Device Physics 413

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ϕ → potential
E → Electric field.
V → Potential difference
F_e → Force on an $\bar{e}$.
$\langle v \rangle$ → avg. velocity in one direction
μ_n → mobility.
τ → avg time b/w collisions.
m^* → effective mass
$J = \frac{I}{A}$ current density.
σ = Conductivity $= \frac{1}{\rho}$ → Depends on E_g & carrier conc. in bands.
E_{tot} = Energy of $\bar{e}$ in nth level.
E_g Si = 1.12 eV.
h = Planck's const $= 6.62 \times 10^{-34}$ Js
ϵ = permittivity.
n = energy level.
E_F = Fermi level.
E = Energy level
n, p = no. of $\bar{e}/h^+$ in CB/VB.
$k = 1.38 \times 10^{-23}$ $m^2 kg\, s^{-2} K^{-1}$

$$\phi = -\int_0^L \vec{E} \cdot \vec{dl}$$

$$F_e = -qE$$

$$\langle v \rangle = \mu_n \vec{E} \quad \text{where} \quad \mu_n = \frac{-q\tau}{m^*}$$

$[cm^2/Vs]$

$$J = n \mu_n \vec{E} q = \sigma \vec{E}$$

In H & other atoms

$$\boxed{E_{tot} = \frac{-13.6}{n^2} (eV)} = \frac{-p^2}{2m}$$

where,

$$p = \frac{m}{2\epsilon} \cdot \frac{q^2}{nh}$$

Carrier distribution (Fermi Dirac Distribution)

$$f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}}$$

Prob. of E_F to be occupied = 50%

E_F
→ Prob. of finding an electron at room temp. = 50%. (Semiconductors)
→ All levels below E_F are filled with $\bar{e}$s at 0K & all levels above are empty. (metals)

when $E - E_F > 3kT$

$$f(E) \simeq e^{\frac{E_F - E}{kT}} \quad \text{(Boltzmann distribution)}$$

$$n = N_c\, e^{\frac{E_F - E_c}{kT}}$$

$$p = N_v\, e^{\frac{E_v - E_F}{kT}}$$

Intrinsic Silicon

n_i is used as a reference point.

$$\boxed{E_F = \frac{E_c + E_v}{2}}$$

$E_F = E_i \quad \& \quad n_i = p_i$

$$\boxed{n_0 p_0 = n_i^2} \ ^*$$

$$\boxed{N_A^- + n^- = N_D^+ + p^+} \ ^*$$

Boron (p) [−ve ions] N_A = conc. of acceptors.

Phos. (n) [+ve ions] N_D = conc. of donors.

n = conc. of electrons

p = conc. of holes.

units (cm^{-3}).

Motion of Carriers

Drift :- Current due to electric field. (minority)

Note:- (q does not have sign)

$$\left.\begin{aligned} J_n &= (n\,q\,\mu_n)\vec{E} = \sigma_n \vec{E} \\ J_p &= (p\,q\,\mu_p)\vec{E} = \sigma_p \vec{E} \end{aligned}\right\} \vec{J} = \vec{J_n} + \vec{J_p}$$

$[\mu_p < \mu_n]$

Diffusion :- Current due to carrier conc. gradient & random [illegible] (majority)

$$J = -q\,D\,\frac{\partial \rho}{\partial x}$$

ρ = charge density

Note :- q is −ve for $\bar{e}$ & +ve for h^+

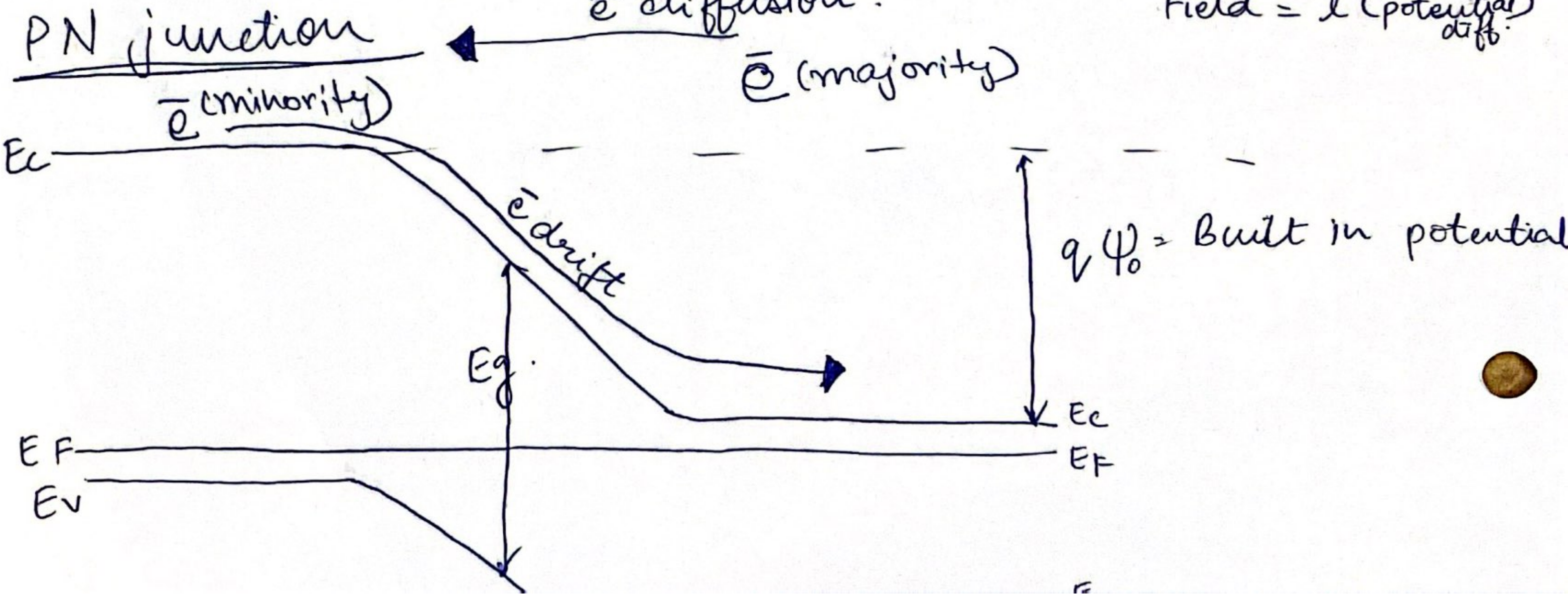
$$\frac{D}{\mu} = \frac{kT}{q}$$

$\propto e^{-\frac{q\psi_0}{kT}}$

Energy = q (potential diff)

Field = ℓ (potential diff)

PN junction



Forward bias $(+V_D)$

$V_T = \frac{KT}{q}$ thermal Voltage.

I_s = const.

$$I_D(V_D) = I_s\left(e^{\frac{V_D}{V_T}} - 1\right)$$

Reverse bias $(-V_D)$

$$I_D(V_D) = I_s\left(e^{\frac{-V_D}{V_T}} - 1\right) \simeq -I_s$$

Junction Capacitance $\boxed{C_j = \frac{\epsilon_s}{x_n + x_p}}$

$$\vec{E} = -\frac{d\phi}{dl} \Rightarrow \phi = -\int \vec{E}\cdot d\vec{l} \quad -③$$

$$\vec{E} = -\frac{1}{\epsilon_s}\int_{x_1}^{x_2} \rho(x)\,dx \quad -②$$

$\rho(x) \rightarrow$ doping conc. $= qN_D - qN_A$ └①

$$\left.\begin{aligned} \phi_p &= \frac{q N_A x_p^2}{2\epsilon_s} \\ \phi_n &= \frac{q N_D x_n^2}{2\epsilon_s} \end{aligned}\right\} \text{From ①, ② \& ③} \quad ④$$

$$\text{Drift} = \text{Diffusion} \Rightarrow \left.\begin{aligned} \phi_p &= -\frac{KT}{q}\ln\frac{N_A}{n_i} \\ \phi_n &= \frac{KT}{q}\ln\frac{N_D}{n_i} \end{aligned}\right\} ⑤$$

Barrier pot.

$$\varphi_0 = |\phi_p| + |\phi_n|$$

& $x_n N_D = x_p N_A$.
(Total charge on both sides)

④ & ⑤

$$\therefore \varphi_0 = \frac{KT}{q}\ln\frac{N_D N_A}{n_i^2} = \frac{1}{2}q\frac{N_A}{\epsilon_s}\cdot x_p^2\left(1 + \frac{N_A}{N_D}\right)$$

Solving

$$x_p = \sqrt{\frac{2\epsilon_s}{qN_A}\cdot\frac{N_D}{N_A + N_D}\cdot\varphi_0}$$

$$x_n = \sqrt{\frac{2\epsilon_s}{qN_D}\cdot\frac{N_A}{N_A + N_D}\cdot\varphi_0}$$

Depletion width $= x_p + x_n$.

$$x = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\varphi_0}$$

$$C_j = \left[\frac{q\epsilon_s}{2\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\varphi_0}\right]^{1/2} = \frac{C_{j0} = C_j \text{ when } V_D = 0}{\sqrt{1 - \frac{V_D}{\varphi_0}}}$$

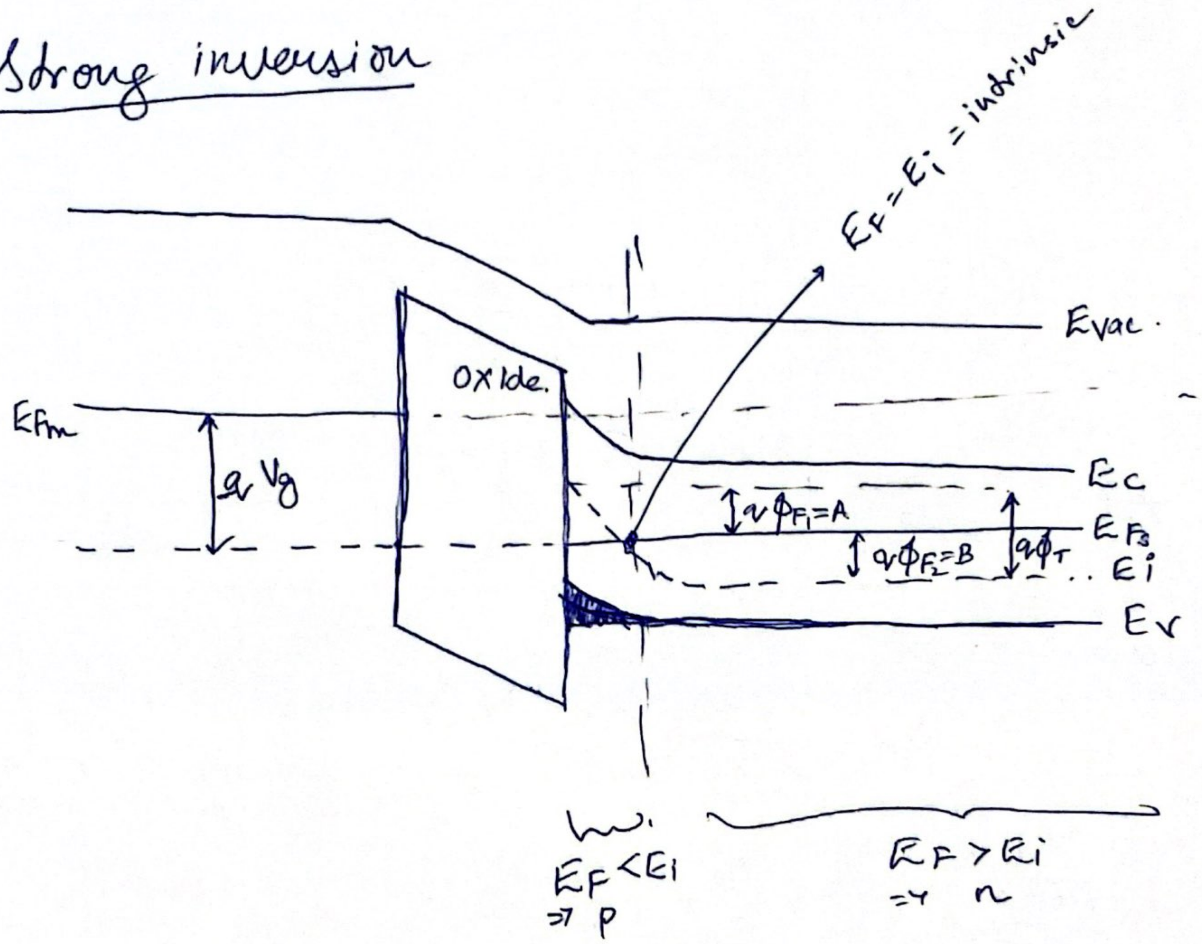
Replace with ($\varphi_0 - V_D$) when biased.

MOS CAP Accumulation; Weak/Strong inversion
$[V_G > 0]$ $[V_G < 0]$ → This is where we operate & channel forms.

For p type → It has n bulk.

$$\boxed{E_{Fm} - E_{Fs} = -qV_G}$$

Strong inversion



Strong inversion. $(V_g = V_{th})$

A → potential of 'p' at oxide surface.
B → potential of 'n' at bulk of substrate } A = B
⇒ $q\phi_{F_1} = q\phi_{F_2} = q\phi_F$

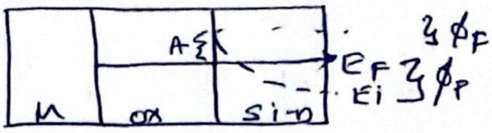
$$\therefore \boxed{\phi_T = 2\phi_F = \frac{2}{q}(E_F - E_i)}$$

Let potential drop across oxide = ϕ_{ox}

$\therefore V_g = \Delta\phi_{ox} + \phi_T$ } → Change in Fermi level of SC is higher than change in E_i, E_c & E_v.
Change in FL ⇒ V_g
Change in E_i, E_c & E_v ⇒ ϕ_T

$$\boxed{\phi_T = \frac{2KT}{q} \ln \frac{N_D}{n_i}}$$

$$\boxed{V_{th} = \phi_T + \Delta\phi_{ox}}$$



$$x_d = \sqrt{\frac{2\epsilon_{si}\phi_T}{qN_D}}$$ width of depletion region in Si.

$$|E_{si}(x=0)| = \sqrt{\frac{2qN_D\phi_T}{\epsilon_{si}}}$$ & $$E_{ox} = \frac{\epsilon_{si}}{\epsilon_{ox}}\sqrt{\frac{2qN_D\phi_T}{\epsilon_{si}}}$$

$E_{ox} \rightarrow \frac{\Delta\phi_{ox}}{t_{ox}}$

$$V_{th} = \phi_T + \Delta\phi_{ox}$$

* $$\boxed{V_{th} = 2\phi_F + \frac{\epsilon_{si}}{\epsilon_{ox}} t_{ox}\sqrt{\frac{2qN_D\phi_T}{\epsilon_{si}}}}$$

where, $\phi_T = 2\phi_F$
$$\left\{\phi_F = \frac{kT}{q}\ln\frac{N_D}{N_i}\right\}$$

* $$\boxed{V_{th} = 2\phi_F + \gamma\sqrt{2\phi_F}}$$

where,
$$\gamma = \frac{\sqrt{2q\epsilon_{si}N_D}}{C_{ox}}$$
$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

In reality,

$$\boxed{V_{th} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F}}$$ or

* $$\boxed{V_{th} = V_{tho} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]}$$

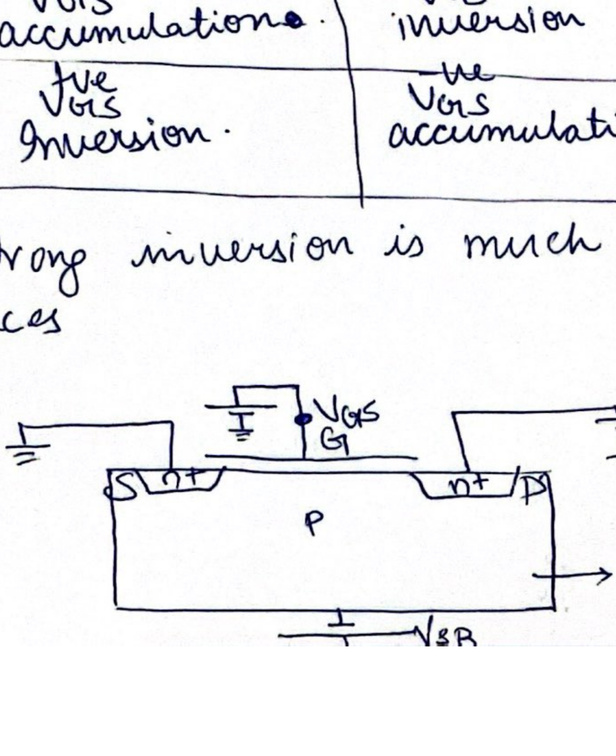
V_{FB} → Flatband voltage if the band is not flat + if carriers inside oxide

$$V_{FB} = \phi_{ms} - \frac{Q_{ss}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

Changes ~~on~~ surface & in oxide

p type mos	n bulk	+ve V_{GS} accumulation.	−ve V_{GS} inversion
n type mos	p bulk	+ve V_{GS} Inversion.	−ve V_{GS} accumulation

In MOSFET strong inversion is much faster
- 2 n^+ sources
- V_{SB}



→ at negative for easy strong inversion.

When $V_{SB} \neq 0$

$$\boxed{V_{th} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F + V_{SB}}}$$ $\Rightarrow$ V_{th} increases.

Current in MOSFET

$V_{GS} > V_{th}$

V_{DS} is small & +ve.

$\Rightarrow$ $$\boxed{I_D = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) V_{DS}}$$ acts as resistor.

V_{DS} is large. $\boxed{V_{DS} \leq V_{GS} - V_{th}}$

Triode *

$$\boxed{I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right]}$$ still not a current source

V_{DS} is even larger $\Rightarrow$ pinch off.

$$\boxed{I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L'} [V_{GS} - V_{th}]^2}$$

$V_{DS} = V_{GS} - V_{th}$

$L = L'$

not ideal since L' depends on V_{DS}. (CLM)

Let $\lambda = 1/V_A = \dfrac{\partial I_D}{\partial V_{DS}} \Big/ I_D = \dfrac{1}{L'} \cdot \dfrac{\partial X_d}{\partial V_{DS}}$

$\boxed{V_{DS} \geq V_{GS} - V_{th}}$

*. $$\boxed{I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [V_{GS} - V_{th}]^2 [1 + \lambda V_{DS}]} \rightarrow \text{CLM}$$

for ideal transistor :- $\lambda \downarrow$ $\Rightarrow$ $\underbrace{L\uparrow \text{ or Doping}\uparrow}_{\text{Both have issues.}}$

CLM Channel length modulation.

$V_{DS}\uparrow \Rightarrow x_d \uparrow \Rightarrow L' \downarrow \Rightarrow$ not ideal.

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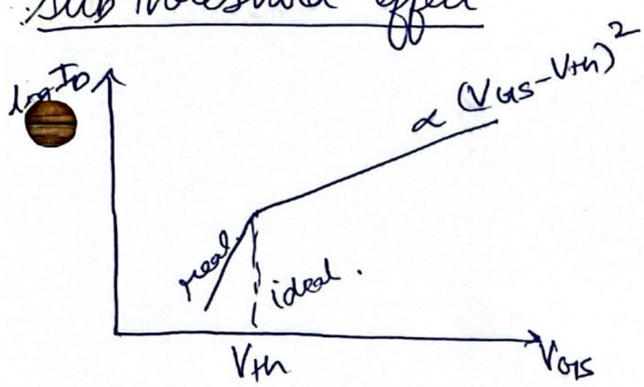
Sub threshold effect



$I_D \propto \frac{W}{L} e^{\frac{V_{GS}}{\eta V_T}}$

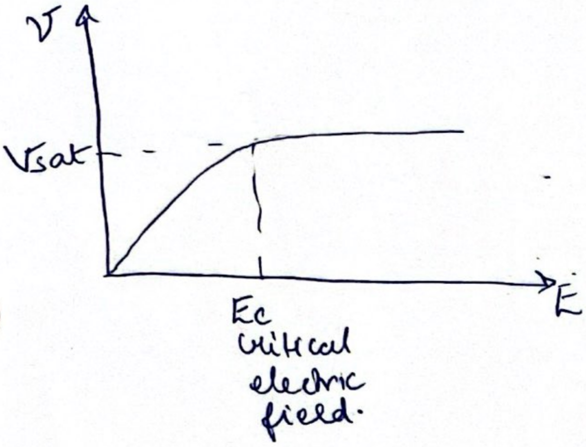
$g_m = \frac{I_D}{\eta V_T}$

We can use it by lowering V_{GS} & increasing $\frac{W}{L}$ => high gain but more parasitic capacitances.

Velocity saturation [Short channel devices]

Drift velocity saturates as $E\uparrow$

$\boxed{v_d = \mu \vec{E}}$ when $E < E_c$



Use $\mu = \frac{\mu_0}{1+\frac{E}{E_c}}$ to model this effect.

$$I_D = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \cdot \frac{(V_{GS}-V_{th})^2}{1+\frac{V_{GS}-V_{th}}{E_c \cdot L}}$$

Assuming. $\frac{V_{GS}-V_{th}}{E_c \cdot L} \gg 1 \Rightarrow I_d = \frac{\mu_n C_{ox}}{2} \cdot E_c W (V_{GS}-V_{th})$

$\therefore I_d \propto (V_{GS}-V_{th}) \Rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = \text{const}.$

General

$I_d \propto (V_{GS}-V_{th})^m$ $\quad 1 \le m \le 2$

small channel (m = 1) large channel. (m = 2)

Effects of Small dimensions

① Velocity saturation.

② Lower V_{th} because S&D depletion regions are significant.

③ Hot electron effect (Thin doping near channel) ④ DIBL. (lower r_o)

Voltage controlled current source

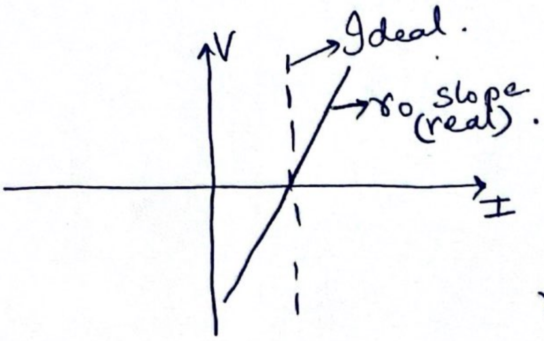
Transconductance

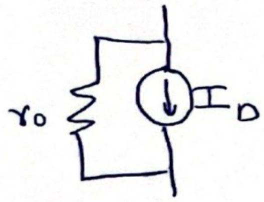
$$\boxed{g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})}$$

ignore λ.

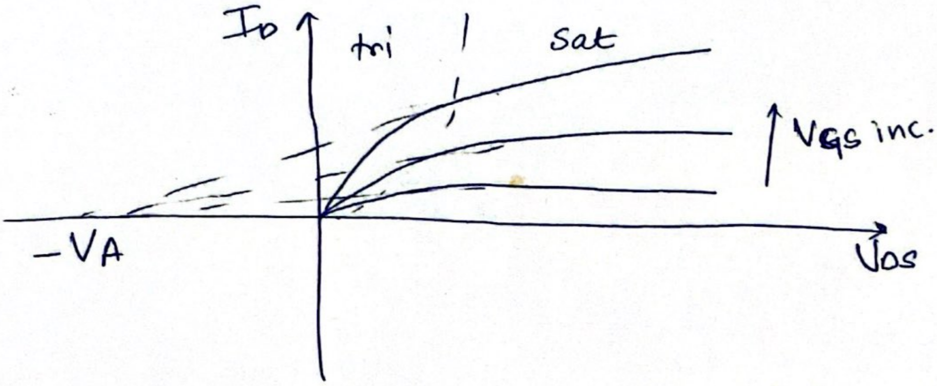
$g_m \propto \sqrt{I_D}$

$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2 (1 + \lambda V_{DS})$ → we can model this nonlinearity as a resistor. r_o



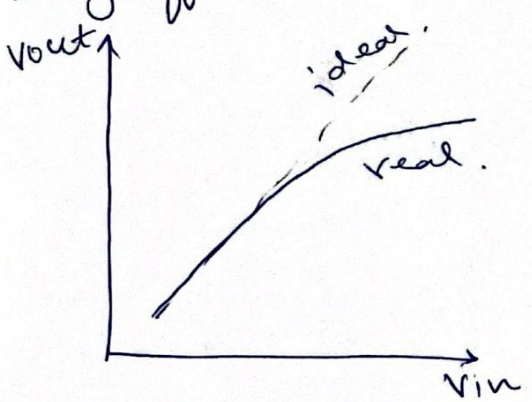


$$r_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{\lambda I_D} = \frac{V_A}{I_D}$$ V_A → Early voltage.



$$\boxed{\lambda = \frac{1}{L'} \left[\frac{dX_d}{dV_{DS}}\right]}$$

Body effect $V_{in}\uparrow \Rightarrow V_{out}\uparrow \Rightarrow V_S\uparrow \Rightarrow V_{SB}\uparrow \Rightarrow V_{th}\uparrow \Rightarrow I_D\downarrow$



I_D is affected by channel length modulation & body effect.

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}$$

$$\boxed{g_{mb} = g_m \left(-\frac{\partial V_{th}}{\partial V_{BS}}\right)}$$

$$g_{mb} = \underbrace{\frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}}_{\chi} \cdot g_m = \chi g_m$$

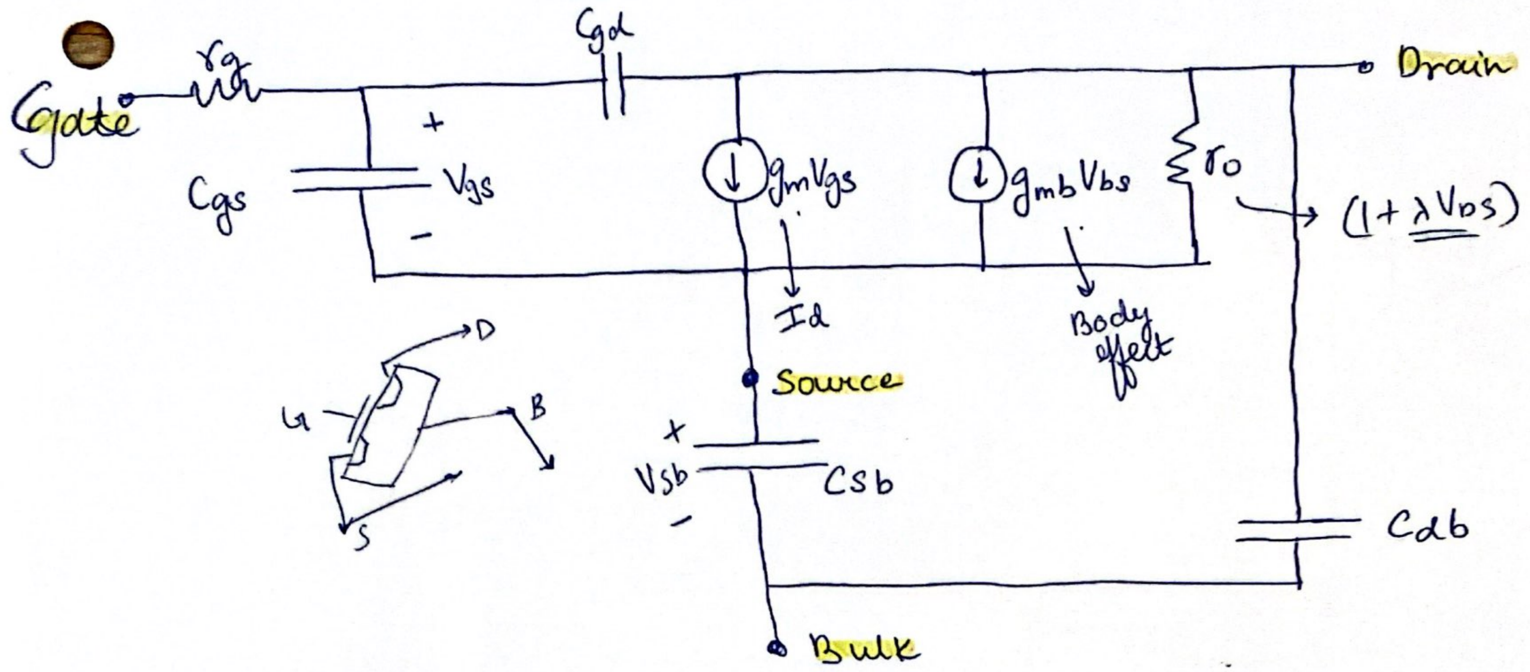
$\gamma = \sqrt{2q \epsilon_{Si} N_D}$

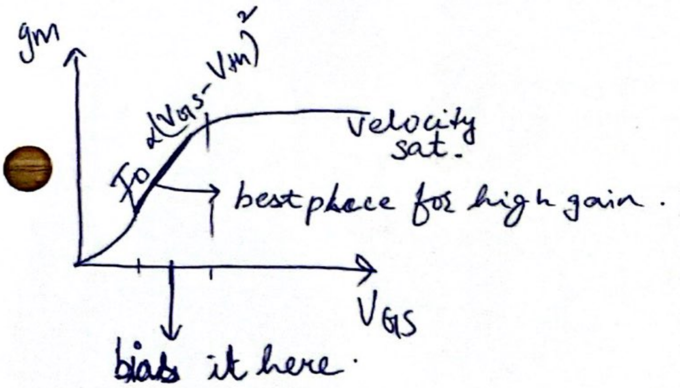
To fix it

→ Connect S & B

→ $\gamma\downarrow \Rightarrow N_d\downarrow$.

Small signal model





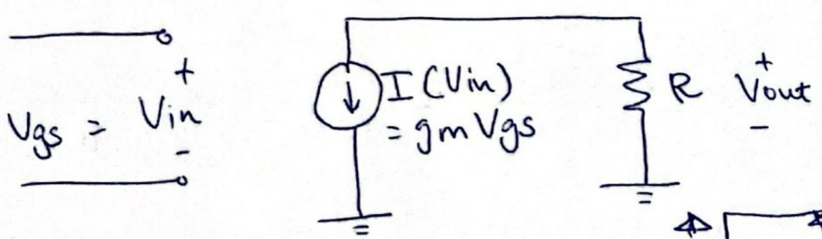
Cut off frequency

when current gain = 1

$$\boxed{f_T = \frac{1}{2\pi}\frac{g_m}{C_{gs}+C_{gd}}}$$

$f_T \propto \frac{V_{sat}}{L}$ → limiting factor of f_T is architecture.

Amplifier Design



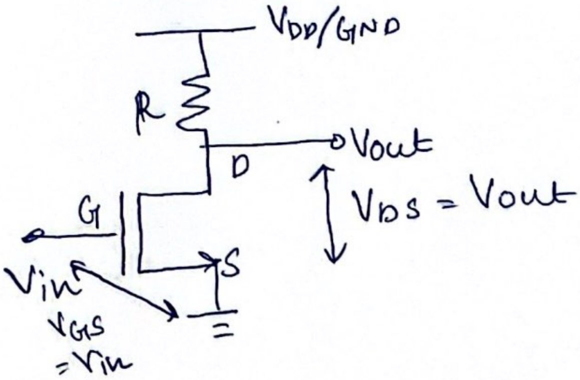
$V_{out} = R\,I(V_{in})$

$V_{out} = R \cdot g_m \cdot V_{in}$

gain

Av	$\frac{V_{out}}{V_{in}} = g_m \cdot R$

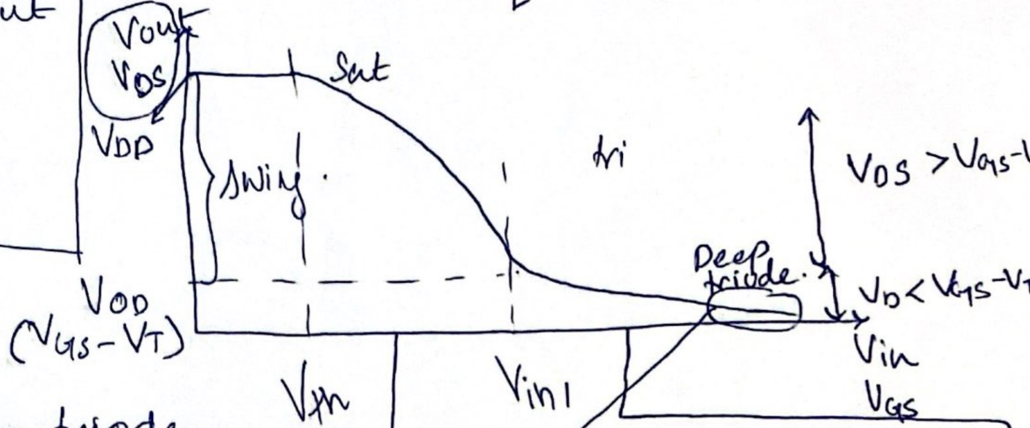
Common Source



Large signal (Always equate currents)

$$V_{out} = V_{DD} - I_D R_D$$

$$= V_{DD} - R_D \left[\frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \cdot (V_{in} - V_{th})^2 \right]$$



$$I_D = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \left[V_{GS} - V_{th}\right]^2$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{th}) V_{DS}\right]$$

→ resist

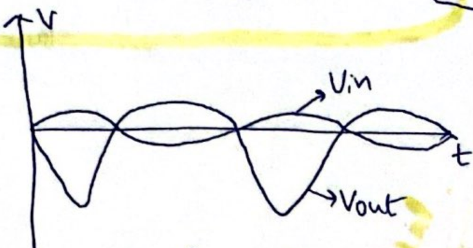
Gain in sat

$$A_v = \frac{\partial V_{out}}{\partial V_{in}}$$

$$\boxed{A_v = -g_m R_D}$$

Deep triode

$$V_{out} = \frac{V_{DD}}{1 + \mu_n C_{ox} \frac{W}{L} R_D (V_{in} - V_{th})}$$



nonlinear gain.

$V_{in} \downarrow \Rightarrow I_D \downarrow \Rightarrow g_m \downarrow \because (V_{GS} \downarrow)$

$$\boxed{A_v \propto \frac{\Delta V_{RD}}{\sqrt{I_D}} \sqrt{\frac{W}{L}}}$$

Tradeoffs → $R_D I_D$ (const)

Const swing :- $\overset{\times n}{I_d \uparrow} \Rightarrow$ gain $\downarrow \Rightarrow \overset{\div n}{R_d \downarrow}$ (const swing) $\Rightarrow \overset{\times \sqrt{n}}{g_m \uparrow} \Rightarrow \underbrace{\overset{\div \sqrt{n}}{g_m R_D \downarrow}}_{\text{gain} \downarrow}$

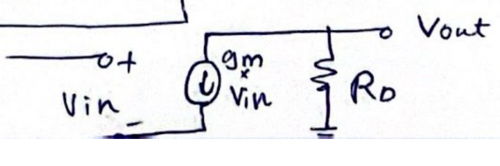
① $\boxed{\text{Tradeoff b/w const. swing \& gain \& size}}$

overcome.

Size effects :- $\overset{\times n}{\frac{W}{L} \uparrow} \Rightarrow \overset{\times n}{I_d \uparrow} \Rightarrow \overset{\times n}{g_m \uparrow} \Rightarrow \overset{\div n}{R_D \downarrow}$ (const swing) $\Rightarrow$ Constant gain. but bulky & slow.

Swing (const) vs Gain vs $\frac{W}{L}$

Small signal model :-



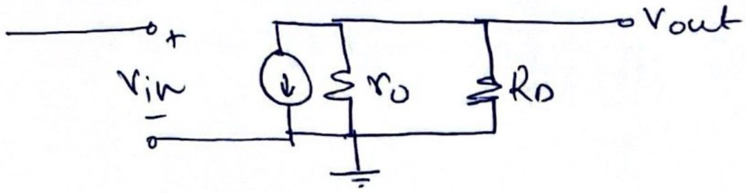
Output r_o

$r_o = \frac{1}{\lambda I_D}$

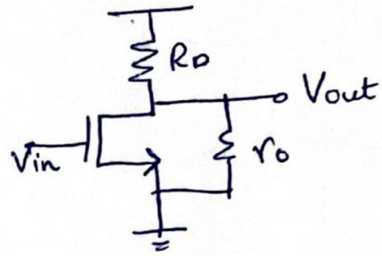
$$\boxed{A_v = -g_m \frac{R_D r_o}{R_D + r_o}}$$

$\Rightarrow r_o$ & R_D are in parallel.

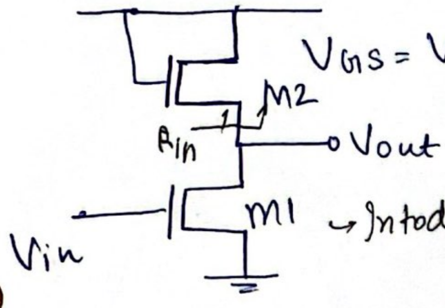
SS



LS



Diode connected transistor



$V_{GS} = V_{DS} \Rightarrow V_{DS} > V_{GS} - V_T \Rightarrow$ only sat. or off

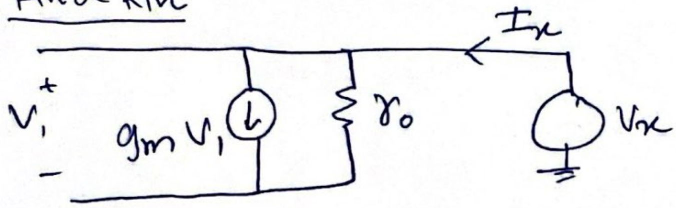
→ In today's technology

$A_v = -g_{m1}\left[\frac{1}{g_{m2}} \| r_{o1} \| r_{o2}\right]$

$$\chi = \frac{\gamma}{2(2|\phi_F| + V_{SB})^{1/2}}$$

$\gamma \rightarrow \frac{\sqrt{2q\epsilon_{Si} N_D}}{C_{ox}}$

M2 Find R_{in}



$R_{in} = \frac{V_x}{I_x} \quad (V_1 = V_x)$

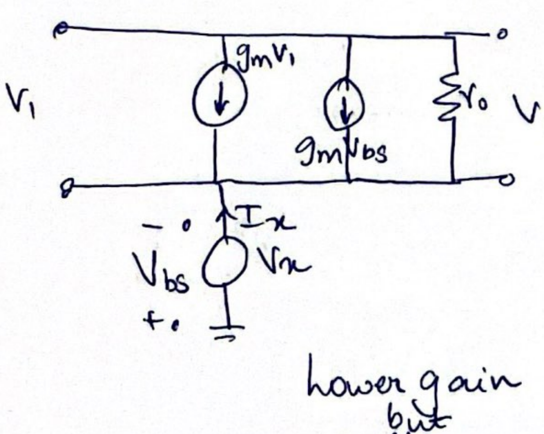
$I_x = g_m V_1 + \frac{V_x}{r_o}$

$R_{in} = \frac{1}{g_m + \frac{1}{r_o}} = \boxed{\left(\frac{1}{g_m} \| r_o\right)}$

$\boxed{R_{in} \simeq \frac{1}{g_m}}$

$r_o \gg \frac{1}{g_m}$

Body effect (Small Signal)



$V_1 = V_{bs} \left\{ \begin{array}{l} \therefore V_{bs} = -V_x \\ V_1 = -V_x \end{array} \right.$

$\boxed{R_{out} = r_{o1} \| R_{in}}$

$\boxed{R_{in} = \frac{1}{g_m + g_{mb} + \frac{1}{r_o}}}$

$\boxed{R_{in} \simeq \frac{1}{g_m + g_{mb}}}$

Lower gain but pretty

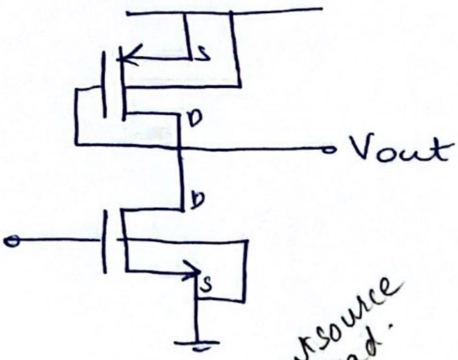
wrt temp → constant

$$\boxed{A_v = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot \frac{1}{1+\chi}}$$

$\chi \rightarrow \frac{g_{mb2}}{g_{m2}}$

$\chi \rightarrow \frac{\partial V_{th2}}{\partial V_{out}}$

Using PMOS to eliminate Body effect



$$A_v = -g_{m_1} \cdot \frac{1}{g_{m_2}}$$

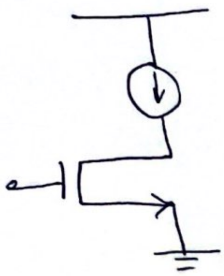
$$A_v = \sqrt{\frac{\mu_n \, (W/L)_1}{\mu_p \, (W/L)_2}}$$

no χ so more linear but temp. change issue

Active load - current source as load.

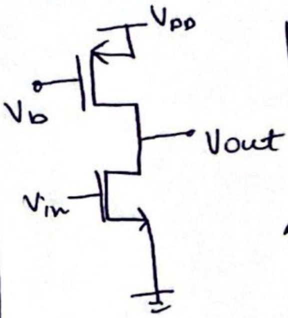
Active load is little different

→ Current source load.



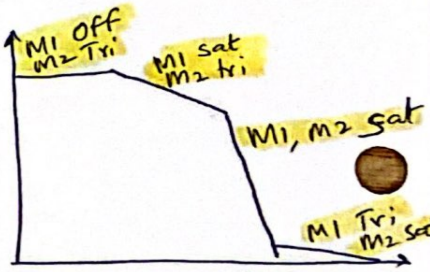
$$\boxed{A_v = -g_m r_o}$$

$$A_v = \frac{2V_A}{V_{OD}}$$



$$\boxed{A_v = -g_m (r_{o1} || r_{o2})}$$

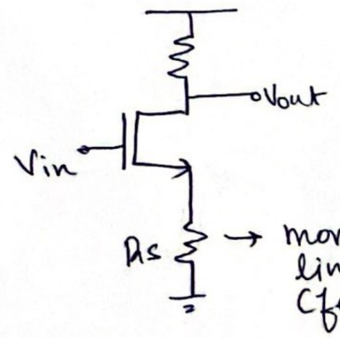
because V_2 for $\uparrow r_{o2}$



For large swing ⇒ $V_b \uparrow$

For high gain ⇒ $W \uparrow$ but slow

Source degeneration



→ more linear (feedback)

$$G_m = \frac{g_m r_o}{R_S + [1 + (g_m + g_{mb}) R_S] r_o}$$

$-G_m [R_D || R_{out}]$ gives this.

$$A_v = \frac{-g_m R_D}{1 + g_m R_D} = \frac{-g_m r_o R_D}{R_D + R_S + r_o + (g_m + g_{mb}) R_S r_o}$$

this

$$\boxed{G_m = \frac{g_m}{1 + g_m R_S}}$$

Trading gain for linearity.

$$= \frac{g_m}{1 + \left(g_m + g_{mb} + \frac{1}{r_o}\right) R_S}$$

→ Rs gives a negative feedback proportional to Rs.

$\Delta V_{in} = \Delta V_{GS} + \Delta V_{RS}$

↳ $R_S g_m \Delta V_{GS}$.

$$\boxed{R_{out} = r_o [1 + (g_m + g_{mb}) R_S]}$$

boosts R_S by $g_m r_o$

↳ To find Rout we ground the gate input.

Story of the transistor

Amplifier design

Common source

$$\boxed{A_v = -g_m R_D}$$

→ Non linear (g_m varies with V_{in})
→ Not very high gain.
→ Swing vs. speed $\left(\frac{W}{L}\right)$ vs. Gain

$$A_v = -g_m (r_o \| R_D)$$

Diode connected MOSFET

$$\boxed{A_v = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \cdot \frac{1}{1+\chi}}$$

→ Lower gain
→ More linear

PMOS Diode connected MOSFET

$$\boxed{A_v = -\sqrt{\frac{\mu_n}{\mu_p} \cdot \frac{(W/L)_1}{(W/L)_2}}}$$

→ Even More linear
→ Depends on process & temp

∴ So far gain has been quite low.
Need large $r_{(AC)}$ but small $R_{(DC)}$.

Active load MOSFET (Ideal)

$$\boxed{A_v = -g_m r_o} \boxed{= \frac{2V_A}{V_{OD}}} *$$

Active load MOSFET (Non ideal) - [PMOS with V_b]

$$\boxed{A_v = -g_{m1}(r_{o1} \| r_{o2})}$$

→ High gain (M_1, M_2 sat)
→ High swing (V_b is high)

∴ How to make it more linear.

Source degeneration

$$\boxed{G_m = \frac{g_m}{1 + g_m R_s}}$$

→ Lower gain but much more linear.

Solid state Equations

$n = N_c e^{\frac{E_F - E_C}{kT}}$ $\quad p = N_v e^{\frac{E_v - E_F}{kT}}$ $\quad np = n_i^2$ $\quad E_F = \frac{E_c + E_v}{2}$

Drift

$\vec{J} = \vec{J_n} + \vec{J_p} = q(n\mu_n + p\mu_p)\vec{E}$

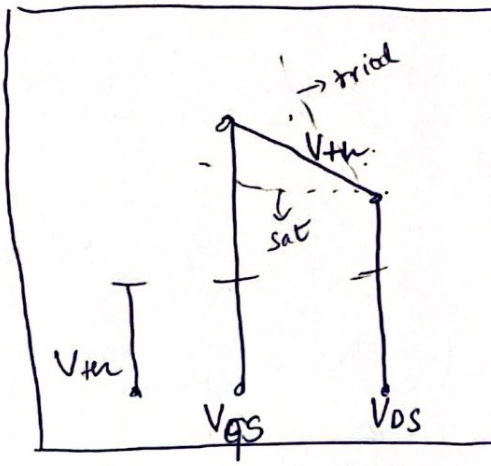
Diffusion

$\vec{J} = -qD\frac{\partial \rho}{\partial x}$; $\frac{D}{\mu} = \frac{kT}{q}$

Barrier potential (PN)

$$\varphi_0 = \frac{1}{2}\frac{qN_A}{\epsilon_s} x_p^2\left(1 + \frac{N_A}{N_D}\right)$$

$$C_{j0} = \left[\frac{q\epsilon_s}{2\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\varphi_0}\right]^{1/2} \qquad C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\varphi_0}}}$$



MOSFET

$V_{TH} = \phi_T + \Delta\phi_{ox}$ where $\phi_T = \frac{2kT}{q}\ln\frac{N_D}{N_i}$

$V_{TH} = V_{FB} + 2\phi_F + \gamma\sqrt{2\phi_F}$ where $\gamma = \frac{\sqrt{2q\epsilon_{si}N_D}}{C_{ox}}$, $C_{ox} \to \frac{\epsilon_{ox}}{t_{ox}}$

Body effect: $V_{TH} = V_{TH0} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]$ $\quad \phi_F = \frac{\phi_T}{2}$

Circuits (calculate gm & then proceed) (Equate currents)

In saturation intrinsic gain: $g_m r_o = \frac{4 I_D}{\mu_n C_{ox}(V_{GS} - V_{th})^3 \lambda \frac{W}{L}}$

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS} - V_{th})V_{DS} - V_{DS}^2\right]$ – Triode.

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[V_{GS} - V_{th}\right]^2(1 + \lambda V_{DS})$ – Sat

$\lambda = \frac{1}{L'}\frac{\partial x_d}{\partial V_{DS}} = \frac{1}{V_p}$

$g_{mb} = \chi g_m = \boxed{\frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}} \cdot g_m} \to \gamma = \frac{\sqrt{2q\epsilon_{si}N_D}}{C_{ox}}$

$\lambda = \frac{1}{r_o I_D}$, $r_o = \frac{V_A}{I_D}$ ↳ Halved if channel length is doubled

$\boxed{g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})} = \boxed{\sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}}\ \boxed{\frac{2 I_D}{(V_{GS} - V_{th})}}$

small signal: $\boxed{r_o = \frac{1}{\lambda_N I_{D \to k(V_{GS} - V_{th})^2}}}$ (saturation)

Large signal: $\boxed{r_o = \frac{1 + \lambda V_{DS}}{\lambda I_D}}$

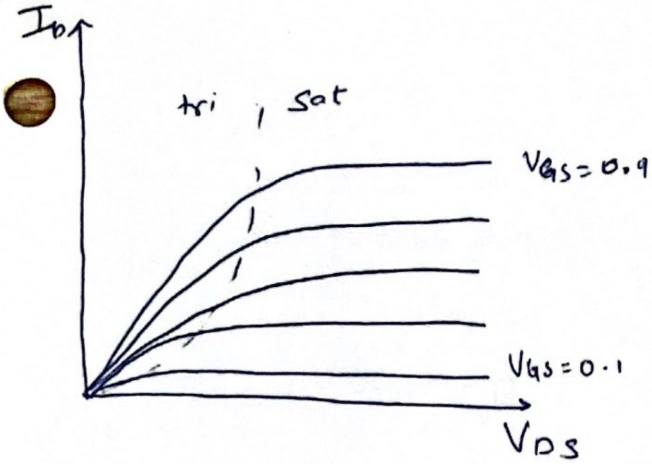
$V_{in} = V_{th} + \left(\frac{2 I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}\right)^{1/2}$

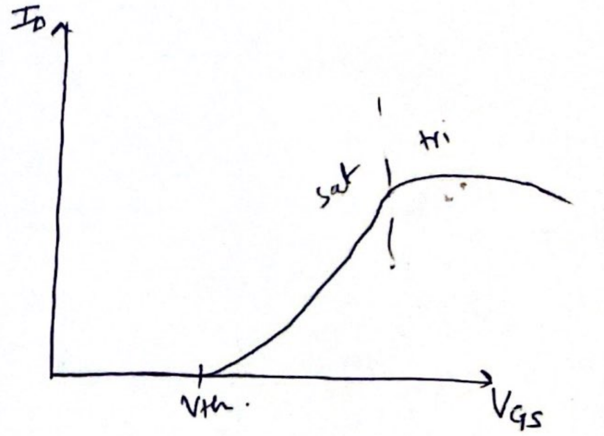
$R_o^{-1} = \frac{\partial I_D}{\partial V_{DS}}$ (triode)

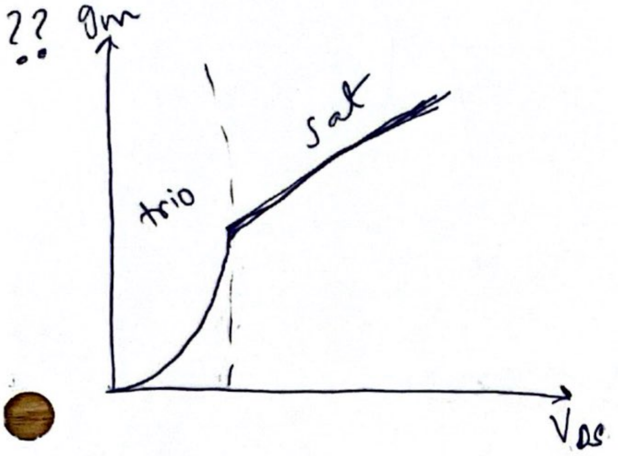
$^{850}\mu_n C_{ox} \frac{8.85\times10^{-14}}{9\times10^{-7}}\left(\frac{\epsilon_{ox}}{t_{ox}}\right)$ 1.34225×10^{-4}

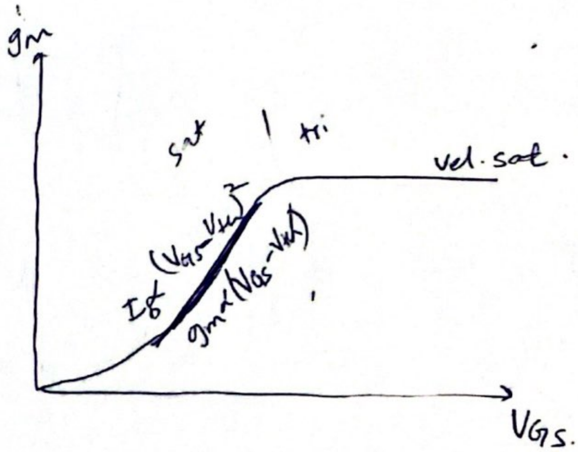
$_{100}\mu_p C_{ox} = 3.835 \times 10^{-5}$

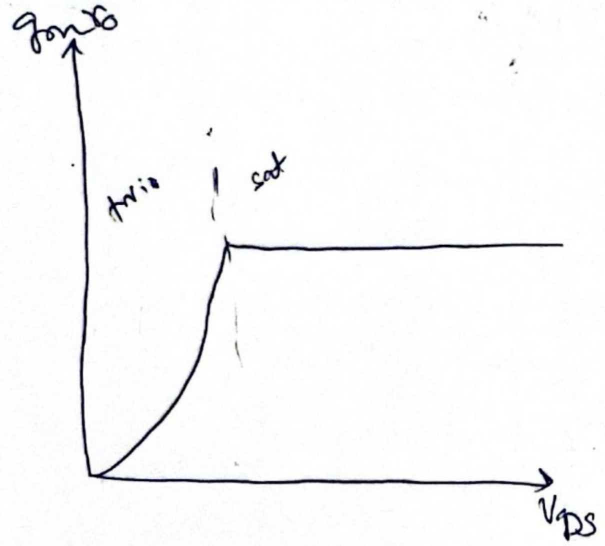
Intrinsic NMOS

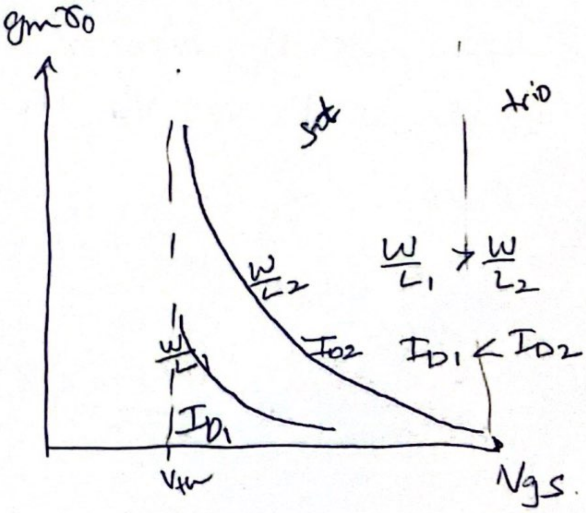












In driode

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right) V_{DS}\,(1+\lambda V_{DS})$$

$$r_o^{-1} = g_o = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)\left\{(V_{GS}-V_{th}-V_{DS})(1+\lambda V_{DS}) + \lambda\left[(V_{GS}-V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right]\right\}$$

$$A_v = -g_m r_o$$

Equivalent circuits

Gm :- Short Vout, find $\frac{i_{out}}{V_{in}}$. $g_m = \frac{\partial I_D}{\partial V_{GS}}$

Rout :- $R_{out} = \frac{V_t}{i_t}$; short Vin ; apply test v_{in} & find $\frac{V_{in}}{i_{in}}$

Rin :- $R_{in} = \frac{V_t}{I_t}$; short Vout. ; apply test Vin & find $\frac{V_{in}}{I_{in}}$

$$V_{th} = V_{th0} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]$$

$$I_x = \frac{C\,dV_x}{dt} \Rightarrow dV_x = \frac{1}{C} \cdot I_x\,dt$$

$$\cancel{\int dV_x = \frac{1}{C}\int_0^t I_x\,dt}$$

1 → Figure out sat. vs. triode vs. off → Write down Eqs.

2 → When does it move to Tri/sat.

3 → What is I_x vs. V_x etc...

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{\mu_n C_{ox}}{2}\frac{W}{L}\left[2V_{DS}\right]$$

Circuits → CS, CD, CS cascode, Inverter, current mirrors, 3 appendices.

$V_{th_0} = 2\phi_F + \gamma\sqrt{2\phi_F}$

$$\boxed{V_{th} = V_{th0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)}$$ Body Effect

$\phi_F = \frac{kT}{q} \ln \frac{N_D}{n_i}$

$\gamma = \frac{\sqrt{2q\epsilon_{si} N_D}}{C_{ox}}$

$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$

TRIODE

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \left[2(V_{GS} - V_{th})V_{DS} - V_{DS}^2\right]$$

SATURATION

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \left[V_{GS} - V_{th}\right]^2 \left[1 + \lambda V_{DS}\right]$$

Channel length modulation

$$g_m = \mu_n C_{ox} \frac{W}{L} \left[V_{GS} - V_{th}\right]\left[1 + \lambda V_{DS}\right]$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D \left[1 + \lambda V_{DS}\right]}$$

$$g_m = \frac{2 I_D}{V_{GS} - V_{th}}$$

$\lambda = \frac{1}{L'} \frac{\partial x_d}{\partial V_{DS}}$

Small signal

$\lambda = \frac{1}{r_o I_D}$

$r_o = \frac{1}{\lambda I_D}$

Large signal

$r_o = \frac{1 + \lambda V_{DS}}{\lambda I_D}$

$r_o^{-1} = \frac{\partial I_D}{\partial V_{DS}}$

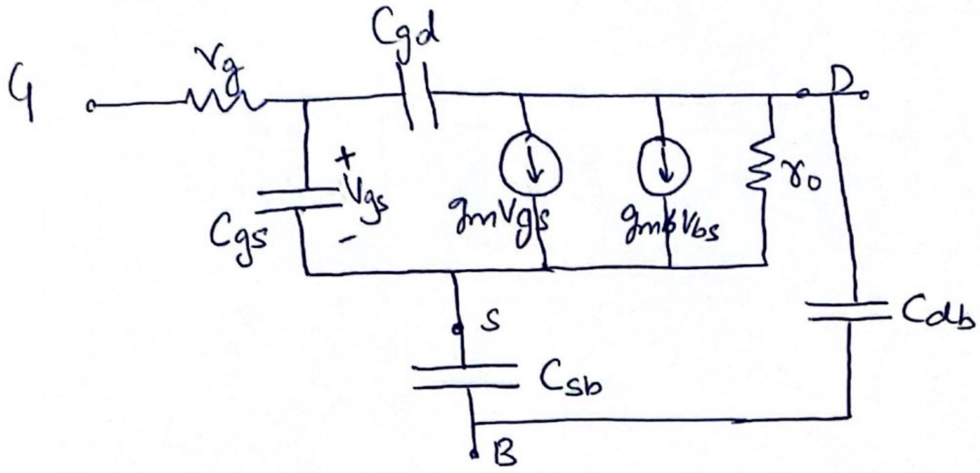
Body effect

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \chi g_m$$

where $\chi = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}$

$\gamma = \frac{\sqrt{2q\epsilon_{si} N_D}}{C_{ox}}$

Small signal model



$$f_T = \frac{1}{2\pi}\frac{g_m}{C_{gs}+C_{gd}}$$

f_T is limited by

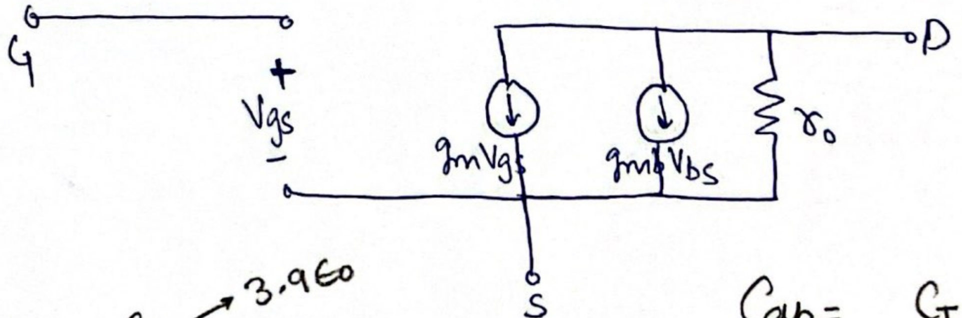
V_{sat} => By architecture

Capacitances	Cgb	Cgd	Cgs
Cutoff	$W L_{eff} C_{ox}$	$W C_{ov}$	$W C_{ov}$
Linear/Triode	0	$W C_{ov} + \frac{W L_{eff} C_{ox}}{2}$	$W C_{ov} + \frac{W L_{eff}}{2} C_{ox}$
Saturation	0	$W C_{ov}$	$W C_{ov} + \frac{2}{3} W L_{eff} C_{ox}$

$$C_{gs} = \frac{2}{3} C_{ox} W L_{eff} + C_{ov} W$$

$$C_{gd} = C_{gdo} \cdot W$$

At low frequencies



$C_{Jn} = 0.56 \times 10^{-3}$

$C_{Jp} = 0.94 \times 10^{-3}$

$C_{JSW_n} = 0.35 \times 10^{-11}$

$C_{JSWp} = 0.32 \times 10^{-11}$

$$C_{ox} = \frac{\epsilon_{SiO_2}}{t_{ox}} \quad \epsilon_{SiO_2} \rightarrow 3.9\epsilon_0$$

$$C_{db} = \frac{C_J \times W \times 1.5\mu m}{\left[1 - \frac{V_D}{PB}\right]^{MJ}} + \frac{C_{JSW}(2W + 3\mu m)}{\left[1 - \frac{V_D}{PB}\right]^{MJSW}}$$

$V_D \rightarrow$ reverse bias Jn. voltage

$V_D = 1$

$PB = 0.9$

$MJ = 0.6$

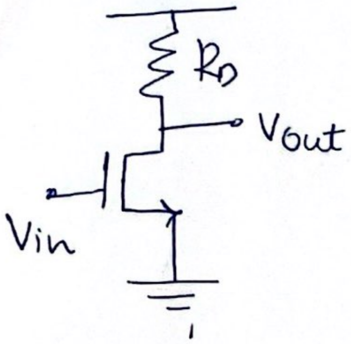
$MJSW = 0.2$

$$L_{eff} = L - 2(L_{ov})$$

$$C_{ov} = C_{ox} \times L_{ov}$$

Common Source (Gate - i/p ; Drain o/p)

(I) Basic

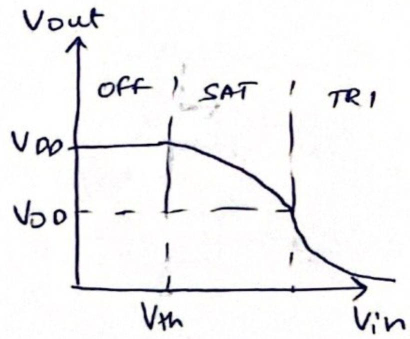


$$\boxed{A_v = -g_m R_D} \quad \lambda = 0$$

$$\boxed{A_v = -g_m (R_D \| r_o)} \quad \lambda \neq 0$$

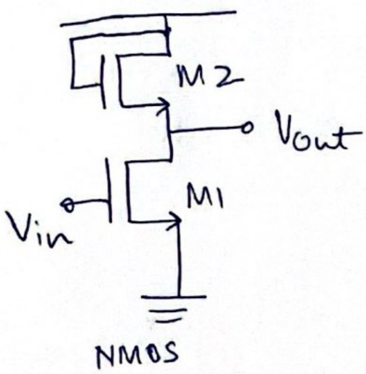
$$\boxed{R_{in} = \infty}$$

$$\boxed{R_{out} = R_D \| r_o}$$



→ High gain
→ Nonlinear
→ High Rin, low Rout

(II) Diode connected

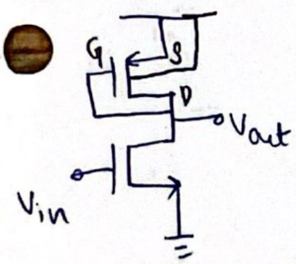


$$\boxed{A_v = \frac{-g_{m1}}{g_{m2}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \times \frac{1}{1+\chi_2}} \quad \chi = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}$$

$$\boxed{R_{in} = \infty}$$

$$\boxed{R_{out} = r_{o1} \,\|\, \left\{\frac{1}{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}}\right\}} \rightarrow \frac{1}{g_m + g_{mb}} \| r_{o2} \| r_{o1}$$

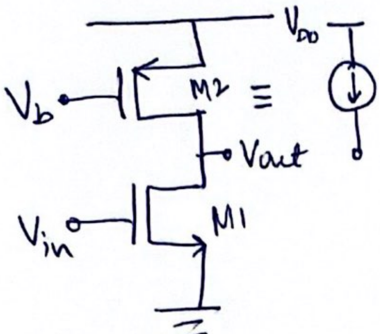
→ More linear!
→ Lower gain!



$$\boxed{A_v = -\sqrt{\frac{\mu_n}{\mu_p} \cdot \frac{(W/L)_1}{(W/L)_2}}}$$

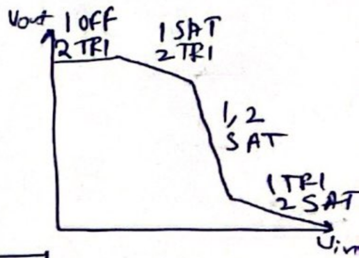
→ No body effect

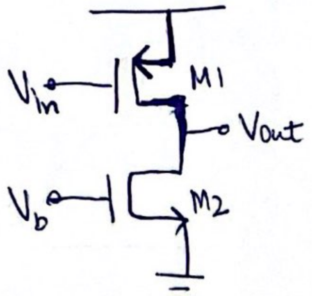
III Active Load



NMOS

$$\boxed{\begin{aligned} A_v &= -g_{m1}(r_{o1} \,\|\, r_{o2}) \\ R_{in} &= \infty \\ R_{out} &= r_{o1} \,\|\, r_{o2} \end{aligned}}$$



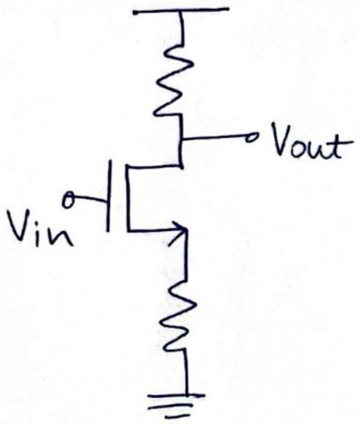


PMOS

$$\boxed{\begin{aligned} A_v &= -g_{m1}(r_{o1} \,\|\, r_{o2}) \\ R_{in} &= \infty \\ R_{out} &= r_{o1} \,\|\, r_{o2} \end{aligned}}$$

→ High gain
→ High swing } Not so linear.

IV Source degeneration



$$\boxed{A_v = \frac{-R_D}{\frac{1}{g_m} + R_s}} \qquad \lambda = 0;\ g_{mb} = 0$$

$$\boxed{A_v = -G_M R_{out}} \qquad \lambda \neq 0;\ g_{mb} \neq 0$$

$$G_M = \frac{g_m}{1 + \left(g_m + g_{mb} + \frac{1}{r_o}\right)R_s} = \frac{g_m r_o}{r_o + (g_m r_o + g_{mb} r_o + 1) R_s}$$

$$R_{out} = \left[r_o\left(1 + (g_m + g_{mb})R_s\right) + R_s\right] \| R_D$$

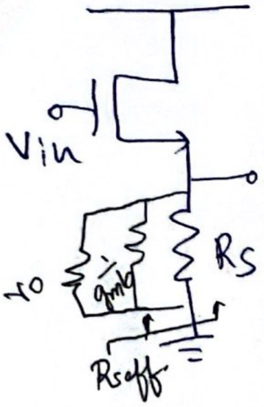
$$\boxed{R_{in} = \infty}$$

→ Lower gain
→ Much more linear.

$$R_{out} \simeq (1 + g_m r_o) R_s + r_o$$
$$\simeq (1 + g_m R_s) r_o + R_s$$

Common Drain (or) Source follower

[Gate i/p ; Source o/p]



gmb is just a resistor of value $\frac{1}{g_{mb}}$ in this case because drain is short

$\Rightarrow R_{seff} = \frac{1}{g_{mb}} \| r_o \| R_s$ substitute here

$$\boxed{A_v = \frac{R_{seff}}{\frac{1}{g_m} + R_{seff}}} \rightarrow \frac{g_m R_s}{1+(g_m+g_{mb})R_s}$$

$R_{seff} = R_s \| r_o \| \frac{1}{g_{mb}}$

$\hookrightarrow$ No $\frac{1}{g_m}$ in this.

$A_v = \frac{1}{1+\chi}$ when $R_s \gg \frac{1}{g_{mb}}$

$$\boxed{R_{out} = \frac{1}{g_m + g_{mb} + \frac{1}{r_o} + \frac{1}{R_s}}} \simeq \frac{1}{g_m+g_{mb}}$$

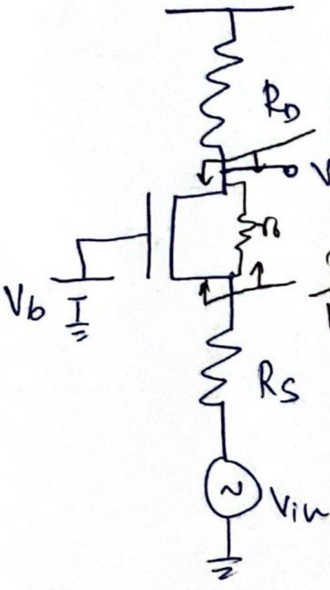
$\boxed{R_{in} = \infty}$

Rseff is resistance looking from ground into the source.

- → High Rin } Buffer.
- → Low Rout } Buffer.
- → Maybe used as a level shifter as well

Common Gate [Source i/p ; Drain output]

$A_v = (g_m + g_{mb} + \frac{1}{r_o})(R_D \| r_o)$ when $R_s = 0$



$\boxed{A_v = +g_m R_D (1+\chi)}$ $\lambda = 0$; $R_s = 0$

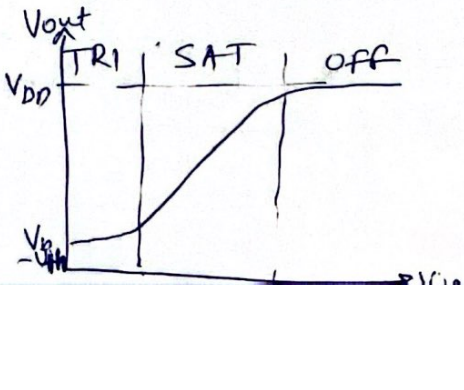
$$\boxed{A_v = \frac{(g_m + g_{mb} + \frac{1}{r_o}) r_o R_D}{R_D + r_o + R_s(1+(g_m+g_{mb})r_o)}}$$ $\lambda \neq 0$, $R_s \neq 0$

$$\boxed{R_{in} = R_s + \frac{r_o + R_D}{1+(g_m+g_{mb})r_o}}$$ Body effect decreases Rin.

$$\boxed{R_{out} = R_D \| \{r_o + (1+g_m r_o)R_s\}}$$

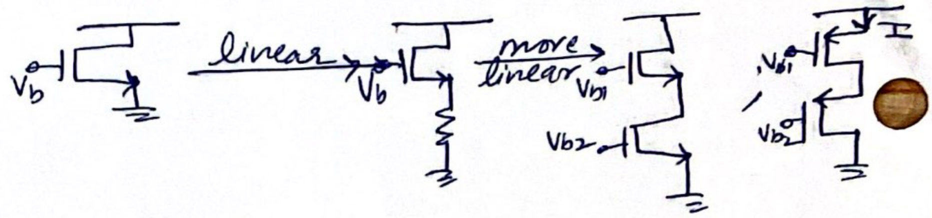
same as source degen.

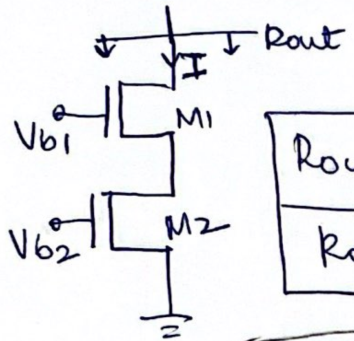
- → High Rout } Impedance matching
- → Low Rin } Impedance matching



Cascode

(I) Current sources

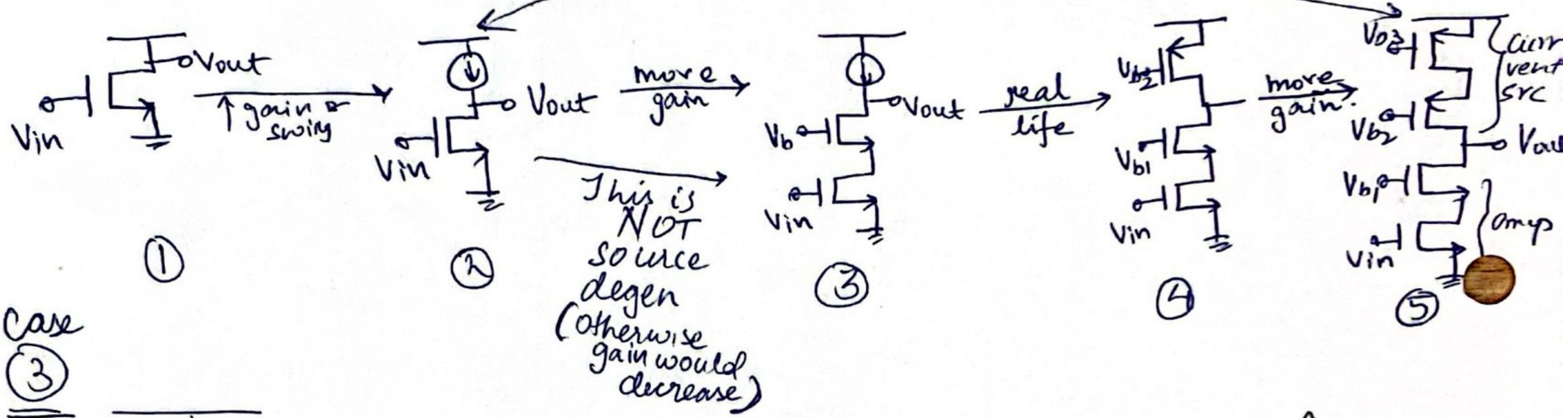




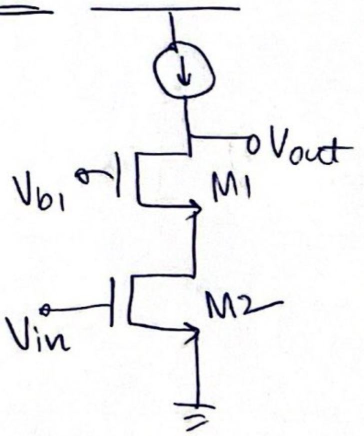
$$\boxed{R_{out} = (1+g_{m1}r_{o1})r_{o2} + r_{o1}}$$

$$\boxed{R_{out} \simeq (g_{m1}r_{o1})r_{o2}}$$

(II) Cascode ~~amplifiers~~ Rout booster → High O/p resistance*



case ③



$$\boxed{R_{out} = (1+g_{m1}r_{o1})r_{o2} + r_{o1}}$$

$$\boxed{G_M \simeq g_{m2}}$$

$$\boxed{A_v = -G_M R_{out}}$$

$$G_M = g_{m2} \cdot \frac{r_{o2}}{r_{o2} + \frac{1}{g_{m1}+g_{mb1}} \| r_{o1}}$$

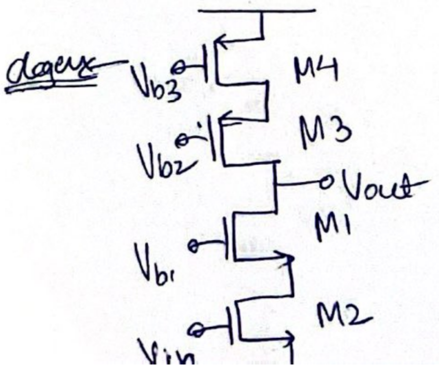
Biasing

$V_{in} > V_{th1}$

$V_{out} > V_{OD1} + V_{OD2}$

O/p swing decreases by V_{OD2}

Case ⑤

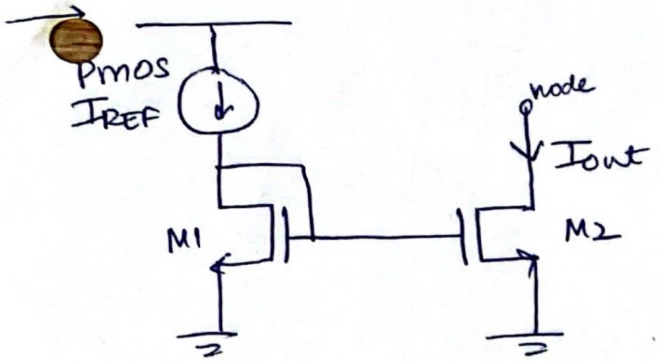


$$\boxed{R_{out} = [(1+g_{m3}r_{o3})r_{o4} + r_{o3}] \| [(1+g_{m1}r_{o1})r_{o2} + r_{o1}]}$$

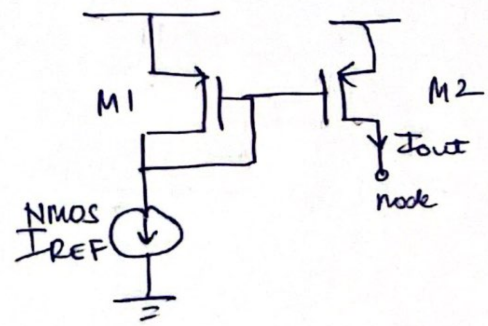
$$\boxed{G_M = g_{m2}}$$

$$\boxed{A_v = -G_M R_{out}}$$

Current Mirrors (To bias) {Ignoring CLM}



PMOS I_{REF} → NMOS bias



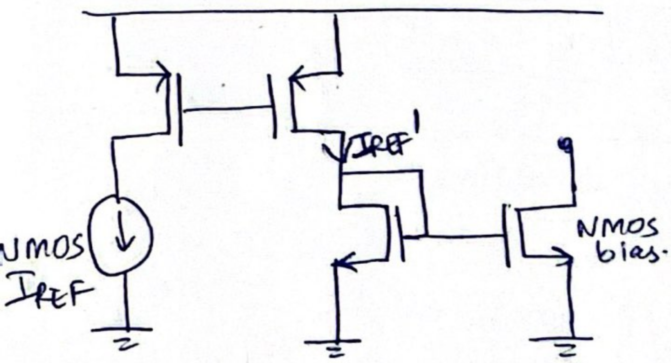
NMOS I_{REF} → PMOS bias.

$$\boxed{I_{out} = \left\{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}\right\} I_{REF}}$$

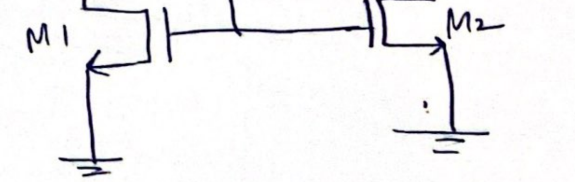
Series ⇒ $L \times 2$ or $W \div 2$

Parallel ⇒ $L \div 2$ or $W \times 2$

→ NMOS SRC → NMOS bias



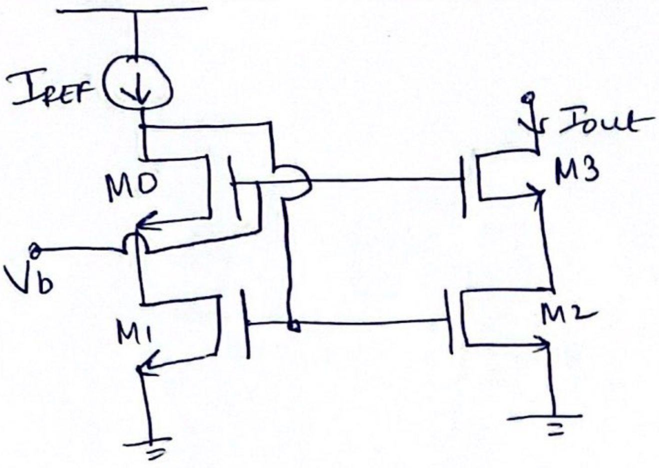
If we do not ignore CLM we must use cascode to ensure $I_{REF} = I_{out}$



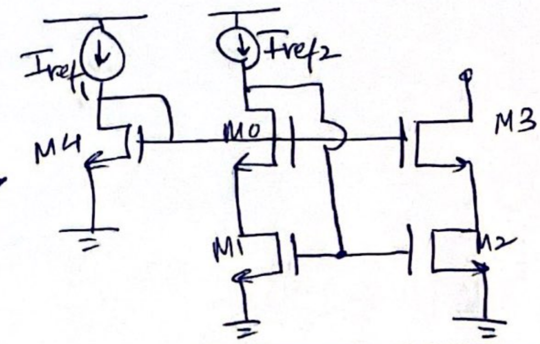
→ M_0 is to properly bias M3 and M3 is to ~~remove~~ reduce CLM because it increases Rout & Iout doesn't change as much

→ This design requires very high V_{DD} ~~bias~~ values.

→ To lower bias values & increase $V_{peak(min)}$

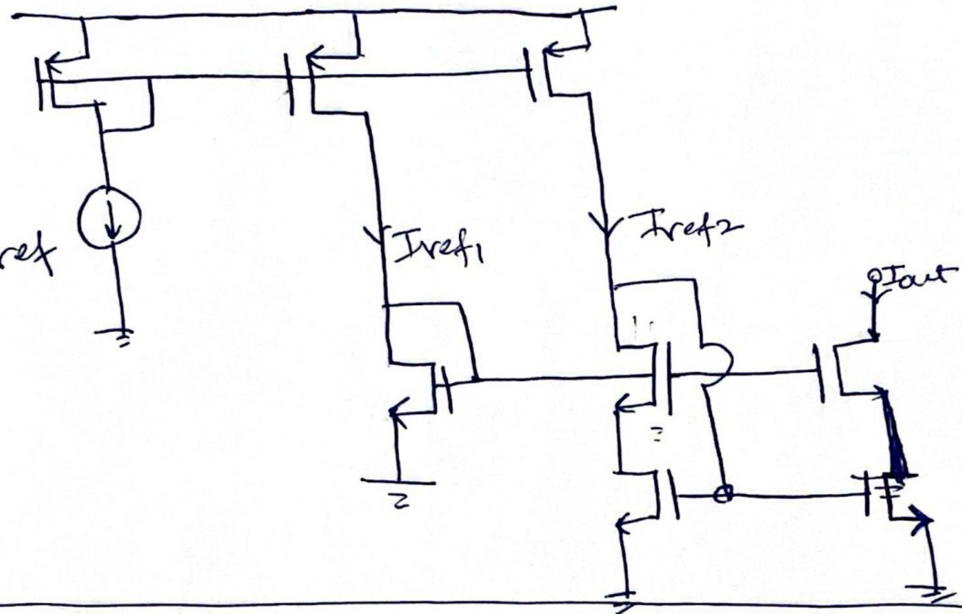


⇒ real life



$$\boxed{\left(\frac{W}{L}\right)_4 = \frac{1}{4}\left(\frac{W}{L}\right)}$$

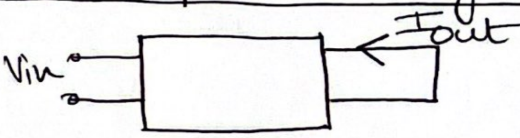
→ Replacing with only one I_{REF}



$\boxed{\text{Wide swing \& low voltage biasing techniques}}$

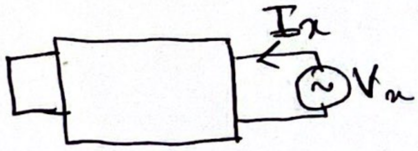
Basic Rules for solving any amplifier circuit

Step a



→ Short o/p to GND.
→ Short all DC sources
→ Find $\frac{I_{out}}{V_{in}} = G_m$

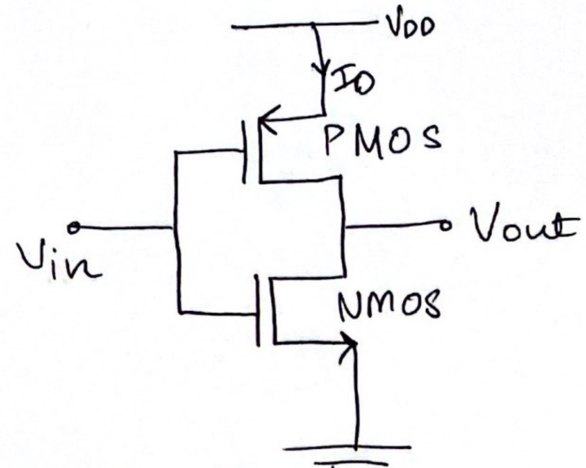
Step b



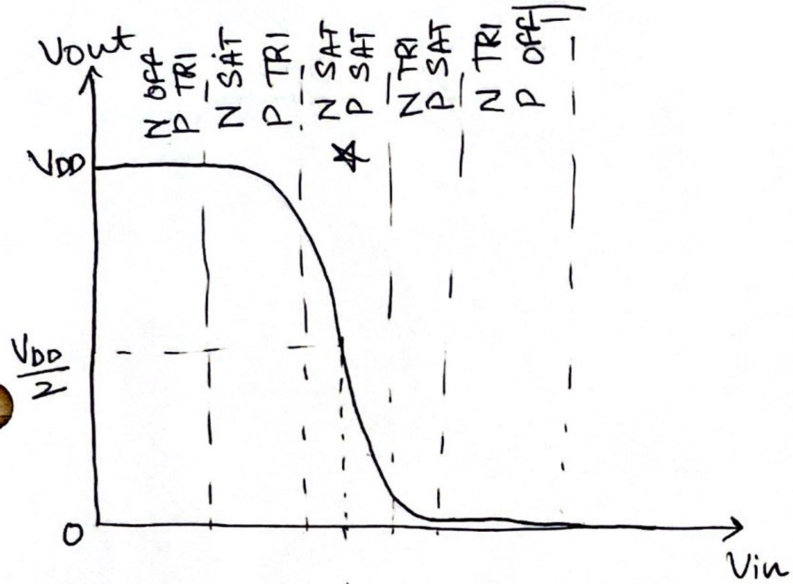
→ Remove all independant sources including Vin
→ Find $R_x = \frac{V_x}{I_x}$.

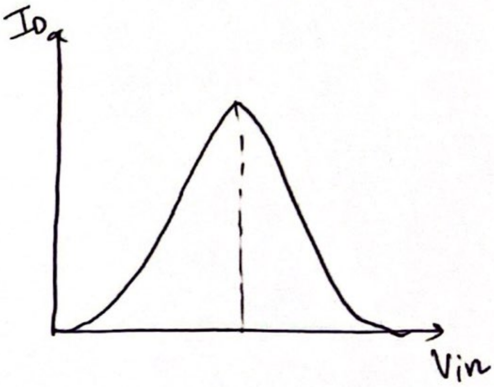
$$\boxed{A_v = -G_m R_x}$$

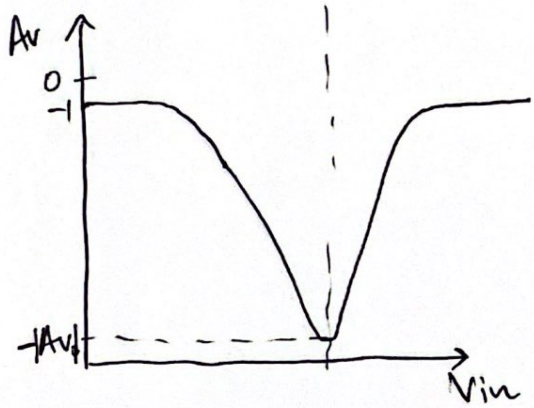
CMOS Inverter → This is called active load!

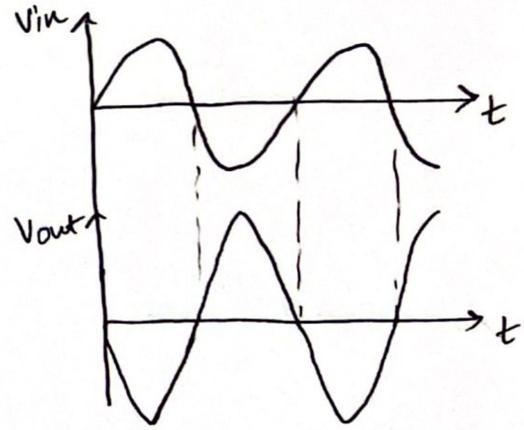


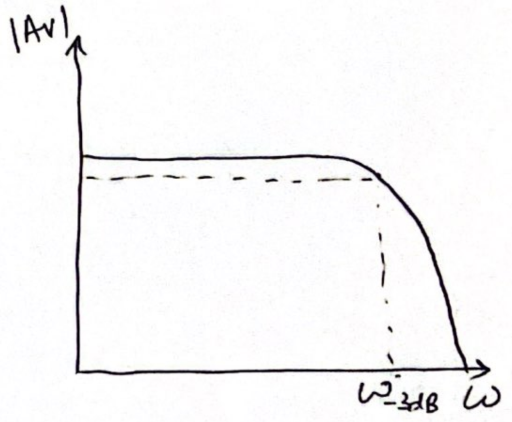
$$A_v = -(g_{m_n} + g_{m_p})(r_{on} \parallel r_{op})$$



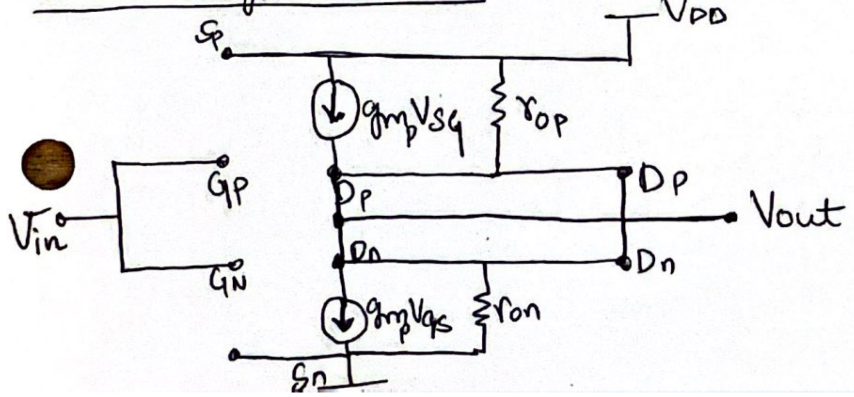




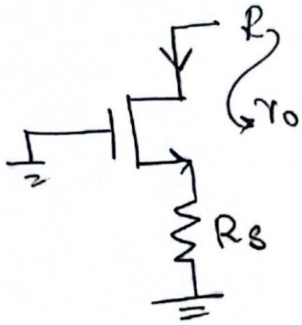




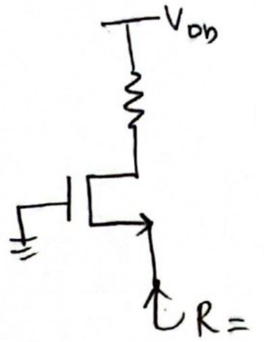
Small signal model



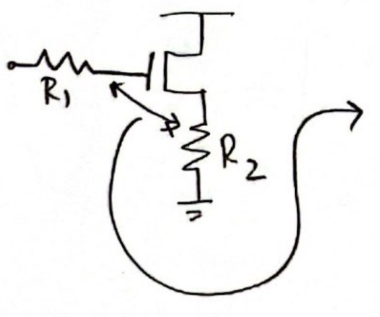
Resistance Equivalents



$$r_o + R_S + (g_m + g_{mb}) r_o R_S \simeq (g_m + g_{mb}) r_o R_S$$

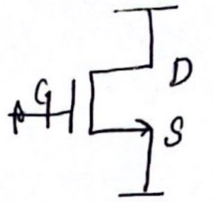


$$R = \frac{R_D + r_o}{1 + (g_m + g_{mb}) r_o} \simeq \frac{1}{g_m + g_{mb}}$$

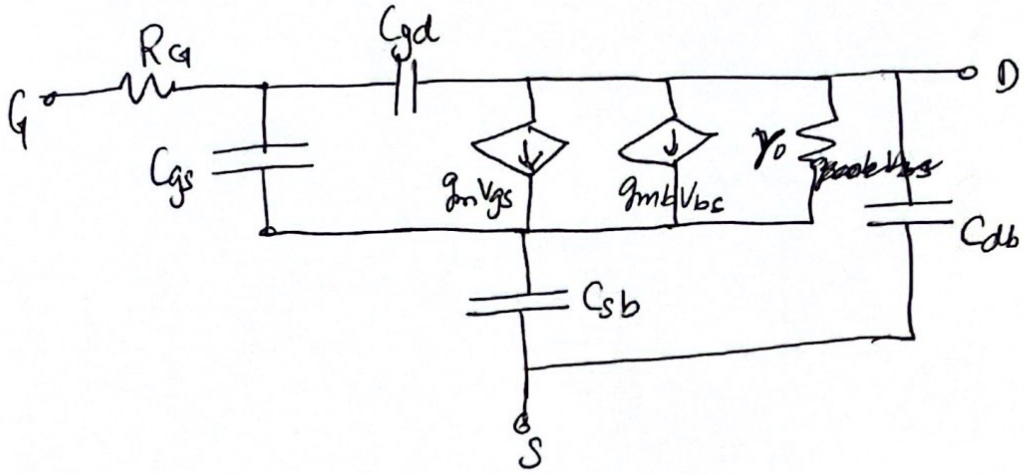


$$R = \frac{\frac{R_1}{R_2} + 1 + g_{mb} R_1 + \frac{R_1}{r_o}}{g_m + \frac{1}{R_2} + g_{mb} + \frac{1}{r_o}} \simeq \frac{1 + g_{mb} R_1}{g_m + g_{mb}}$$

MOSFET summary.



π model



$$I_{D\,\text{triode}} = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_{tn}) V_{DS} - \frac{V_{DS}^2}{2}\right]$$

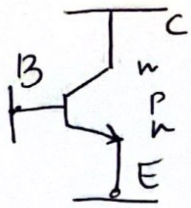
$$I_{D\,\text{sat}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{tn})^2 (1 + \lambda V_{DS}) \qquad \lambda = \frac{1}{V_A}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{tn}) \qquad g_{mb} = \chi g_m, \quad \chi = \frac{\gamma}{2\sqrt{2\phi_F + V_{SB}}}$$

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} = \frac{2 I_D}{(V_{GS} - V_{tn})}$$

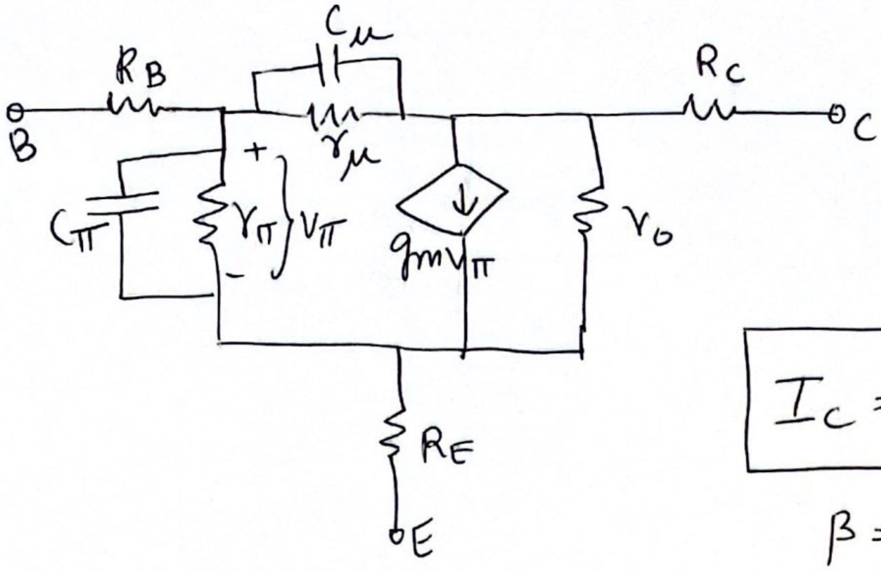
$$r_o = \frac{1}{\lambda_N I}$$

BJT Summary.



Small signal model.

Π

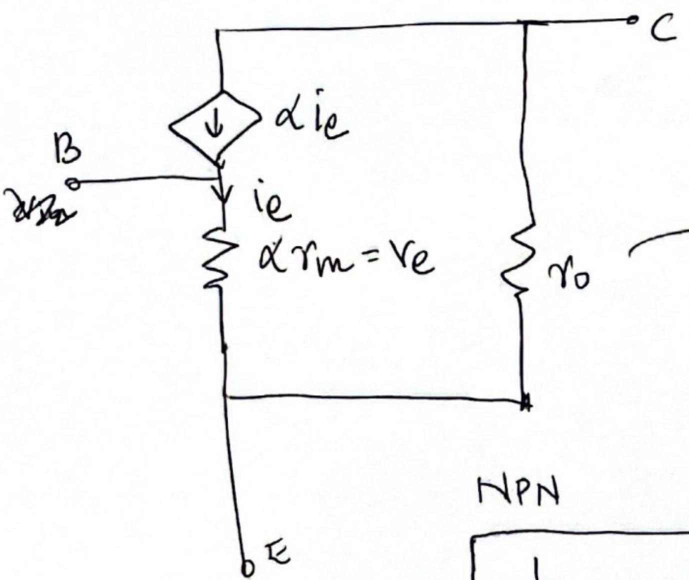


$$\boxed{I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)}$$

$$\beta = \frac{\alpha}{\alpha - 1} \quad ; \quad \frac{1}{\alpha} - \frac{1}{\beta} = 1$$

$\alpha \sim 1$
$\beta \sim 100$

T model



$$\boxed{\begin{aligned} I_C &= \beta I_B \\ I_C &= \alpha I_E \end{aligned}}$$

$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \underbrace{\frac{V_{CE}}{V_A}}_{\text{Early Effect}}\right)$$

NPN

$$\boxed{\frac{1}{r_m}} = \boxed{g_m = \frac{I_{C,Q}}{V_T}} = \boxed{\frac{i_c}{v_{be}}}$$

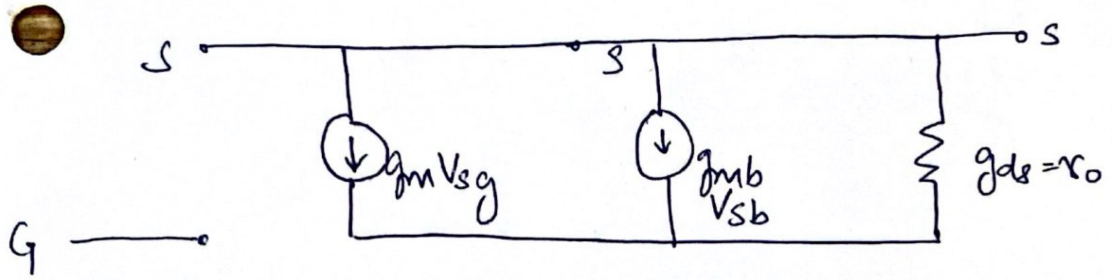
$$\boxed{r_\pi = \frac{\beta}{g_m} = \beta r_m} \rightarrow \left[\frac{\partial I_B}{\partial V_{BE}}\right]^{-1}$$

$$\boxed{r_o = \frac{V_A}{I_{C,Q}}} \rightarrow \left[\frac{\partial I_C}{\partial V_{CE}}\right]^{-1}$$

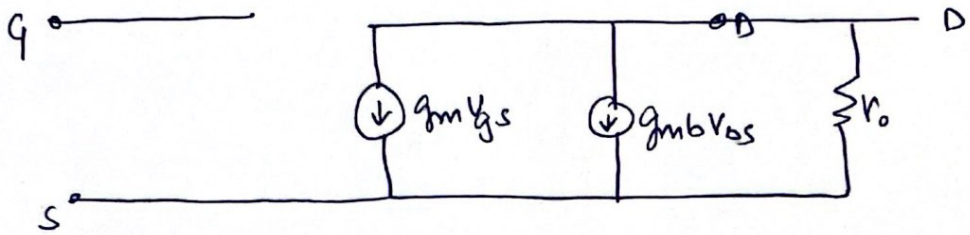
$$r_\mu = \left[\frac{\partial I_B}{\partial V_{CB}}\right]^{-1}$$

$$\boxed{r_\mu \simeq \beta r_o}$$

PMOS



or



TRIODE

$$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} \left[2(V_{gs} - V_{th}) V_{DS} - V_{DS}^2 \right]$$

$V_{GS} < V_{th}$

$V_{DS} > V_{GS} - V_{th}$

SAT

$$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} \left[V_{gs} - V_{th} \right]^2 \left[1 + \lambda V_{DS} \right]$$

$V_{GS} < V_{th}$

$V_{DS} \leq V_{GS} - V_{th}$

Laplace Transform

$$L[f(t)] = \int_0^{\infty} f(t) e^{-st} dt = F(s) \quad ; \quad s = \sigma + j\omega$$

$\delta(t)$	—	1
1	—	$1/s$
t	—	$1/s^2$
t^n	—	$\frac{n!}{s^{n+1}}$
t^a	—	$\frac{\Gamma(a+1)}{s^{a+1}}$
e^{-at}	—	$\frac{1}{s+a}$
$\cos\omega t$	—	$\frac{s}{s^2+\omega^2}$
$\sin\omega t$	—	$\frac{\omega}{s^2+\omega^2}$

$$L[f'(t)] = sL[f(t)] - f(0)$$

$$L[f''(t)] = s^2 L[f(t)] - sf(0) - f'(0)$$

$$L\left[\int f(t')dt'\right] = \frac{1}{s} L[f(t)]$$

differential $\Rightarrow$ $\times s$ − something

integral $\Rightarrow$ $\div s$

$$L[e^{at} f(t)] = F(s-a)$$

$$\left.\begin{aligned} y(t) &= x(t) * h(t) \\ \Rightarrow Y(s) &= H(s) \cdot X(s) \end{aligned}\right\} \text{for LTI}$$

Inverse

$$f(t) = L^{-1}[F(s)] = \frac{1}{2\pi i} \lim_{t\to\infty} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s) e^{st} ds$$

→ Converts a differential equation to an algebraic equation.

Frequency Response {3 methods, CD, CS, Cascode, cascade, BW enhance mode, Diff pair,

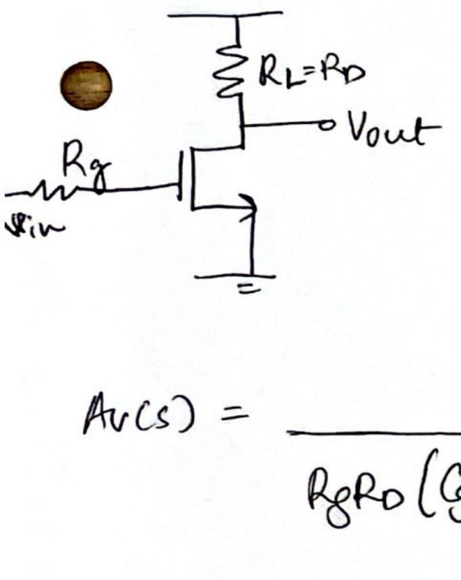
(I) Direct approach

- Draw small signal model with the caps.
- Solve KVL and KCL to find $\frac{V_{out}(s)}{V_{in}(s)} = A_v(s)$
- Find poles and zeroes (ω_p, ω_z)
- Make assumptions to solve for ω_p if needed.

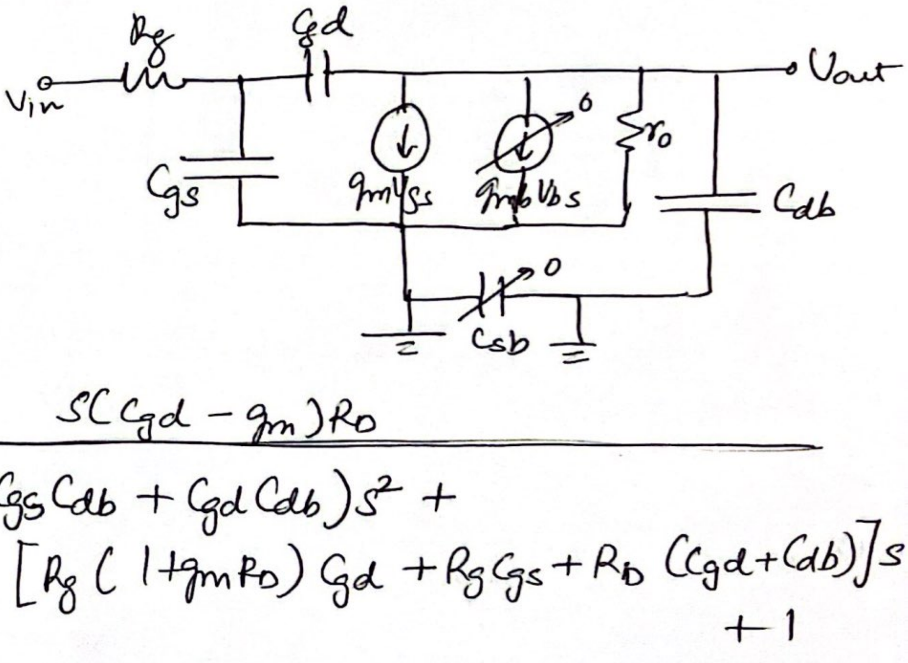
$$\omega_{p1} \ll \omega_{p2} \; ; \; C_{gs} \gg C_{gd}(1+g_m R_D) \; ; \; g_m R_D \gg 1.$$

<u>Note:-</u> ω_z is usually very high so it can be ignored.

Common Source



$\Rightarrow$



$$A_v(s) = \frac{s(C_{gd} - g_m)R_D}{R_g R_D (C_{gs}C_{gd} + C_{gs}C_{db} + C_{gd}C_{db})s^2 + [R_g(1+g_m R_D)C_{gd} + R_g C_{gs} + R_D(C_{gd}+C_{db})]s + 1}$$

$\omega_z = \frac{g_m}{C_{gd}}$; Assume $\omega_{p1} \ll \omega_{p2}$

dom pole $\omega_{p1} = \frac{1}{R_g(1+g_m R_D)C_{gd} + R_g C_{gs} + R_D(C_{gd}+C_{db})} \xrightarrow{C_{gs} \gg} \frac{1}{R_D(C_{gd}+C_{db})}$

$\omega_{p2} \approx \frac{R_g(1+g_m R_D)C_{gd} + R_g C_{gs} + R_D(C_{gd}+C_{db})}{R_g R_D(C_{gs}C_{gd} + C_{gd}C_{db} + C_{gs}C_{db})} \xrightarrow{g_m R_D \gg 1} \frac{g_m}{C_{gs}\left(1+\frac{C_{db}}{C_{gd}}\right)}$

→ A simpler direct approach

> Assume no zeroes :- $A_v(s) = \dfrac{A_{v_0}}{(\tau_{p_1}s+1)(\tau_{p_2}s+1) * \cdots (\tau_{p_n}s+1)}$

> $D(s) = b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0$

$b_1 = \sum_{i=1}^{n} \tau_{p_i}$; $b_0 = 1$

Dominant pole assumption :- ⇒ only b_1 & b_0 affect ω_{-3dB}

$\therefore D(s) \simeq b_1 s + b_0 = b_1 s + 1$

$$\therefore \boxed{\omega_{-3dB} \simeq \frac{1}{b_1} \simeq \frac{1}{\sum \tau_p}}$$

II Open circuit time constant method

> Find R_{jo} facing capacitor C_j when all other caps are open-removed.

> $\tau_{jo} = R_{jo}C_j$; ⇒ $\boxed{\omega_{-3dB} = \dfrac{1}{\sum_{j=1}^{n} R_{jo}C_j}}$

> For a cap with gate & drain on two sides

$R = R_{left} + R_{right} + g_m R_{left} R_{right}$

For common source

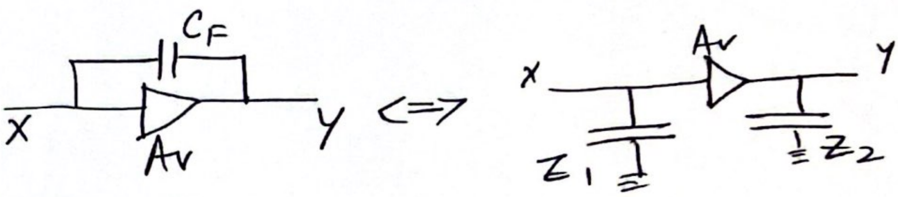
$$\omega_{3dB} = \frac{1}{R_g(1+g_m R_D)C_{gd} + R_g C_{gs} + R_D(C_{gd}+C_{db})}$$

III Associating poles to nodes

- Assign a pole to each node and find RC time constant

$$\omega = \frac{1}{\Sigma RC}.$$

Miller effect



$$\boxed{Z_1 = \frac{Z}{1 - A_v}} \qquad \boxed{Z_2 = \frac{Z}{1 - \frac{1}{A_v}}}$$

where. $Z = \frac{1}{sC_F}$

$$\Rightarrow \boxed{C_{F_1} = C_F(1 - A_v)} \qquad \boxed{C_{F_2} = C_F\left(1 - \frac{1}{A_v}\right)}$$

Two stage.

- Coupling caps lower bandwidth by introducing an ω_{-3dB} (low)
- To find ω_{-3dB}(low) use $$A_v(s) = \frac{K_n s^n}{(s+s_1)(s+s_2)+ \dots (s+s_n)}$$
- Approximating to dominant pole

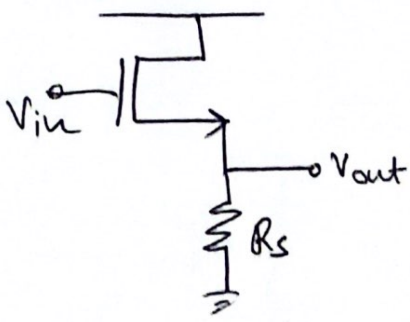
$$A_v(s) = \frac{K_n s}{s + \sum_{i=1}^{n} s_i}$$

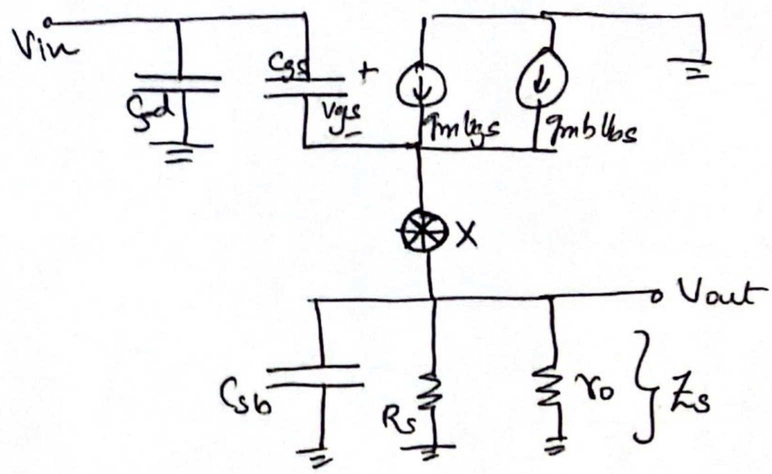
and $$\boxed{\omega_{-3dB} = \sum_{i=1}^{n} s_i}$$

Recipe!

- Find R_{is} facing the capacitor C_i when all other caps are short (because they are at higher frequencies).
  → C_i are caps contributing to the lower ω_{-3dB}.
- $$\boxed{\omega_{-3dB\,low} = \sum_{i=1}^{n} \frac{1}{R_{is} C_i}}$$

Common Drain

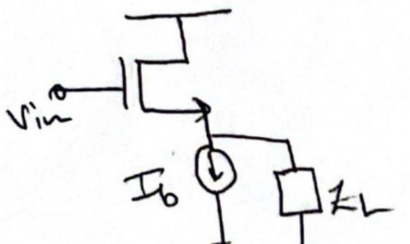




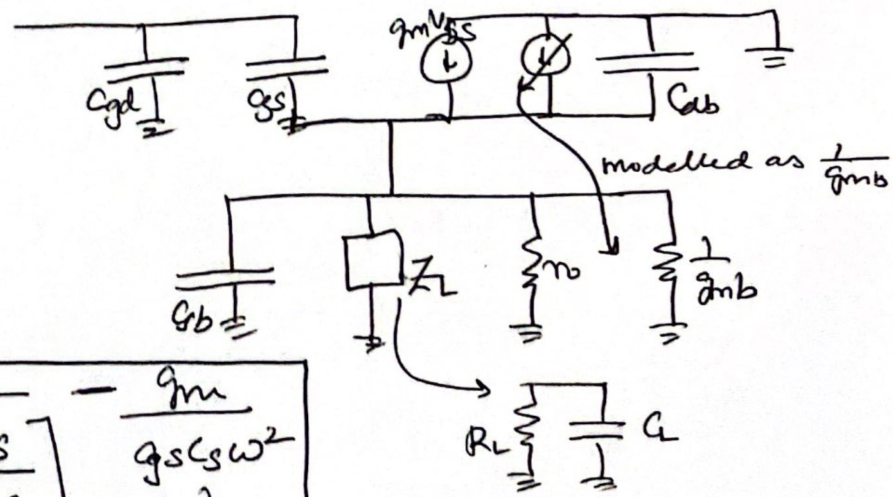
Direct method

$$A_v = \frac{SC_{gs} + g_m}{SC_{gs} + \frac{1}{Z_S} + g_m + g_{mb}} \Rightarrow \quad S_z = \frac{-g_m}{C_{gs}} \qquad S_p = \frac{-g_m}{C_{gs} + C_{sb}}$$

> Rin and Rout have interesting behaviours



$\Rightarrow$



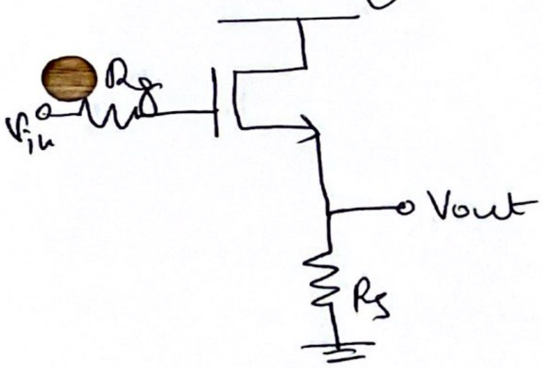
$$\boxed{Z_{in}'(j\omega) = \frac{1}{j\omega\left[\frac{C_{gs} C_S}{C_{gs} + C_S}\right]} - \frac{g_m}{C_{gs} C_S \omega^2}}$$

A negative resistor ⇒ active element & it can compensate for a lossy circuit. However, it is a strong fn of frequency.

Where $C_S = C_{Sb} + C_{load}$.

$Z_{in} = Z_{in}' \parallel \frac{1}{SC_{gd}}$.

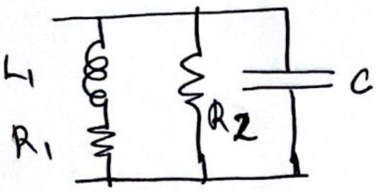
∴ Rout of C.D



$$Z_{out} = Z_{out}' \parallel r_o \parallel \frac{1}{g_{mb}} \parallel \frac{1}{sC_{sb}}$$

$$Z_{out}' = \frac{R_g(C_{gs}+C_{gd})s+1}{(R_g C_{gd} s+1)(sC_{gs}+g_m)}$$

Z_{out}' can be modelled as the following RLC circuit



$$C = \frac{C_{gs}C_{gd}}{C_{gs}+C_{gd}}$$

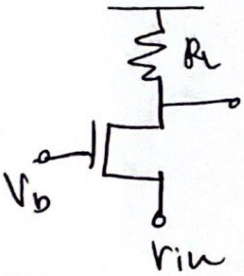
$$R_1 \simeq \frac{1}{g_m}$$

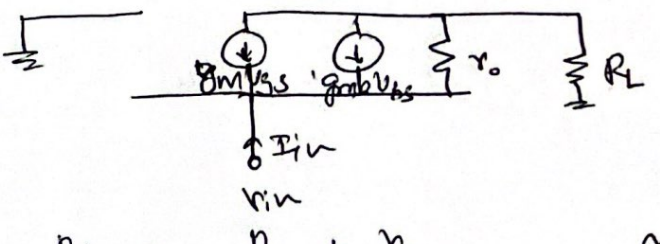
$$R_2 = \frac{R_g(C_{gs}+C_{gd})}{C_{gs}\left(\frac{C_{gs}}{C_{gs}+C_{gd}}\right)+g_m R_g C_{gd}}$$

$$L = \frac{R_g(C_{gs}+C_{gd})}{g_m}.$$

∴ It has an inductive element in it

Common Gate





$$R_{in} = \frac{R_L + r_o}{1+(g_m+g_{mb})r_o} \simeq \frac{1}{g_m+g_{mb}}$$

where $r_o \gg R_L$.

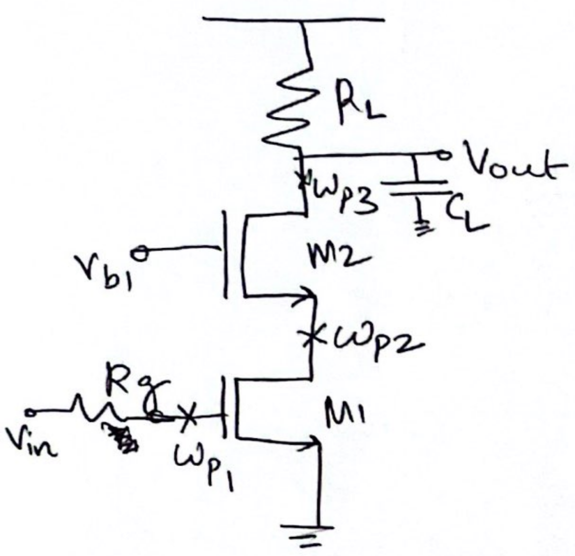
↳ Independent of R_L *

$$\boxed{A_{v_0} = \frac{R_L}{R_s + \frac{1}{g_m+g_{mb}}}}$$

$$\boxed{\omega_{p1} = \frac{1}{\left[R_s \parallel \frac{1}{g_m+g_{mb}}\right](C_{gs}+C_{sb})} \qquad \omega_{p2} = \left[R_L(C_{gd}+C_{db})\right]^{-1}}$$

$$A_v = \frac{A_{v_0}}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)}$$

Cascode



$$\omega_{p1} = \frac{1}{R_g\left[C_{gd1}\left(1 + g_{m1}\frac{1}{g_{m2}+g_{mb2}}\right) + C_{gs1}\right]}$$

$$\omega_{p1} \simeq \frac{1}{R_g\left[2C_{gd1} + C_{gs1}\right]}$$ when M_1, M_2 are identical

$$\omega_{p2} = \frac{1}{\left[r_{o1} \,||\, \frac{1}{g_{m2}+g_{mb2}}\right] \times \left\{C_{gd1}\left(1 + \frac{g_{m2}+g_{mb2}}{g_{m1}}\right) + C_{db1} + C_{gs2} + C_{sb2}\right\}}$$

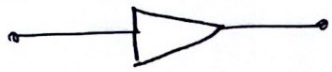
When active load is used ω_{p2} remains same since ω_{p3} acts as a short when we reach frequency of ω_{p2}.

* dominant
$$\omega_{p3} = \frac{1}{(R_L \,||\, \text{Large})\left\{C_{gd2} + C_L + C_{db2}\right\}}$$

* Usually output pole is dominant

Frequency $\leftrightarrow$ Time

Amplifier Single pole



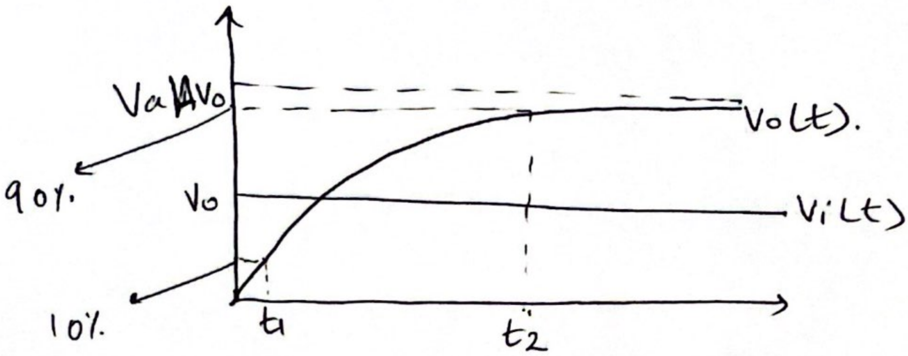
$$A_v = \frac{A_{vo}}{\frac{s}{\omega_p} + 1}$$

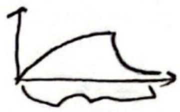
Step i/p

$\therefore$ $v_i(t) = V_a\, u(t)$

$$v_o(t) = V_a A_{vo}[u(t)] - V_a A_{vo}[e^{-\omega_p t} u(t)]$$

$$\boxed{V_o(t) = V_a A_{vo}\left[1 - e^{-\omega_p t}\right] u(t)}$$





$t_r = t_2 - t_1$

$$\boxed{t_r = \frac{2.2}{\omega_{-3dB}}}$$

$\omega_{-3dB} \rightarrow$ rps

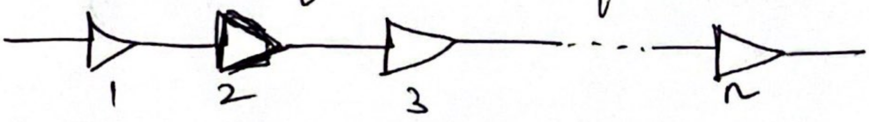
For single pole or dominant pole systems.

$\Rightarrow$ $\boxed{t_r = \frac{0.35}{f_{Hz}}}$

$t_{r+f} = \frac{0.7}{f_{Hz}} \Rightarrow f_{Hz} = \frac{0.7}{t_{r+f}}$

$\boxed{f_{Hz} \geq 0.7\,\text{data rate}}$

Cascade of n amplifiers



$$A_{v_{tot}} = \left[\frac{A_{vo}}{\frac{s}{\omega_p} + 1}\right]^n$$

$$\boxed{\omega_{-3dB} = \omega_p\sqrt{2^{1/n} - 1}}$$

$n = 2 \Rightarrow \omega_{-3dB} = 0.64\,\omega_p$

$\Rightarrow$ ~~Gain~~ BW drops by 36% *

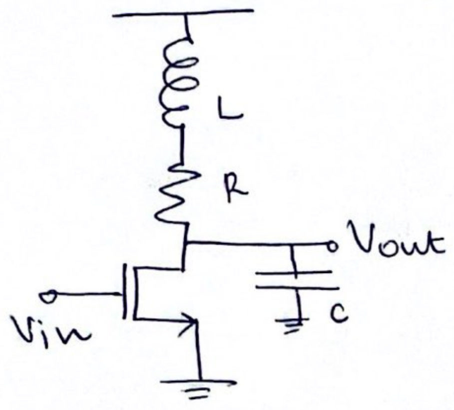
$\Rightarrow$ Gain is squared *

$$\boxed{t_{r_{tot}} = \sqrt{t_{r_1}^2 + t_{r_2}^2 + \dots + t_{r_n}^2}}$$

$\rightarrow$ when all stages have only one pole

Bandwidth Enhancement

① Shunt Peaking



$$Z_L(s) = \frac{R\left[\frac{sL}{R}+1\right]}{s^2LC+sRC+1}$$

$$|Z_L(j\omega)| = R\sqrt{\frac{(\omega\tau)^2+1}{\left(1-\frac{\omega^2\tau}{\omega_1}\right)^2+\left(\frac{\omega}{\omega_1}\right)^2}}$$

where, $\tau = \frac{L}{R}$

$\omega_1 = \omega_{3dB}$ without L

$\omega_1 = \frac{1}{RC}$

Scenario (i)

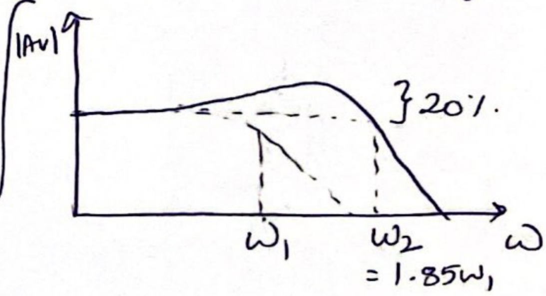
Maximum Bandwidth $\Rightarrow \frac{\partial Z_L}{\partial \tau} = 0$ & find τ

Substitute τ in $|Z_L|$ & $= \frac{1}{\sqrt{2}}\cdot \underbrace{R}_{Z_L(0)}$.

We get, $\boxed{\tau = \frac{1}{\sqrt{2}\,\omega_1}}$ & $\boxed{BW = 1.85\,\omega_1}$*

85% higher

However we get Shunt Peaking.



Scenario (ii)

We force peaking to happen after ω_1

$\Rightarrow \left| Z_L(j\omega)\right|\Big|_{\omega=\omega_1} = R$ we get $\boxed{\tau = \frac{1}{2\omega_1}}$ & $\boxed{BW \simeq 1.8\,\omega_1}$*

Scenario (iii)

No peaking $\Rightarrow \frac{\partial Z_L}{\partial \tau}$ does not change sign. $\Rightarrow \boxed{\tau = \frac{1}{(1+\sqrt{2})\omega_1}}$ & $\boxed{BW = 1.72\,\omega_1}$*

The cost of doing this is the phase response!

↳ (Check notes on

· Scenario (iv)

Need to make sure t_g (group delay) vs. ω is flat or atleast not changing sign. $\}\Rightarrow \frac{dt_g}{d\omega}$ does not change sign.

$\Rightarrow \boxed{\tau = \frac{1}{3.1\omega_1}}$ & $\boxed{BW = 1.6\omega_1}$

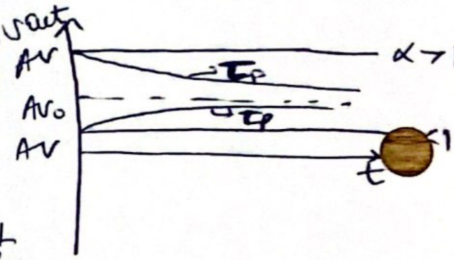
(II) Adding a Zero to the Transfer Function

$$A_v = A_{vo} \cdot \frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_p}\right)}$$

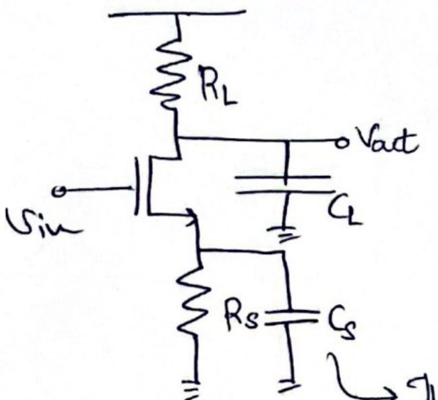
However if there is design mismatch $\Rightarrow$?? $\boxed{\tau_z = \alpha \tau_p.}$

The bandwidth may increase but settling time will also increase.

$$A_v = A_{vo}\left[1+(\alpha-1)e^{-t/\tau_p}\right]$$



How can we implement it?



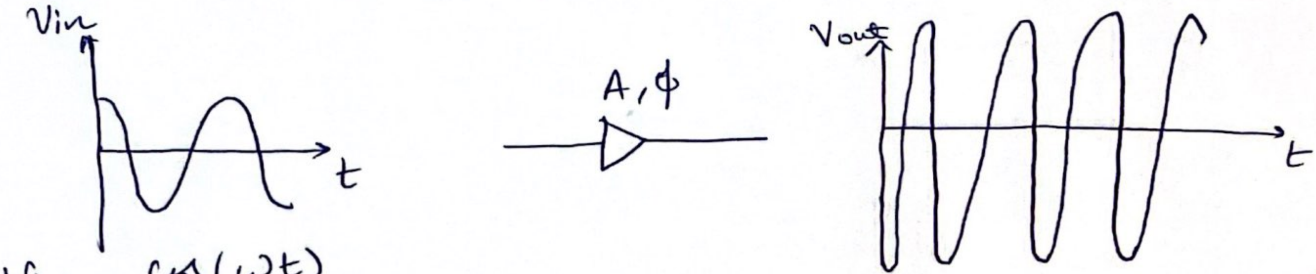
$$A_v = \frac{-Z_L}{\left[\frac{1}{sC_s} \| R_s\right] + \frac{1}{g_m}}$$

$Z_L \rightarrow R_L \| \frac{1}{sC}$.

If $R_s \gg \frac{1}{g_m} \Rightarrow \boxed{A_v = \frac{-Z_L}{Z_s}}$

C_s $\hookrightarrow$ This gives a '0' But in reality it also gives a pole!

Phase Response



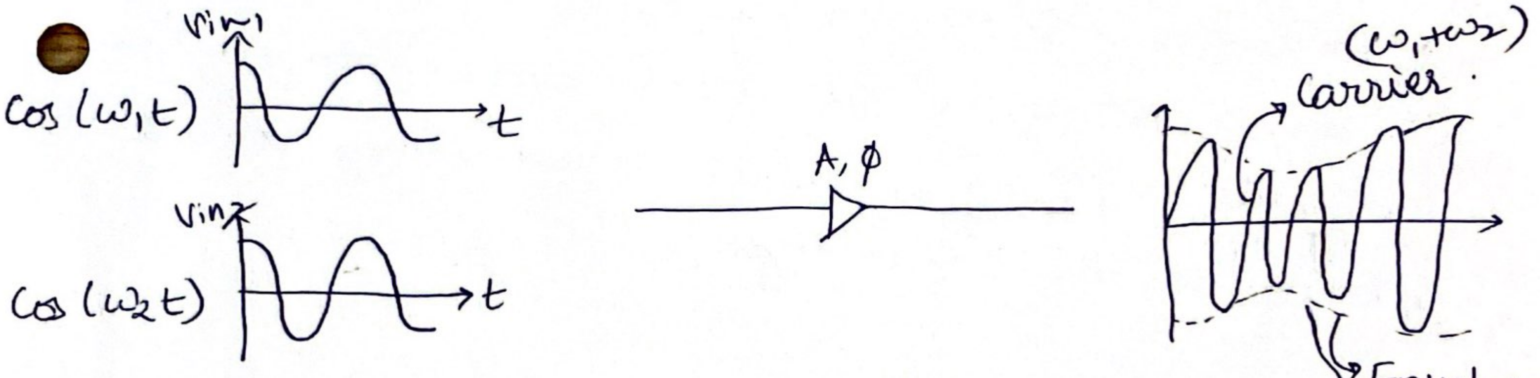
$V_{in} = \cos(\omega t)$

$V_{out} = A\cos(\omega t + \phi)$
$= A\cos(\omega(t - t_d))$

Phase Delay

Here $\boxed{t_d = \dfrac{-\phi}{\omega}}$ → Amount of delay in signal introduced by the system.

2 tone signal



$V_{out}(t) = 2|A| \underbrace{\cos\left(\frac{\omega_1-\omega_2}{2}t + \left(\frac{\phi_1-\phi_2}{2}\right)\right)}_{\text{Envelope}} \underbrace{\cos\left(\frac{\omega_1+\omega_2}{2}t + \left(\frac{\phi_1+\phi_2}{2}\right)\right)}$

Envelope (Information) $(\omega_1-\omega_2)$

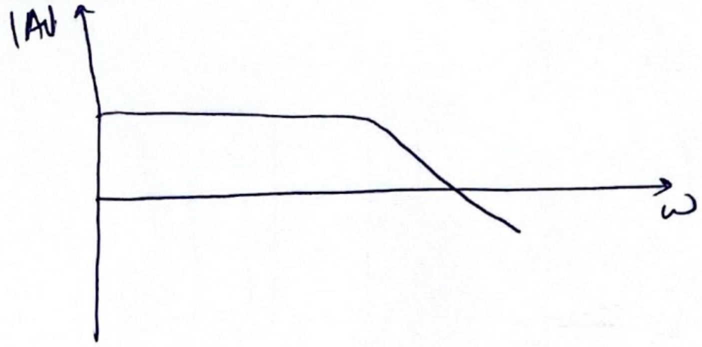
Group delay

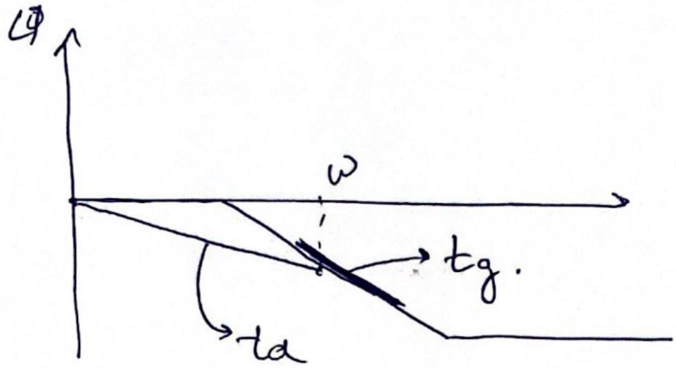
$\boxed{t_g = \dfrac{-(\phi_2-\phi_1)}{\omega_2-\omega_1}}$

∵ Envelope carries the information we care about.

- For multitone signals. $\boxed{t_g = \dfrac{-d\phi}{d\omega}}$ Slope of ϕ over ω. (Bode plot).

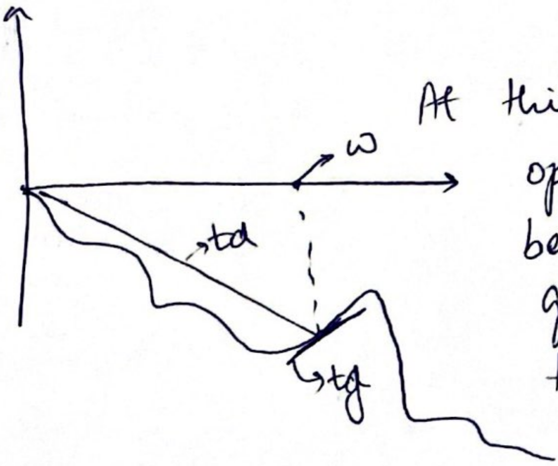
Bode plot





At some ω, the group delay is slope of line. & phase delay is $\frac{\angle\phi(\omega)}{\omega}$

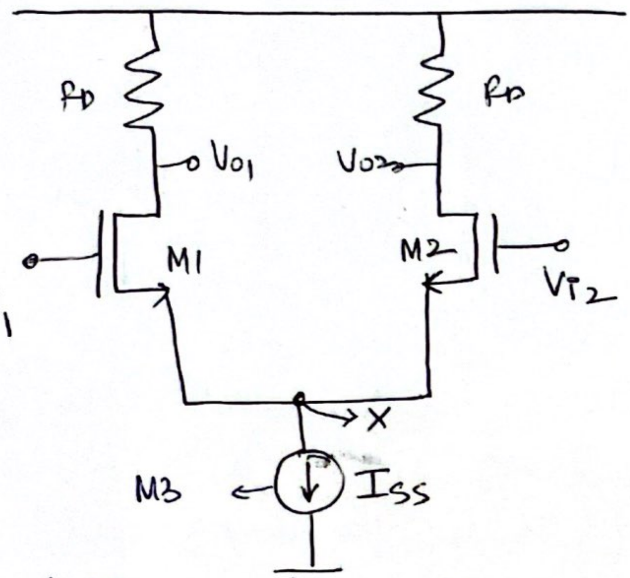
→ In pursuit of higher and higher bandwidth in $|A_v|$ vs. ω we should not ignore how phase response changes. Ideally we want $t_d = t_g$ at all ω. However they should atleast be the same sign.

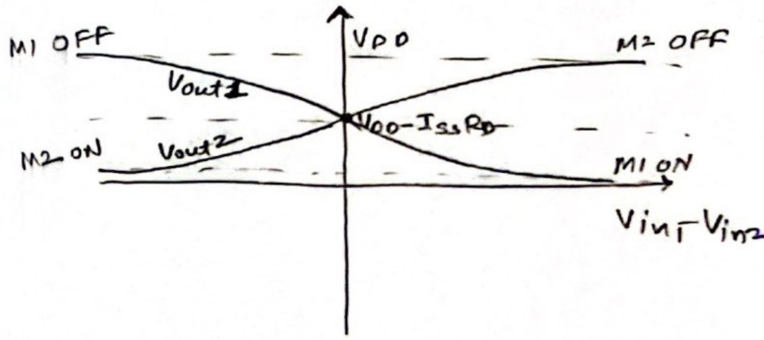


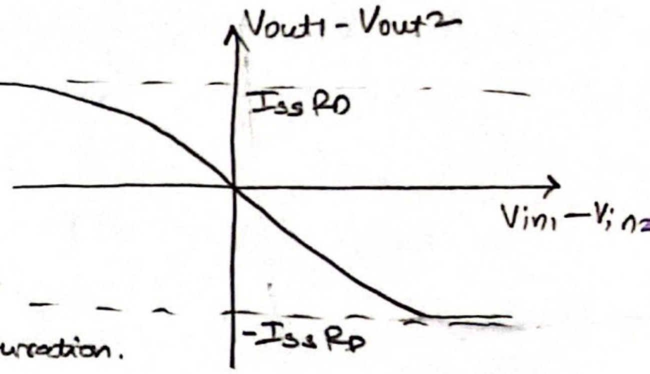
At this frequency the td & tg have opposite sign. ⇒ Each signal has been shifted forward but the group has shifted backward in time.

Differential Pair

> Immune to noise.
> Double the swing.
> Output/gain not a strong function of input. ⇒ more constant & stable gain.





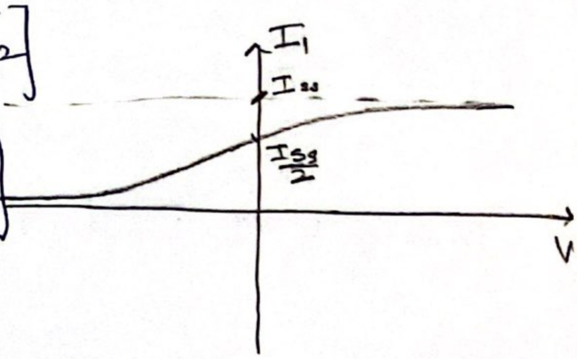


Range of i/p common mode bias

$\rightarrow \underbrace{V_{GS_{1,2}} + V_{OD3}}_{\text{to keep } M_1, M_2 \text{ ON}} < V_{CM} < \underbrace{V_{DD} - \frac{I_{SS}R_D}{2} + V_{th}}_{\text{To stay in saturation.}}$

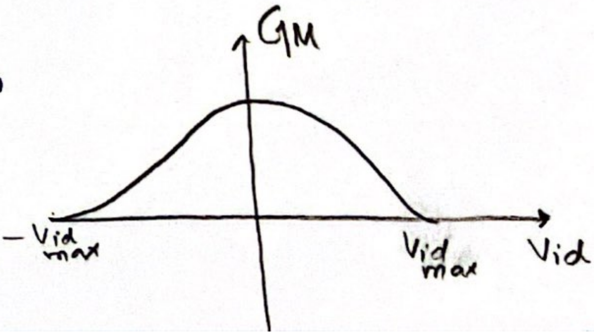
$$I_1 - I_2 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[V_{id}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{id}^2}\right]$$

$$\rightarrow I_{1,2} = \frac{I_{SS}}{2} \pm \frac{1}{4}\mu_n C_{ox}\frac{W}{L}\left[V_{id}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{id}^2}\right]$$



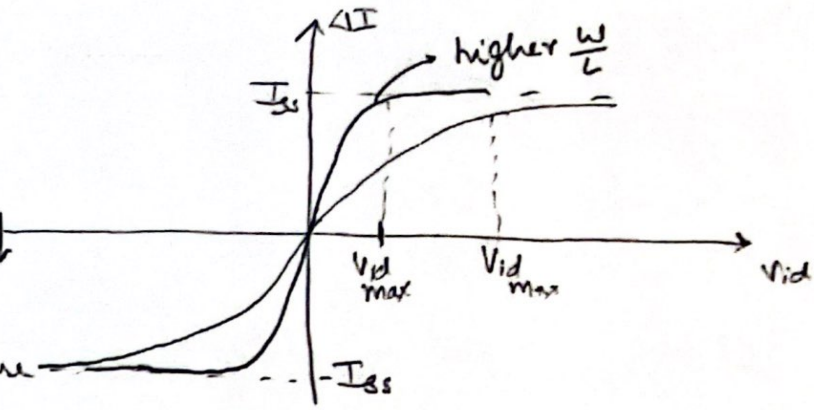
$$\rightarrow G_M = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2V_{id}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{id}^2}}$$

$$\rightarrow V_{id_{max}} \xrightarrow{} \sqrt{2}\cdot V_{OP} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \quad \text{To keep both transistors ON}$$



→ Note:- When i/p is differential 'X' is a virtual ground & the circuit can be split into regular circuits to be analysed.

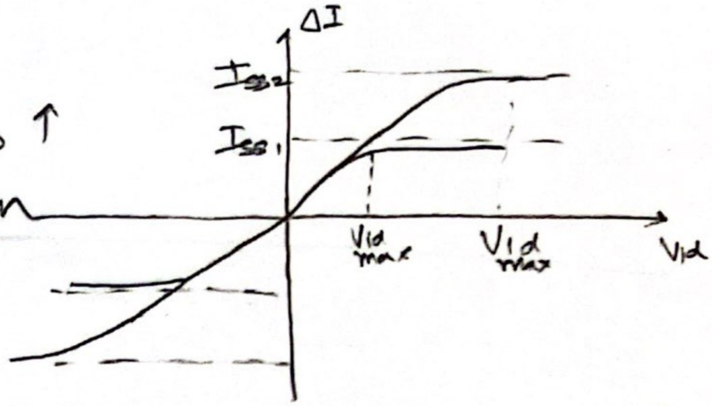
Observations



① $\frac{W}{L} \uparrow \Rightarrow V_{id\,max} \downarrow$

gain ↑ but swing at i/p ↓

⇒ All current flows through one transistor faster because it is bigger.



② $I_{ss} \uparrow \Rightarrow V_{id\,max} \uparrow$

gain is same, swing at i/p ↑

→ Higher power consumption

→ When Vin changes by small amount, all the change appears across the V_{GS} and V_x acts as a virtual ground. This is the beauty of this topology. Remember this is only for differential inputs and small signals.

→ $\boxed{A_{v_d} = -g_m (R_{out})}$

Same gain as single side? Not really since current consumption is double. If power is constant gain ↓ by $\sqrt{2}$.

* CM input DM output ⇒ $A_{v_{cm\,dm}} = 0$

→ DM input CM output ⇒ $A_{v_{dm\,cm}} = 0$

} Ideally.

→ CM input CM output ⇒ * $\boxed{A_{cm\,cm} = \dfrac{-R_D}{\frac{1}{g_m} + 2R_{SS}}}$ *

→ Ideally should be 0 since $R_{ss} \rightarrow \infty$ and we dont have a virtual ground.

$$SNR = \frac{(Av_{DMDM} \cdot Vin_{DM})^2}{(Av_{CMDM} \cdot Vin_{CM})^2}$$

Mismatch

Resistor mismatch

- $R_1 = R$, $R_2 = R + \Delta R$.

$\Rightarrow$ $$\boxed{Av_{CMDM} = \frac{\Delta R_D}{R_D} \cdot Av_{CMCM}}$$

g_m mismatch

> $g_{m1} = g_m$; $g_{m2} = g_m + \Delta g_m$.

$$\boxed{Av_{CMDM} \cong \left|\frac{\Delta g_m}{g_m}\right| \cdot Av_{CMCM}}$$

Δg_m is small.

→ In case of threshold mismatch g_m changes.

→ Common mode rejection ratio CMRR

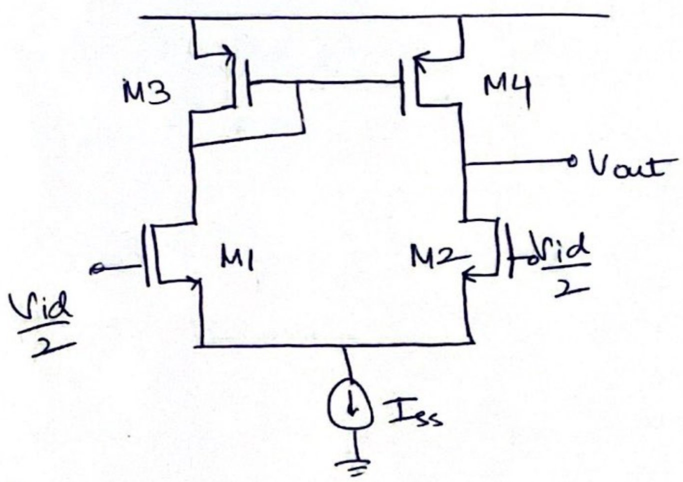
$$\boxed{CMRR = \frac{Av_{DMDM}}{Av_{CMDM}}}$$

How well can the circuit reject common signals like noise.

Ideally → ∞.

- Active Load Differential Amplifier ⇒ Single ended output, Higher gain

→ Very difficult to bias 3 current sources. So we copy current from one line to another and only bias I_{SS}.



$$\boxed{G_m = g_{m1}}$$

$$\boxed{R_{out} = r_{o2} \parallel r_{o4}}$$

$$\boxed{Av_{DM} = g_{m1}(r_{o2} \parallel r_{o4})}$$

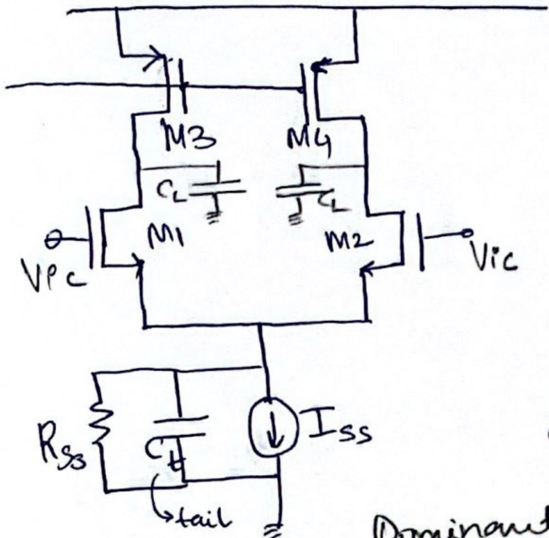
So far so good

$$Av_{CM} = \frac{\frac{1}{2g_{m3,4}} \parallel \frac{r_{o3,4}}{2}}{R_{SS} + \frac{1}{2g_{m1,2}}}$$

- $$\boxed{Av_{CM} \approx \frac{\frac{1}{2g_m}}{R_{SS}}}$$

→ Not so good.

Frequency Response of Diff. Amp



Assuming we could bias it

$$Av_{CMDM} = \frac{\Delta g_m \cdot R_D (R_{SS} C_t s + 1)}{(R_D C_L s + 1)(R_{SS}(g_{m1} + g_{m2}) + 1 + R_{SS} C_t s)}$$

$R_D = r_{o1} \| r_{o3}$

$R_{SS} = R_{SS}$

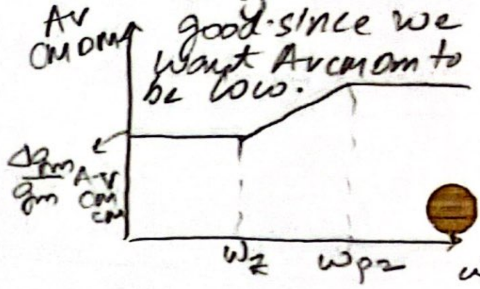
$\therefore \quad \omega_z = \dfrac{1}{R_{SS} C_t}$

Dominant Pole "let's ignore it" $\leftarrow \omega_{p_1} = \dfrac{1}{R_D C_L}$

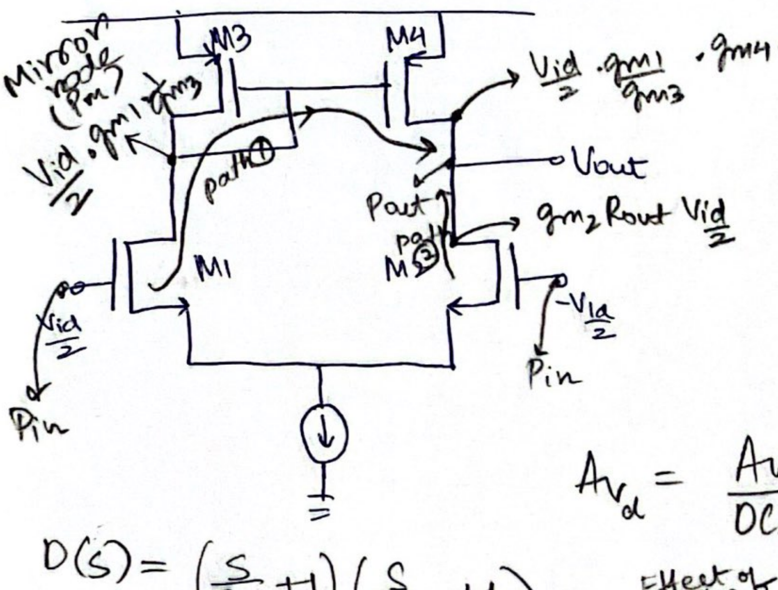
$\omega_{p_2} \simeq \dfrac{g_{m1} + g_{m2}}{C_t}$

tail capacitor has added a pole and a zero. Not good since we want Av_{CMDM} to be low.

→ The tail cap causes significant problems when we look at Av_{CMDM}.



Active load



mirror pole

$$P_M = \frac{1}{\frac{1}{g_{m3}}\left(2C_{gd_1} + C_{db_1} + C_{db_3} + C_{gs_3} + C_{gs_4} + C_{gd_4}\right)}$$

since miller effect & gain of M1 is −1

No miller effect since output pole is at low freq. & gain is 0 when we reach P_m

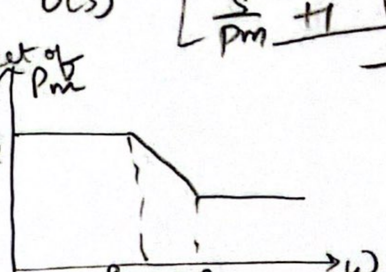
$$Av_d = \frac{Avdo}{D(s)} \cdot \left[\frac{\frac{s}{2P_m} + 1}{\frac{s}{P_m} + 1}\right]$$

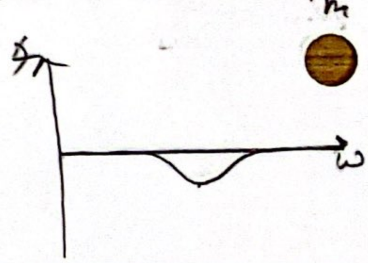
→ zero at $2P_m$

→ pole at P_m

$$D(s) = \left(\frac{s}{P_{in}} + 1\right)\left(\frac{s}{P_{out}} + 1\right)$$

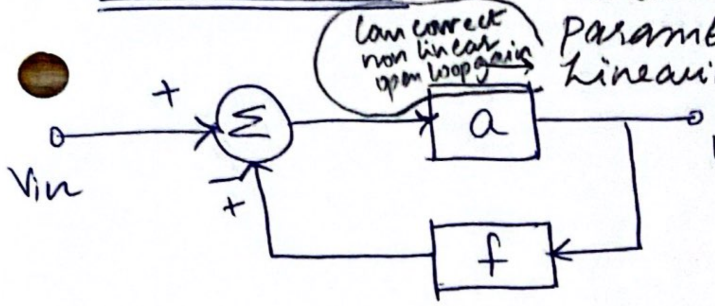
→ P_m & $2P_m$ are less than a decade apart so there is a small dip in phase response, which is annoying.





Feedback

→ Higher bandwidth.
→ Desensitizing the gain w.r.t transistor parameters. as long as gain is high.
→ Linearization!



$$\boxed{\frac{V_{out}}{V_{in}} = \frac{a}{1+af}}$$

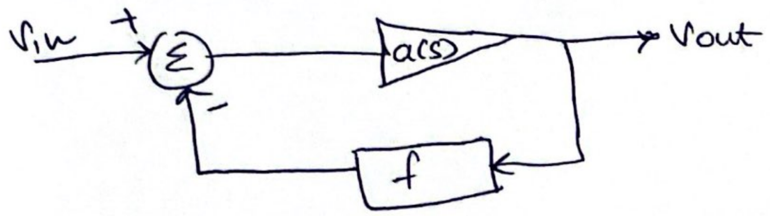
af → loop gain (T)

$$A = \frac{a}{1+af}$$

→ Relative variation in open loop gain has a small effect on total gain $\left\{ \frac{dA}{A} = \frac{da}{a}\left(\frac{1}{1+af}\right) \right\}$

→ Variation in feedback gain has a larger effect so we try to keep feedback as a passive element $\left\{ \frac{dA}{A} = \frac{df}{f}\left(\frac{-af}{1+af}\right) \right\}$

Frequency Response

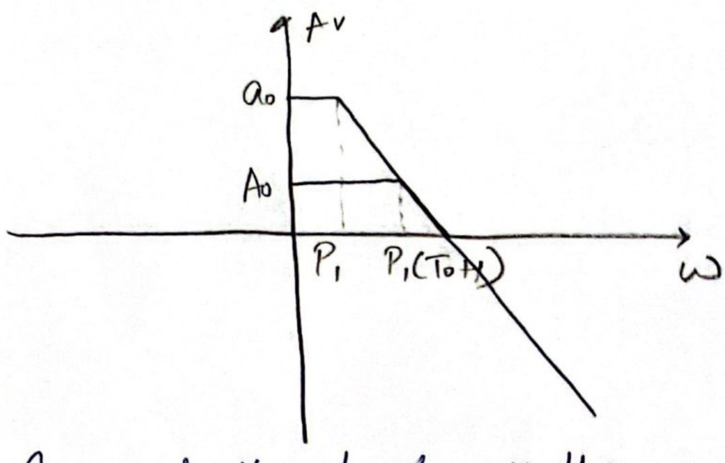


Ideal case of only one pole.

$$\boxed{A(s) = \frac{A_0}{\frac{s}{P_1(T_0+1)} + 1}}$$

$$A_0 = \frac{a_0}{a_0 f + 1}$$

$$a(s) = \frac{a_0}{\frac{s}{P_1} + 1}$$

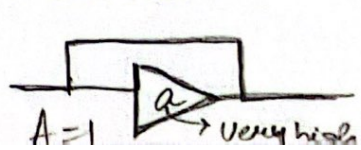


→ Adding feedback can compromise gain for B.W.

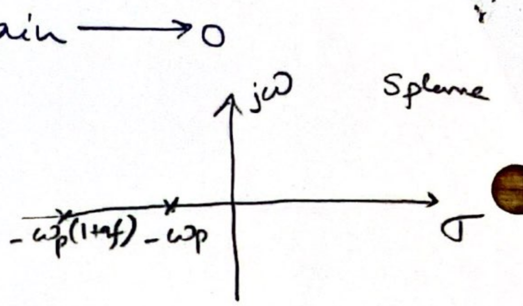
$a_0 \longrightarrow \frac{a_0}{1+T}$

$BW \longrightarrow BW(1+T)$

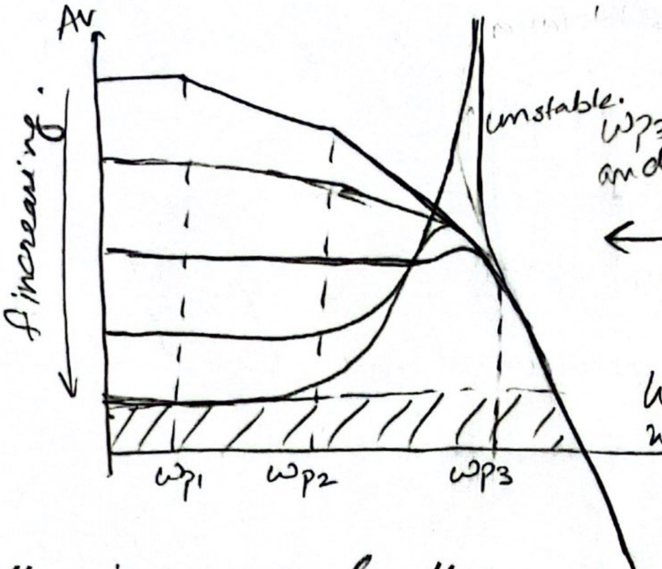
⇒ Ideal buffer trades all the gain for B.W

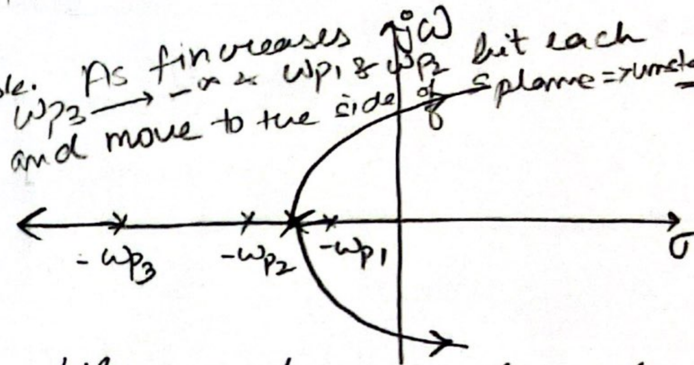


→ As $af \to \infty$, pole $\to -\infty$ & gain $\to 0$



→ However in reality we have more than one pole and as f ↑ we start seeing shooting in the gain and system becomes unstable.

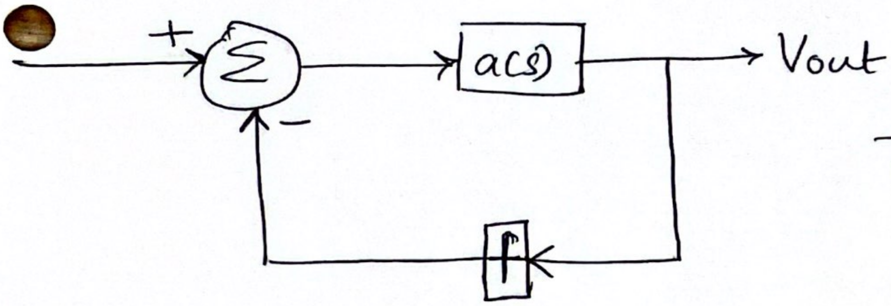




When we have complex poles we start seeing ringing.

→ As we further increase f the peaking might go down but the time domain response actually goes to ∞.

Stability



$T = af \rightarrow$ loop gain

$A = \dfrac{a}{1+af}$

Marginally stable

$|af| = 1$

$\angle af = \pm 180^\circ$

Therefore we want to avoid

$|af| > 1$ & $\angle af = -180^\circ$

$\downarrow$ 0dB, $\downarrow$ -180°

Unstable

$|af| > 1$

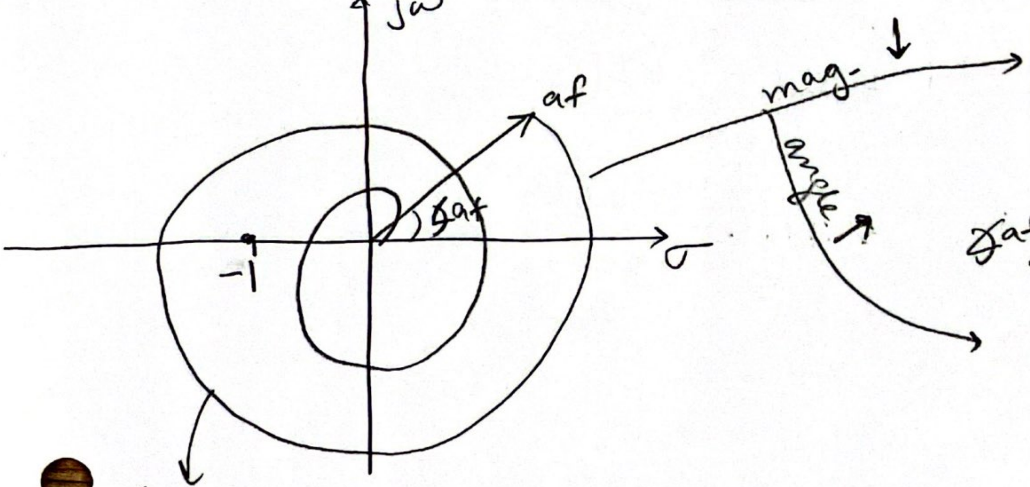
$\angle af = -180^\circ$

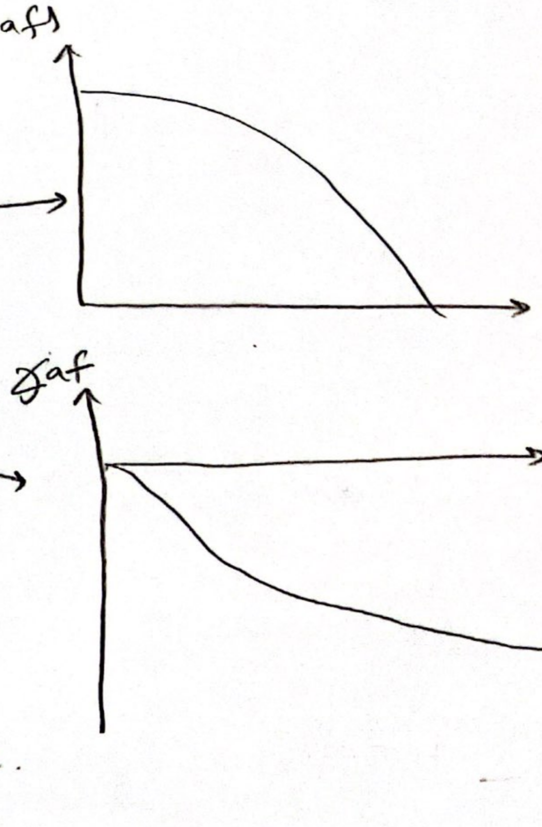
Stable

$|af| < 1$

$\angle af = \pm 180^\circ$.

On s plane :-

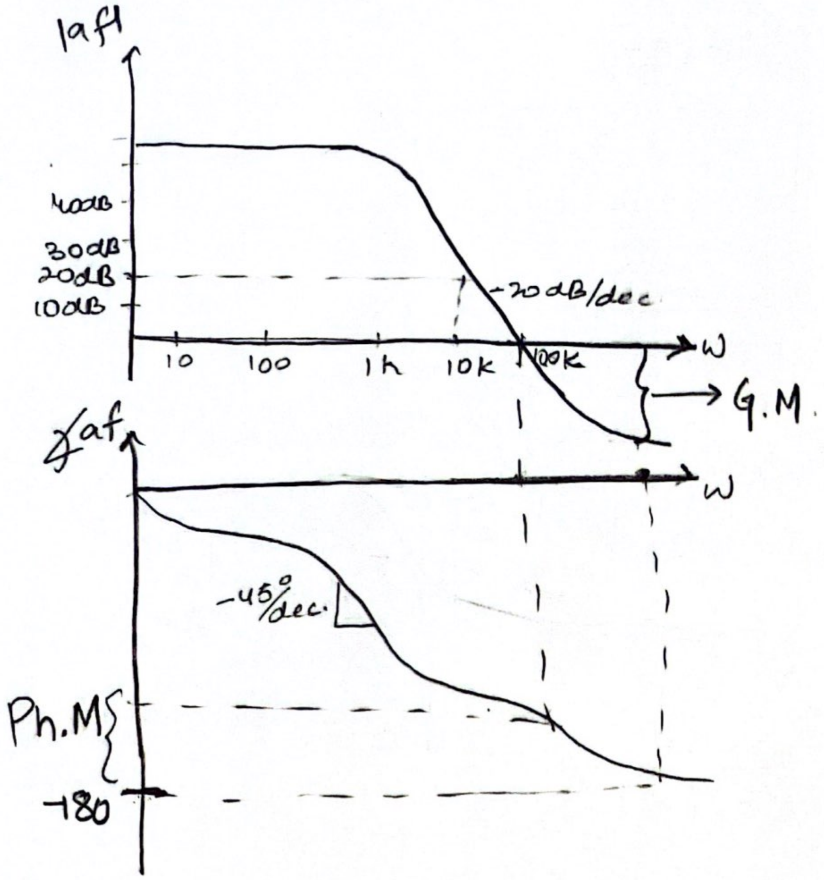




→ If it encircles $\underline{1} \Rightarrow$ unstable.

→ Only true if open loop system is stable.

Gain and Phase Margins



Gain Margin

At $\measuredangle af = -180°$ how much room is left before hitting the dreadful 0dB.

$$\boxed{G.M = \frac{1}{|af| \text{ at } \omega_{180°}}}$$

Phase margin

At $|af| = 1$ how much phase is left before hitting the dreadful $-180°$.

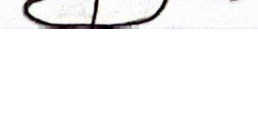
$$\boxed{Ph.M = 180° + \measuredangle af \text{ at } \omega_{0dB}}$$

→ For a system $A(s) = \dfrac{A_0}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\left(\frac{s}{\omega_{p3}}+1\right)}$

$$\left.\begin{aligned} |A(j\omega)| &= 20\log\frac{\omega}{\omega_{p1}} + 20\log\frac{\omega}{\omega_{p2}} + 20\log\frac{\omega}{\omega_{p3}} \\ \measuredangle A(j\omega) &= -\tan^{-1}\frac{\omega}{\omega_{p1}} - \tan^{-1}\frac{\omega}{\omega_{p2}} - \tan^{-1}\frac{\omega}{\omega_{p3}} \end{aligned}\right\}$$

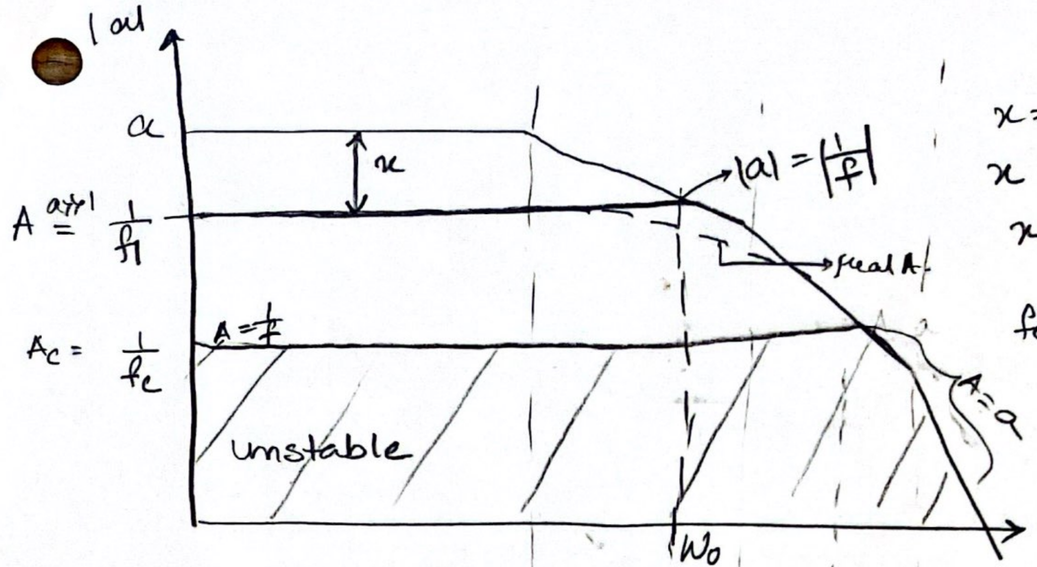
When poles are atleast 1 decade apart. Not sure otherwise.

→ GM & Ph.M may be high but small change in PVT could make the system unstable. Need to look at S plane to get the full picture.



→ If it turns a little it becomes unstable

→ In our 3 pole system



$x = 20\log|a| - 20\log\left|\frac{1}{f}\right|$

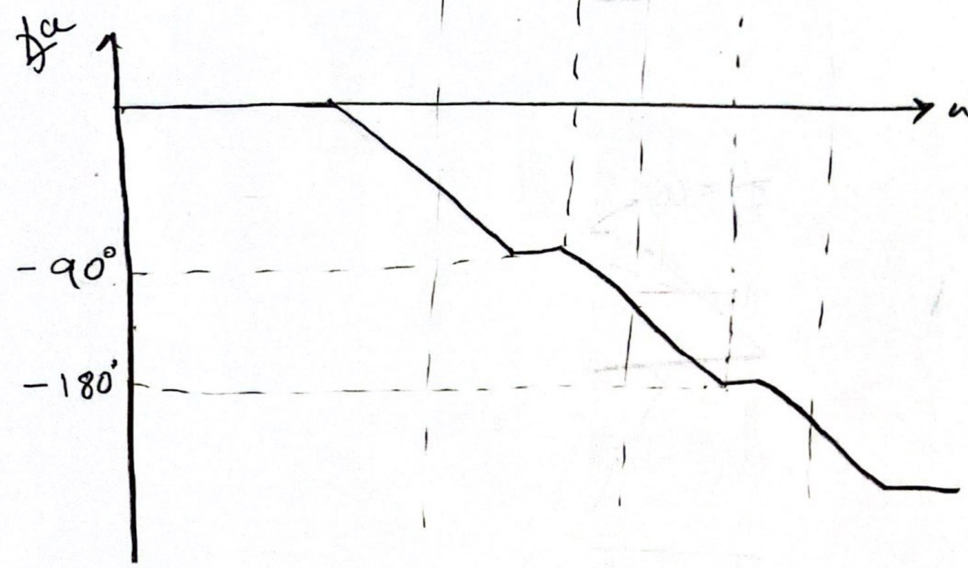
$x = 20\log|af|$

$x = 20\log|T|$

f_c = critical feedback at which Ph. M = 0°.

$f > f_c \Rightarrow$ unstable.

At $f = f_1$, Ph.M = 90°



$A \cong \frac{1}{f}$ when a is large or when ω is small

$A \cong a$ when a is small or when ω is large.

<u>At ω_0</u> :- $|af| = 1$ and $|A| = \frac{|a|}{|1 + a(j\omega)f|}$

<u>Assume Ph. M at ω_0 = 90°</u>

$$|A| = \frac{|a|_{\omega_0}}{|1 + a(j\omega_0)f|}$$

$\measuredangle a(j\omega_0)f = -90°$

$|a(j\omega_0)f| = 1$

$\Rightarrow |A| = \frac{|a|}{\sqrt{2}} \Rightarrow$ magnitude of A < a which is expected

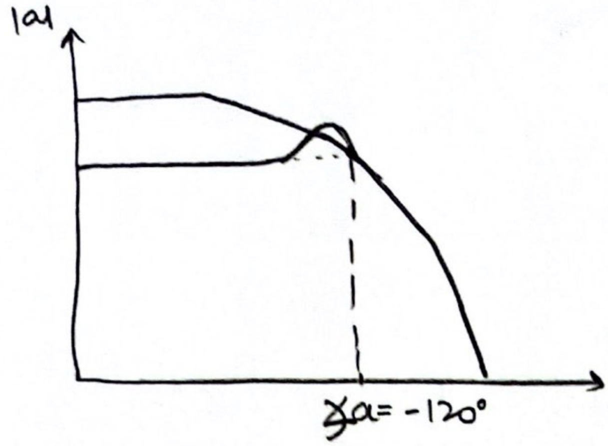
⇒ it is 3dB lower.

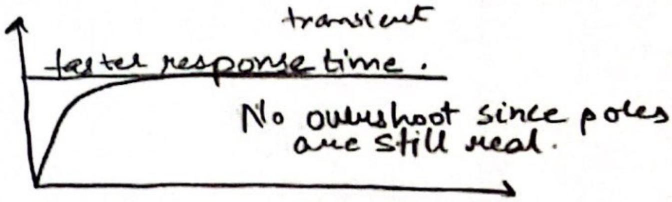
⇒ System is well behaved ⇒

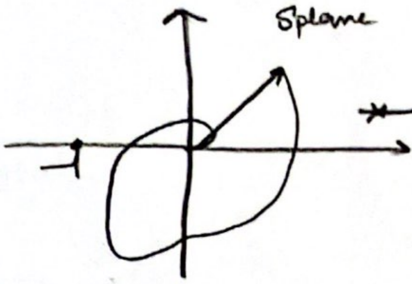
→ **Ph. M = 60°**

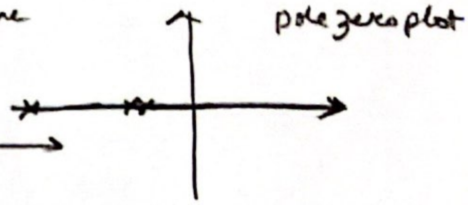
$\Rightarrow |A|_{\omega_{at\ Ph.M=60°}} = \dfrac{|a|}{\left|1+1e^{-j(120°)}\right|} = \underline{\underline{|a|}}$

⇒ There is 3dB peaking so that |A| and |a| are same.





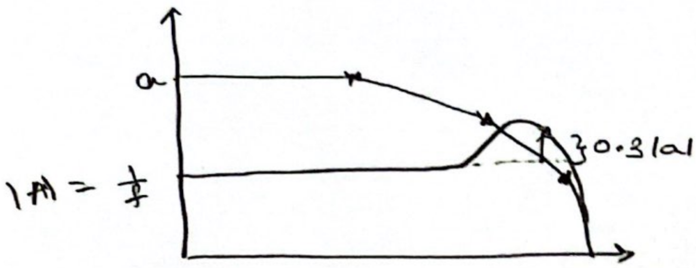


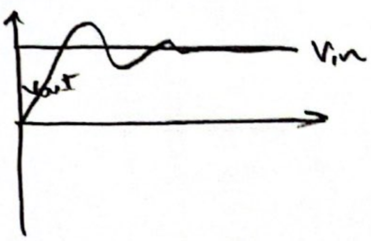


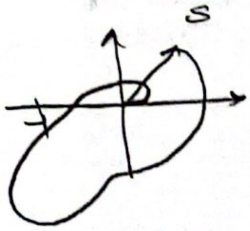
→ **Ph. M = 45°** → popular case. $\measuredangle a = -135°$

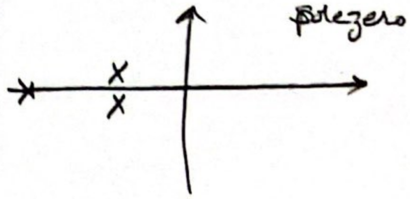
$|A| = \dfrac{|a|}{\left|1+1\cdot e^{-j(135°)}\right|}$

$|A| = 1.3\,|a|$



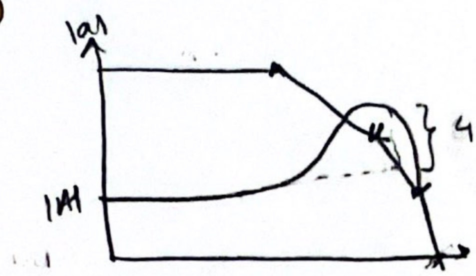


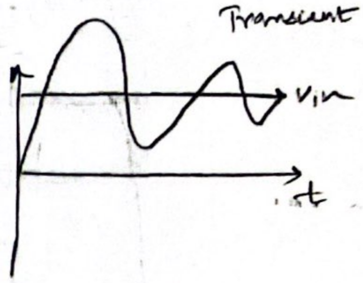


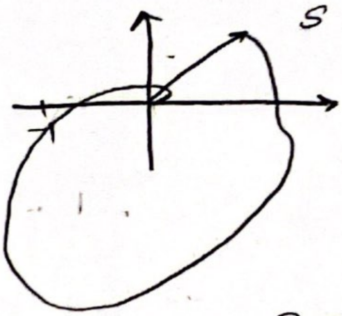


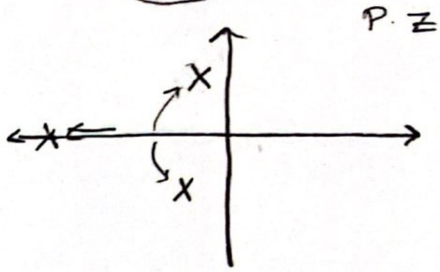
Phase margin = 30° $\measuredangle a = -150°$

$|A| = 5|a|$

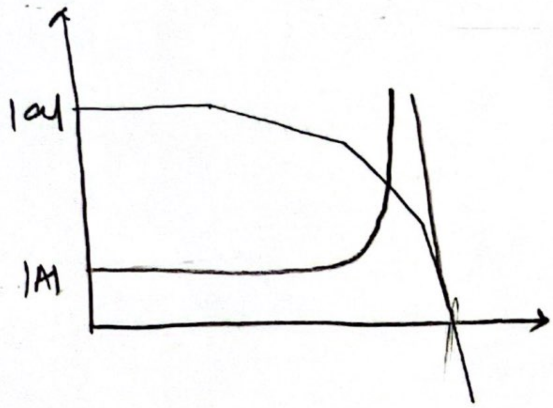


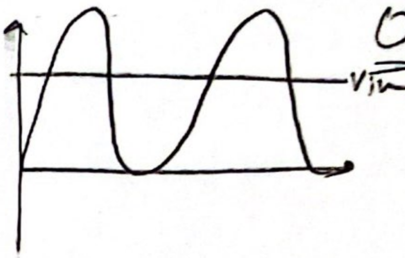




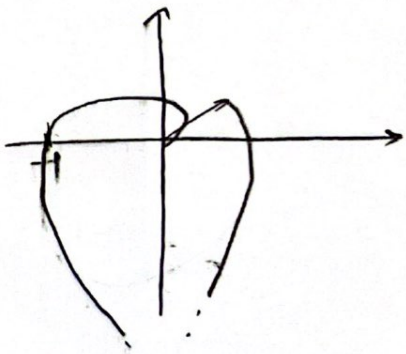


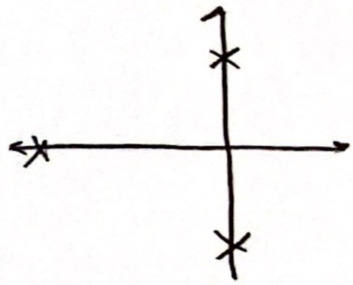
Phase Margin = 0°





Oscillator!





→ We need to change 'a' so that even at max feedback $f=1$ we do not lose stability. Solution:- Give up the BW.

Frequency Compensation.

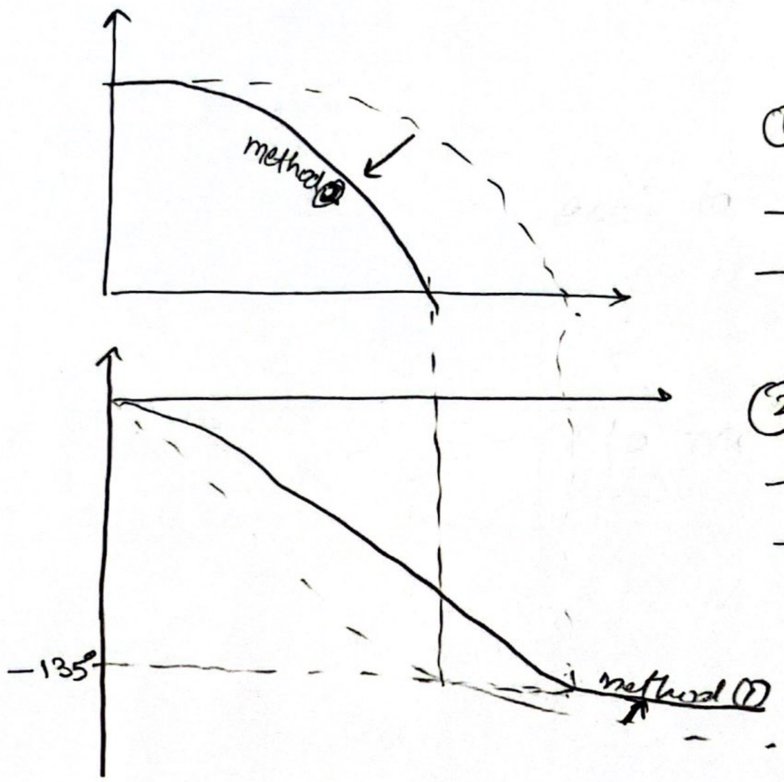
→ Let's say we want Ph.M = 45° when f=1. } ⇒ $|af|_{w_0} = 1$ & $\angle af|_{w_0} = -135°$

<u>Method ①</u> Decrease |phase| to meet −135° when |af|=1

→ Better since B.W is not lost. However much harder to implement.

<u>Method ②</u> Decrease gain to meet 1 when $\angle af = -135°$.

→ Easier to implement.



<u>Influence of Zeroes.</u>

① Left half plane zero.

→ Increase both |a| & $\angle a$.

→ May or may not work since we need |a|↓ & $\angle a$↑

② Righ half plane zero.

→ Increase |a| & decreases $\angle a$

→ Really bad since we want ↓|a| & ↑$\angle a$.

→ How to implement? Add an RC stage that has a low frequency pole.

<u>Steps to add P_{new}</u> :-

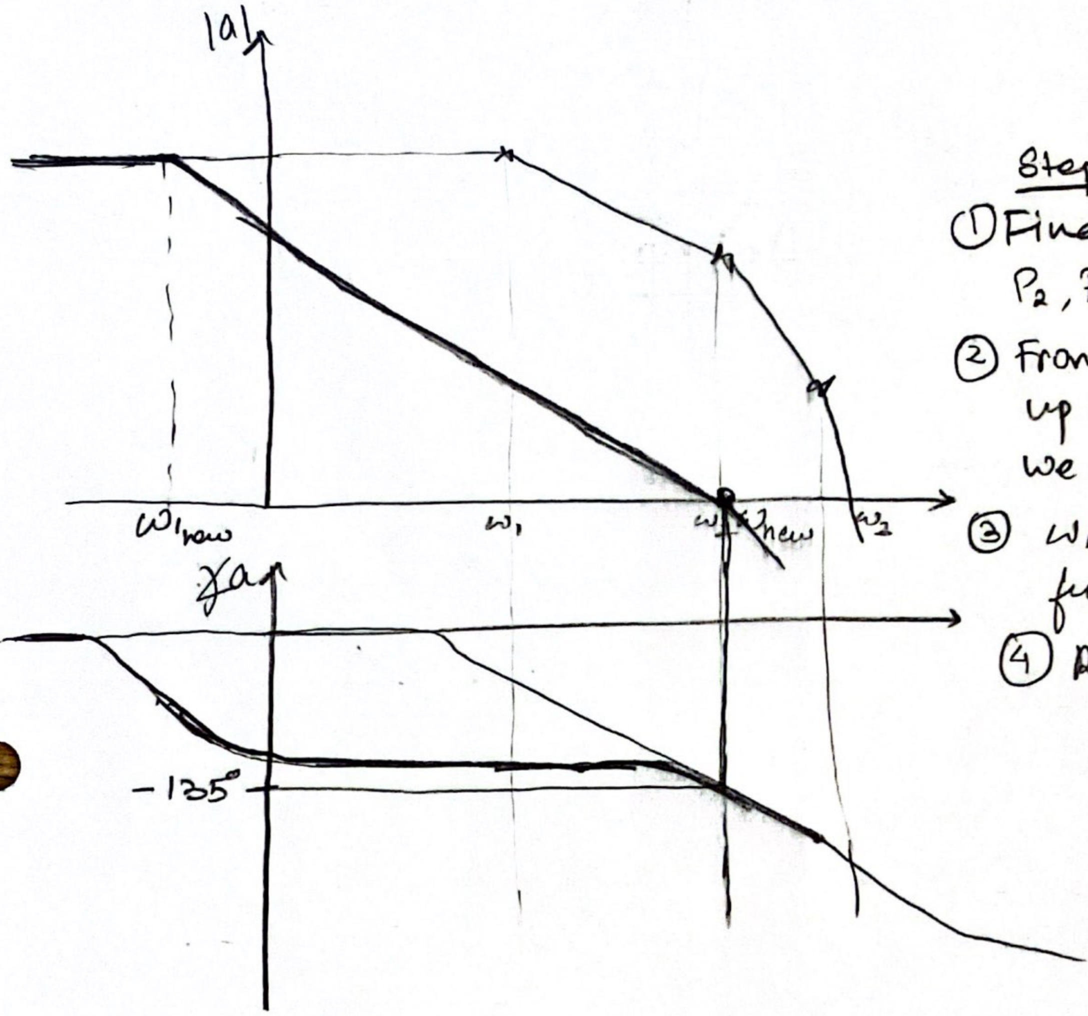
① Find w_0 where current poles give −45° since new pole should give −90° ⇒ total = −135

② From w_0 in |a| trace back @ −20 dB/dec & see where it hits |a| ⇒ w_{new}.

③ Add a pole at w_{new} & redraw |a| & $\angle a$.

Another (Better) Implementation

Lower P_1 such that at $\omega|_{|af|=1}$ we hit $\angle a = -135^\circ$.

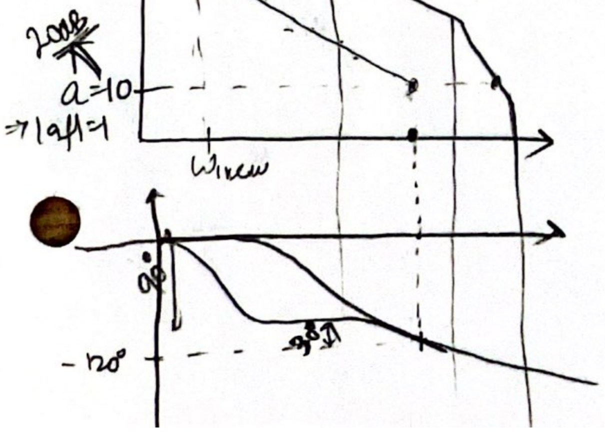


Steps:-

1. Find ω_0 at which $P_2, P_3 \ldots$ give -45°.
2. From this ω_0 we move up at -20dB/dec until we hit $|a|$ @ ω_{1new}.
3. ω_{1new} is the new pole frequency of ω_1.
4. Redraw $|a|$ & $\angle a$.

→ Here ω_2 is the 3dB BW of $|A|$ & unity BW of $|a|$.

→ If $f = 0.1$ & Ph.M $= 60^\circ$.



Steps:-

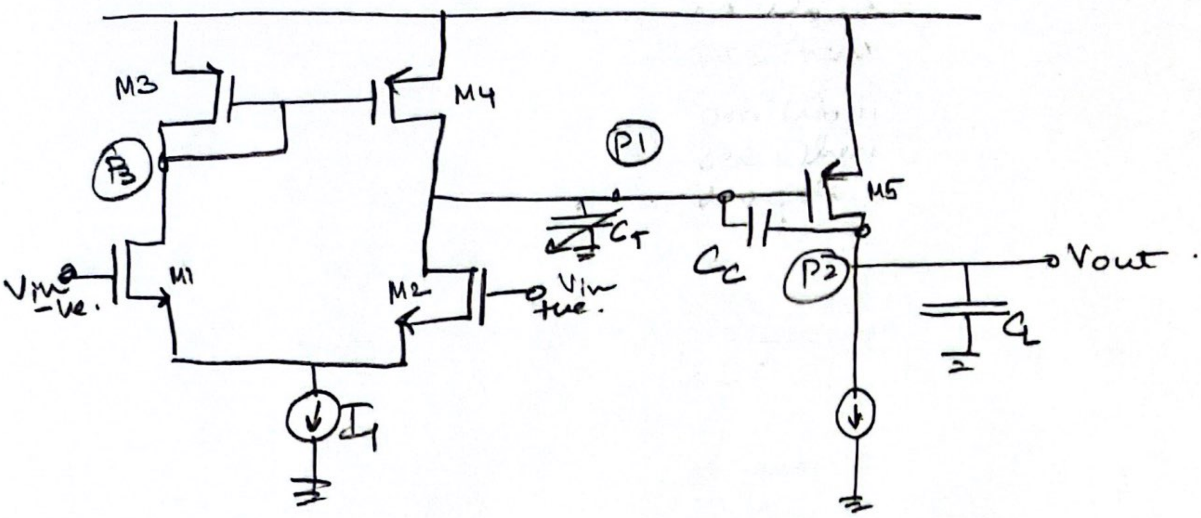
1. Find ω_0 at which $P_2, P_3 \ldots$ give -30°.
2. From this freq. we move from $|af| = 10$ to $|a|$. Wherever it hit $|a|$ is our new ω_{1new}.

If poles are atleast 1 decade apart:-

$$\angle A(s) = -\tan^{-1}\frac{\omega}{\omega_{p1}} - \tan^{-1}\frac{\omega}{\omega_{p2}}$$

$$|A(s)| = 20\log\frac{\omega}{\omega_{p1}} + 20\log\frac{\omega}{\omega_{p2}}$$

Circuit implementation : opamp example.



P1: $\frac{1}{R_1C_1}$ where $R_1 = r_{o2} \parallel r_{o4}$;

no miller since high gain

$C_1 = C_{gd_2} + C_{db_2} + C_{gd_4} + C_{db_4} + C_{gd_5}(1+g_{m_5}r_{o5}) + C_{gs_5}$

P2: $\frac{1}{R_2C_2}$ where $R_2 = r_{o5}$

$C_2 = C_L + C_{gd_5} + C_{db5}$

high gain ⇒ no miller.

P3:- High ⇒ Don't care.

We want to ↑ P_1 ⇒ ↑ C_1 ⇒ add a capacitor C_T

$\frac{1}{R_1C_T}$

→ Assuming P_1 ($\frac{1}{R_1C_1}$) was 10KHz & we now want it at 1.1 Hz.
C_T needs to be 10K times larger than C_1 ⇒ not realistic to be on chip.

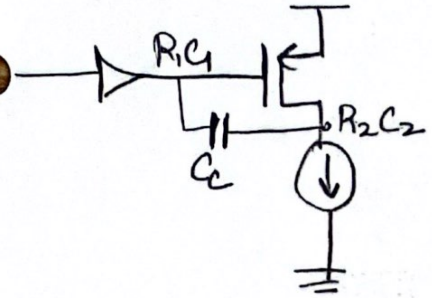
→ But observe miller effect at C_{gd_5} ⇒ why not increase that by adding a compensation cap there?

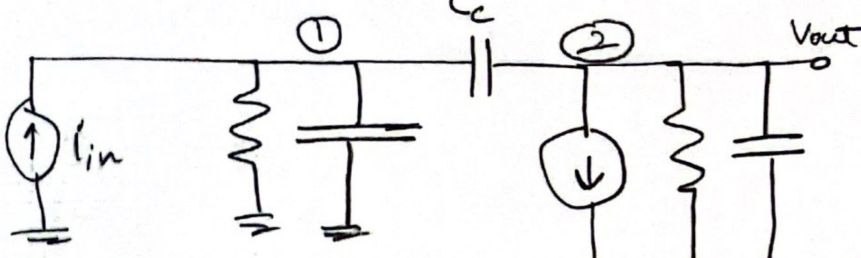
→ Notice this C_c should increase C_2 or lower P_2 ⇒ ↓ BW of (A)

→ In reality it actually moves P_2 to higher frequencies

→ This is called pole splitting.

Pole Splitting :- Why does it happen?





$A_1 = -g_{m_I} R_1$ $\qquad$ $A_2 = -g_{m_{II}} R_2$

$$\frac{V_{out}}{i_{in}} = \frac{R_1R_2\left[C_cS - g_{m5}\right]}{1 + S\left[R_1(C_c+C_1) + R_2(C_c+C_2) + R_1R_2g_{m5}C_c\right] + R_1R_2S^2\left[C_cC_1 + C_cC_2 + C_1C_2\right]}$$

notice $S_z = \frac{g_{m5}}{C_c}$ $\longrightarrow$ right half plane zero $\rightarrow$ very bad!
will deal with it later.

Assuming dominant pole :-

$$P_1 = \frac{1}{R_1(C_c+C_1) + R_2(C_c+C_2) + R_1R_2g_{m5}C_c}$$

If $C_c \gg C_1, C_2$ & $g_{m5}R_1R_2 \gg R_1, R_2$

$$\boxed{P_1 = \frac{1}{R_1\underbrace{R_2g_{m5}}C_c}}$$

$\rightarrow$ C_c is boosted by gain of 2^{nd} stage.

What happened to P_2?

Assuming dominant pole :-

$$P_2 = \frac{g_{m5} C_c}{C_c(C_1+C_2)+C_1C_2}$$

$$\boxed{P_2 \simeq \frac{g_{m5}}{C_1+C_2}}$$ if $C_c \gg C_1, C_2$

Earlier it was $\frac{1}{R_2 C_2}$. It has moved to higher freq. $\Rightarrow$ Great!

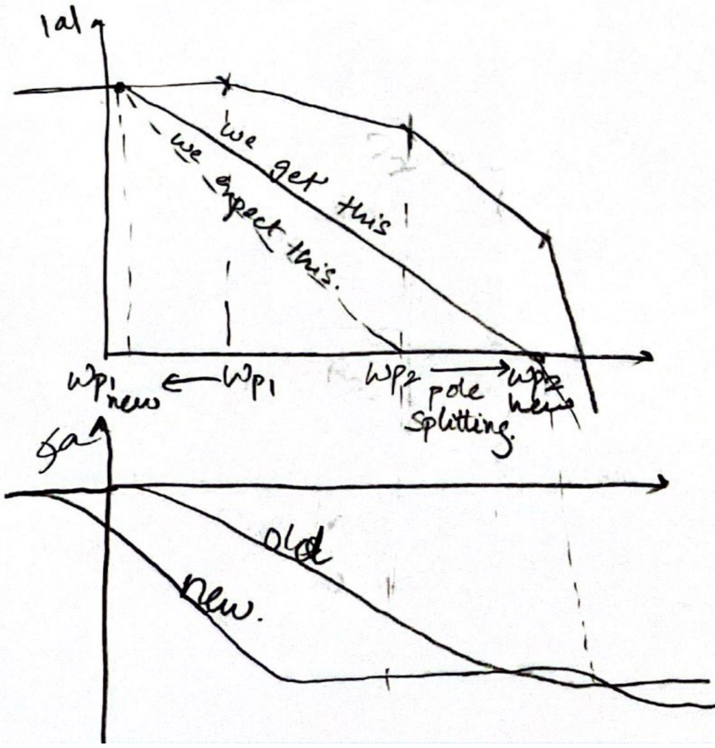
W/o C_c :- $P_1 = \frac{1}{R_1C_1}$; $P_2 = \frac{1}{R_2C_2}$

W C_c :- $P_1 = \frac{1}{R_1C_1(g_{m5}R_2)}\downarrow$; $P_2 = \frac{g_{m5}}{C_1+C_2}\uparrow$

Becomes much higher since: $R_2 \gg \frac{1}{g_{m5}}$

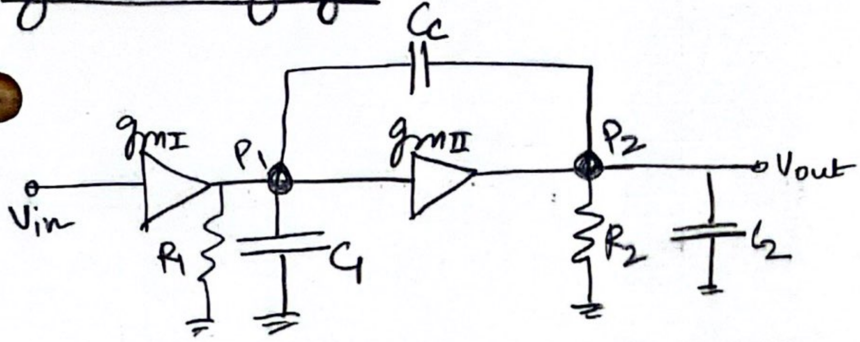
$\Rightarrow$ we regain some lost B.W

(However not all is well since now we have a R.H.P zero) [This is the zero that pushes P_2 to higher frequencies



ω_{-3dB} of A = $\omega_{p2_{new}}$.

Generalizing



$$P_1 = \frac{1}{R_1 (R_2 g_{mII}) C_c}$$

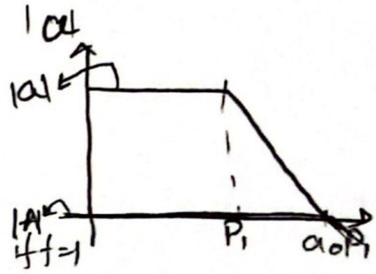
$$z = \frac{g_{mII}}{C_c}$$

$$P_2 = \frac{g_{mII} C_c}{C_1 (C_1 + C_2)}$$

→ System behaves as a single pole system with P_1 as dominant pole.

⇒ GBW product is constant.

→ $A = \dfrac{a}{1+af}$ where $a \cong \dfrac{a_0}{1+\frac{s}{P_1}}$



→ gain of $a = a_0$; BW of $a = P_1$

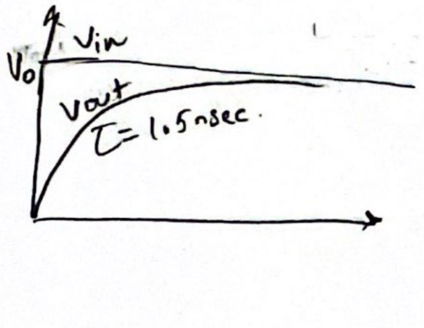
→ gain of $A = 1$; BW of $A = a_0 P_1$ if $f=1$ since GBW = constant

Ex:- If $|a| = 10^5$ & $P_1 = 1KHz$.

After $f=1$ $|A| = 1$ & $BW = 10^8 = 100MHz$..

→ If $V_{in} = V_0 \cdot u(t)$ {step} ⇒ $V_{in}(s) = \frac{V_0}{s}$

O/p would be exponential with $\tau = \frac{1}{2\pi RC} \simeq 1.5nsec.$ where $RC = 10^8 M$



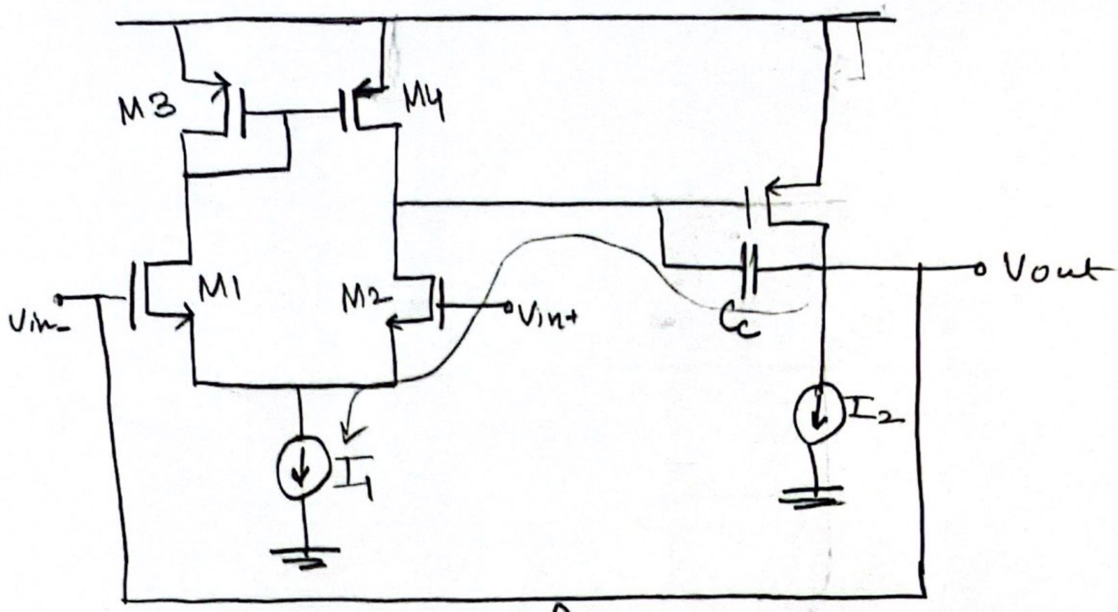
→ However this is only true when V_0 is small.

→ For a large V_0 we see a linear rise in V_{out}.} But why?

Answer:- Does not behave like a single pole system anymore. ⇒ Slew Rate.

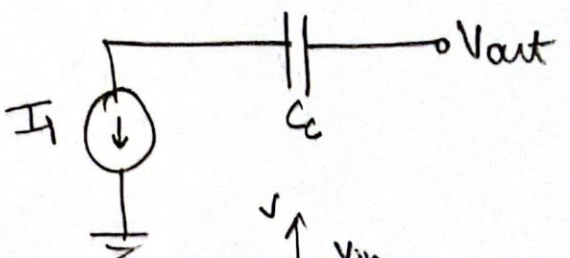
SLEW RATE

Opamp example.



f = 1

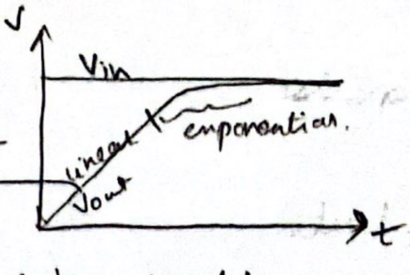
- Initially $V_{in}^{+} = V_{out} = V_{in}^{-} = 0$
- Let's apply $V_{in}^{+} = V_{DD}$ (largest i/p) :- at $t = 0^{+}$: M2 ON
  M1, M3, M4 {Very OFF}
- Max V_{in} for it to be differential is $\sqrt{2}\, V_{OD}$ but here we applied much larger i/p. ⇒ M1 would be completely off since M_2 pulls all I_1.
- Now we have a Cap in series with a current source.



$$\Rightarrow \quad I_1 = C_c \cdot \frac{dV}{dt}$$

$$\Rightarrow \quad \left.\frac{dV_{out}}{dt} = \frac{I_1}{C_c}\right\} \text{ Slew rate !}$$

$$\Rightarrow \quad V_{out} = \frac{I_1}{C} \cdot t \Rightarrow \text{linear increment.}$$



Slope of this line is slew rate

→ This linear rise continues until Vin- ↑ due to feedback & $|V_{id}| < \sqrt{2}\, V_{OD}$ → Diff amp starts working. ⇒ Then it is exponential.

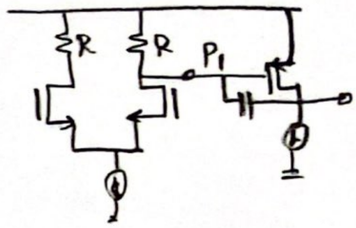
$\Rightarrow$ ∴ Slew Rate $= \dfrac{I_1}{C_c}$

- B.W may show I but S.R. is the real deal because that's how the system actually responds.

→ This is bad so we need to increase the slew rate.

→ How? Increase I_1? It doesn't work much.

→ To increase $I_1 \Rightarrow R\downarrow \Rightarrow P_1 \uparrow$ but we can't change P_1 since we designed it. To keep P_1 constant we must $C_c \uparrow \Rightarrow SR \downarrow$.



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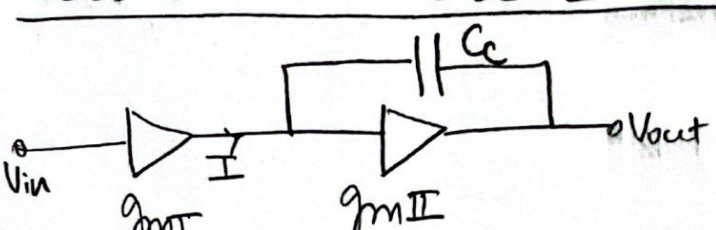
→ Let's take a closer look at S.R

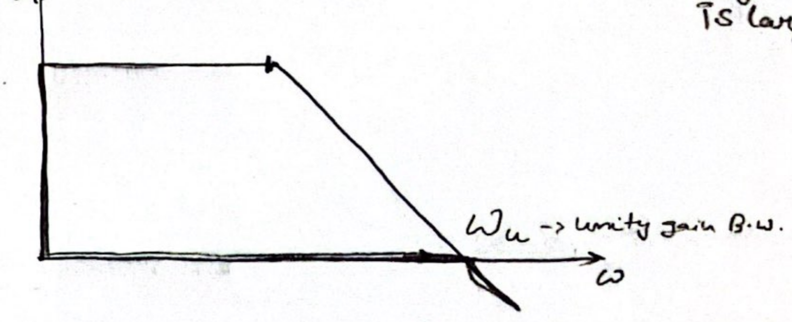


$$\frac{\Delta I}{\Delta V_{in}} = g_{mI}$$

$$\frac{\Delta V_{out}}{\Delta I} = R_{out} \simeq \frac{1}{sC_c}$$

if gain of II is large.

$$\Rightarrow \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{g_{mI}}{sC_c}.$$



At ω_u:- $\left|\dfrac{\Delta V_{out}}{\Delta V_{in}}\right| = 1$

-

$$\Rightarrow \left|\frac{g_{mI}}{sC_c}\right| = 1 \Rightarrow \left|\frac{g_{mI}}{j\omega_u C_c}\right| = 1 \Rightarrow \boxed{C_c = \frac{g_{mI}}{\omega_u}}$$

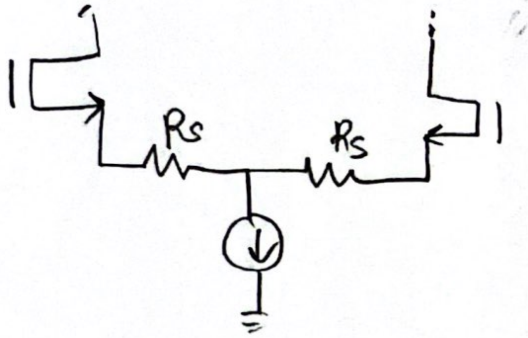
$$\rightarrow \boxed{C_c = \frac{g_{mI}}{\omega_u}}$$ → Not very precise since we are assuming Ph.M is constant. But it gives insight.

$$\rightarrow S.R = \frac{I_1}{C_c} = \frac{I_1}{g_{mI}} \cdot \omega_u$$

$\frac{\Delta V_{out}}{\Delta t}$

① → We could increase B.W :- But this needs to done during original design and not during frequency compensation.

② → For a fixed I_1, need to lower g_{mI} ⇒ Degeneration!

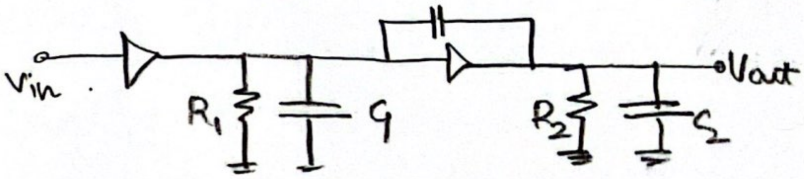


$$\Rightarrow g_m = \frac{g_m}{1+g_m R_S} \Rightarrow$$ Gain ↓, Noise ↑ } Bad

But SR ↑ } Good

→ Lesson :- Design such that g_{mI} is very low from the start.

→ Dealing with that annoying RHP zero.



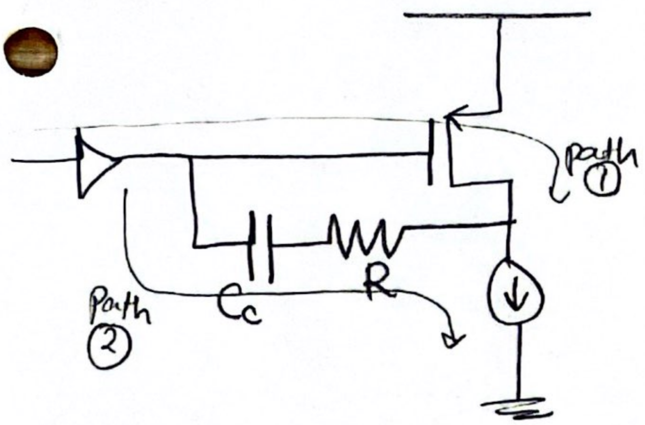
$$\left\{ \begin{array}{l} z = \frac{g_{mII}}{C_c} \\ P_2 = \frac{g_{mII}}{C_1+C_2} \quad \text{if } C_c \gg C_1, C_2 . \\ \omega_u = \frac{g_{mI}}{C_c} \end{array} \right.$$

$$\boxed{\frac{z}{\omega_u} \simeq \frac{g_{mII}}{g_{mI}}}$$ → We push z atleast one decade after ω_u.

$$\Rightarrow \frac{g_{mII}}{g_{mI}} > 10$$

Also notice, $\frac{P_2}{\omega_u} = \frac{g_{mII}}{g_{mI}} \cdot \frac{C_c}{C_1+C_2}$ ⇒ We make $g_{mII} \gg g_{mI}$ so that we can also increase P_2 ⇒ ω_{300} of $|A|$.

Another way to fix the RHP zero.



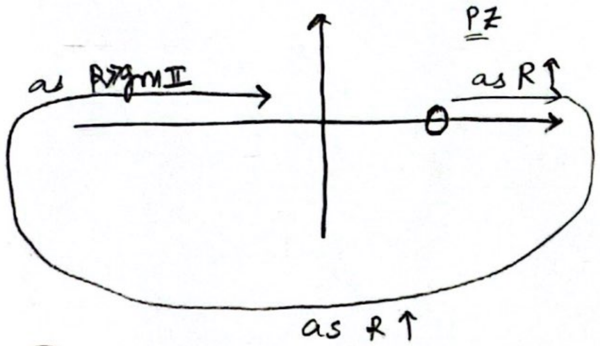
$$z_{w/oR} = \frac{g_{mII}}{C_c}$$

$$z_{with\,R} = \frac{1}{C_c\left(\frac{1}{g_{mII}} - R\right)}$$

at $R = \frac{1}{g_{mII}}$ z is at $+\infty$

at $R > \frac{1}{g_{mII}}$ z is at $-\infty$

at $R \gg \frac{1}{g_{mII}}$ z is in LHP.



→ Intuitively,

Path① has 180° phase shift.

Path② w/o R has 90° phase shift → adding a zero.

Path② w R has <90° phase shift → reducing the effect of the zero.