightarrow E = \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} = \frac{\sin(n\psi/2)}{\sin(\psi/2)}$

As, $\psi \to 0$; $E_{max} = n$,

$$\boxed{E_{norm} = \frac{\sin(n\psi/2)}{n\sin(\psi/2)}}$$

→ Array Factor?

Lec 16

→ N Isotropic point sources of Equal Amplitude and Spacing

- $E = \dfrac{1-e^{jn\psi}}{1-e^{j\psi}} = \dfrac{\sin(n\psi/2)}{\sin(\psi/2)}$

As $\psi \to 0$, $E_{max} = n$, $E_{norm} = \dfrac{\sin(n\psi/2)}{\sin(\psi/2)}$

- As $n\uparrow$, no. of side lobes increase & HPBW $\downarrow \Rightarrow D\uparrow$
- $\psi = 0 \Rightarrow$ Array factor is highest.

Broadside Array.

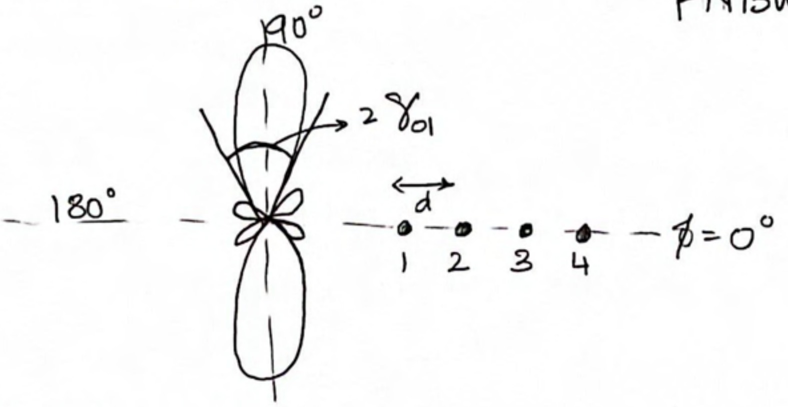
$\psi = \dfrac{2\pi d}{\lambda}\cos\phi + \delta$; $\delta = 0$; $d = \dfrac{\lambda}{2}$ and $n = 4$.

$\psi = \pi\cos\phi$ $\quad E = \dfrac{\sin(n\psi/2)}{\sin(\psi/2)}$

	ϕ	ψ	E
	$0°$	π	0
FNBW →	$60°$	$\pi/2$	0
	$90°$	0	1

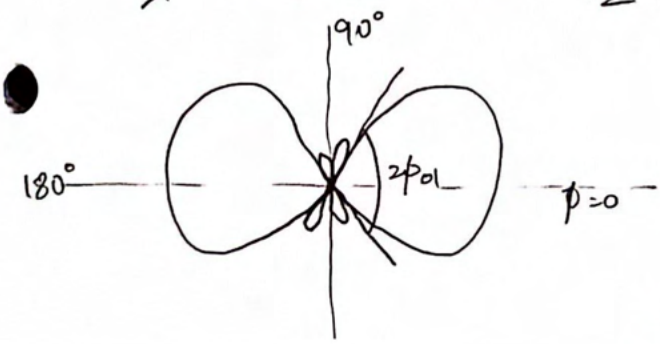
Radiation Pattern

$FNBW = 2\gamma_{01} = 60°$



Endfire Pattern

$\psi = \dfrac{2\pi d}{\lambda}\cos\phi + \delta$ For $d = \dfrac{\lambda}{2}$; $\phi = 0° \,\&\, \psi = 0 \Rightarrow \delta = -\pi$; $\psi = \pi(\cos\phi - 1)$



ϕ	ψ	E
$0°$	0	1
$60°$	$-\pi/2$	0
$90°$	$-\pi$	0

$FNBW = 120°$

How to increase directivity?

Increased Directivity Endfire Array (IDEA)

$\psi = d_r(\cos\phi - 1) \rightarrow$ Endfire.

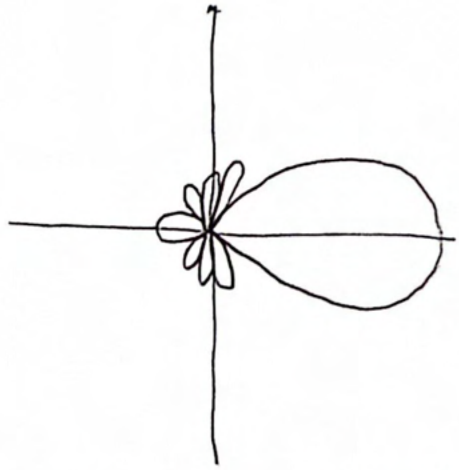
$\psi = d_r(\cos\phi - 1) - \frac{\pi}{n} \rightarrow$ Increased directivity endifire array.

additional phase.

$$E_{norm} = \sin\left(\frac{\pi}{2n}\right) \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

Array Factor.

Parameter	ordinary endfire	IDEA
HPBW	69°	38°
FNBW	106°	74°
Directivity	11	19



› Adding some phase creates a sharper roll off in the Enorm function.

(a) Phase for increased directivity.

$\delta = -0.6\pi$

(b) Phase for ordinary endfire.

$\delta = -0.5\pi$

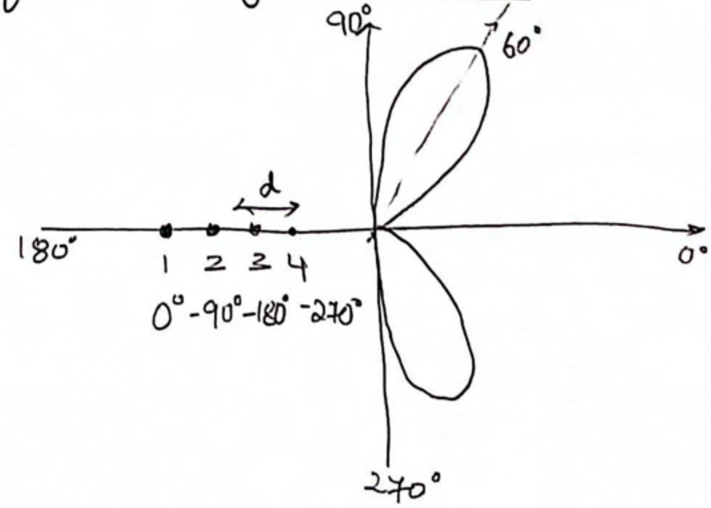
Array with Maximum Field in any Arbitrary Direction

For Beam Maxima at $\phi = 60°$

$\psi = 0 = d_r \cos 60° + \delta$

For $d = \lambda/2$, $d_r = \pi$

$\delta = -\frac{\pi}{2}$



› Conical radiation pattern

› SLL → Side lobe level.

$$E_{norm} = \frac{\sin(n\psi/2)}{n\sin(\psi/2)} \rightarrow$$ nulls are where num $= 0$ since den $\neq 0$.

Direction of Nulls:

$$\sin\left(\frac{n\psi}{2}\right) = 0 \rightarrow \frac{n\psi}{2} = \pm k\pi \quad \text{where } k = 1, 2, 3$$

$$\psi = \pm \frac{2k\pi}{n}$$

For Broadside Array, $\delta = 0$

$$\frac{2\pi d}{\lambda}\cos\phi_0 = \pm\frac{2k\pi}{n} \rightarrow \phi_0 = \pm\cos^{-1}\left(\frac{k\lambda}{nd}\right) \quad \text{— (1)}$$

Null direction and Beamwidth between first nulls for linear arrays of n isotropic point sources of equal amplitude & spacing:

Type of Array	Null Directions (array of any length)	Null Directions (Long array)	Beamwidth b/w first nulls (long array)
General case	$\phi_0 = \arccos\left[\left[\pm\frac{2k\pi}{n} - \delta\right]\frac{1}{d_r}\right]$	—	—
Broadside	* notice this is not ϕ_0 as given earlier in Eq (1) $\gamma_0 = \arcsin\left(\pm\frac{k\lambda}{nd}\right)$	$\gamma_0 = \pm\frac{k\lambda}{nd}$	$2\gamma_{01} \cong \frac{2\lambda}{nd}$
Ordinary endfire	$\phi_0 = 2\arcsin\left(\pm\sqrt{\frac{kd}{2nd}}\right)$	$\phi_0 = \pm\sqrt{\frac{2k\lambda}{nd}}$	$2\phi_{01} = 2\sqrt{\frac{2\lambda}{nd}}$
Endfire with increased directivity	$\phi_0 = 2\arcsin\left[\pm\sqrt{\frac{\lambda}{4nd}(2k-1)}\right]$	$\phi_0 = \pm\sqrt{\frac{\lambda}{nd}(2k-1)}$	$2\phi_{01} = 2\sqrt{\frac{\lambda}{nd}}$

Directions of Max SLL for Arrays of N Isotropic Point Sources

$$\sin\frac{n\psi}{2} = \pm 1 \rightarrow \frac{n\psi}{2} = \pm\frac{(2K+1)\pi}{2} \quad \text{where } K = 1,2,3$$

$$\psi = \pm\frac{(2K+1)}{n}\pi$$

SLL mag: $$AF = \left|\frac{\sin\frac{n\psi}{2}}{n\sin\frac{\psi}{2}}\right| = \left|\frac{1}{n\sin\left(\frac{(2K+1)\pi}{2n}\right)}\right|$$

For very large n:

$$AF = \left|\frac{1}{n\times\left(\frac{(2K+1)\pi}{2n}\right)}\right| = \frac{2}{(2K+1)\pi} = 0.212 \text{ for } K=1 \text{ (First SLL)}$$

$$= -13.5\,dB$$

→ SLL = −10dB ⇒ 10% power goes to sidelobes.

Type of array:	Direction of minor lobe maxima.
General case.	$\phi_m = \arccos\left[\left(\pm\frac{(2K+1)\pi}{n} - \delta\right)\frac{1}{d_r}\right]$
Broadside	$\phi_m \simeq \arccos\left[\pm\frac{(2K+1)\lambda}{2nd}\right]$
Ordinary endfire	$\phi_m \simeq \arccos\left[\pm\frac{(2K+1)\lambda}{2nd} + 1\right]$
Endfire with increased directivity	$\phi_m \simeq \arccos\left[\frac{\lambda}{2nd}\left[1 \pm (2K+1)\right] + 1\right]$

HPBW of Array.

- $AF = \left| \dfrac{\sin \frac{n\psi}{2}}{n \sin \frac{\psi}{2}} \right| = 1/\sqrt{2}$

For large n, HPBW is small: $AF \simeq \left| \dfrac{\sin \frac{n\psi}{2}}{n \sin \frac{\psi}{2}} \right| = 1/\sqrt{2}$

Solution

$n\psi/2 = 1.3915$

For Broadside: $\psi = \frac{2\pi d}{\lambda} \cos\phi = 2.783/n.$

$\cos\phi = \sin(90-\phi) = 1.3915/(\pi n d/\lambda) = 0.443/L_\lambda$ (radian)

$\Rightarrow$ HPBW $\simeq 2 \times (90 - \phi) = 50.8°/L_\lambda$ $\hookrightarrow \frac{nd}{\lambda}$

- Lec 17

> If ψ goes upto 360° instead of upto 180°, it creates another maxima called grating lobe which we must avoid.

> $\psi = \frac{2\pi d}{\lambda}(\cos\phi - \cos\phi_m) < 2\pi$ where ϕ_m is direction of max radiation.

$\phi_m = 0° \Rightarrow$ Endfire

$\phi_m = 90° \Rightarrow$ Broadside

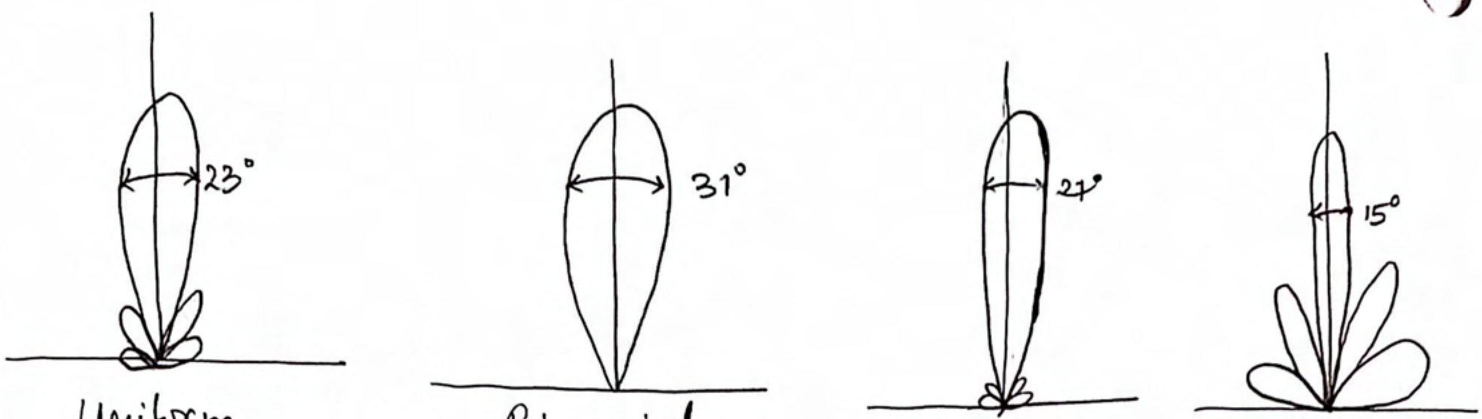
$\frac{d}{\lambda} < \dfrac{1}{\cos\phi - \cos\phi_m} \rightarrow \frac{d}{\lambda} < \dfrac{1}{1 + |\cos\phi_m|}$

For Broadside Array: $\frac{d}{\lambda} < 1 \rightarrow \boxed{d < \lambda}$ usually $d \simeq 0.5\lambda$

For Endfire : $\boxed{d < \frac{\lambda}{2}}$ usually $d \simeq 0.25\lambda$

-

Radiation Pattern of Broadside Array with Nonuniform amplitude
(5 elements with spacing = $\lambda/2$, Total length = 2λ)



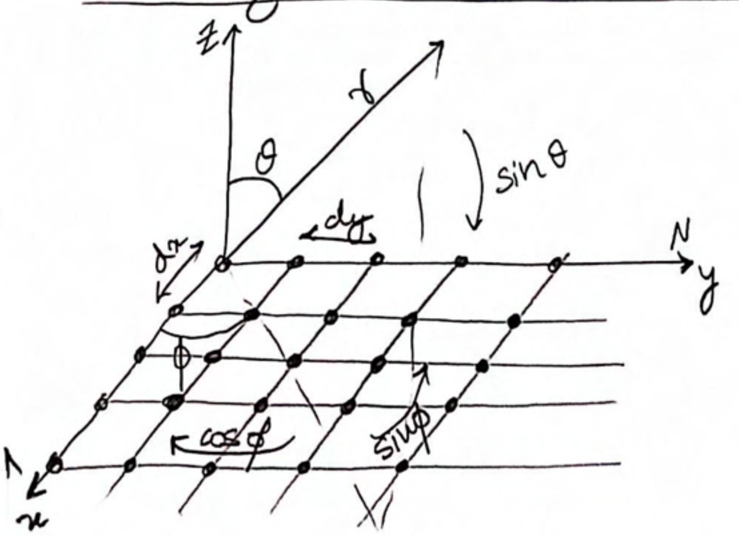
Uniform	Binomial	Optimum	Edge
	→ not used much → low gain → High HPBW	→ practical.	
1 1 1 1 1	1 4 6 4 1	1, 1.6, 1.9, 1.6, 1	1 0 0 0 1
SLL < −13dB	No SLL	SLL ≤ −20dB	Grating lobes. ($d = 2\lambda$!)

→ All sources are in phase.

→ If d is small we use space factor instead of array factor.

→ Could use square or circular arrays too.

Rectangular Planar Array



$$AF_n(\theta,\phi) = \left\{\frac{1}{M}\cdot\frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{N}\cdot\frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right\}$$

where, $\psi_x = k d_x \sin\theta\cos\phi + \beta_x$

$\psi_y = k d_y \sin\theta\sin\phi + \beta_y$

$\beta_x = -k d_x \sin\theta_0 \cos\phi_0$ for $\psi_x = 0$

$\beta_y = -k d_y \sin\theta_0 \sin\phi_0$ for $\psi_y = 0$

> Find AF for each element in one direction & use the array as an element that is repeated in the other direction.

> β_x & β_y are phase difference in x & y direction.

Lec 18 Planar Arrays.

- > Broadside ⇒ $\theta_0 = 0 \Rightarrow \beta_x, \beta_y = 0$ → all elements should be inphase.
  > For desired θ_0 & ϕ_0 find β_x & β_y! → phased array.

  > $$\tan\phi_0 = \frac{\beta_y d_x}{\beta_x d_y} \quad \& \quad \sin^2\theta_0 = \left(\frac{\beta_x}{k d_x}\right)^2 + \left(\frac{\beta_y}{k d_y}\right)^2 \text{ where } k = 2\pi/\lambda_0$$

Principle maximum $(m=n=0)$ & grating lobes $(m=n=1)$ can be found by:

$$k d_x (\sin\theta\cos\phi - \sin\theta_0\cos\phi_0) = \pm 2m\pi \qquad m = 0,1,2\ldots$$
$$k d_y (\sin\theta\sin\phi - \sin\theta_0\sin\phi_0) = \pm 2n\pi \qquad n = 0,1,2\ldots$$

- **Directivity of Array (Rectangular)**

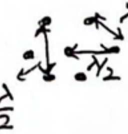
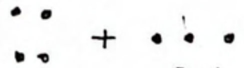
$$\boxed{D = \pi D_x D_y \cos\theta_0}$$

For Broadside $D = \pi D_x D_y$

Directivity of circular array

Assume 100% η

$$G = \frac{4\pi A}{\lambda^2}, \quad A = \pi a^2 \Rightarrow D = \left(\frac{2\pi a}{\lambda}\right)^2$$

Hexagonal array 7 elements  Group into → Equilateral Δ^{le} array 

2×2 + 3×1

- $$AF_n(\theta,\phi) = \left\{\frac{1}{M}\frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right\}$$

Total array factor = Array factor of Group1 + Group2

$$\psi_x = \frac{2\pi}{\lambda} d_x \sin\theta\cos\phi \qquad \psi_y = \frac{2\pi}{\lambda} d_y \sin\theta\sin\phi$$

$$AF_1(\theta,\phi) = \frac{1}{3}\frac{\sin\left(\frac{3}{2}\psi_x\right)}{\sin\frac{\psi_x}{2}} \qquad d_x = \lambda/2$$
group 1

$$AF_2(\theta,\phi) = \left\{\frac{1}{2}\frac{\sin\left(\frac{2}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{2}\frac{\sin\left(\frac{2}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right\} \qquad d_x = \frac{\lambda}{2}, \quad d_y = \frac{\sqrt{3}\lambda}{2}$$
group 2

$$AF_{total} = AF_1 + AF_2$$

> More complex arrays can be grouped together similarly.

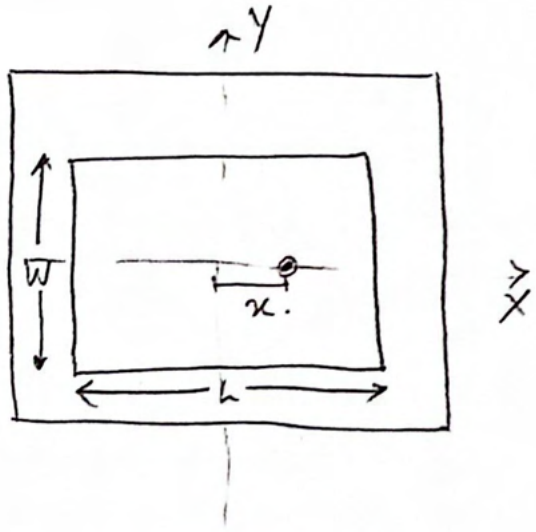
> Circular arrays can be reduced to hexagonal + rectangular arrays.. or use Bessel functions (complex) → Balanis book.

> In array to avoid grating lobes $d < 0.7\lambda$ since diagonal elements are spaced at $\sqrt{2}.d$ (in rectangular). Or use a hexagonal array.

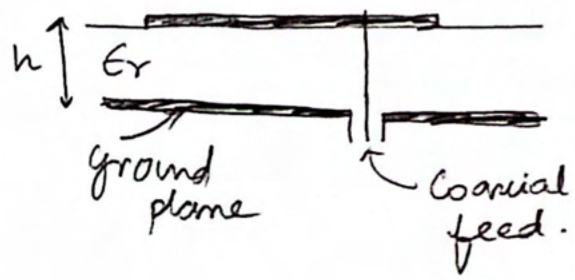
Lec 19 Microstrip Antenna.

> Rectangular Microstrip Antenna (RMSA)

> L → gives fres.
> W → gives radiation (Gain & BW)
> Matching network not required.
> Need small ϵ_r large thickness & width. This increases the fringing fields.



> Substrates: Glass Epoxy. (FR4)
→ $\epsilon_r \simeq 4.4$
→ Losstangent = 0.02 (pretty high).
→ Very cheap.

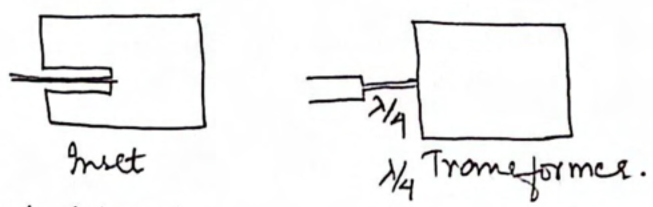


> Best substrate is Air → Low $\epsilon_r = 1$
→ Loss tangent = 0
→ Free

> Can have almost any kind of polarization.

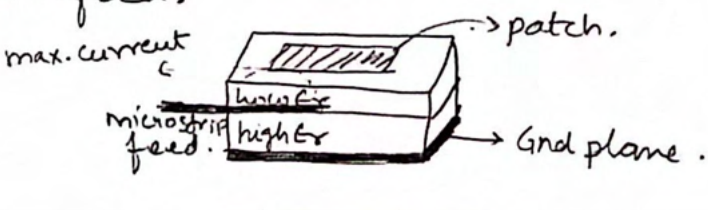
Disadvantages
> Narrow bandwidth. > Low power handling capacity > Gain ≤ 30dB.
> Polarization purity is difficult.

Feed types. 1) Coaxial feed.
2) Microstrip line feed. → For matching use inset feed or λ/4 transformer.



3) Electromagnetically coupled feed.

> Max. current on feed line is at the edge of the patch & it is magnetically coupled.



4) **Aperture coupled feed** → not very good.

Resonance Frequency.

$L_e = L + 2\Delta L$ } due to
$W_e = W + 2\Delta W$ } fringing.

$$\Delta L \cong \frac{h}{\sqrt{\epsilon_e}}$$

$$f_0 = \frac{c}{2\sqrt{\epsilon_e}}\left[\left(\frac{m}{L}\right)^2 + \left(\frac{n}{W}\right)^2\right]^{1/2}$$

for fundamental mode TM_{10}
$m=1$; $n=0$

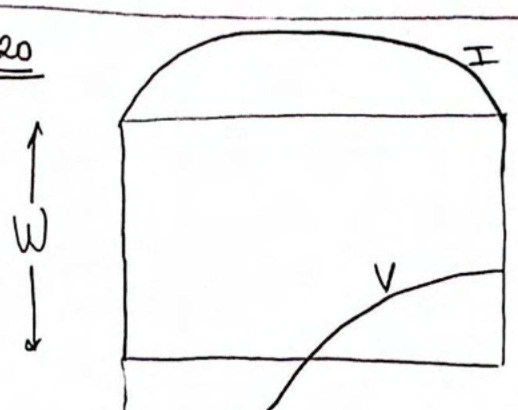
ϵ_e (effective) $\cong \dfrac{\epsilon_r+1}{2} + \dfrac{\epsilon_r-1}{2}\left[1 + \dfrac{10h}{W}\right]^{-1/2}$

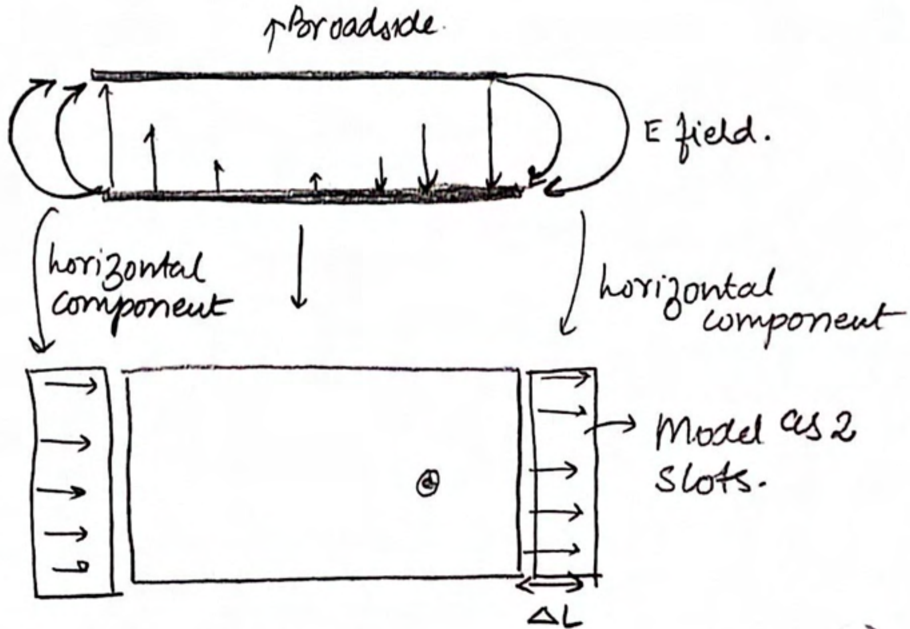
$$W = \frac{c}{2f_0\sqrt{\frac{(\epsilon_r+1)}{2}}}$$

> Smaller or larger W can be taken if needed. since BW $\propto$ W & gain $\propto$ W.

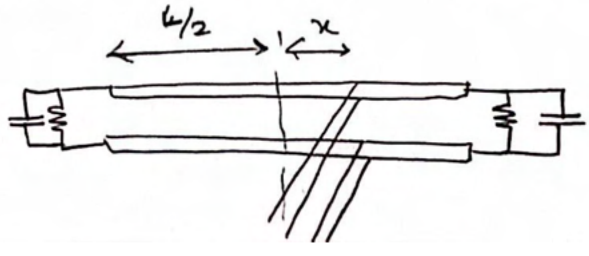
> $L_e = L + 2\Delta L = \dfrac{\lambda_0}{2\sqrt{\epsilon_e}} = \dfrac{c}{2f_0\sqrt{\epsilon_e}}$ → [Choose x b/w $L/6$ to $L/4$]

Lec 20





> The fringing fields' horizontal components add up in the broadside direction & radiate.
> Could model as 2 slots or a T-line with fringe fields as a cap + radiation resistance.



> Since part of the field is in the air $\epsilon_e \neq \epsilon_r$.

> Design steps

> From ϵ_r and f_o find 'W'.

> From W find ϵ_e.

> Then $L_e \rightarrow L$.

Example: Design RMSA for WiFi. (2.4 - 2.483 GHz) → $f_o = 2.445$ GHz

Choose $\epsilon_r = 2.32$, $h = 0.16$ cm, $\tan\delta = 0.001$

Sol:

$$W = \frac{c}{2f_o\sqrt{\frac{\epsilon_r+1}{2}}} = 4.77\text{cm}$$, let W = 4.7cm.

$$\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1+\frac{10h}{W}\right]^{-1/2} = 2.23$$

$$L_e = \frac{c}{2f_o\sqrt{\epsilon_e}} = 4.11\text{cm}.$$

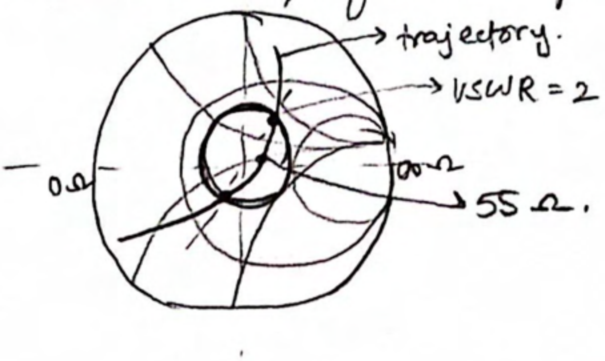
$$L = L_e - 2\Delta L = 4.11 - \frac{2\times 0.16}{\sqrt{2.23}} = 3.9\text{cm}$$

> Majority of the time using this error < 1%.

> Increase W to meet gain & BW. But $W\uparrow \Rightarrow f_o\downarrow$ → $W\uparrow \Rightarrow$ radiation ↑ also impedance ↓

> Increase h/λ_o to ↑BW, reduce h to ↑f_o

> On smith chart draw VSWR = 2 circle & make sure the plot cuts this circle at its diameter so that BW is maximum, but matching would not be perfect, impedance ≃ 55 Ω.



> Despite the 2 slots being $\frac{\lambda}{2}$ apart this is different from $\frac{\lambda_0}{2}$ we saw in array theory. Recall $\lambda = \lambda_0/\sqrt{\epsilon_e}$.
> Feed probe should be small orelse it radiates.

Lec 21

$\frac{h}{\lambda_0}\uparrow \Rightarrow BW\uparrow$ But $\frac{h}{\lambda_0} \leq \frac{0.3}{2\pi\sqrt{\epsilon_r}}$ to reduce surface waves.
↳ could cause total internal reflection.

→ BW vs. surface wave tradeoff.

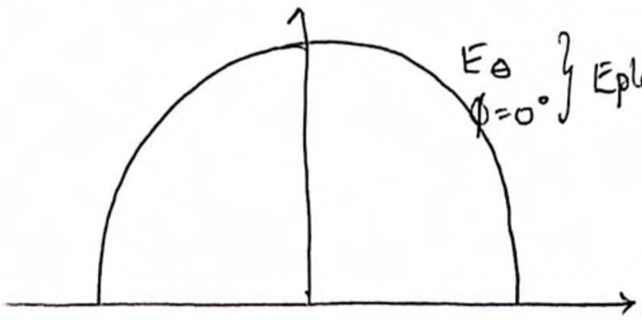
→ Probe (feed) diameter matters. (SMA vs Ntype)

→ $\tan\delta \uparrow \Rightarrow$ impedance ↓ BW ↑ but efficiency ↓ & Gain ↓.

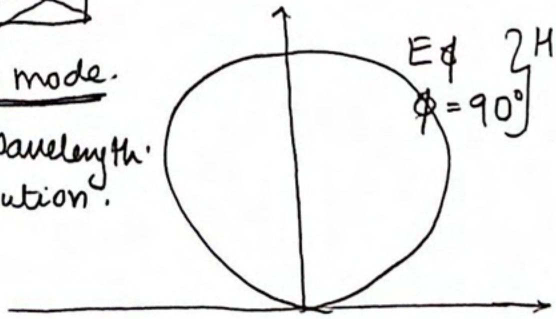
→ $\epsilon_r \uparrow \Rightarrow$ Gain ↓, BW ↓ since L ↓ & W ↓ therefore air is best!

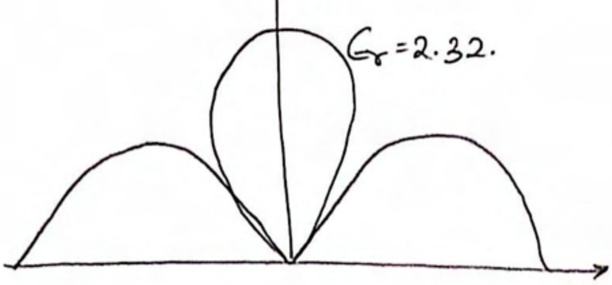
→ $\epsilon_r \downarrow \Rightarrow$ L ↑ & W ↑ ⇒ fringing ↑ & Aperture area ↑ ⇒ BW ↑ & gain ↑.

→ Radiation pattern



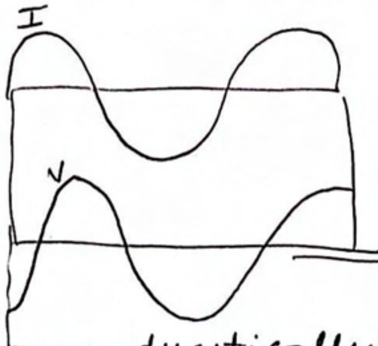
TM10 mode.
⇒ one ½ wavelength distribution.

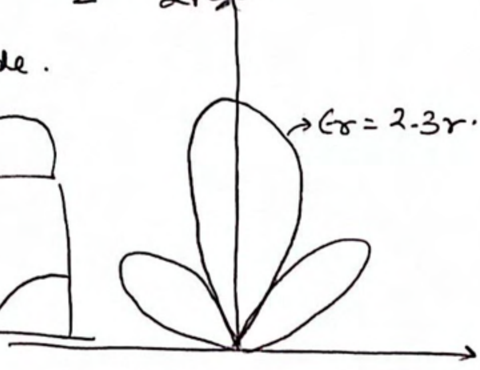




$L_e = \frac{3\lambda}{2} = \frac{3\lambda_0}{2\sqrt{\epsilon_e}}$.

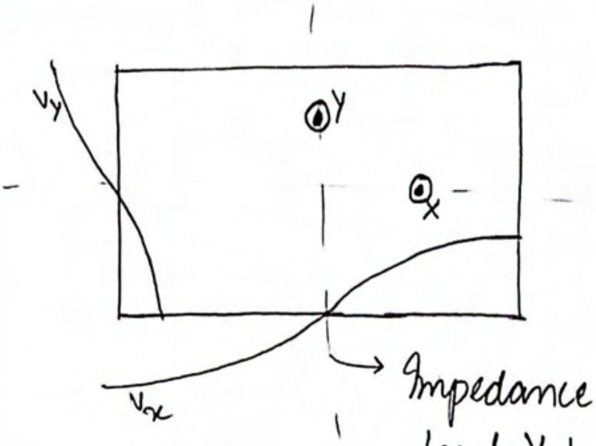
TM30 mode.

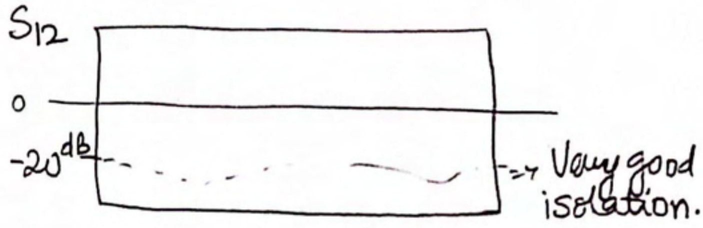




> when $\epsilon_r = 9.8$ the pattern changes drastically in TM_{30} mode.
> $TM_{xy} \Rightarrow (x=1) \Rightarrow \frac{\lambda}{2}$ variation along length.
> TM_{11} $(y=1) \Rightarrow \lambda/2$ variation along width.

Dual Polarization (TM_{10} & TM_{01})





→ Impedance on this line is 0. ⇒ X feed does not load Y feed. W & h are changed for Y.

→ 2 resonance frequencies for 2 orothogonal modes. For square patch, 2 orthogonal modes would exist at the same frequency.

⇒ Orthogonal polarization ⇒ Horizontal & Vertical polarization.

Finite ground plane

> $L_g = L + 6h + 6h$ & $W_g = W + 6h + 6h$ ⇒ results would be almost same as ∞ ground plane. Only F/B ratio is lower.
  F/B ↑ ⇒ larger or cavity shaped.

> $L = L_g$ ⇒ We get a bidirectional antenna but Gain ↓.

Lec 22

> Second order modes cancel on broadside direction ⇒ not used.

> Stick to first order & use arrays to ↑ gain.

> As $\epsilon_r \downarrow$ ⇒ % BW ↑ ; $\epsilon_r \downarrow$ ⇒ $\eta \uparrow$ ⇒ Keep ϵ_r as low = 1.

> As $h \uparrow$ ⇒ % BW ↑.

> To use air as substrate, use a shorting metal post to support in the middle since the field there is 0. It also provides a dc short which is good.

CMSA → Circular-

$$f_0 = \frac{K_{nm}\, c}{2\pi a_e \sqrt{\epsilon_e}}$$

a_e ← effective radius.

Where K_{nm} is the m^{th} root of the derivative of the Bessel fn of order n.

← Variation of field is sinusoidal along circumference & we need sin(sin) or sin(cos) ⇒ use Bessel fns

$$f_{0 \text{ for } TM_{11}} = \frac{8.791}{\left[\underbrace{(a + h/\sqrt{\epsilon_r})}_{a_e}\sqrt{\epsilon_e}\right]}$$

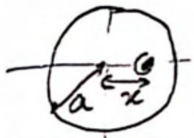
in GHz. a & h are in cm. In general $\epsilon_e < \epsilon_r$

$$a \simeq \frac{8.791}{(f_0\sqrt{\epsilon_e})} - \frac{h}{\sqrt{\epsilon_r}}$$

fund. → TM_{11}

TM_{xy}	K_{nm}
TM_{11}	1.84118
TM_{21}	3.05424
TM_{02}	3.83171
TM_{12}	5.33140

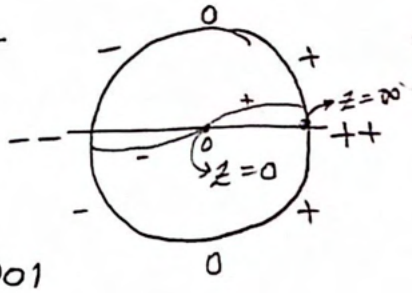
> choose x b/w 0.3a to 0.5a.



Lec 23

$TM_{\underset{x\,y}{11}}$ ⇒ $(X=1) \to \frac{\lambda}{2}$ variation along circumference.

$(Y=1) \to \frac{\lambda}{2}$ variation along diameter.



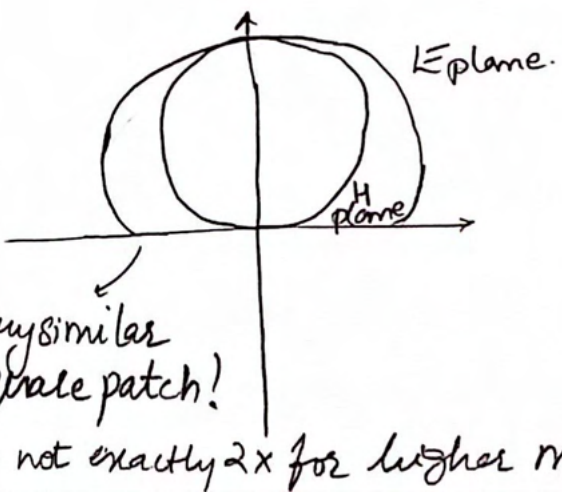
Eg: $a = 3\,cm$, $h = 0.318\,cm$; $\epsilon_r = 2.55$, $\tan\delta = 0.001$

Let $x = 0.3$, $a = 0.9\,cm$.

$$f_0 = \frac{8.791}{\left[(3 + 0.318/\sqrt{2.55})\sqrt{2.45}\right]} = 1.756\ GHz.$$

> For a given mode & substrate $f_0 \cdot a_e$ is a constant!
⇒ $a_e \uparrow \Rightarrow f_0 \downarrow$.

→ E_θ & E_ϕ are functions of bessel functions.



→ Only difference is that it is not an efficient radiator for higher order harmonics since K_{nm} is not exactly $2\times$ for higher modes.

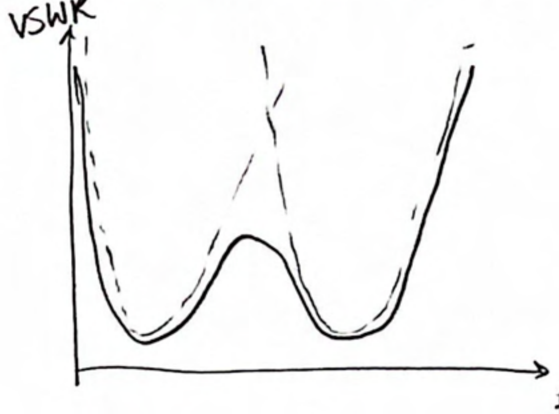
> N way power divider can be designed using the TM_{02} mode.
↳ at around 20 min of the video.

Lec 24.

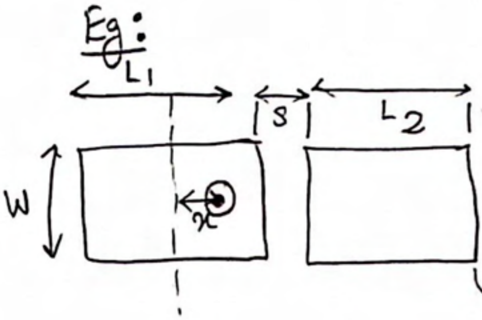
> Semicircular MSA. > Equilateral triangular MSA

Lec 25 Broadband Microstrip Antenna.

> Use multiple patches with 2 different f_0 that are close.



Eg:



→ Parasitic patch, Excited by patch 1

→ Impedance is lowered when parasitic patch is added ⇒ need to move the feedpoint towards the edge.
$L_2 < L_1$ to get higher f_0.

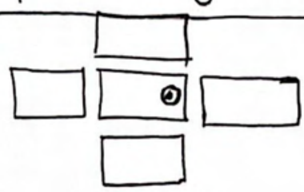
→ $S\downarrow \Rightarrow$ Coupling $\uparrow \Rightarrow$ wide band [loop size on smith chart is wider]
→ But need to look at radiation pattern

→ Using 3 Gap Coupled RMSA →

Radiation max is broadside & symmetric.

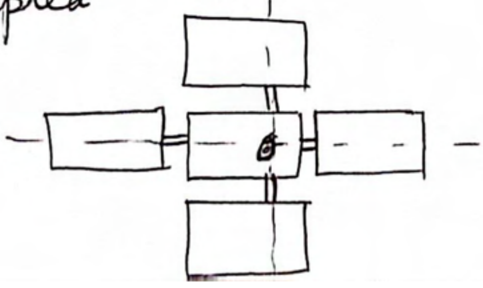
> H plane radiation pattern remains same.
> Could also use 3 gap coupled along H plane.

Lec 26 > Four Edge Coupled

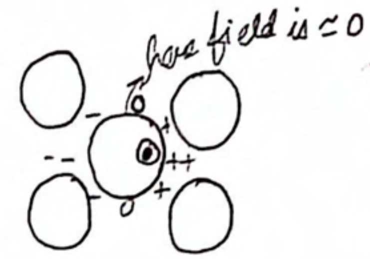


> Produces 2 loops on the smith charts.
⇒ Very wide band. (18%)
↳ optimize the lengths accordingly to put the 2 loops inside VSWR = 2 circle.

> Could couple using a metal or connecting strip → called directly coupled

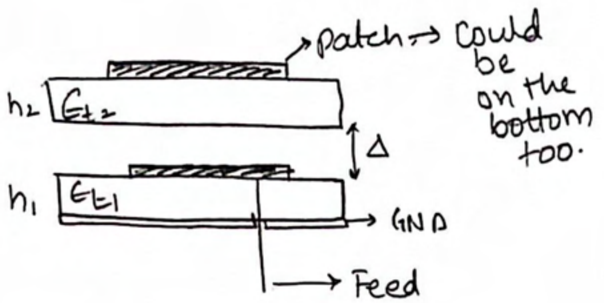


- In array all patches are in phase. But here since we have optimized for BW ⇒ The phases of these patches are not same.



- gap coupled Circular MSA : Another option.
  → Coupling area is very low ⇒ gap must be much lower.
- Broadband microstrip antennas book.

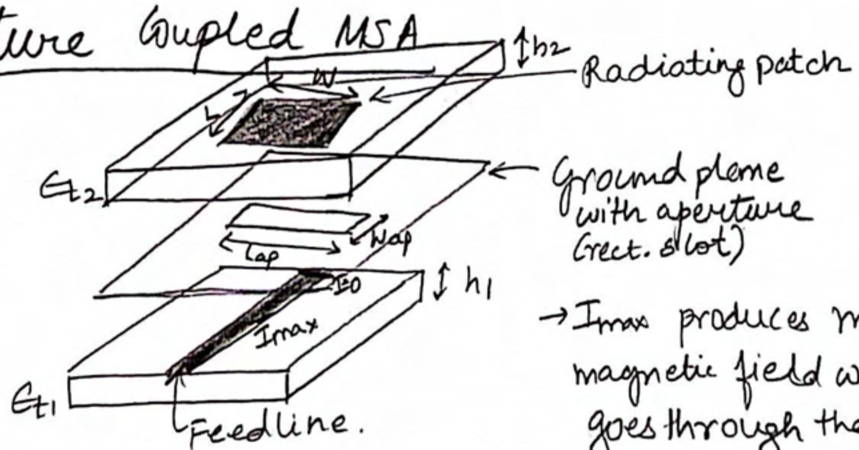
Lec 27 Electromagnetically coupled MSA



- top patch should have higher f_0
- Stronger coupling ⇒ loop formed on smith chart is larger.

VSWR < 2 ⇒ 11% power reflected.
VSWR < 1.5 ⇒ 4% power reflected.

Aperture Coupled MSA



→ I_{max} produces max magnetic field which goes through the slot & couples to patch.

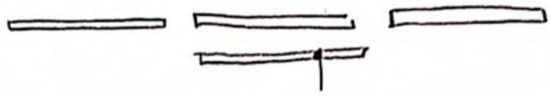
→ Slot shape and dimensions matters a lot. → Hour glass shape gives ↑ coupling.

→ Adv : Large BW, design flexibility

Dis : Back radiation is high, multi layer substrate, alignment.

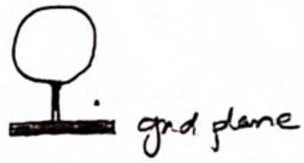
Lec 28 — Stacked Planar MSA 1B3T

1 bottom & 3 top patches



- > Very high Bandwidth.
- > Combines planar and stacked concepts.
- > Could use 1B2T as well or 1B4T.
- > Can get high gain & BW.

> For very broadband use circular monopole antenna.



> Multiple modes are excited $\Rightarrow$ Higher BW.

> Parallel ground plane does not exist (it is at ∞) $\Rightarrow$ Higher BW.

> Radiation pattern is not constant over entire B.W.

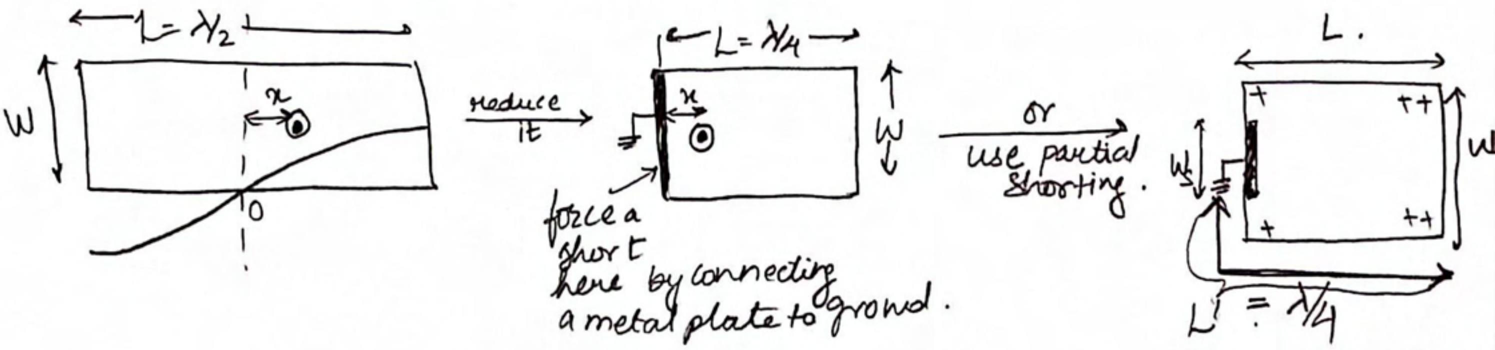
Lec 29 — Compact MSA

> L_{eff} needs to be $\lambda/2 \Rightarrow$ at low f it is large.

Approaches

1) Substrate with higher ϵ_r but BW & $\eta \downarrow$
2) Shorting post at appropriate location.
3) Cutting slots at appropriate location.

> Compact Shorted Rectangular MSA



$$L' = L_e + \frac{W_e - W_s}{2}$$

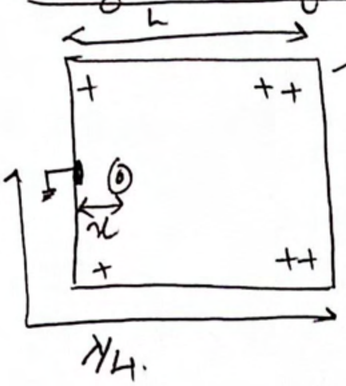
$$f_0 = \frac{30}{4\left[L_e + (W_e - W_s)/2\right]\sqrt{\epsilon_e}} \text{ in GHz}$$

[Le & We are in cm]

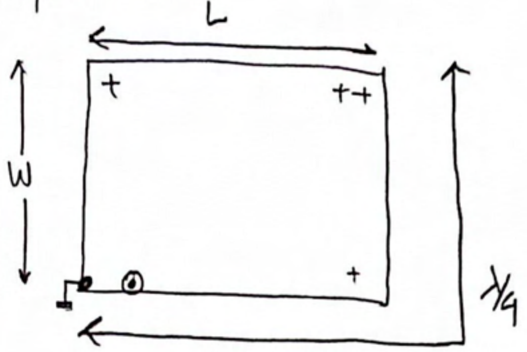
$\rightarrow \frac{W_s}{W} \uparrow \Rightarrow f_0 \uparrow$ & $Z_{in} \downarrow\downarrow$

$\hookrightarrow \Rightarrow$ for small $\frac{W_s}{W}$ need to shift x to match.

→ Single shorting post



→ also called planar inverted F antenna. (PIFA)



$$f_0 = \frac{30}{4(L_e + W_e/2)\sqrt{\epsilon_e}} \qquad f_0 = \frac{30}{4(L_e + W_e)\sqrt{\epsilon_e}}$$

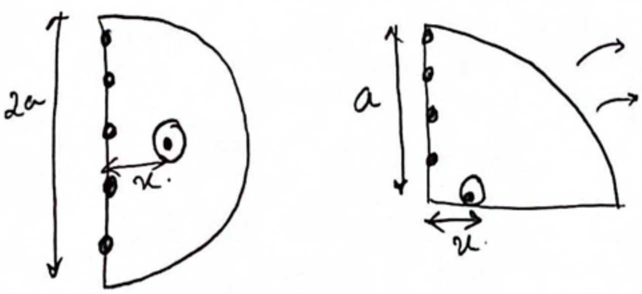
→ η is low but BW ↑↑

→ To improve η ⇒ use air, supporting structure changes f_0.

→ Gain is lower since only one "slot" is radiating. ⇒ wider coverage.

→ Used in mobile phones. (wide beam & high BW)

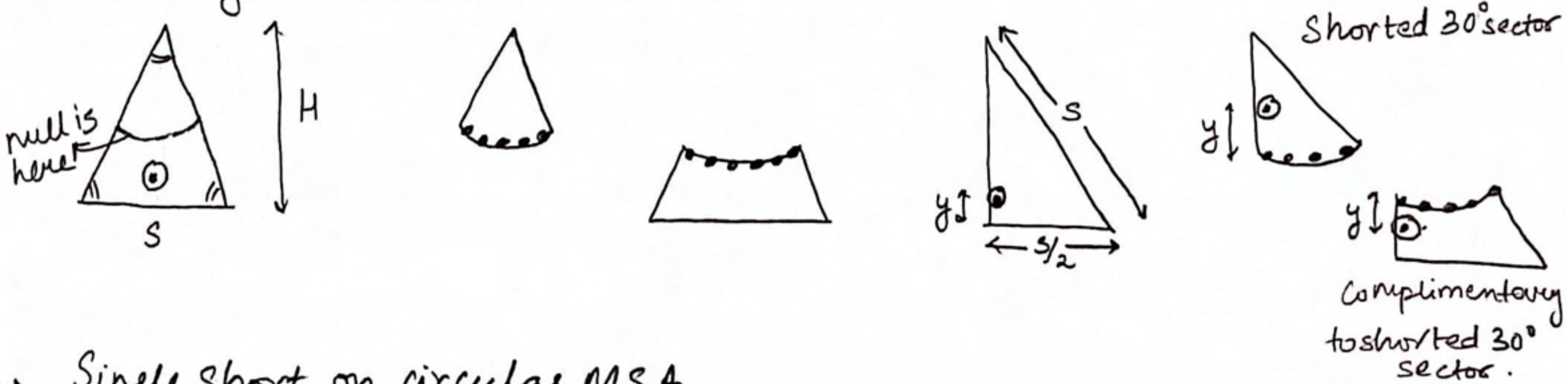
→ Could use a circular MSA in shorting post configuration.



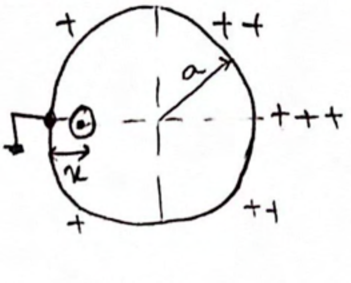
→ Very area (÷ 4)

→ BW is lower but not much.

→ Triangular MSA (shorting post)

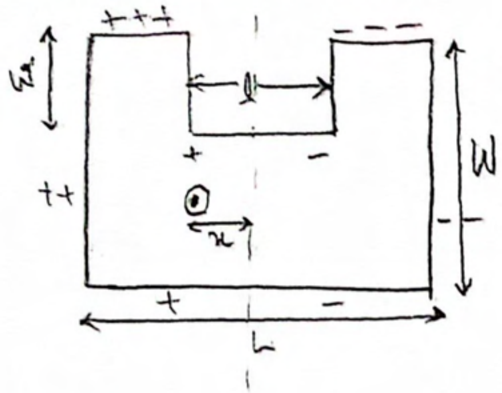


→ Single short on circular MSA



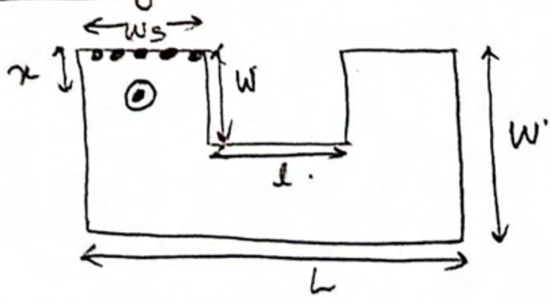
$$f_0 = \frac{8.791}{a_{e1}\sqrt{\epsilon_e}} \text{ GHz.}$$

C shaped MSA



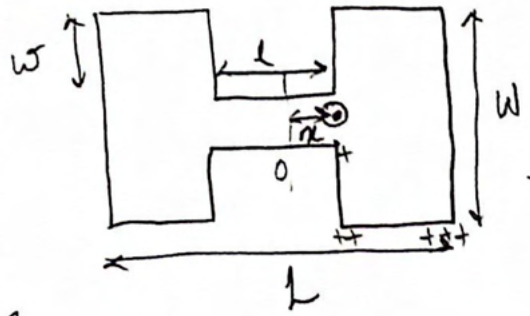
- $W \& l \uparrow \Rightarrow f_0 \downarrow \& x \downarrow$ also $BW \downarrow \& D \downarrow \& \eta \downarrow$
- Keep ϵ_r low.
- Probably one of the worst structure.
- low BW, D & η. Even f_0 is low.

Shorting it.



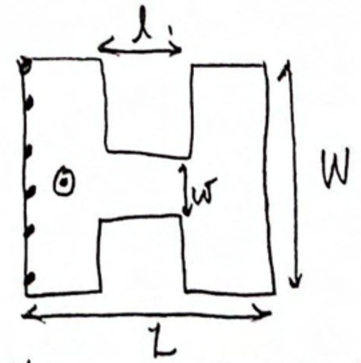
- Combination of short & slot.
- Compact!

H shaped MSA



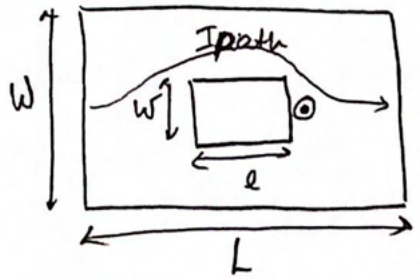
short

→ η is higher



→ Some of the fields get cancelled so it's efficiency is slightly lower.

Rectangular ring MSA

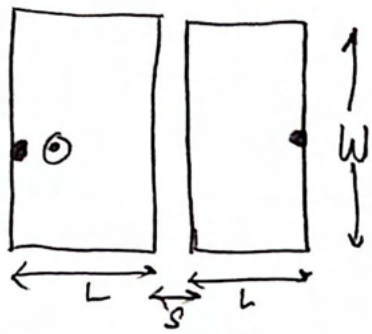


- The idea behind all these compact antennas is that the effective length for current path is larger & $L_{eff} = \lambda/2$ so the size can be reduced.
- BW is lower.
- η is higher.

- To increase BW, use broadband techniques [planar & stacked].

Lec 31 > Basically using broadband techniques for compact structures.

>

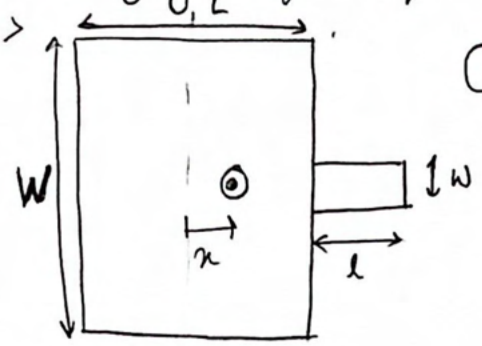


> Higher BW than rectangular antenna.

Lec 32 Tunable antennas

> Changing f_0 after fabrication.

>



① Stub tuning.

> New f_0 is smaller.

$$f_0 = \frac{c}{2(L_e + \Delta l_1)\sqrt{\epsilon_e}}.$$

where

$$\Delta l_1 = W_e \cdot l_e / W_e$$

zero axis also shifts

⇒ need to redo match → Can also be used to change the impedance match.

> To prevent changing x, add stubs on both sides.

> To increase f_0, cut a notch!

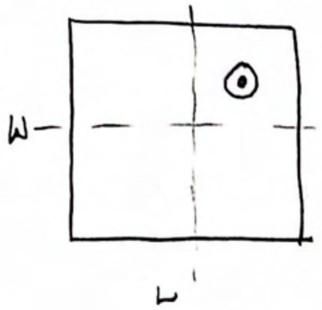
② > Could use shorting concept as discussed earlier.

f_0 depends on $\frac{W_s}{W}$.

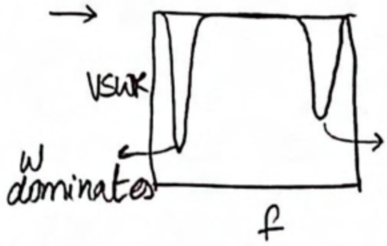
> Could use diodes as shorting nodes to turn them on & off and tune accordingly. Could use the tuning voltage of the VCO to tune the antenna as well in FMCW. This is great since now the antenna can be designed to have narrowband & high gain.

> Varactor diodes or PIN diodes are used.

Single feed dual band.

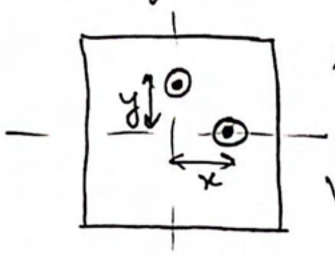


→ Orthogonal polarization.
→ At different frequencies E field is excited along L & W.
→

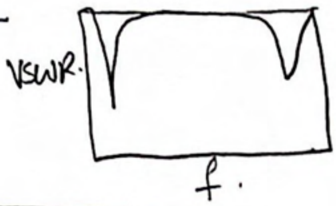


⇒ 2 resonant frequencies.

→ ## Dual feed dual band.



> Isolation between two feeds is good due to null axis

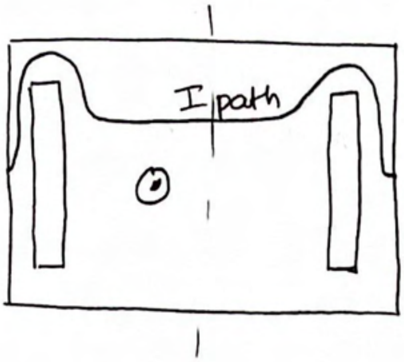


> Orthogonal polarization.

Lec 33

→ ## Dual band Slotted RMSA

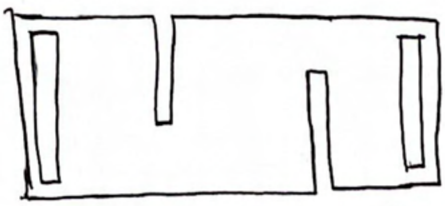
> Need dual band with same polarization



→ TM_{10} & TM_{30} modes exist.
→ The path is more significantly different after slots are cut for the TM_{30} mode.
→ Without slots the frequency ratio b/w TM_{30} & TM_{10} is 3
→ After slots the f ratio is $\simeq 1.57$.

→ Slot dimensions & location strongly affect the frequency ratio.

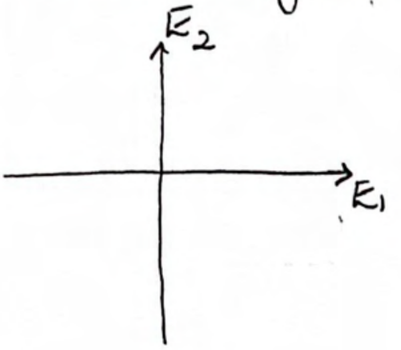
→ Could increase the path length for current by adding notches.



→ Makes structure more compact.

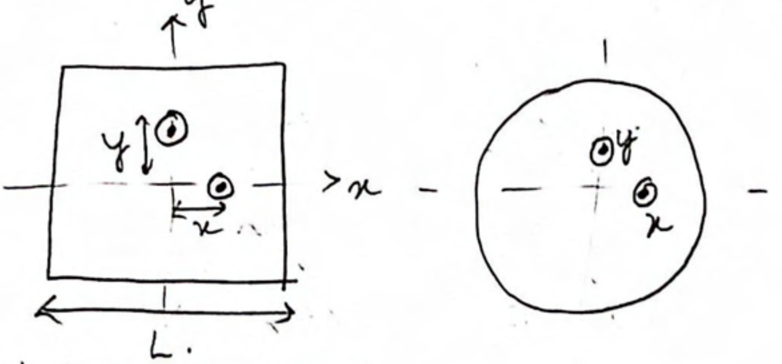
→ Could also change notch dimensions for further tuning.

Circularly Polarized MSA

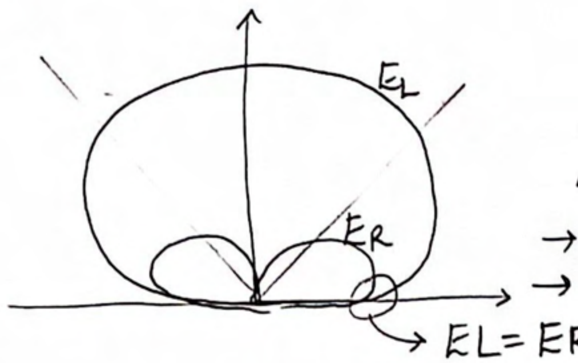


→ Need to feed them with 90° phase difference.

→ Why do we need?
- Rx antenna could change orientation.
- But power Rx is lower. [3dB lower power]



- Isolation is good.
- x & y must be fed at 90° offset in phase.
- Axial ratio must be 0.
- x & y should be equal?
- $\angle x = 0°$ & $\angle y = 90° \Rightarrow$ LHCP.



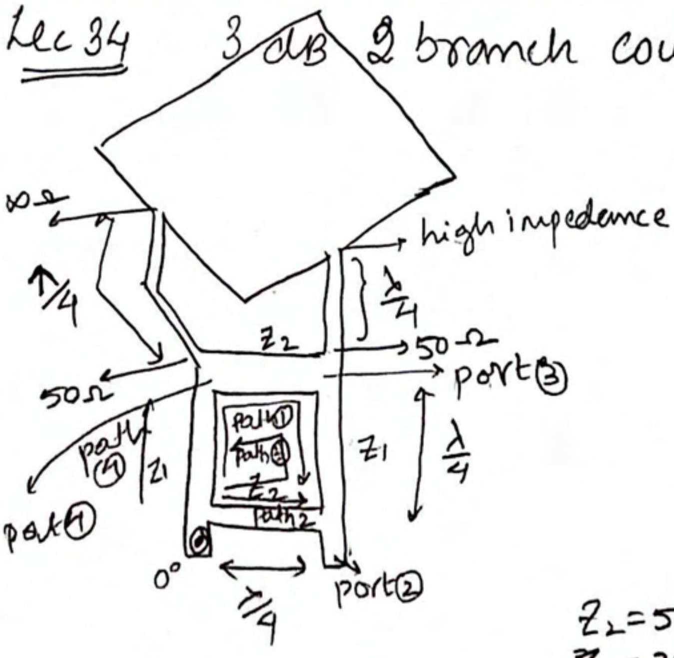
$E_L \rightarrow$ LHCP
$E_R \rightarrow$ RHCP
} Need isolation b/w E_L & E_R to be high.

→ In broadside E_L max & E_R min ⇒ good.

→ $E_L = E_R$ ⇒ Not circularly polarized.

- Axial ratio = $\dfrac{\text{Major axis}}{\text{Minor axis}}$ in dB.
  - → Circular polarized ⇒ 0 dB
  - → Linearly polarized ⇒ ∞ dB.
- Circular ⇒ ARBW ≤ 3dB

Lec 34 ~~3 dB~~ 2 branch coupler feed. for circular polarization.

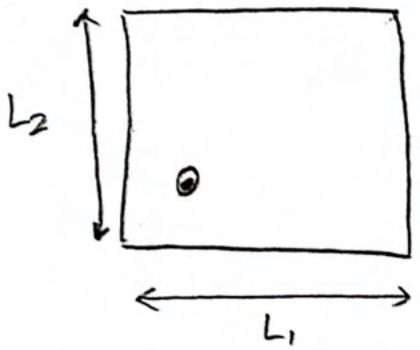


- path ① & path ② are 180° out of phase ⇒ no power goes to port 2.
- path ③ & ④ are also 180° out of phase. But metal in path ④ is wider so although they cancel, total power there has phase of 90° & is not fully cancelled
- Both paths leading to port ③ are 180°
- ⇒ Port ③ & ④ are at 90° ⇒ circular polarization

$Z_2 = 50\Omega \Rightarrow$ Z_1 & Z_2 along with widths are chosen s.t. half power goes to Port ③ & ④ each.
$Z_1 = 35.35\Omega$

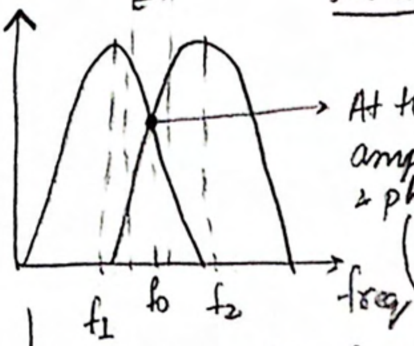
Variations

① $L_1 > L_2$ by small amount

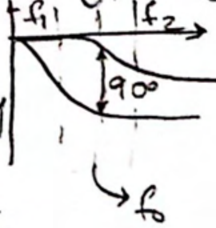


In this range axial ratio $\leq 3dB$

The two amplitudes may be unequal at $90°$ phase diff if f_1 & f_2 are not chosen well.



→ At this point the two amplitudes are equal & phase diff $= 90° \Rightarrow$ circularly polarized



> Why can't we use a square?

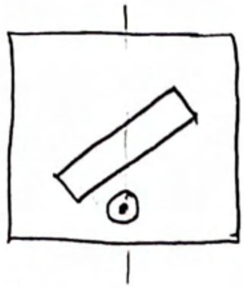
> By choosing f_1 & f_2 correctly we can get $90°$ phase difference between the 2 modes. This cannot be done in square since the two modes are in phase at all frequencies.

> Add stubs, or notches to change one length.

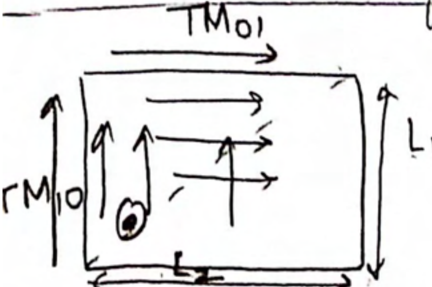
②

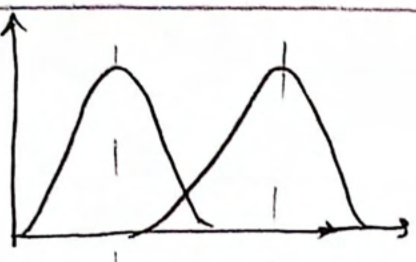
$L_{1\,eff} > L_2$

Diagonals are different.



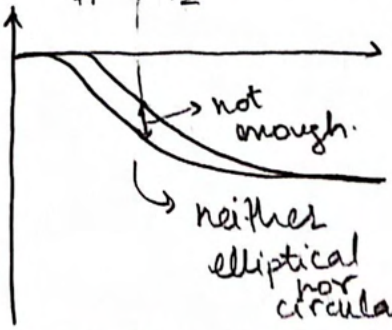
> Axial ratio BW of all these cases is very low.



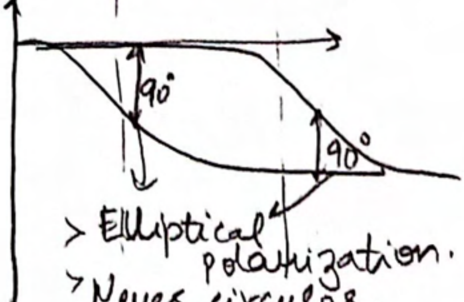


> TM01 is at f_2 & it is due larger dimension of L_2

> smaller dimension ⇒ excited at lower frequency.



neither elliptical nor circular



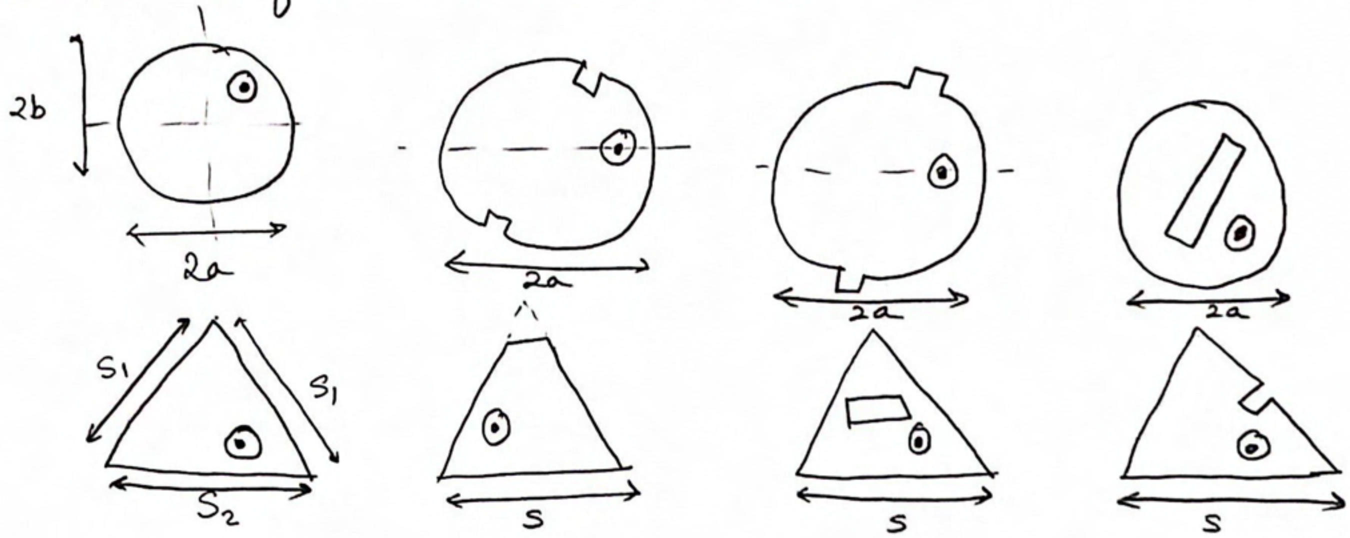
> Elliptical polarization.
> Never circular.

→ Somewhere in the middle we get circular polarization.

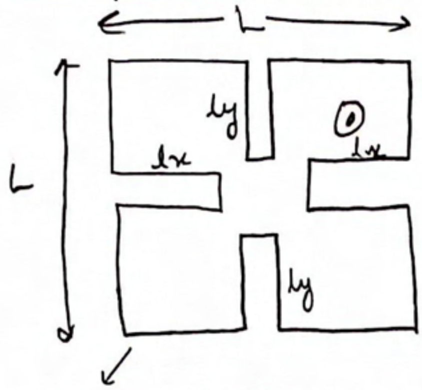
> If L_1 is fixed & we change L_2 ⟹ there is an optimum value of L_2 for which AR < 3dB.

Variations of CMSAs & ETMSAs

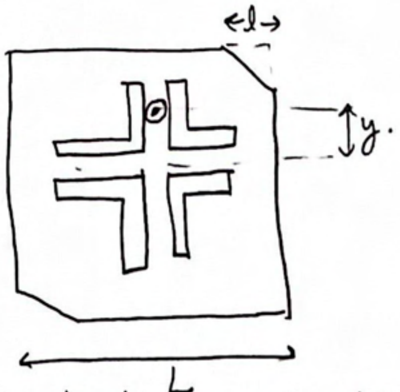
↳ Equilateral.



Compact CP square MSA

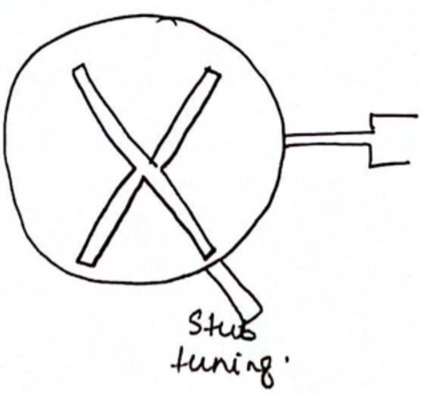


> Modified H shape in 2 axis. More path lengths.

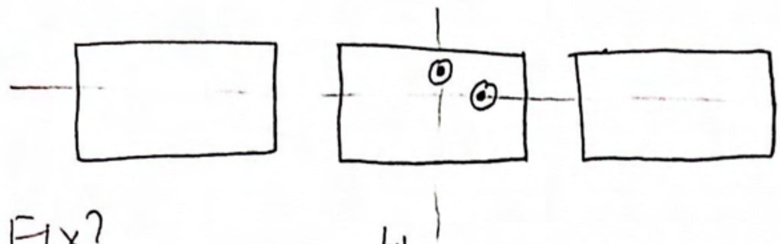


> Application: GPS 1575±10MHz.
> Very compact & needs CP.

Circular

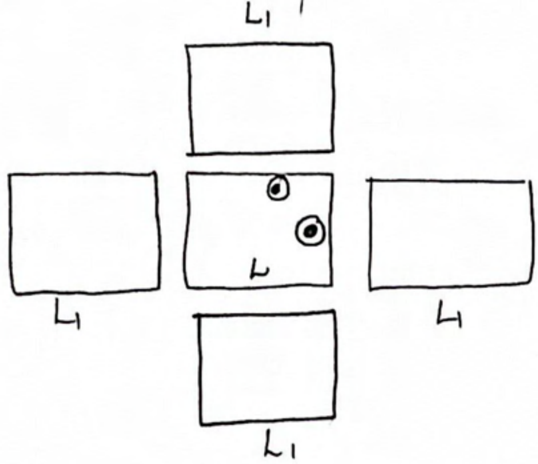


Broadband CP MSA



> Not symmetric & coupling is different for the 2 feeds.

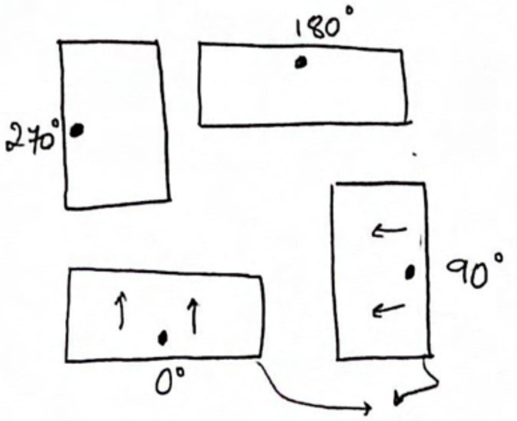
> Fix?



> Very broadband 25%
> Can be used as 3 dB branched couplour.

Lec 36

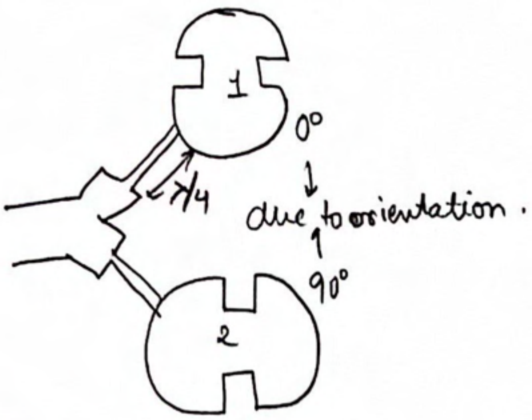
>

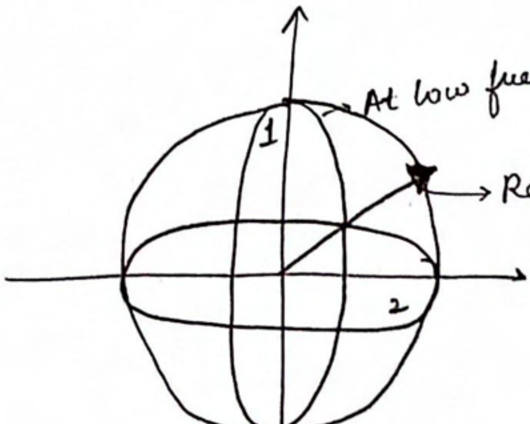


> Very broad band CP

Both are dominantly linearly polarized ⇒ at a large distance the polarization is ↻

>





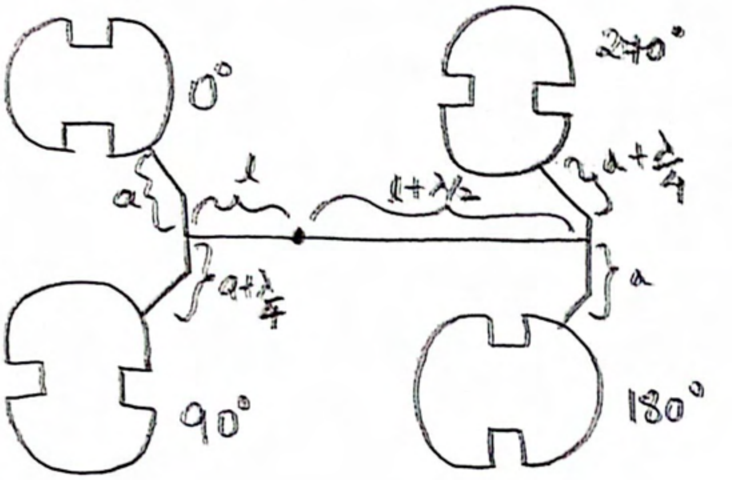
At low frequency (1) has ↑↑ field

Resultant is a perfectly circular antenna

> As f ↑, both become circularly polarized ⇒ circular.

> As f ↑↑, (2) is & (1) is ⇒ still circular
⇒ Very broadband.

> Could 4 elements → Sequentially rotated array



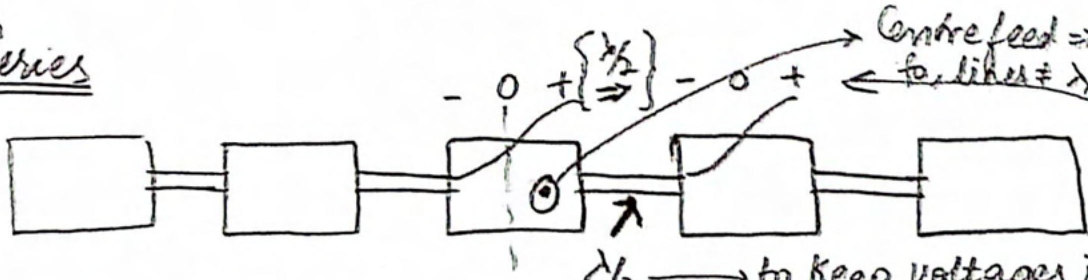
→ Also no need for $\lambda/4$ transformer since impedance of the 4 ends up in parallel to the feed.

> Could do 8 → 16 ... patches. VSWR & AR BW are better.
> Disadvantage: Gain is lower since phases dont add.

Lec 37 Feed Networks.

> Parallel feed 1-2-4-8-16... aka Corporate feed.
> Series feed → :) reduced feed length, (low loss), :(Narrow BW
:) lower sidelobe. :(Beam tilt with f

Series



Centre feed ⇒ for f different from f_0, lines ≠ $\lambda/2$ ⇒ delay on both sides is equal & beam tilt from both sides is equal & cancelled.

$\lambda/2$ → to keep voltages in phase.

> Odd no. of elements for symmetry.
> Lines must be $\simeq \lambda/2$ (due to fringing field it is not $= \lambda/2$).
> Impedance of lines? Since it is $\lambda/2$ line does Z_0 of line not matter? → Since $\lambda/2$ line ⇒ $Z_L = Z_S$. No because if W↑, fringing fields ↓ ⇒ effective length changes ⇒ $Z_S \neq Z_L$.

> As no. of elements are added loading effects propagate back on the line ⇒ designing is hard.

> MM wave: Lot of bands exist that have high absorption.

> 34-36 GHz → Least absorption.
> 60 GHz → Very high absorption.
> 94 GHz → Lower absorption.
> 140 GHz → " "
> 220 GHz → " "

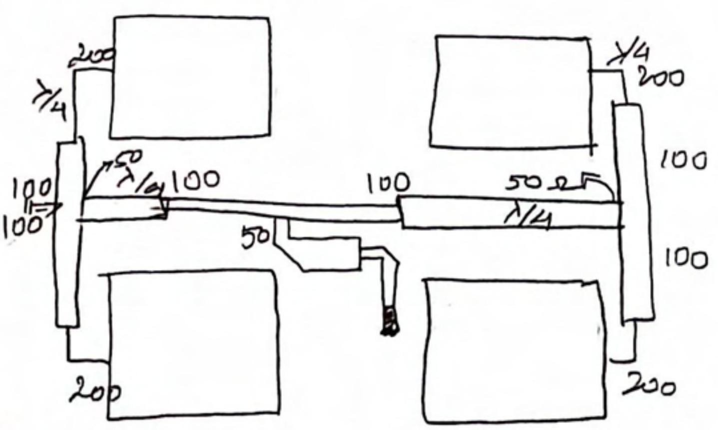
> Why is 60-67 GHz so popular? Exactly due to path loss. Signal can be kept bounded.
> High data rate confidential communication.

> Water resonates ≃ 24 GHz, 184 GHz, 350 GHz.
> O_2 ≃ 60 GHz & 130 GHz.

Lec 38

> Central feed of series array gives a natural amplitude variation or taper & recall arrays with tapered feed have lower side lobe levels.

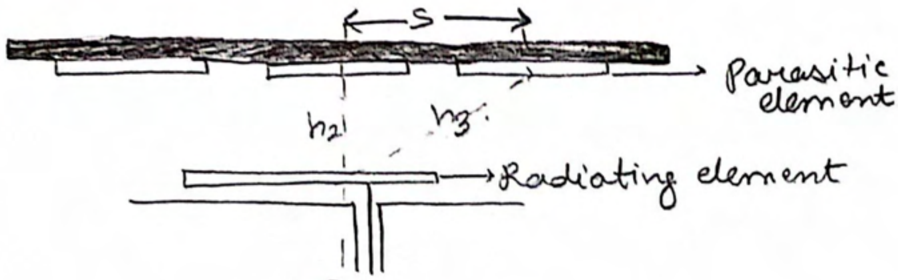
Corporate feed



> Use λ/4 line to go up & down in impedance.
> Modular ⇒ can extend the array further × 2 ...

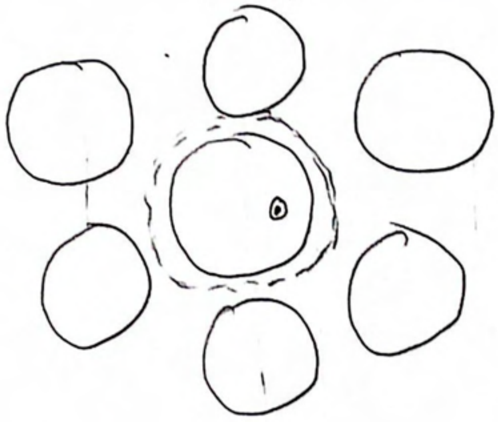
Lec 39 Space Fed MSA Array.

> To avoid large feed network (loss)



> $h_3 > h_2 \Rightarrow$ phase delay exists

> So make dimensions of ③ shorter to compensate.



> Low sidelobes. → natural taper!

> High frequency since dimensions are very small & feed is minimum.

> It is narrowband.

> orthogonal polarization requires very complex feed networks which are simplified here since only main element defines polarization

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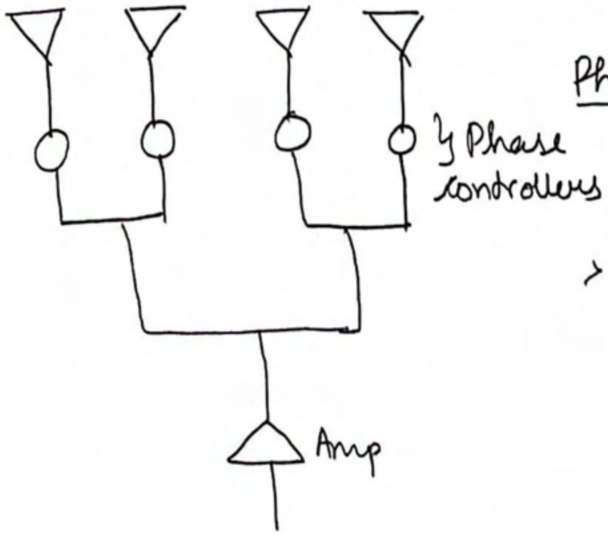
Phased Arrays. [Hot topic of research]

>



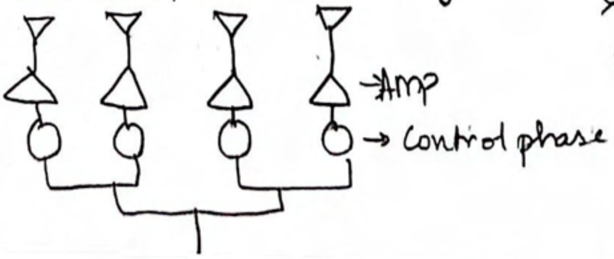
Phase: 0° 0° 0° 0° $\Rightarrow$

0° -10° -20° -30° $\Rightarrow$

> Phase controllers are quite complex.

- Analog or Digital.
  - Analog ↳ Varactor diodes
  - Digital ↳ PIN diodes.

> Active phased arrays

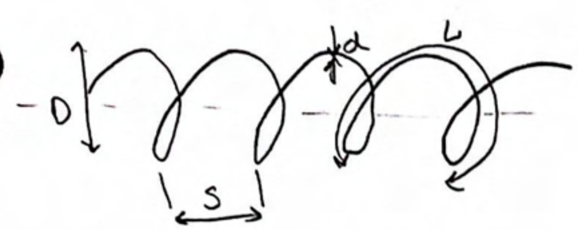


> High power!. But it is only a TX antenna.

> For bidirectional use T · R module. Which is a bidirectional amp with good isolation.
(T → TX, R → RX)

> Very expensive

Lec 40 Helical Antenna

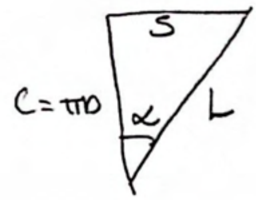


Total wire length $= nL$ — length of a single turn

$A =$ Total axial length $= nS$

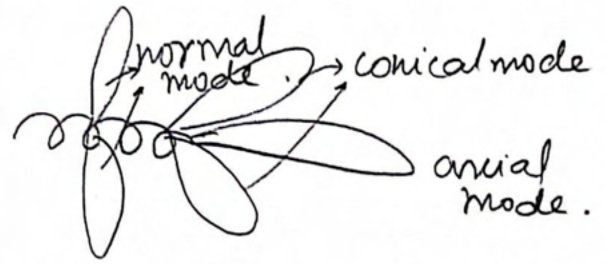
$L = \sqrt{S^2 + C^2}$ where $C = \pi D$

$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) = \tan^{-1}\left(\frac{S}{C}\right)$



$\alpha = 0° \Rightarrow$ loop antenna. $(S=0)$

$\alpha = 90° \Rightarrow$ linear antenna $(D=0)$.

→ 

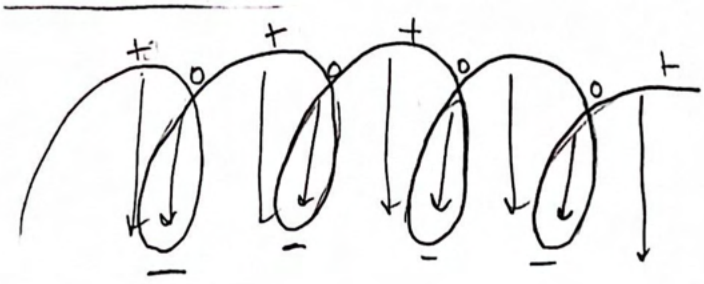
normal mode : $C = \pi D \ll \lambda$ → small diameter.

Axial mode : $C \approx \lambda$ ⟶ large diameter

not used ← Conical mode : $C \simeq n\lambda,\ n = 2, 3 \ldots$

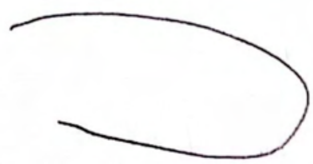
> Can easily generate circular polarization. (α is from 12°-14°)

Axial mode. $C = \lambda \Rightarrow$ each turn is in phase.

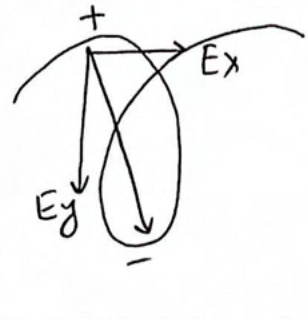


Field ↓ L

⇒ Radiation

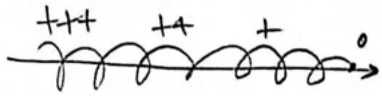


> Think of it as an endfire array of n elements.

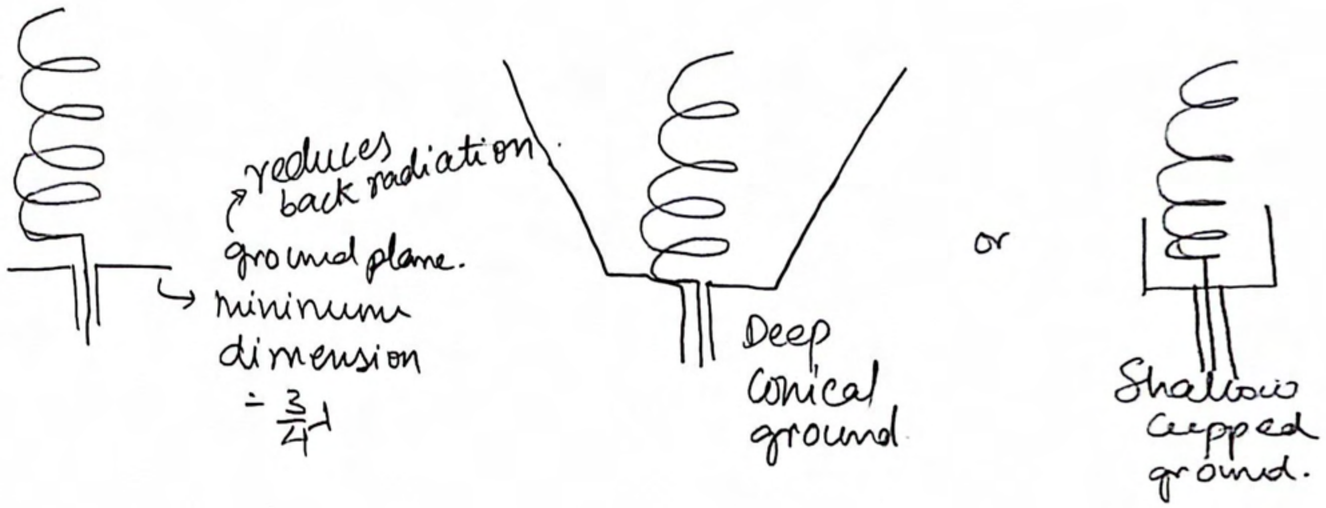


Since field is diagonal, by choosing the rise angle (α) & pitch correctly we can get $|E_x| = |E_y|$ & a phase difference of 90°

> Normal mode is like a monopole antenna. since ifield (↳ current) varies from +++ → ++ → + → 0 as we move along. ($\because C << \lambda$)



Axial mode.



> <u>Conductor diameter</u>

> Thicker diameter is self sustaining but weight↑. But it can handle very high power.

> Axial mode ⇒ diameter doesn't affect

Normal mode ⇒ BW ∝ diameter

> It is a travelling wave structure since little current is reflected back from open end. Most of power gets radiated before.

⇒ R is high.

$$R = 140^* C_\lambda \ \Omega \quad \text{for axial feed.}$$

<u>Lec 41</u> $R \cong 150/\sqrt{C_\lambda} \ \Omega$ for peripheral or circumferential feed.

> Imaginary part ≃ 0.

> These are valid for a) $0.8 \leq C_\lambda \leq 1.2$

b) $12^\circ \leq \alpha \leq 14$ → (circular)

c) $n \geq 4$ ⇒ or else reflected current would be large.

> Wide bandwidth

Matching.

1> Tapered transition.
2> Tapered microstrip.

> Don't use $\lambda/4$ → Kills BW.

> Axial mode endfire ⇒ $HPBW \simeq \dfrac{52}{C_\lambda \sqrt{n S_\lambda}}$ $BWFN = \dfrac{115}{C_\lambda \sqrt{n S_\lambda}}$

$$D = \frac{3200}{(HPBW)^2} \Rightarrow \boxed{D = 12 C_\lambda^2 n S_\lambda} \ \& \ \boxed{Gain = \eta \times D}$$

η → 60%

> For 1GHz ; $\lambda = 30cm$ ⇒ $C_\lambda = 1.05 \times 30$
$S_\lambda = 0.2366 \times 30$
$\eta = 60\%$
$\alpha \simeq 12.7°$

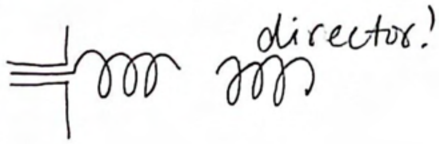
Preffered values

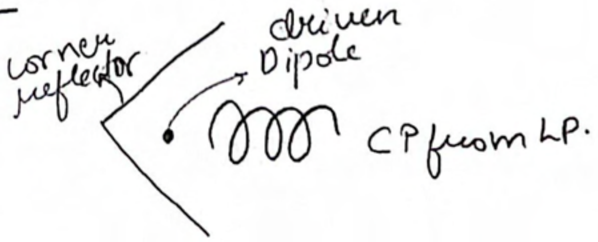
$$\Rightarrow \quad n = \frac{251.19}{12 \cdot [(1.05) \times 30]^2 [0.2366 \times 30]^2}$$ if desired $D = 251.19$

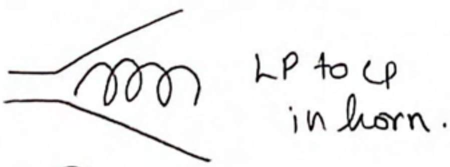
Lec 42

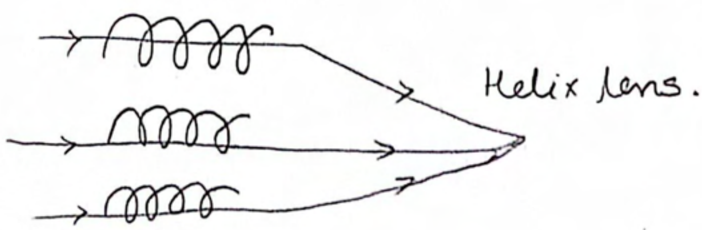
> If n is too large you can use an array.

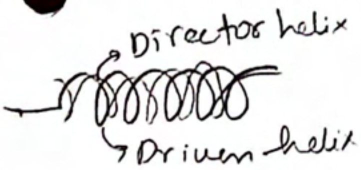
> Helix as a Parasitic Element







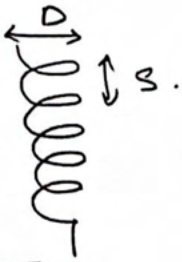




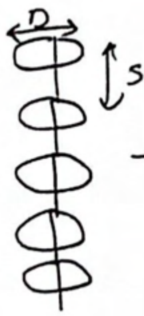
Lec 43

Normal mode

$$AR = \frac{|E_\theta|}{|E_\phi|} = \frac{2S_\lambda}{C_\lambda^2}$$



$\Longleftrightarrow$



→ Combination of small loops & small dipoles. Which are orthogonal.

Circular $\Rightarrow$ $C_\lambda = \sqrt{2S_\lambda}$

> It can receive both linear in horizontal & vertical polarizations

> Good for communications indoor.

> Wirelength should be > $\lambda/4$ since it is a slow wave structure. It depends on the pitch.

→ Feed here for matching
→ very low impedence!

} This shifts the resonance frequency.

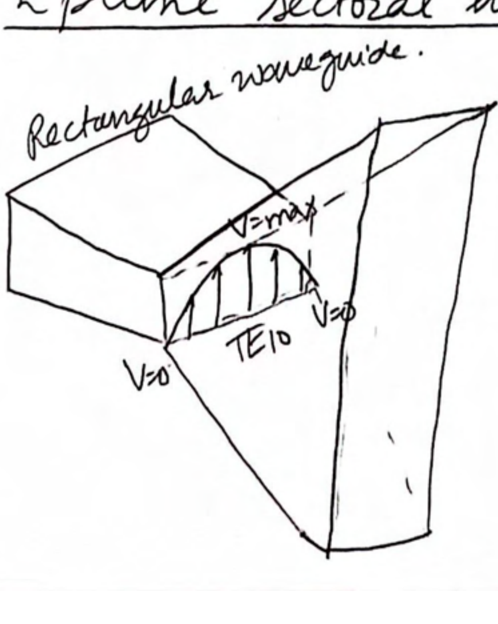
Lec 44.

> As ground plane size ↑ f_0 ↓.

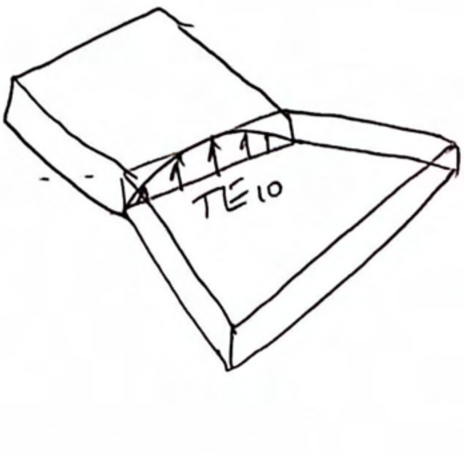
> As wire radius ↓ f_0 ↓. → pretty sensitive for normal mode.

Lec 45 Horn Antenna.

E plane sectoral horn



H Plane Sectoral horn

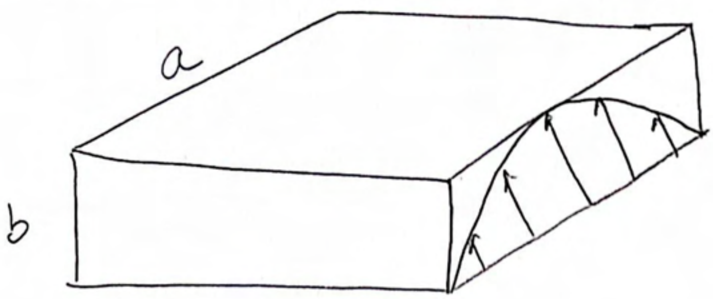


> Rectangular waveguide feed is used.
 TE10 mode => $\frac{1}{2}$ wavelength along horizontal.

> Dimension expanded in E plane ↕ => E plane horn
> Dimension expanded in H plane ⇔ => H plane horn.
> " " in both planes => Pyramidal horn.

> Circular => Conical Horn. (uses circular wave guide).

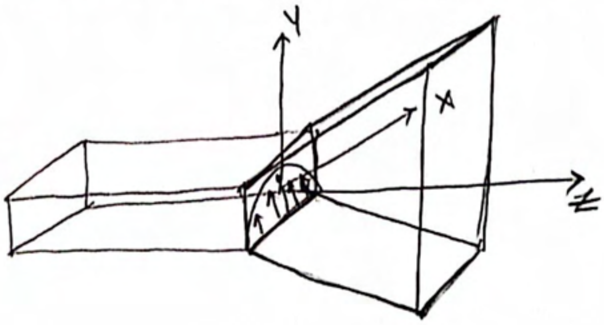
Rectangular Waveguides.



TE10 mode.

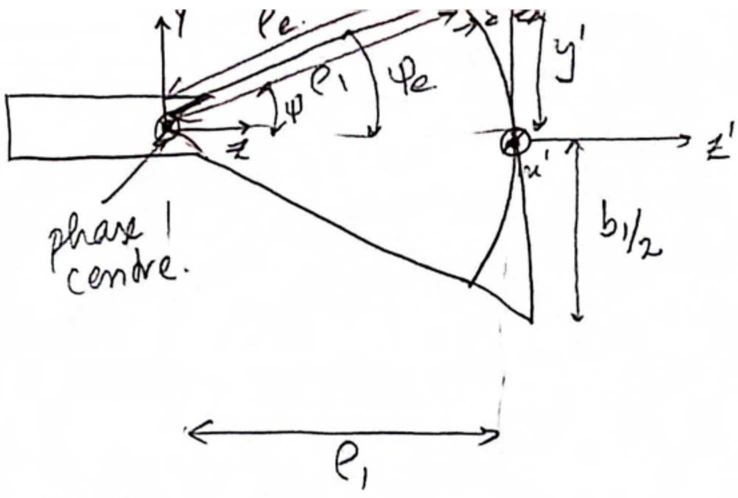
→ E field varies sinusoidally along a.
→ No variation along b.

> Cutoff wavelength = 2a = λ_{cutoff}.
> Cutoff frequency = c/λ_{cutoff}.
> High pass filter response.
> Need to use waveguide beyond 1.3 times f_{cutoff}.
> At higher frequencies different modes get excited, therefore TE10 mode is sustained in a particular range.



> E only exists in y direction.

$$E_y'(x', y') = \underbrace{E_1 \cos\left(\frac{\pi}{a}x'\right)}_{\text{Amplitude distribution}} \underbrace{e^{-j\frac{k y'^2}{2\rho_1}}}_{\text{Phase distribution}}$$

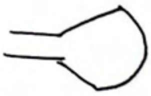


<u>Phase error</u>

$$\delta(y') = -\rho_1 + \rho_1\left[1 + \left(\frac{y'}{\rho_1}\right)^2\right]^{1/2}.$$

$$\Rightarrow \boxed{\delta(y') \simeq \frac{1}{2}\left(\frac{y'^2}{\rho_1}\right)}$$

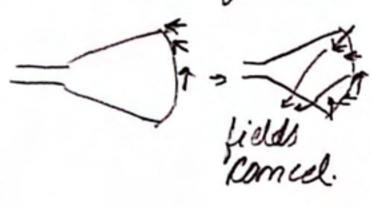
$\delta(y') \times K$ gives it in angle form. $\Rightarrow$ phase error is $e^{\frac{-jKy'^2}{2\rho_1}}$

> Even when horn is cut like  the phase error exists because we look at plane waves.

> Could use a lens antenna at the end to compensate the phase.

> For fixed ρ_1, if $b_1 \uparrow$, Directivity has an optimum value.

> Max Directivity at $\boxed{b_1 = \sqrt{2\lambda\rho_1}}$ $\longrightarrow$ Since phase error $\uparrow$ & fields start cancelling



> Max phase error at $y' = \frac{b_1}{2}$

$\Rightarrow$ $\delta_{max} = 2\pi S$ where $S = \frac{b_1^2}{8\lambda\rho_1}$

$\Rightarrow$ $\delta_{max} = 90^\circ \longrightarrow$ max phase error.

$\hookrightarrow$ not recommended to operate here. } Phase error should be $\leq 45^\circ$

> $\delta_{max} = 90° \Rightarrow$ efficiency is low since 40% of power is radiated along null directions.

Lec 46 H plane sectoral horn

$$E_y' = E_2 \cos\left(\frac{\pi}{a_1} x'\right) e^{-j\frac{k}{2}\left(\frac{x'^2}{\rho_2}\right)}$$

Max. phase error $= x' = a_1/2$

$\delta_{max} = 2\pi t$, where, $t = \frac{a_1^2}{8\lambda\rho_2}$

Max directivity: $a_1 = \sqrt{3\lambda\rho_2}$

Again there is an optimum.

$\delta_{max} = 2\pi t$ where $t = \frac{a_1^2}{8\lambda\rho_2}$

design for $\delta \in [45° \to 90°]$

$$t_{optimum} = \left.\frac{a_1^2}{8\lambda\rho_2}\right|_{a_1 = \sqrt{3\lambda\rho_2}} = \frac{3}{8} \Rightarrow \delta_{max} = 135°$$

not recommended

Pyramidal horn

Large phase error.

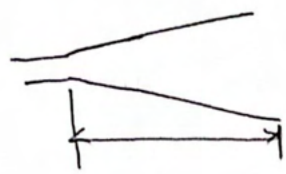
$$E_y'(x'y') = E_0 \cos\left[\frac{\pi}{a_1} x'\right] e^{-j\left[k\left(\frac{x'^2}{2\rho_2} + \frac{y_1^2}{2\rho_1}\right)\right]}$$

Condition for physical realisation: $P_e = P_h$.

Where, $P_e = (b_1 - b)\left[\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}\right]^{1/2}$

$P_h = (a_1 - a)\left[\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}\right]^{1/2}$

Neck to mouth.



→ Phase centres are different for both.

Design Procedure.

> Choose the right waveguide

> Use directivity curves of E&H planes to design for D.

> Or use its aperture to find directivity. Gain

$$G_0 \cong \underset{\eta = 50\%}{\frac{1}{2}} \underbrace{\left[\frac{4\pi}{\lambda^2} a_1 b_1\right]}_{\text{Directivity}}$$

> $a_1 = \sqrt{3\lambda \rho_h}$ assuming $\rho_2 = \rho_1$.

$b_1 = \sqrt{3\lambda \rho_e}$ " } if aperture << length.

> $$\rho_e = (b_1 - b)\sqrt{\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}}$$

$$\rho_h = (a_1 - a)\sqrt{\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}}$$

Lec 47

Pyramidal horn design steps.

1. $\chi = \chi_1 = x\,(\text{trial}) = \dfrac{G_0}{2\pi\sqrt{2\pi}}$ → initial guess of χ
2. $\rho_e = \chi\lambda$, $\rho_h = \dfrac{G_0^2}{8\pi^3} \cdot \dfrac{1}{\chi} \cdot \lambda$
3. $a_1 = \sqrt{3\lambda\rho_2} = \sqrt{3\lambda\rho_h} = \dfrac{G_0}{2\pi}\sqrt{\dfrac{3}{2\pi\chi}}\,\lambda$

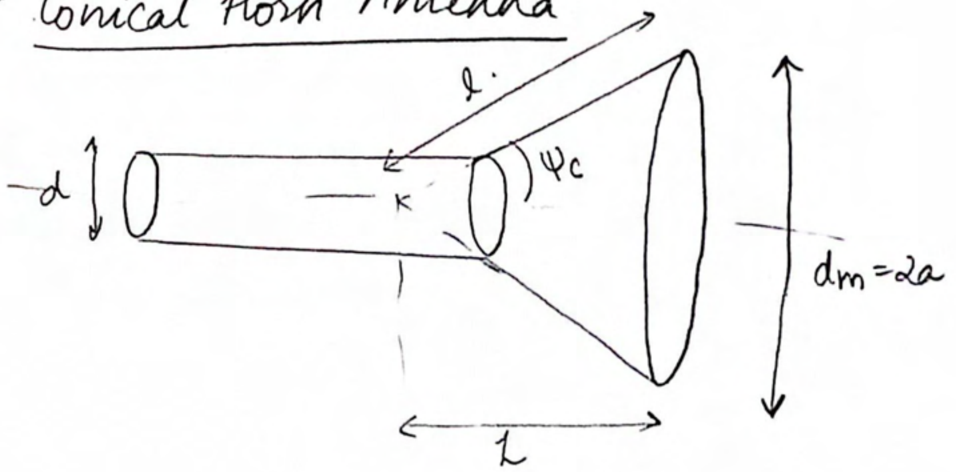
   $b_1 = \sqrt{2\lambda\rho_1} = \sqrt{2\lambda\rho_e} = \sqrt{2\chi\lambda}$
4. ρ_e, ρ_h.

> Kraus' book has a graph that makes design simpler.

> Refer to the video for feed information.

Lec 48

> Conical Horn Antenna



> As $d_m \uparrow$, Directivity increases (but it has an optimum)

> $S = \frac{d_m^2}{8\lambda l}$ = maximum phase deviation (in λ)

> Gain is optimum when $d_m \simeq \sqrt{3\lambda l}$

Thus,

$$S\Big|_{\text{optimum gain}} = \frac{d_m^2}{8l\lambda}\Bigg|_{d_m=\sqrt{3\lambda l}} = \frac{3\lambda l}{8\lambda l} = \frac{3}{8} \Rightarrow \delta_{max} = 135^\circ$$

> But the phase error is too high at this optimum $\Rightarrow$ not recommended

> Refer to the video for feed types.

> Tapered or Corrugated horn antennas are used for high power symmetric radiation.

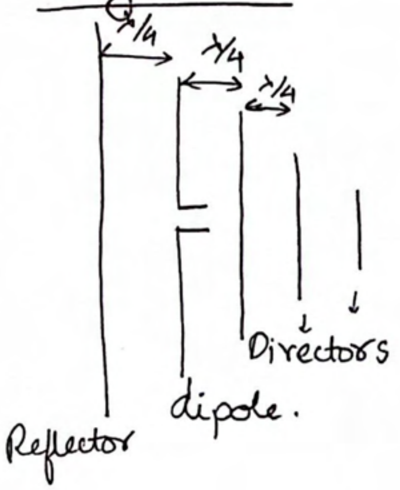
Lec 49

> Corrugated horn antenna.
- E plane wave is symmetrical.

> Broadband horn antenna (with fairly constant gain)

Lec 50 Yagi Uda & Log Periodic antennas.
(High gain) (High BW)

Yagi Uda.



Only dipole ⇒ Gain ≅ 2dB
With reflector ⇒ gain ≅ 5dB

→ $l + d = 0.48\lambda$

→ $L_{ref} > l > L_{dir}$

→ Endfire configuration. 3 elements ⇒ 1 director ⇒ gain ≅ 7dB.

→ Spacing ≅ $\lambda/4$ ⇒ phase diff is ≅ $\pi/2$ ⇒ Endfire.

> More directors ⇒ more gain.

> All directors may be of equal length.

> Director length : $(0.4 - 0.44)\lambda$

> Feeder length : $(0.44 - 0.48)\lambda$ → Usually folded dipole

> Reflector length : $(>0.5)\lambda$

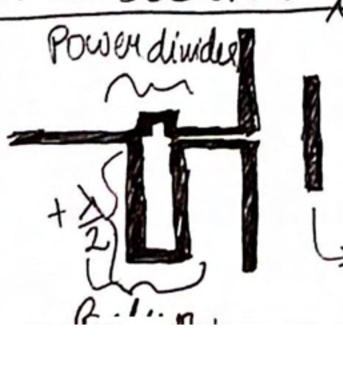
> Reflector-feeder spacing : $(0.2 - 0.25)\lambda$

> Director spacing : $(0.25 - 0.3)\lambda$

> No. of elements ↑ ⇒ D ↑

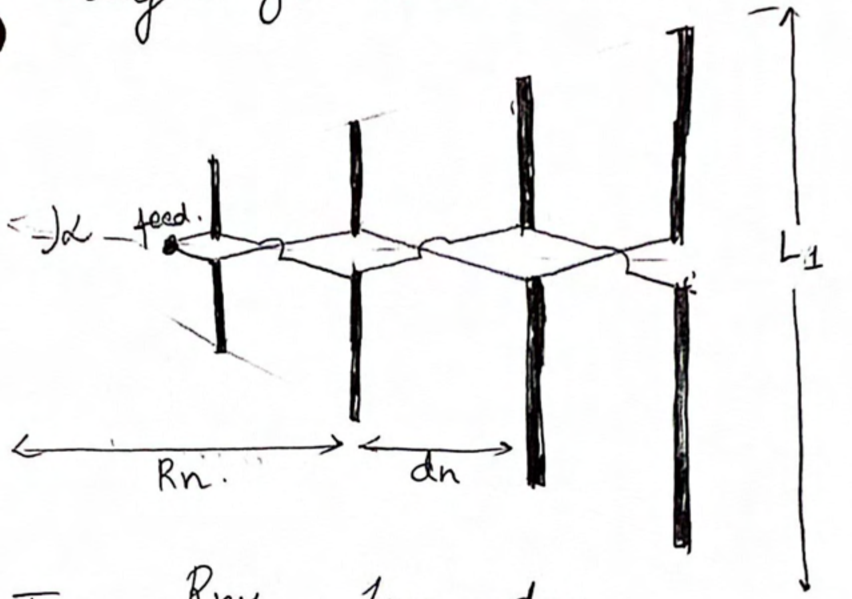
Lec 51

* Broadband Quasi Yagi Uda.



Log Periodic antenna

> Very high bandwidth.



> N numbers of dipoles with alternating fields.

> Spacing $\simeq \frac{\lambda}{2}$ since phase diff. is 180°?

> $\tau = \dfrac{R_{n+1}}{R_n} = \dfrac{L_{n+1}}{L_n} = \dfrac{d_{n+1}}{d_n}$

$\tan\frac{\alpha}{2} = \dfrac{L_n/2}{R_n} = \dfrac{L_{n+1}/2}{R_{n+1}}$

> Even diameters of these dipoles must vary with τ.

> space factor $\sigma = \dfrac{d_n}{2L_n}$ $\quad d_n = R_n - R_{n+1}$

$\Rightarrow R_n = \dfrac{L_n}{2\tan(\alpha/2)}$

$d_n = R_n - \tau R_n = (1-\tau)R_n$

$\sigma = \dfrac{d_n}{2L_n} = \dfrac{1-\tau}{4\tan(\alpha/2)}$

$\alpha = 2\tan^{-1}\left(\dfrac{1-\tau}{4\sigma}\right)$

Design

- Let f_L & f_U be low f & high f required.

- $L_1 \simeq \frac{\lambda_L}{2}$

  $L_N \simeq \frac{\lambda_U}{2}$

These two steps combine Yagi Uda & Log periodic.

- Add one large dipole for λ_L which acts as a reflector to increase gain at low. freq.
- Add a few small dipoles in front for λ_U, to act as directors to increase gain at high freq.

Lec 52

- Look at τ vs. σ graph to choose optimum values for a given gain.
- Calculate α from τ & σ.
- Choose L_1 & L_n. & Keep multiplying with τ till $L_1 \xrightarrow{\text{reaches}} L_n$.
- Add a reflector & directors if needed.
- Diameters should be b/w $\lambda/100$ & $\lambda/20$. Choose accordingly.

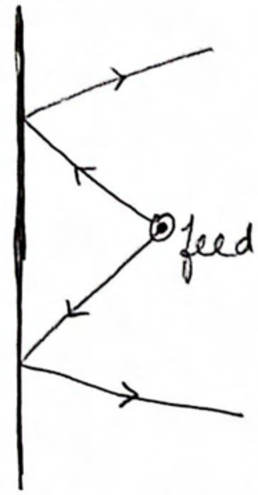
Lec 53 — Lec 55

↳ On IE3D

Lec 56 Reflector Antennas.

> Very very high gain!

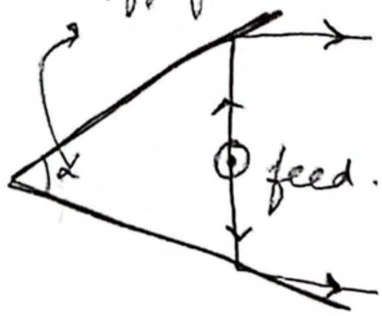
> Flat reflectors



Planar

If $\alpha = 90°$, any rays falling on it are reflected back in the same direction

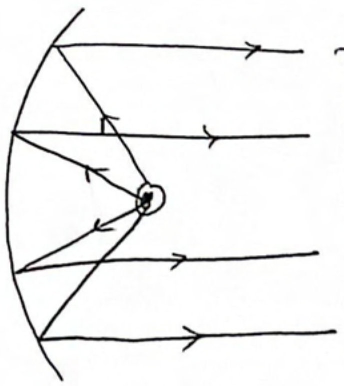
Great for radar detection.



Corner

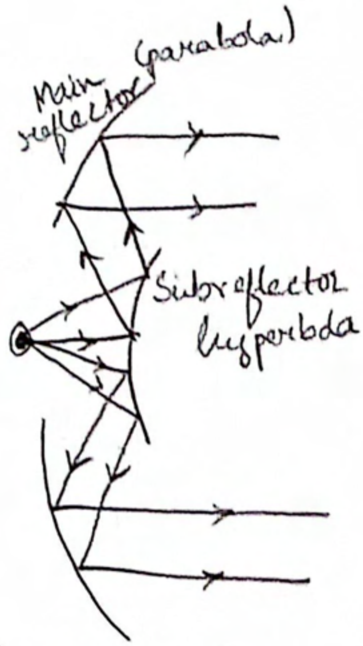
Used in ships for very good RCS.

> Curved reflectors



All have same phase.

> Prime Focus reflector. (parabolic)



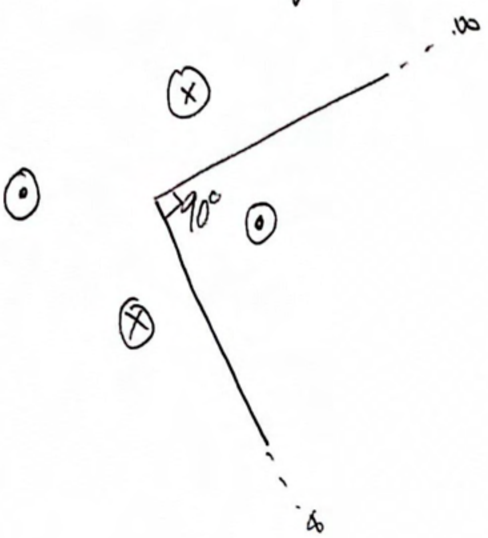
> Cassegrain reflector.

> For very high power feed to avoid blockage & connections → in case of offset feed

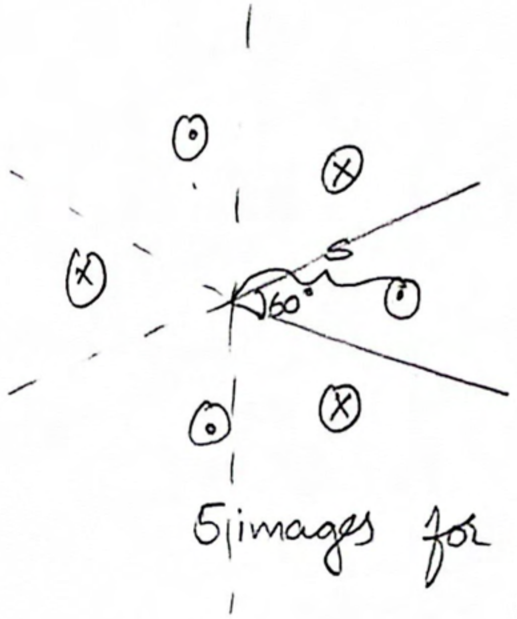
> Subreflectors still cause blockage.

Lec 57

Corner reflector:



3 images.



5 images for 60°.

> No. of images = $\frac{360}{\alpha} - 1$

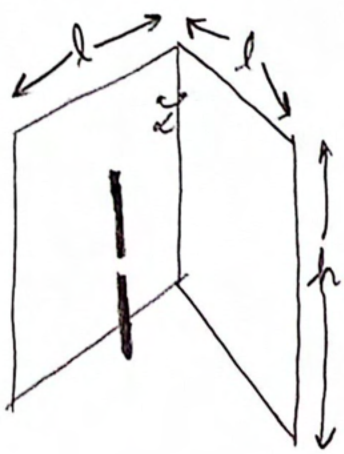
> Smaller $\alpha \Rightarrow$ higher gain.

> Total field is sum of contribution of feed & images.

> Use array theory!

> As $S \uparrow$ pattern splits from broadside to conical. So keep S b/w 0.1λ to 0.5λ (since reflector size is $< \infty$).

> $h \approx \lambda$; $l \geq 2 \times S$;



> $S \ll 0.7\lambda$ for $\alpha = 90°$

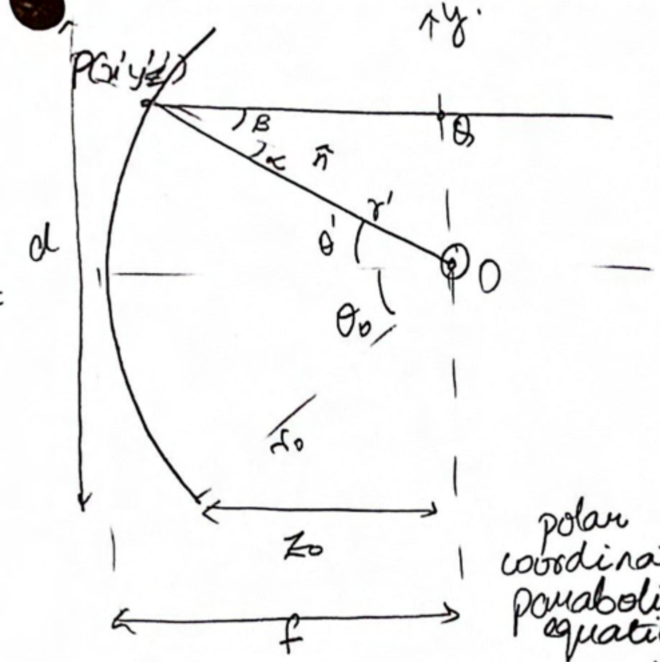
$S \ll 0.95\lambda$ for $\alpha = 60°$

$S \ll 1.2\lambda$ for $\alpha = 45°$

$S \ll 2.5\lambda$ for $\alpha = 30°$

> Try to keep S low since ground plane is never ∞ & l would be small.

Parabolic reflector



$>$ $OP + PQ = \text{constant} = 2f$. $\Rightarrow$ in phase

$>$ $OP = r' \qquad PQ = r' \cos\theta'$

$\Rightarrow r'(1+\cos\theta') = 2f$

$$r' = \frac{2f}{1+\cos\theta'}$$

polar coordinate parabolic equation $\leftarrow$ $\boxed{r' = f \sec^2\left(\frac{\theta'}{2}\right).}$ $\qquad \theta \le \theta_0$

$$\because r' + r'\cos\theta = \sqrt{(x')^2+(y')^2+(z')^2} + z' = \underline{2f}$$

$>$ $\theta_0 = \tan^{-1}\left(\frac{d/2}{z_0}\right)$

$$\theta_0 = \tan^{-1}\left|\frac{\frac{d}{2}}{f - \frac{d^2}{16f}}\right| = \tan^{-1}\left|\frac{\frac{1}{2}\left(\frac{f}{d}\right)}{\left(\frac{f}{d}\right)^2 - \frac{1}{16}}\right|$$

$$\boxed{f = \frac{d}{4}\cot\left(\frac{\theta_0}{2}\right)}$$

f/d	0.4	0.5	0.6	0.7	0.8	1.0
θ_0	64.0	53.1	46.2	39.3	34.7	28.1

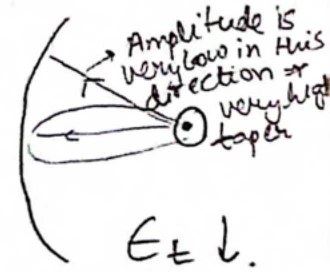
53.1: Generally chosen for prime focus

28.1: Chosen for Cassegrain.

Aperture efficiency.

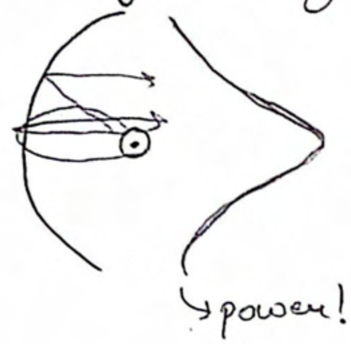
$$\epsilon_{ap} = \underbrace{\epsilon_s \cdot \epsilon_t}_{\text{design}} \cdot \underbrace{\epsilon_p \cdot \epsilon_x \cdot \epsilon_b \cdot \epsilon_r}_{\text{fabrication}}.$$

ϵ_s - spillover } Depends strongly on radiation pattern of feed.
ϵ_t - taper }
ϵ_p - phase
ϵ_x - polarization → imperfect polarization.
ϵ_b - Blockage → Feed blocks radiation. (use offset feed)
ϵ_r - random error → manufacturing doesn't give perfectly smooth surface.

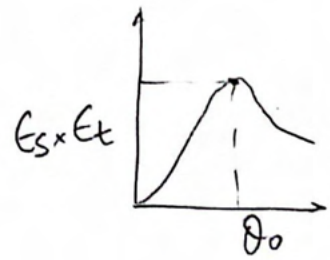


$$\text{Gain} = \epsilon_{ap} \cdot D_u = \epsilon_{ap} \cdot \frac{4\pi}{\lambda^2} \cdot A_p.$$

> Taper high ⇒ the plane of radiation after reflection has a nonuniform steep taper. From array theory ⇒ low gain



> Product of ϵ_s & ϵ_t is aperture efficiency if the rest are ignored.
> It has an optimum!



Lec 59

> Phase efficiency :- Parabola is not perfect. ⇒ phase of reflected wave is not all equal. At higher frequencies ⇒ problematic.
> Could use parabolic cylindrical antenna (airports).