EE 117B/217B Microwave and Wireless Measurements

Lab 2 - Time Domain Measurements Using VNAs

Fall 2026

CAUTION: READ THIS PAGE CAREFULLY BEFORE STARTING THE LAB

This class uses the FieldFox Microwave Analyzer N9918B as the primary instrument. It serves as the Vector Network Analyzer (VNA) and the Spectrum Analyzer (SA). Along with the FieldFox, this class also uses several other RF instruments and equipment. Please handle the instruments very carefully as they are **very expensive** and delicate. The most vulnerable parts of the instruments are the RF PORTS (APC-3.5 connectors). These connectors are protected by the small Female-Female Port Savers, which are the silver ferrules attached to the ports. **DO NOT REMOVE THE PORT SAVERS FROM THE VNA PORTS FOR ANY EXPERIMENT.**

All connections are to be made through these Port Saver Connectors. If you have never handled RF instrumentation before, ask the instructor to demonstrate how to make connections and handle these connectors, calibration standards, and cables. Always hold the cable with one hand (non-dominant) and rotate the end nuts of the SMA cables with the other (dominant) hand. Once you make a soft connection by hand, use the torque wrench provided to complete the connection. Note that whenever SMA connections are made, you must only rotate the end nuts and not the components; this ensure the internal pin enters and exits the jack in a straight trajectory. As far as pushing the control knobs and keys on the instrument are concerned, no special care is needed (they are fail-safe). The worst that can happen is that the instrument may lock up. In that case, press the `Preset` key or ask the instructor for help.

Caution for this particular lab

This lab uses printed circuit boards, which require their own handling precautions. Handle all boards by their edges. The board-mounted SMA launches are soldered to a thin substrate and will lift the trace if stressed. To be safe, **do not torque the board-mounted connectors to the torque wrench limit**; use a mild torque with the wrench instead. Ask the instructor to show you when you get to this section in the experiments.

Introduction

In Lab 1 we became familiar with the Vector Network Analyzer, its noise reduction and coherent subtraction capabilities, and coaxial calibration (1-port OSL and 2-port SOLT). This lab extends those skills into the *time domain*. The same swept-frequency S-parameters measured by the VNA can be transformed to the time domain, where the position and type of each discontinuity along a device becomes directly visible. This enables two powerful capabilities: (1) *time gating*, which isolates a response of interest and removes unwanted reflections; and (2) *time domain reflectometry* (TDR), which locates and quantifies discontinuities along a line. You will use both: TRL to calibrate the connectors *away* and measure what is behind them, and then TDR to look directly *at* those same connectors and read a parasitic model off the screen.

This lab also introduces *TRL calibration*, a technique better suited than coaxial SOLT to devices fabricated on substrates, PCBs, and ICs, because it references the measurement to a fabricated transmission line and moves the reference plane onto the board. In this lab the TRL calibration is performed **manually**: you will collect the raw (uncorrected) responses of the Thru, Reflect and Line standards and solve for the error terms yourself, following the procedure given in Appendix A.

Objectives

Building on the S-parameter measurements and coaxial calibration of Lab 1, the purpose of this lab is to introduce the time-domain capabilities of the VNA and the calibration techniques appropriate to non-coaxial media. At the end of this lab you will:

1. Become familiar with the time-domain transform of the VNA and understand how bandwidth, number of frequency points, and windowing shape the recovered response.
2. Use time gating to isolate the response of a single discontinuity, and understand the trade-offs it introduces in the frequency domain.
3. Perform a TRL calibration manually and use it to de-embed a device fabricated on a substrate.
4. Extract the values of lumped passive elements from a de-embedded measurement, accounting for the board trace left between the reference plane and the component.
5. Read a lumped parasitic model of a connector directly off a TDR step response, and extract element values by integrating the area under the trace.

This lab is divided into four parts: (1) Time domain and time gating, (2) Manual TRL calibration and de-embedding, (3) Extraction of passive elements in the frequency domain, and (4) Extraction of the same elements, and of the connectors themselves, directly from the TDR step response.
Since 3 students are performing the lab together, please ensure that you **take turns on performing the experiments.**

Equipment List

Table 1 and Fig. 1 list the instruments, standards, and devices under test used throughout this lab. Verify that all items are present at your bench and in your Lab Box before beginning, and consult the instructor if anything is missing or damaged.

Lab Report

Each lab comes with a Lab Questionnaire. As you go through this Lab Manual, you will encounter several questions along the way. When you do, go over to that question in the Lab Questionnaire and answer it accordingly. The list of questions and answers will be turned in as your Lab Report. All students in one lab group will submit one combined lab manual. **It is strongly recommended that you complete the post-processing parts of the lab also during the designated lab hours since you may need to retake measurements at times.**

Equipment	**Part Number**	**Qty**
Keysight FieldFox Microwave Analyzer	N9918B	1
SMA coaxial cable(s) / extension cables (1 m)	NA	2
Extra long SMA cable (delay line, ∼30 ft)	NA	**shared**
RF high-pass filter (HPF)	SHP-20+	1
10-dB attenuator (pad)	FW-10A+	1
Calibration kit (3.5 mm)	TOJOIN 3.5 mm cal kit	1
Female-Female SMA adapters	NA	2
TRL/TDR calibration substrate (THRU, REFLECT, LINE and DUT structures)	NA	1
Torque wrench	(shared)	1
USB flash drive	Sandisk	1
PC workstation with MATLAB / Python (`scikit-rf`)	(shared)	1

Table 1: Equipment used in the VNA Lab 2 experiments.

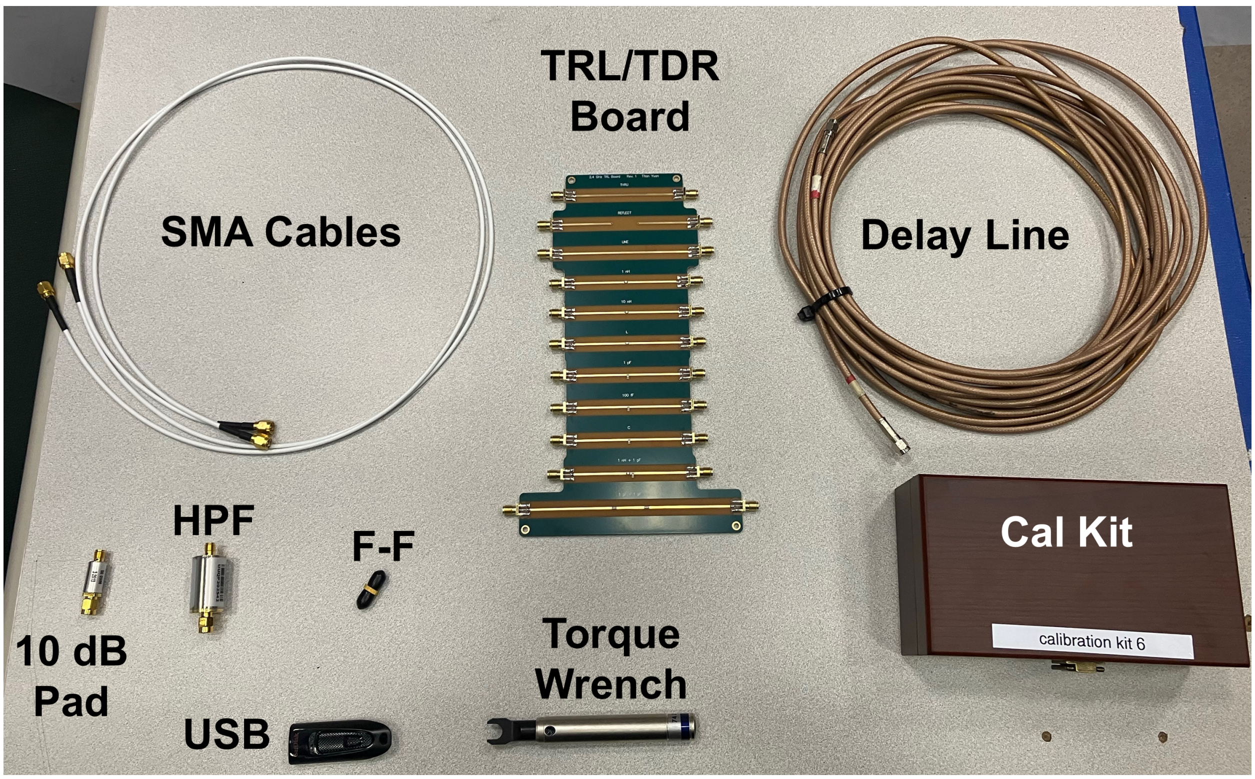

Figure 1: Components used in this lab (VNA Time Domain).

1 Introduction to Time Domain and Time Gating

The purpose of this section is to familiarize students with the time-domain transform of the FieldFox and, in particular, to make explicit how the *gating parameters* (span, shape, and position) reshape the recovered frequency response. This builds directly on the time-domain gating introduced conceptually in the Lecture.

1.1 Time Domain Experiments

To get started, once the FieldFox is turned on, press `Mode` and select `NA`. Set the frequency range to 1 GHz – 5 GHz, with 801 points and an IF bandwidth of 10 kHz. Set up the time-domain transform as follows:

`Meas Setup` → `Transform` → `Start Stop Time`
→ `Start Time`: −20 ns and `Stop Time`: 80 ns → `Done`
→ `Transform On`

Then set the measurement and the display scale as follows:

`Measure` → S_{11}
`Scale/Amptd` → `Scale` → 10 dB/div
→ `Reference Level` → 0 dB
→ `Ref Pos` → 0 DIV

Store the measured trace in memory and display both the live data and the memory trace.

`Trace` → `Math and Memory` → `Data->Mem`
→ `Data and Memory`

(a) Connect one 1 m SMA cable to Port 1. Compare the live data with the stored memory trace and explain the observed differences. How many peaks do you see in the memory and data traces and what is their individual origin (**Q1.1.1**)? Make sure to look at Data and Memory traces separately to correctly identify all the peaks. Once you are done, store the current trace in memory while continuing to display both the data and memory traces.

(b) Attach a 10 dB attenuator to the end of the extension cable. Display only `Data`. Explain the origin of the different peaks in this case, including the changes in each of their magnitudes (**Q1.1.2**).

(c) **Replace** the attenuator with the extra long SMA cable (delay line). Please share the delay line with your neighboring bench. Can you explain the origin of the two large peaks (**Q1.1.3**)? Do you see the reflection from the end of the delay line (**Q1.1.3**)?

(d) Make the `Stop Time` 100 ns. Now we see the reflection from the end of the delay line. Can you estimate roughly the one-way loss in the delay line (**Q1.1.4**)?

(e) From the location of the major peak and the peak corresponding to the beginning of the delay line within the time domain trace, determine approximately the physical length of the delay line. Assume that the effective dielectric constant of the insulating material inside the delay line is 2 (Teflon) (**Q1.1.5**). (Note, when using a marker, the `Dist.(Refl)` given assumes free space and gives the one way distance).

(f) Can you explain the origin of the two smaller peaks in negative time (**Q1.1.6**)?

(g) Increase the `Stop Time` to 200 ns. Decrease the `Number of Points` to 101. Increase it step by step to 201, 401, 601, 801, 1001, 1601, 4001 and 10001. At what point do you start seeing the "correct" time domain response (**Q1.1.7**)? This is when the time domain response does not change upon increasing the number of points further.

(h) Set the `Number of Points` to what you got from the previous step. Did the peaks at negative time disappear? Comment on the relationship between the two small peaks near 200 ns and the peaks you saw earlier in negative time (**Q1.1.8**)?

(i) Disconnect the delay line and set the frequency range to 1–2 GHz. Set `Stop Time` = 40 ns, and `Number of Points` to 801. Connect the HPF to the end of the 1 m SMA cable, followed by another 1 m SMA cable terminated by the 10 dB pad. So you should have: 1 m SMA → HPF → 1 m SMA → 10 dB pad. Explain the features of the observed responses. How many large distinct peaks do you see (**Q1.1.9**)?

(j) Increase the `Stop` frequency to 5.0 GHz. Note that in effect you are increasing the measurement RF bandwidth (or effectively decreasing the width of the equivalent time domain pulse). How many large distinct peaks do you see and what are their origins. Save this trace and plot it with your result (**Q1.1.10**)? Include peaks that are closely spaced as distinct peaks. Take your time to identify the peaks correctly. You can disconnect and reconnect different sections to get a better insight.

EE 217B: Temporal Resolution
Disconnect all cables. Zoom into the peak around 0 ns and measure the 6 dB width of the peak using markers (try the `Delta` marker). This is the spatial/temporal resolution. Compare it with the theoretical value obtained from the bandwidth (1–5 GHz) and estimate the Beta-factor of the Kaiser-Bessel window (**Q1.1.11**). Try and find the Beta-factor in the VNA settings and check with your result. Move the band to span from 30 kHz to 4 GHz and change `Transform Settings` to `Low Pass Impulse`. Note that the start frequency changes a little but the bandwidth is still approximately 4 GHz. Now measure the temporal resolution. For the same bandwidth, why is the temporal resolution different for the two cases (**Q1.1.12**)?

1.2 Time Gating

`Preset` the VNA, and set the stop frequency to 12 GHz with 801 points. Connect the 1 m SMA cable between ports 1 and 2. Set the measurement to S21 and apply Time Gating as follows:

`Meas Setup` → `Transform` → `Transform ON`
→ `Gating ON`
→ `Gating Start Stop`
→ set `Center` to gate the S21 peak and `Span` to 1.5 ns
→ `Done`

Turn OFF the Time Domain Transform. You are now looking at the Gated S21 response in Frequency Domain. While staying in Frequency Domain, change the Gate Center frequency by a small amount (using the rotary dial). As you change the center frequency do you see the band edges going beyond 0 dB? Congratulations, you have made an amplifier from a wire! You can now

claim the (Ig) Nobel Prize! Before you do that, please explain why the band edges go beyond 0 dB (**Q1.2.1**).
Next, connect the following setup: 1 m SMA → HPF → 1 m SMA → 10 dB pad. Go to `Bandpass Impulse` mode. Set frequency span to 1–3 GHz, 801 points and observe time domain from −20 ns to 40 ns. Isolate the response of the 10 dB pad by applying time domain gating.

(a) Switch back to frequency domain by turning off the Time Domain Transform. You are now looking at the gated response of the 10 dB attenuator. Compare this with the ungated response by toggling `Gating ON` and `OFF` while remaining in the frequency domain. Plot both the results (**Q1.2.2**). Note that the band edges are lower due to the gating. Also note that when `Gating` is `OFF`, at several frequencies the response is higher than −20 dB despite the presence of 20 dB attenuation in S_{11}, but we don't see this in the Gated response. Can you explain how it is possible for the response at some frequencies to be greater than −20 dB, when there is at least 20 dB of attenuation in the signal path (**Q1.2.3**)? We will explore this further.

(b) Next, change the gate span to see how the response changes. **Do this while staying in the frequency domain.** The band edge distortion slowly improves as you increase the span. As soon as you capture the next reflection peak you will see a strong interference (ripple) in the frequency domain. You can check this by toggling on and off the Time Domain Transform. Think about the relationship between the ripple in frequency domain and the peaks in time domain. Calculate the time separation between the two time domain peaks by measuring the ripple period in frequency domain and compare with the peak spacing measured directly in time domain (**Q1.2.4**).

EE 217B: Time Domain and Time Gating Manually
Turn Gating and Transform OFF. Collect the complex S-parameter data of the current setup as an S1P file. Use AI to implement your own Time Domain Transform and Time Gating. Plot the **gated** Time Domain response and Frequency Domain response of the 10 dB pad and compare with the results obtained from the gating applied internally in the FieldFox (**Q1.2.5**).

Reminder: Take turns in doing the experiments.

2 TRL Calibration and De-Embedding

Coaxial SOLT calibration places the reference plane at the ends of the coaxial cables. When measuring devices on PCBs, chips or substrates we want to remove the probes, wirebonds, pads and connectors as well. The Thru-Reflect-Line (TRL) calibration does this: it moves the reference plane onto the board.

TRL needs only *partially* known standards — a Thru, a Reflect whose value need not be known (only its approximate sign), and a Line of approximately known extra length. The calibration solves for the reflect coefficient and the line's propagation constant along the way. Its accuracy rests on a fabricated length of transmission line rather than on lumped-element models, which is why it is preferred for substrates, PCBs and ICs. The complete derivation is in Appendix A; you will implement it yourself.

2.1 Step 1: Setup and First-Tier Calibration

1. For the given Line length of 17 mm, what is the Stop Frequency up to which the TRL calibration is valid? Assume validity up to 160° phase difference between the Line and Thru standards. Set this value as your f_{stop} (**Q2.1.1**).

2. Preset the VNA, then set it up as follows:

   `Mode` → `NA`
   `Freq/Dist` → `Stop` → f_{stop} GHz
   `Sweep` → `Resolution` → 1601
   `Meas Setup` → `Transform` → `Transform Settings`
   → `Stimulus` → `Low Pass Impulse` → `Done`

   `BW` → `IF BW` → 10 kHz
   `Meas Setup` → `Power Level` → −15 dBm

   Selecting `Low Pass` sets the start frequency to $f_{\text{start}} = f_{\text{stop}}/N$, which makes the frequencies harmonically related. Recall that this is needed for the Low Pass time domain response.

3. Perform a standard full 2-port SOLT calibration at the ends of the two extension cables, exactly as in Lab 1. **Do not disconnect or move the cables much after this point.** Do you really need this calibration — explain why or why not (**Q2.1.2**)?

4. Verify it by connecting the two cables through the coaxial THRU and confirming $|S_{21}| \approx 0$ dB and $|S_{11}| < -30$ dB across the band. Report $|S_{21}|$ after `Autoscale` (**Q2.1.3**).

2.2 Step 2: Connect the TRL Standards

Ask the instructor for the TRL board and also ask them to show you how to carefully torque the connectors onto the board. **Do not torque the board connectors to the wrench limit**; use a mild torque. Measure and save each of the following as a Data (S2P) file, **full 2-port data in every case. Check that the FF is saving the S2P file to USB**.

1. **Thru.** Save as `File_T.s2p`. Check S_{21} and S_{11}/S_{22}: does it look like an ideal THRU? If not, why not (**Q2.2.1**)?

2. **Reflect.** Save as `File_R.s2p`. Only S_{11} and S_{22} are used; the transmission terms are meaningless because the two halves are isolated. From the Smith chart, is this a short or an open, and how can you tell (**Q2.2.2**)?

3. **Line.** Save as `File_L.s2p`. The length difference $\Delta\ell$ between LINE and THRU is 17 mm.

4. **Thru again.** Re-connect and save as `File_T_2.s2p`. This independently re-connected thru is what you will verify the calibration against.

5. **DUTs.** The DUTs are the next 7 traces (3 inductors, 3 capacitors and one LC). Save each DUT trace as `File_1.s2p`, `File_2.s2p` and so on, in the order the structures appear on the board.

2.3 Step 3: Solve for the Error Terms

Following Appendix A, write a script (MATLAB, or Python with `scikit-rf` for file I/O) that:

1. converts the measured THRU and LINE to cascade matrices, Eq. (3);
2. forms $\mathbf{M} = \mathbf{T}_L \mathbf{T}_T^{-1}$, Eq. (8);
3. takes the eigenvalues of $\mathbf{M}$ to get γ, Eq. (11), and hence ε_{eff}, Eq. (12). Plot α and β versus frequency and report ε_{eff} (**Q2.3.1**);
4. takes the eigenvectors to get e_{00} and $r_1 = e_{01}e_{10}/e_{11}$, Eqs. (14)–(15);
5. repeats on the port-reversed THRU and LINE, Eq. (16), for e_{33} and r_2;
6. uses the THRU reflection for $P = e_{11}e_{22}$, Eq. (19);
7. uses the REFLECT at both ports for e_{11}/e_{22}, Eqs. (21)–(22);
8. separates e_{11} and e_{22}, Eq. (23), resolving the sign with the known approximate reflect (Section A.10), then back-substitutes, Eqs. (25)–(26);
9. recovers Γ_R from Eq. (24) and plots it on a Smith chart. Does it match the physical standard (**Q2.3.2**)?

Plot the magnitude of all seven independent error terms versus frequency (**Q2.3.3**): e_{00}, e_{11}, $e_{01}e_{10}$ for port 1; e_{33}, e_{22}, $e_{23}e_{32}$ for port 2; and $e_{10}e_{32}$ common to both.

2.4 Step 4: Verification

1. Apply Eq. (27) to `File_T_2.s2p`. A correct calibration must return $S_{11} = S_{22} = 0$ and $S_{21} = S_{12} = 1$. Overlay $|S_{11}|$, $|S_{21}|$ and $|S_{22}|$ raw and de-embedded, plot the residual, and report the worst-case deviation in dB (**Q2.4.1**). If this fails, the error terms are wrong and you need to repeat the TRL calibration or debug your code, more carefully.
2. Measure the THRU trace in the time domain and gate out the launch reflections. Which method removes the launch more cleanly, and why (**Q2.4.2**)?

Reminder: Take turns in doing the experiments.

3 Extraction of Passive Elements

Apply your TRL correction to the seven files, then:

(a) Extract the value of the capacitors and inductors by properly modeling the DUT. Note that the DUT is not just a series inductor or a shunt capacitor since there is a small section of unknown transmission line on each side. There are essentially 3 unknowns (left line length, cap/ind value, right line length). Given that you know the values of the inductors and capacitors for 4 elements, extract the unknown line lengths from these and use them to find the value of the unknown capacitor and inductor (**Q3.1.1**).

(b) What is the error in the extracted passive components in the known cases (**Q3.1.2**)?

(c) What is the self resonance frequency of the different inductors and capacitors (**Q3.1.3**)?

#	Structure	Nominal value	File
1	Series inductor	1 nH	`File_1.s2p`
2	Series inductor	10 nH	`File_2.s2p`
3	Series inductor	*unknown*	`File_3.s2p`
4	Shunt capacitor	1 pF	`File_4.s2p`
5	Shunt capacitor	100 fF	`File_5.s2p`
6	Shunt capacitor	*unknown*	`File_6.s2p`
7	Series 1 nH + shunt 1 pF	1 nH, 1 pF	`File_7.s2p`

Table 2: The seven structures on the passive-element substrate.

(d) For the seventh LC element, which resembles a bondwire inductor followed by a pad capacitor, can you find a way to extract both the L and C independently (**Q3.1.4**)?

EE 217B: Masking (Device 8). Device 8 is two nominally identical 1 pF series capacitors. Measure the distance between them and set the VNA bandwidth sufficiently large enough to separate the two capacitors (**Q3.1.5**). Transform its S_{11} to the time domain (Appendix B) and gate each capacitor separately. The second reads low: the first has already reflected part of the wave before it arrives. Recover the true value with the masking correction,

$$|\Gamma_2| = \frac{|\hat{\Gamma}_2|}{1 - |\hat{\Gamma}_1|^2}, \tag{1}$$

and check the corrected $|\Gamma_2|$ against the time-gated Γ of the single 1 pF capacitor (device 4). Report $|\hat{\Gamma}_1|$, $|\hat{\Gamma}_2|$, the corrected $|\Gamma_2|$, and the comparison (**Q3.1.6**).

Reminder: Take turns in doing the experiments.

4 Time Domain Reflectometry

In this part we will try to extract the passive component values using TDR. Set the VNA to the following settings:

`Freq/Dist` → `Stop` → 12 GHz
`Sweep` → `Resolution` → 1601
`Meas Setup` → `Transform` → `Transform Settings`
→ `Stimulus` → `Low Pass Step` → `Done`
→ `Transform On`
`Measure` → S_{11} → `Format` → `Real`
Set the scale as needed

Check if the start frequency is as expected. Note that the `Real` format is needed to accurately capture the sign of the step response. Switch the format to `Imaginary` once. It should be zero to within the noise. Explain why low-pass mode forces this and band-pass mode does not (**Q4.1.1**).

With the THRU connected, widen the time axis and find the board. It sits at the round-trip delay of the extension cables, about 9–11 ns for the 1 m cables here. Report where yours appears

and confirm it against the cable length (**Q4.1.2**). Then set `Start Time` = 9 ns, `Stop Time` = 11 ns and save each trace as a Data (CSV) file.
If the first-tier calibration is still on, its reference plane is at the cable ends and the board appears near $t = 0$ instead. Turn the calibration off, or preset the device and re-enter the frequency settings.

(a) Connect the THRU line and observe the time domain response of the two SMA connectors. From this response can you estimate what kind of lumped element model one can use for the SMA connectors? (Hint: Recall from the lecture the shape of the step response for different passive components) (**Q4.1.3**).

(b) Connect the Reflect standard and determine the type of Standard (Short vs Open) from the step response (**Q4.1.4**).

(c) Connect the next 6 passive elements and estimate their values from the area under the curve for each, using the area rule of Appendix B (**Q4.1.5**).

(d) Lastly, check the seventh trace and see if it agrees with your intuition of how the step response should look for that particular LC combination. This should help figure out the SMA connection too (**Q4.1.6**)!

(e) Isolate the series cap step response for each capacitor independently by first applying time gating and then looking at the step response. Do they look the same? If not, why not (**Q4.1.7**)?

Despite the temptation to just collect data here and use AI to do the post-processing for you, I highly recommend you go through and process the data yourself with your team. You are welcome to take a quick break, grab dinner or whatever, and come back to finish processing the data. At least, finish processing some of the data now before leaving so you can be confident you have good measurements. To become a strong RF engineer you need to be good at both the experimental setup and understanding the data processing deeply.

If you did indeed finish processing the data and all experiments, congratulations! You have completed the lab. Please disconnect and put all components safely back into the box just as you received them. Get checked off by the instructors to make sure everything has been returned before you leave.

Appendices

A The TRL Calibration Procedure

This appendix gives the complete TRL solution, step by step, in the form you will implement in Section 2. It mirrors what we discussed in the Lecture.

A.1 The Eight-Term Error Model

After the first-tier SOLT calibration, all remaining systematic error is lumped into two *error boxes*: **X** between the port-1 reference plane and the DUT, and **Y** between the DUT and the port-2 reference plane. Using the error-adapter notation of Lecture 4, their scattering matrices are

$$\mathbf{S}_X = \begin{bmatrix} e_{00} & e_{01} \\ e_{10} & e_{11} \end{bmatrix}, \qquad \mathbf{S}_Y = \begin{bmatrix} e_{22} & e_{23} \\ e_{32} & e_{33} \end{bmatrix}. \tag{2}$$

Physically, e_{00} is the port-1 directivity, e_{11} the port-1 source match, and the product $e_{01}e_{10}$ the port-1 reflection tracking; e_{33}, e_{22} and $e_{23}e_{32}$ are the port-2 equivalents. Only the *products* $e_{01}e_{10}$, $e_{23}e_{32}$ and $e_{10}e_{32}$ are ever observable — never the individual transmission terms — which is why the model has **seven** independent unknowns.

A.2 Cascade (T) Parameters

Because the DUT sits between two error boxes, the algebra is far simpler in cascade parameters, which multiply. For any two-port,

$$\mathbf{T} = \frac{1}{S_{21}} \begin{bmatrix} -\det \mathbf{S} & S_{11} \\ -S_{22} & 1 \end{bmatrix}, \qquad \det \mathbf{S} = S_{11}S_{22} - S_{12}S_{21}. \tag{3}$$

The inverse conversion, needed at the end, is

$$\mathbf{S} = \frac{1}{T_{22}} \begin{bmatrix} T_{12} & \det \mathbf{T} \\ 1 & -T_{21} \end{bmatrix}. \tag{4}$$

Applying Eq. (3) to the two error boxes,

$$\mathbf{T}_X = \frac{1}{e_{10}} \underbrace{\begin{bmatrix} -(e_{00}e_{11} - e_{01}e_{10}) & e_{00} \\ -e_{11} & 1 \end{bmatrix}}_{\tilde{\mathbf{T}}_X}, \qquad \mathbf{T}_Y = \frac{1}{e_{32}} \underbrace{\begin{bmatrix} -(e_{22}e_{33} - e_{23}e_{32}) & e_{22} \\ -e_{33} & 1 \end{bmatrix}}_{\tilde{\mathbf{T}}_Y}. \tag{5}$$

The tilde matrices $\tilde{\mathbf{T}}_X$, $\tilde{\mathbf{T}}_Y$ contain *only observable products*; the bare e_{10} and e_{32} have been factored out and will reappear together at the very end as $e_{10}e_{32}$.

A measurement of any device $\mathbf{T}_D$ placed between the error boxes gives

$$\mathbf{T}_M = \mathbf{T}_X\,\mathbf{T}_D\,\mathbf{T}_Y = \frac{1}{e_{10}e_{32}}\,\tilde{\mathbf{T}}_X\,\mathbf{T}_D\,\tilde{\mathbf{T}}_Y. \tag{6}$$

A.3 The Three Standards

- **Thru** ($\mathbf{T}_D = \mathbf{I}$, ideal zero length): $\mathbf{T}_T = \mathbf{T}_X \mathbf{T}_Y$.
- **Line** (length difference $\Delta\ell$ relative to the thru, propagation constant γ):

$$\mathbf{\Lambda} = \begin{bmatrix} e^{-\gamma\Delta\ell} & 0 \\ 0 & e^{+\gamma\Delta\ell} \end{bmatrix}, \qquad \mathbf{T}_L = \mathbf{T}_X\,\mathbf{\Lambda}\,\mathbf{T}_Y. \tag{7}$$

- **Reflect**: an identical, unknown Γ_R at each port.

A.4 Step 1 — Eliminate the Port-2 Box

Form the product of the measured line with the inverse of the measured thru:

$$\mathbf{M} \;\equiv\; \mathbf{T}_L\,\mathbf{T}_T^{-1} = \mathbf{T}_X\mathbf{\Lambda}\mathbf{T}_Y\,(\mathbf{T}_X\mathbf{T}_Y)^{-1} = \mathbf{T}_X\,\mathbf{\Lambda}\,\mathbf{T}_X^{-1}. \tag{8}$$

$\mathbf{T}_Y$ has cancelled completely. Equation (8) says the *measurable* matrix $\mathbf{M}$ is similar to the diagonal line matrix $\mathbf{\Lambda}$. Note that the scalar prefactors cancel too, so $\mathbf{M} = \tilde{\mathbf{T}}_X\mathbf{\Lambda}\tilde{\mathbf{T}}_X^{-1}$ equally — you may use either form.

A.5 Step 2 — Eigenvalues Give the Propagation Constant

Since $\mathbf{M}$ is similar to $\mathbf{\Lambda}$, its eigenvalues *are* the diagonal entries of $\mathbf{\Lambda}$:

$$\lambda_1 = e^{-\gamma\Delta\ell}, \qquad \lambda_2 = e^{+\gamma\Delta\ell}. \tag{9}$$

For a 2×2 matrix these follow in closed form from

$$\lambda_{1,2} = \frac{(M_{11}+M_{22}) \mp \sqrt{(M_{11}-M_{22})^2 + 4M_{12}M_{21}}}{2}. \tag{10}$$

The $\mp$ ambiguity in Eq. (10) can be resolved by computing γ for both cases and checking which one is closer to the expected result (which you can get from simulations or by using an online calculator: the line is a microstrip line at 50 Ω, the board is Rogers RO4350B with a thickness of 1.52 mm). Once λ_1 is identified, the propagation constant follows directly:

$$\boxed{\gamma = \alpha + j\beta = \frac{1}{\Delta\ell}\ln\!\left(\frac{1}{\lambda_1}\right)} \tag{11}$$

choosing the branch with $\alpha = \mathrm{Re}(\gamma) \geq 0$, since a passive line must attenuate. From the imaginary part,

$$\boxed{\varepsilon_{\mathrm{eff}} = \left(\frac{\beta c}{\omega}\right)^2} \qquad \text{and} \qquad v_p = \frac{\omega}{\beta}. \tag{12}$$

The line has characterized itself — this is a genuine measurement of the substrate, obtained for free from the calibration. Note that with f_{stop} set as in Q2.1.1, $\beta\Delta\ell$ stays below 180° across the whole sweep, so the logarithm in Eq. (11) never wraps.

A.6 Step 3 — Eigenvectors Give the Port-1 Terms

Right-multiplying Eq. (8) by $\tilde{\mathbf{T}}_X$ turns it into an eigenvector statement:

$$\mathbf{M}\,\tilde{\mathbf{T}}_X = \tilde{\mathbf{T}}_X\,\boldsymbol{\Lambda} \quad\Longrightarrow\quad \mathbf{M}\mathbf{x}_i = \lambda_i\mathbf{x}_i, \tag{13}$$

where $\mathbf{x}_1, \mathbf{x}_2$ are the two *columns* of $\tilde{\mathbf{T}}_X$. Eigenvectors are defined only up to scale, so normalize each so its second entry is 1. From the top row of $(\mathbf{M} - \lambda_i\mathbf{I})\mathbf{x}_i = 0$,

$$x_1 = \frac{M_{12}}{\lambda_1 - M_{11}}, \qquad x_2 = \frac{M_{12}}{\lambda_2 - M_{11}}. \tag{14}$$

Comparing with the columns of $\tilde{\mathbf{T}}_X$ in Eq. (5): the second column normalized is $[\,e_{00},\,1\,]^T$ and the first column normalized is $[\,e_{00} - e_{01}e_{10}/e_{11},\,1\,]^T$. Therefore

$$\boxed{e_{00} = x_2} \qquad \text{and} \qquad \boxed{r_1 \equiv \frac{e_{01}e_{10}}{e_{11}} = x_2 - x_1} \tag{15}$$

The directivity is obtained *outright*; the tracking is obtained only as a *ratio* to the still-unknown source match e_{11}.

A.7 Step 4 — The Port-2 Terms by Port Reversal

The formal route is to build $\mathbf{M}' = \mathbf{T}_T^{-1}\mathbf{T}_L = \mathbf{T}_Y^{-1}\boldsymbol{\Lambda}\mathbf{T}_Y$ and take left eigenvectors. The easier route to implement is to **reverse the ports** of the measured thru and line and re-run Steps 1–3 unchanged. Port reversal of a measured 2-port is

$$S'_{11} = S_{22}, \quad S'_{22} = S_{11}, \quad S'_{21} = S_{12}, \quad S'_{12} = S_{21}. \tag{16}$$

Applying Eqs. (8)–(15) to the reversed data yields

$$e_{33} = x'_2, \qquad r_2 \equiv \frac{e_{23}e_{32}}{e_{22}} = x'_2 - x'_1. \tag{17}$$

After Step 4 both directivities are known and two unknown scalars remain: e_{11} and e_{22}.

A.8 Step 5 — The Thru Gives the Product $e_{11}e_{22}$

The port-1 reflection of the measured thru is the one-port response of box $\mathbf{X}$ terminated in box $\mathbf{Y}$:

$$S_{11}^T = e_{00} + \frac{e_{01}e_{10}\,e_{22}}{1 - e_{11}e_{22}}. \tag{18}$$

Substituting $e_{01}e_{10} = e_{11}r_1$ and writing $P \equiv e_{11}e_{22}$,

$$S_{11}^T - e_{00} = \frac{r_1 P}{1 - P} \quad\Longrightarrow\quad \boxed{P = e_{11}e_{22} = \frac{S_{11}^T - e_{00}}{r_1 + \left(S_{11}^T - e_{00}\right)}} \tag{19}$$

The reverse thru reflection S_{22}^T gives the same P and is a useful consistency check. Because the thru is symmetric it fixes only the *product*; the *ratio* of e_{11} to e_{22} is still unknown.

A.9 Step 6 — The Reflect Gives the Ratio e_{11}/e_{22}

Let Γ^m_{R1} and Γ^m_{R2} be the measured raw reflections of the reflect standard at ports 1 and 2 (i.e. S_{11} and S_{22} of `File_R.s2p`). Referring Γ^m_{R1} back through box $\mathbf{X}$ using the one-port correction formula,

$$\Gamma_R = \frac{\Gamma^m_{R1} - e_{00}}{e_{11}\big(\Gamma^m_{R1} - e_{00}\big) + e_{01}e_{10}}. \tag{20}$$

Substituting $e_{01}e_{10} = e_{11}r_1$ and factoring e_{11} out of the denominator isolates the unknown:

$$\Gamma_R = \frac{W_1}{e_{11}}, \qquad W_1 \equiv \frac{\Gamma^m_{R1} - e_{00}}{\big(\Gamma^m_{R1} - e_{00}\big) + r_1}, \qquad W_2 \equiv \frac{\Gamma^m_{R2} - e_{33}}{\big(\Gamma^m_{R2} - e_{33}\big) + r_2}, \tag{21}$$

and identically $\Gamma_R = W_2/e_{22}$ at port 2. Both W_1 and W_2 are computed entirely from measured data and quantities already known. Because the *same* physical Γ_R appears at both ports,

$$\boxed{\frac{e_{11}}{e_{22}} = \frac{W_1}{W_2}} \tag{22}$$

A.10 Step 7 — Separate, Choose the Sign, Back-Substitute

Combining the product from Eq. (19) with the ratio from Eq. (22):

$$\boxed{e_{11} = \pm\sqrt{P\,\frac{W_1}{W_2}}, \qquad e_{22} = \pm\sqrt{P\,\frac{W_2}{W_1}}} \tag{23}$$

Resolving the sign. This is the *only* place the reflect's prior knowledge is used. Evaluate

$$\Gamma_R = \frac{W_1}{e_{11}} \tag{24}$$

for both sign choices and keep the one whose value matches the physical standard: $\Gamma_R \approx -1$ for a short, $\Gamma_R \approx +1$ for an open. Only the approximate *phase* of the reflect is needed — never its exact value. The chosen root also delivers Γ_R itself as a by-product, which is a valuable check on the calibration (**Q2.3.2**).

With e_{11} and e_{22} now known individually, back-substitute:

$$e_{01}e_{10} = e_{11}\,r_1, \qquad e_{23}e_{32} = e_{22}\,r_2. \tag{25}$$

Finally the transmission tracking follows from the thru transmission, $S^T_{21} = e_{10}e_{32}/(1 - e_{11}e_{22})$:

$$\boxed{e_{10}e_{32} = S^T_{21}\,\big(1 - P\big)} \tag{26}$$

All seven independent error terms are now determined: e_{00}, e_{11}, $e_{01}e_{10}$, e_{33}, e_{22}, $e_{23}e_{32}$, $e_{10}e_{32}$.

A.11 Step 8 — Correcting a Measured DUT

Invert the cascade of Eq. (6). Using the tilde matrices, whose entries are all known,

$$\boxed{\mathbf{T}_D = \big(e_{10}e_{32}\big)\,\tilde{\mathbf{T}}_X^{-1}\,\mathbf{T}_M\,\tilde{\mathbf{T}}_Y^{-1}} \tag{27}$$

with the explicit inverses (note $\det \tilde{\mathbf{T}}_X = e_{01}e_{10}$ and $\det \tilde{\mathbf{T}}_Y = e_{23}e_{32}$):

$$\tilde{\mathbf{T}}_X^{-1} = \frac{1}{e_{01}e_{10}} \begin{bmatrix} 1 & -e_{00} \\ e_{11} & -(e_{00}e_{11} - e_{01}e_{10}) \end{bmatrix}, \tag{28}$$

$$\tilde{\mathbf{T}}_Y^{-1} = \frac{1}{e_{23}e_{32}} \begin{bmatrix} 1 & -e_{22} \\ e_{33} & -(e_{22}e_{33} - e_{23}e_{32}) \end{bmatrix}. \tag{29}$$

Convert $\mathbf{T}_D$ back to S-parameters with Eq. (4). Note that the reflection parameters S_{11}, S_{22} are ratios within $\mathbf{T}_D$ and are therefore independent of the scalar $e_{10}e_{32}$; only the *transmission* parameters need it. This is why $e_{10}e_{32}$ is required for S_{21} but not for S_{11}.

B The Area Method for Extracting Parasitics

Part 4 asks you to read element values off a step response by integrating the area under it. This appendix derives that rule.

B.1 The Area Rule

Take a series impedance $Z = j\omega L$ embedded in an otherwise matched Z_0 line. Looking in from one side the shunt path is the far half of the line, Z_0, so the element sees Z in series with $2Z_0$ and

$$\Gamma(\omega) = \frac{Z}{Z + 2Z_0} = \frac{j\omega\tau}{1 + j\omega\tau}, \qquad \tau \equiv \frac{L}{2Z_0}. \tag{30}$$

A TDR measures the response to a *step*, which in the frequency domain is division by $j\omega$. Inverse-transforming $\Gamma/(j\omega)$ gives, for $t > 0$,

$$\rho(t) = e^{-t/\tau}. \tag{31}$$

The trace jumps up the instant the step reaches the inductor — a series inductance is momentarily an open — and then decays as the current builds. Integrating,

$$\int_0^\infty \rho(t)\,dt = \tau = \frac{L}{2Z_0} \qquad \Longrightarrow \qquad \boxed{L = 2Z_0 \int \rho(t)\,dt} \tag{32}$$

Repeating for a shunt admittance $Y = j\omega C$, where the element sees $Z_0/2$ and $\Gamma = -j\omega\tau/(1 + j\omega\tau)$ with $\tau = CZ_0/2$, gives $\rho(t) = -e^{-t/\tau}$ and

$$\boxed{C = -\frac{2}{Z_0} \int \rho(t)\,dt} \tag{33}$$

Three things are worth noticing. Nothing was linearised and the element was not assumed small, so Eqs. (32) and (33) are *exact*. The **sign** of the area identifies the type on its own: a positive excursion is a higher impedance than Z_0 and therefore a series inductance, a negative one is a lower impedance and therefore a shunt capacitance. And the integral is taken relative to the *local* baseline either side of the feature, not relative to zero — a board trace that is not exactly 50 Ω sits at a small non-zero ρ, and that offset would otherwise accumulate across the window.