EE 117B/217B Microwave and Wireless Measurements

Lab 1 - Introduction to Vector Network Analyzers

Fall 2026

CAUTION: READ THIS PAGE CAREFULLY BEFORE STARTING THE LAB

This class uses the FieldFox Microwave Analyzer N9918B as the primary instrument. It serves as the Vector Network Analyzer (VNA) and the Spectrum Analyzer (SA). Along with the FieldFox, this class also uses several other RF instruments and equipment. Please handle the instruments very carefully as they are **very expensive** and delicate. The most vulnerable parts of the instruments are the RF PORTS (APC-3.5 connectors). These connectors are protected by the small Female-Female Port Savers, which are the silver ferrules attached to the ports. **DO NOT REMOVE THE PORT SAVERS FROM THE VNA PORTS FOR ANY EXPERIMENT.**

All connections are to be made through these Port Saver Connectors. If you have never handled RF instrumentation before, ask the instructor to demonstrate how to make connections and handle these connectors, calibration standards, and cables. Always hold the cable with one hand (non-dominant) and rotate the end nuts of the SMA cables with the other (dominant) hand. Once you make a soft connection by hand, use the torque wrench provided to complete the connection. Note that whenever SMA connections are made, you must only rotate the end nuts and not the components; this ensure the internal pin enters and exits the jack in a straight trajectory. As far as pushing the control knobs and keys on the instrument are concerned, no special care is needed (they are fail-safe). The worst that can happen is that the instrument may lock up. In that case, press the `Preset` key or ask the instructor for help.

Introduction

Vector Network Analyzers are capable of measuring the steady state reflection and transmission characteristics (S-parameters) of devices and networks. This is accomplished by using a precise microwave signal generator (usually a sweep oscillator) and a very sensitive quadrature receiver. The magnitude and phase of the signal transmitted through the device or reflected from its input are compared with those of the incident signal generated (reference) by the RF source at individual frequency points. The receiver may have more than one channel to measure the reference, transmitted, and reflected signals simultaneously. We will study the internal architecture and operation of the VNA, including calibration, in more detail in the upcoming lectures.

Objectives

Although we have not covered the VNA architecture and calibration in detail yet, at this point you are familiar with S-parameters and a list of several Linear Time Invariant (LTI) components. The purpose of this lab is to familiarize you with the VNA at an operational level. At the end of this lab you will:

1. Become familiar with the VNA and its various settings.
2. Understand VNA calibration at an operational level.
3. Measure S-parameters of several LTI components.
4. Use a one-port reflection measurement to determine a material property.

Since 3 students are performing the lab together, please ensure that you **take turns on performing the experiments.**

Equipment List

Table 1 and Figs. 1 and 2 list the instruments, standards, and devices under test used throughout this lab. Verify that all items are present at your bench and in your Lab Box before beginning, and consult the instructor if anything is missing or damaged.

Lab Report

Each lab comes with a Lab Questionnaire. As you go through this Lab Manual, you will encounter several questions along the way. When you do, go over to that question in the Lab Questionnaire and answer it accordingly. The list of questions and answers will be turned in as your Lab Report. All students in one lab group will submit one combined lab manual. **It is recommended that you complete the post-processing parts of the lab also during the designated lab hours so you can work together with your team.**

Equipment	**Part Number**	**Qty**
Keysight FieldFox Microwave Analyzer	N9918B	1
SMA coaxial cable(s) / extension cables	NA	2
DC Supply Banana Cables	NA	2
RF high-pass filter (HPF)	SHP-20+	1
Calibration kit (3.5 mm)	TOJOIN 3.5 mm cal kit	1
50 Ω matched load(s)	NA	1
Isolator	MICA 7Y213	1
Bandpass filter	VBFZ-5500-S+	1
10-dB attenuator (pad)	FW-10A+	2
RF amplifier	ZX60-123LN-S+	1
DC bias power supply	Agilent E3631A	1
Directional coupler (0.5–8.0 GHz)	KRYTAR 158010	1
Male-Male SMA adapters	NA	2
Female-Female SMA adapters	NA	2
Torque wrench	(shared)	1
USB flash drive	Sandisk	1
Open-ended SMA dielectric probe	NA	1
Bench vice	NA	1
Glass beaker (50–100 mL)	NA	2
Distilled water	(ask instructor)	—
Isopropyl alcohol (liquid under test)	(ask instructor)	—
Digital Multimeter	(shared)	1

Table 1: Equipment used in the VNA Lab 1 experiments.

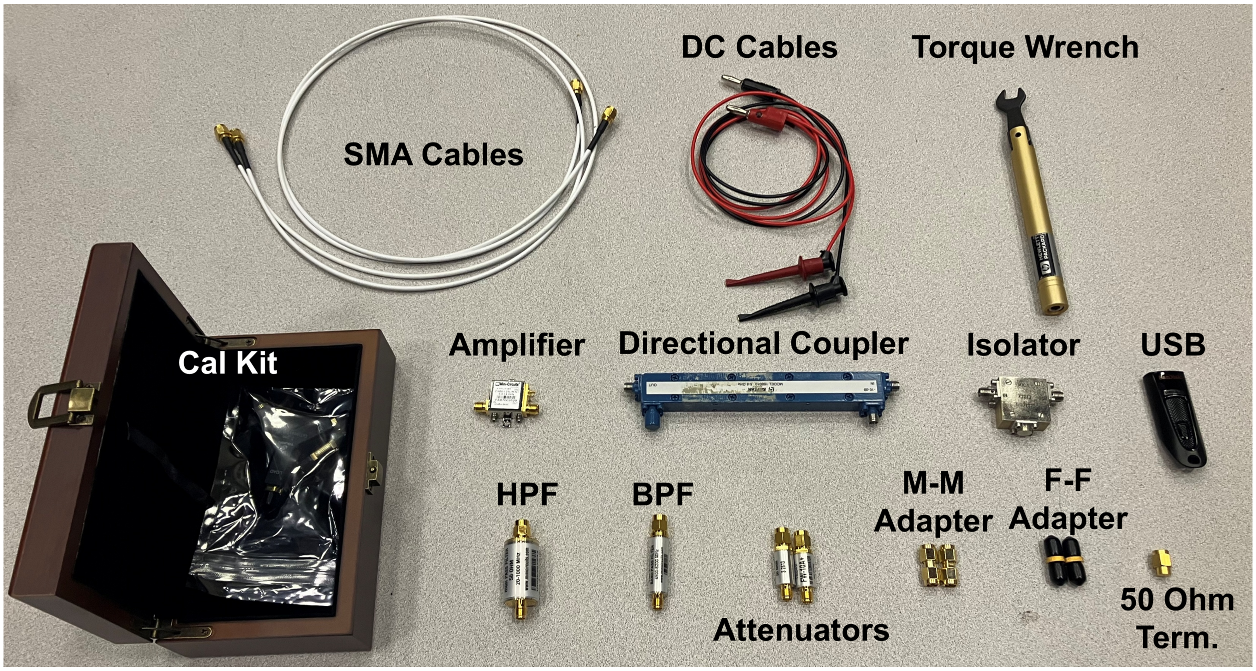

Figure 1: Components used in the first 5 VNA experiments.

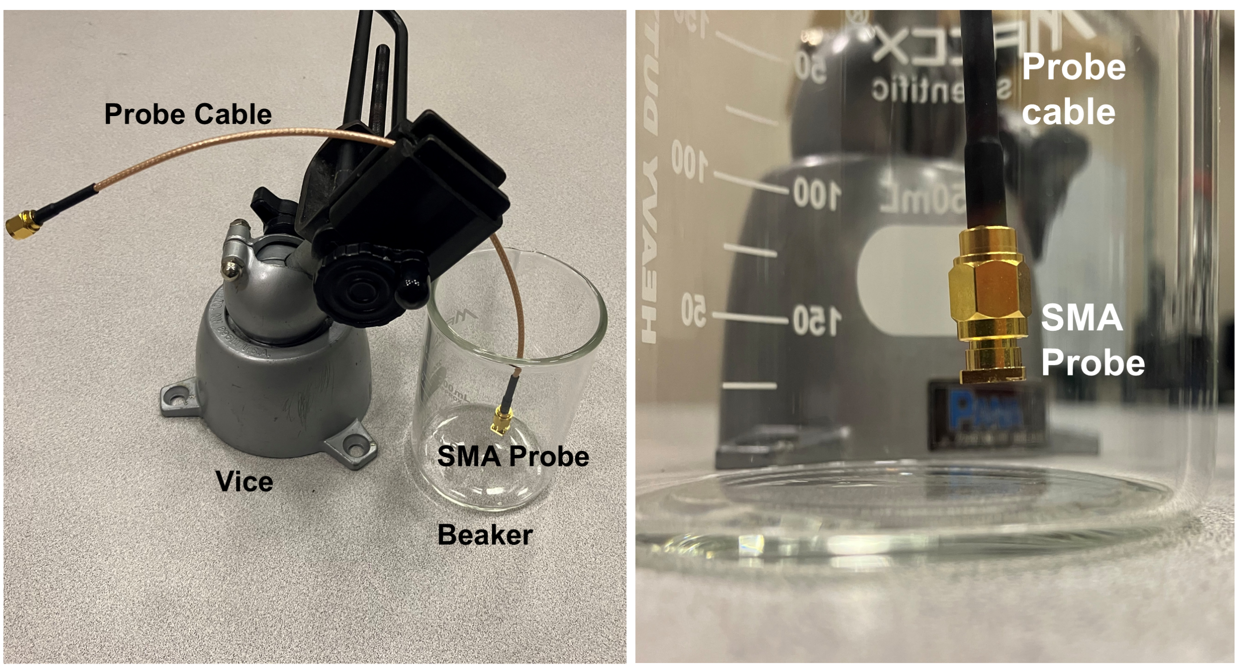

Figure 2: Components used in the 6th VNA experiment.

1 Familiarization with Network Analyzer

The purpose of this section is to familiarize the students with the signal processing principles and capabilities of Keysight Network Analyzers. These include: (a) different formats for displaying the desired responses of a test device (Magnitude plot, Phase plot, Smith Chart, Polar plot, etc.), (b) different noise reduction techniques, (c) display options (e.g. display of multiple traces and trace storage via VNA internal memory), and (d) coherent subtraction capability. In next week's lab we will look at the time domain capabilities of the VNA.

1.1 Initial Setup

To get started, once the FieldFox is turned on, press `Mode` and select `NA`.
The first step is to setup the VNA for measuring the transmission coefficient (S_{21}) of a device over 401 equally spaced frequency points spanning a frequency range of 0.1 MHz to 100.0 MHz. In order to do this perform the following:

`Freq/Dist` → `Start` → 0.1 MHz
→ `Stop` → 100.0 MHz
`Sweep` → `Resolution` → 401
`Meas Setup` → `Power Level` → -15 dBm
`Measure` → S_{21}
`Scale/Amptd` → `Scale` → 10 dB/div
→ `Reference Level` → 0 dB
→ `Ref Pos` → 0 DIV

Next we want to display the magnitude of the measured response in logarithmic scale and examine various ways to reduce noise-induced fluctuations in the response. The RF highpass filter (HPF) will be used as a test device. Connect the HPF directly to port 1 and use an SMA cable to connect the Female port of the HPF to Port 2 of the FieldFox (See CAUTION on page 1 if making connections for the first time). Then perform the following:

`Measure` → `Format` → `Log Mag`

Markers are placed using the `Marker` key:

`Marker` → `Marker 1` → enter the frequency → `Enter`

Set marker 1 to 0.1 MHz and record the magnitude of S_{21} in dB (**Q1.1.1**). Move the marker to 100 MHz and record the value (**Q1.1.2**).

1.2 Noise Reduction techniques

The VNA can be instructed to reduce the noise-induced fluctuations in the measured response of a device using three different noise reduction techniques: (a) smoothing, (b) coherent averaging, and (c) reduction of the IF bandwidth. Read Appendix A for more information about how each technique works. Only coherent averaging and reduction of IF bandwidth techniques are examined below.

1.2.1 Coherent Averaging

(a) Since additive thermal noise is a zero-mean process, its effect on measurements can be reduced simply by averaging many independent measured responses of a device under test. Coherent averaging refers to the averaging of the complex response for each frequency point in the selected frequency range (see Appendix A). To average coherently 8 traces, press the following keys in sequence:

`BW` → `Average` → 8 → `Enter`
→ confirm "Avg 8" is displayed on the left side of the screen

Record the data of HPF response to make a plot after the averaging process is complete. Use the following instructions to save the plot into a PNG file (**Q1.2.1**).

First connect a USB drive to the FieldFox USB ports on the top right.
`Save/Recall` → `Device` → `USB`
→ `File Type` → `Picture (PNG)`
→ `Save` → name the file `Q121` → `Done`

Selecting `More` under `File Type` also lets you save the underlying data as S2P or CSV files.

Set marker 1 to 0.1 MHz and record the magnitude of S_{21} in dB (**Q1.2.2**). Move the marker to 100 MHz and record the value (**Q1.2.3**). Compare with the previously recorded values when averaging was not used.

Once again place the marker at 0.1 MHz. Note that the value fluctuates with time. Note down the value over 20 observations, take their average and report the value when the VNA number of averages is set to 1 and to 8. You can use the `Run/Hold` button to hold the measurement. Compare them and report the results. How much noise suppression has been achieved by averaging 8 measurements? What's your expectation? (**Q1.2.4.A–D**).

(b) Restore the average to 1. The Network Analyzer is also capable of coherently subtracting one trace from another (subtraction is performed in complex domain). This feature turns out to be very useful in many applications. Note that the VNA cannot perform subtraction if the two traces were collected at different frequency spans or with different numbers of frequency points. To examine coherent subtraction, first, store the trace in memory (see below), then disconnect the HPF from the SMA cable and connect it back again, start a new measurement and apply subtraction to test the repeatability of the connections of SMA connectors.

`Trace` → `Math and Memory` → `Data->Mem`
After reconnecting the HPF → `Data Math` → `Data - Mem`

Set the marker to 100 MHz and plot the magnitude of S_{21} in dB (**Q1.2.5**).

(c) Increase the averaging factor to 16 and repeat the previous steps in making the subtracted HPF measurement. Note that the average number is displayed on the left side of the screen. Wait for it to complete before making changes to the measurement system. Observe the effect of the number of samples being averaged. Set the marker to 100 MHz and plot the magnitude of S_{21} in dB for the Data-Memory Trace (**Q1.2.6**).

(d) Set the average back to 1. Increase the Stop Frequency to 10 GHz. Perform the same repeatability test. Place the marker at 10 GHz and plot the magnitude of S_{21} in dB for the Data-Memory Trace. Why is the repeatability worse at higher frequencies? (**Q1.2.7**).

(e) Plot `Data - Mem` and send `Data->Mem`. The resulting plot is almost equal to the noise floor since both data and memory are nearly identical. Now lightly touch the SMA cable with your finger. What did you notice on the plot? A good cable should bring the `Data - Mem` plot back to the noise floor after bending or flexing. Based on your observation, could you comment on the phase stability of this cable (**Q1.2.8**)?

Restore the initial setup by reducing the Stop Frequency to 100 MHz and displaying the measured data only:

`Trace` → `Math and Memory` → `Data Math` → `Math Off`

1.2.2 IF BW technique

Another way to reduce the effect of noise on the measured response is to decrease the IF bandwidth (see Appendix A). Decrease the IF bandwidth from its default value of 10 kHz to 100 Hz.

`BW` → `IF BW` → 100 Hz

Set one marker to 0.1 MHz and a second marker to 100 MHz. Turn the marker table on (`Marker` → `Marker Table` → `On`) and save the plot (**Q1.2.9**). Compare this response with the case when averaging was used. How much noise suppression has been achieved, and what's your expectation (**Q1.2.10**)? Observe the increase in the sweep time (Swp) displayed on the screen. You can see the live sweep with the tiny arrow that moves across the trace. Any idea why the arrow sweeps across the screen for the sweep time and stops at the end for the same amount of time (**Q1.2.11**)? Answering this question requires you to understand the internal architecture of the VNA. Once you are finished, set the IF bandwidth back to 10 kHz:

`BW` → `IF BW` → 10 kHz

1.3 Phase Response

Set the stop frequency to 1 GHz. To display the phase response of the device as a function of frequency, set the display format to phase.

`Measure` → `Format` → `Phase`

Place two markers at 0.2 GHz and 0.8 GHz.

`Marker` → `Marker 1` → 0.2 GHz
→ `Marker 2` → 0.8 GHz

Record the phase difference between the signals at 0.2 GHz and 0.8 GHz (**Q1.3.1**). Note that the phase displayed by the VNA flips many times between ±180° as a function of frequency. This phase wrapping is not a true characteristic of the device under test (the HPF in this case). Recall that the device is connected to the VNA via long coaxial cables that are many wavelengths long. Hence, the total phase detected by the VNA is the sum of the phase response of the device and the phase delay caused by the propagation through the coaxial cables, i.e. a factor e^{-jkl}, where k is the wave number of the coaxial waveguide mode and l is the total length of the cables. To remove the phase delay caused by propagation through the cables, an electrical delay can be applied. First remove the HPF and directly connect the coaxial cable between the two ports. To find the amount of electrical delay to be applied, perform the following:

`Scale/Amptd` → `More` → `Electrical Delay`, then turn the dial to change it

Increase the value of the electrical delay so as to unwrap the phase of the measured signal (fine tune until the trace becomes flat over the measured spectrum). What is the electrical delay value τ_d (**Q1.3.2**)? Applying an electrical delay multiplies the measured response by $e^{+j\omega\tau_d}$, which cancels the propagation factor e^{-jkl} when $\tau_d = l/v_p$, where v_p is the phase velocity in the cable. Keep this electrical delay and add back the HPF. Plot the resulting phase response and record the phase difference between the signals at 0.2 GHz and 0.8 GHz. Is the phase response linear with frequency (**Q1.3.3**)? Save the phase response data and plot the Group Delay versus frequency (**Q1.3.4**). Before leaving this part of the experiment, go back to Log Mag format and reset the electrical delay to 0 s.

`Scale/Amptd` → `More` → `Electrical Delay` → 0 s

Reminder: Take turns in doing the experiments.

2 Calibration

In order to make accurate measurements, almost every instrument needs a calibration procedure to remove the effects of the systematic errors. For the Network Analyzer, the calibration process compensates for cable losses, connector reflections, and internal instrument imperfections. Appendix B discusses the sources of error in network analyzer-based measurements and describes different calibration techniques for correcting for these measurement errors. It should be noted that correcting for errors in the response of a test device will work only if both the calibration and the measurement of the device are performed using identical stimulus setup (i.e. same power level, number of points, frequency span, IF bandwidth, etc.). Hence, test engineers should plan their experiment before performing the instrument calibration. In this section, we will examine two techniques for calibrating the network analyzer: (a) full 1-port calibration when test device is fed by coaxial cables, and (b) full 2-port standard calibration (coaxial feed). At the end of this Lab you will do an experiment with your own unique calibration. In the next lab we will look at a different calibration technique (TRL) which is better suited for PCBs, ICs and substrates.

2.1 Full 1-port Calibration (Coaxial Feed)

One-port calibration is required for accurate measurement of reflection coefficient only (S_{11} or S_{22}). In order to fully calibrate one port of the network analyzer, three independent loads are needed. These loads are the open, short, and matched loads. In this section, full 1-port calibration will be performed. The internal calibration procedure of the network analyzer is performed by both 117B and 217B students.

The **217B students** will repeat the calibration procedure but do it manually. That is, only raw data corresponding to the three standard loads and the test device will be collected and the students will perform the rest of the calibration procedure using a personal computer or a workstation.

2.1.1 Internal Calibration

Setup the network analyzer as described in the previous section (Section 1.1). Increase the Stop Frequency to 10 GHz. You could turn off Marker Table if it is still on. Disconnect the extension cable and HPF so nothing is connected to the VNA. At this point, the calibration plane is at Port 1 (i.e. at the port saver). Connect an SMA cable to Port 1. The calibration procedure we will perform next will move the calibration plane to the end of the SMA cable. Configure the VNA to measure the reflection coefficient S_{11}.

`Measure` → S_{11}
`Cal` → `Mechanical Cal` → `Change Cal Type` → `1-Port OSL (Port 1)`

Because the SMA cables present a male interface, the standards that mate to it are the <u>female</u> open, short, and load. You can configure the connector type and gender as follows:

`Change DUT Connector` → `3.5 mm`
→ `Change Gender` → `Female` → `Next`
→ select the appropriate calibration kit (`85052D`)
→ `Start Calibration`

Connect the open, short, and load standards one at a time and follow the calibration wizard until the calibration is complete. **CAUTION: Remember to only turn the male nuts and not the full calibration standard piece. Also please use a torque wrench to tighten the connections and DO NOT OVER-TIGHTEN the torque wrench. If you have not used a torque wrench before, please ask the TA.**

Save the calibrated state so that you can return to it if the instrument is preset by accident:

`Save/Recall` → `File Type` → `State` → `Save`

At this point, the calibration procedure is done and the network analyzer is in correction mode. It will correct for errors in further measurements and display the corrected responses.

`Measure` → `Format` → `Smith`

Attach the short standard and plot the response. Comment on the result (**Q2.1.1**).
Store this corrected trace in memory before proceeding, so that you can overlay it against the uncorrected response:

`Trace` → `Math and Memory` → `Data->Mem`
→ `Data and Memory`

Now switch the error correction off:

`Cal` → `User Cal` → `Off`

Plot the response on the previous chart and compare the results (both data and memory are displayed) (**Q2.1.2**).

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- Switch off correction mode on the VNA (`Cal` → `User Cal` → `Off`). All data collected below must be *raw*, i.e. uncorrected.
- Set `Measure` → S_{11}. Connect the short standard to the end of the SMA cable and save the S-parameter data as an S1P file.
- Repeat the same procedure, described above, for the open and matched loads.
- Connect the test device (female short) to the SMA cable and collect its raw data with the same stimulus settings. Before proceeding, switch the correction back on and also save the internally corrected response of the same test device to a CSV file, so that you have a reference to compare against.
- Based on the information provided in Appendix B, solve for E_D, E_S, and E_R and correct for the errors in the test device response. Compare the corrected response with the one from the internal calibration part. Also provide a plot of the error factors (**Q2.1.3**).

2.2 Full 2-port Calibration (Coaxial Feed)

Two-port calibration is needed to measure scattering parameters (S-parameters) of two port devices accurately. Before proceeding, preset the state by pressing `Preset` → `Mode Preset`. Connect the two SMA cables to the two ports and perform a 2-port calibration as described in the steps below. **Make sure to attach calibration standards at the end of the two cables, so in essence you are calibrating up to the end of the cables.** Once you do this you should not disconnect or change the cables, otherwise the calibration will not be accurate anymore.
The far ends of both extension cables are male, so the wizard is configured for a female DUT connector and you will use the <u>female</u> open, short, load and thru standards. Follow the sequence listed below for a 2-port calibration:

`Freq/Dist` → `Start` → 1.6 GHz
→ `Stop` → 2.2 GHz
`Sweep` → `Resolution` → 401
`Meas Setup` → `Power Level` → −15 dBm
`Measure` → S_{21}
`Cal` → `Mechanical Cal` → `Change Cal Type` → `Full 2-Port SOLR (unknown thru)`
→ `Change DUT Connector` → `3.5 mm`
→ `Change Gender` → `Female` → `Next Port 2`
→ `Change Gender` → `Female` → `Next`
→ select the calibration kit (85052D) → `Next`
→ select the calibration kit (85052D) → `Finish`
→ `Start Calibration`

Connect the open, short, load, and thru standards as prompted by the calibration wizard. Be patient and careful when connecting and disconnecting the standards so you don't have to repeat the calibration.
Once the calibration is done, verify it before moving on. With the thru connected between the two cables, plot the S_{21} magnitude and the S_{11} and S_{22} magnitudes (you may need to autoscale),

and record the worst-case values over the band. Then replace the thru with the open and short standards on Port 1 and check that the S_{11} Smith Chart traces sit on the expected points. Report the plots and state whether the calibration meets your expectations (**Q2.2.1**).

At this point, the VNA is fully calibrated and ready for further measurements. If you change the span of the VNA, the calibration will be interpolated onto the new span. This is indicated by the `Cal ON U*` displayed on the screen. Interpolation can introduce significant errors. For example, it could make S11 or S22 greater than 0 dB for a completely passive device. The user must be very careful about this. This problem is especially bad if the calibration is performed on a large span with few points and then later the span is reduced drastically. So if a measurement looks suspicious, return to the original settings under which calibration was done.

3 Isolator, Filters, and Attenuator Measurements

3.1 Isolator

Isolators are widely used in RF and microwave systems to protect sources and amplifiers from reflections due to downstream impedance mismatches. In this section, we measure the isolator's insertion loss, isolation, and matching bandwidth/VSWR.

Using the calibrated VNA, characterize the isolator as follows:

1. Measure the forward transmission coefficient (S_{21}). Determine the minimum insertion loss at the center frequency of the operating band (**Q3.1.1**).
2. Measure the reverse transmission coefficient (S_{12}). Determine the isolation at the center frequency of the operating band (**Q3.1.2**).
3. Measure the input reflection coefficient (S_{11}). Determine the input matching bandwidth using the criterion $S_{11} < -20$ dB. Report the lower and upper cutoff frequencies (**Q3.1.3**).
4. Verify the input matching by displaying the VSWR:

   `Measure` → `Format` → `VSWR`

   Compare the VSWR response with the bandwidth obtained from the S_{11} measurement and state the VSWR value that corresponds to $S_{11} = -20$ dB (**Q3.1.4**).
5. Save the S2P file and plot all S-parameters in one figure. Is this device reciprocal (**Q3.1.5**)?
6. Borrow another isolator from a neighboring station and connect them back to back. Is this any different than an attenuator? Is this device reciprocal (**Q3.1.6**)?

Reminder: Take turns in doing the experiments.

3.2 Filters

Recalibrate the VNA over the span of 0.3 GHz – 15.0 GHz with 801 points to capture the passband and both stopband skirts of the bandpass filter. Connect the BPF with a F-F adapter to the two SMA cables. Then characterize it as follows:

1. Measure the transmission coefficient (S_{21}). Determine the center frequency and the minimum insertion loss within the passband (**Q3.2.1**).

2. Determine the 3-dB bandwidth. Report the lower and upper 3-dB cutoff frequencies (**Q3.2.2**).
3. Determine the stopband attenuation at 4.3 GHz and 6.8 GHz and compare with the datasheet table (**Q3.2.3**).
4. Determine the roll-off on both sides of the passband by calculating the attenuation slope (dB/GHz) between the 3-dB cutoff frequency and the corresponding 20-dB attenuation frequency (**Q3.2.4**).
5. Measure the input reflection coefficient (S_{11}). Determine the input matching bandwidth using the criterion $S_{11} < -10$ dB (**Q3.2.5**).
6. Display the response in the VSWR format and verify that it is consistent with the measured input matching bandwidth. Compare the passband VSWR with the datasheet value (1.3 typ., 2.1 max.) (**Q3.2.6**).
7. Save the S2P file and plot all S-parameters in one figure (**Q3.2.7**).

3.3 Attenuator

In this section, the performance of a 10-dB attenuator is characterized.

1. Measure the forward transmission coefficient (S_{21}). Determine the average attenuation over the measured frequency range and the peak-to-peak flatness (**Q3.3.1**).
2. Measure the input reflection coefficient (S_{11}). Report the worst-case return loss over the band and comment on how it compares with the isolator and the filter (**Q3.3.2**).
3. Measure the reverse transmission coefficient (S_{12}). Compare the measured S_{21} and S_{12} responses. Is this component reciprocal (**Q3.3.3**)?
4. Compare the measured characteristics of the attenuator with those of the isolator. Discuss the fundamental differences between these two components (**Q3.3.4**).

4 Amplifier Measurements

The amplifier under test operates over 0.5 – 12 GHz so the current span (0.3 GHz to 15 GHz) is sufficient. Handle it carefully as it is very ESD sensitive. **DO NOT TOUCH the DC supply pins with your bare hands.** With the present setup, connect the RF amplifier between the two extension cables. Make sure that the input port of the amplifier is connected to Port 1 of the VNA. **Bias the amplifier carefully.** Set the DC supply to the bias voltage marked on the amplifier (12 V) *and set a current limit* before connecting it. Always test the supply voltage value using a Digital Multimeter before connecting it up to the amplifier. Once you are confident the values are correct, connect the Amplifier to the VNA and the DC supply. Then turn on the DC supply.
Observe the behavior of the frequency response (`Measure` $\rightarrow S_{21}$). Switch the DC bias power supply off and on to verify that the amplifier is operating properly.
The 2-port calibration was performed at -15 dBm; before recording data, verify that the amplifier is not being driven into compression by lowering the power level to -25 dBm and confirming that the measured gain does not change. Measure the S-parameters and report the following:

1. What are the gain, S_{21} bandwidth, input bandwidth, output bandwidth, reverse isolation and DC power consumption (**Q4.1.1**)?
2. Plot the S-parameters that you measured vs the one provided by the manufacturer (which you can obtain from the mini-circuits website; download the S2P file) (**Q4.1.2**).
3. From the measured S-parameters, extract and plot the Group Delay versus Frequency (**Q4.1.3**).

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Based on the acquired data, compute and plot the Rollett stability factor ($K - \Delta$ Test) as a function of frequency. Is this amplifier unconditionally stable (**Q4.1.4**)? Next, plot the input and output stability circles on a Smith Chart and identify the stable and unstable regions (see Appendix C) (**Q4.1.5**).

Reminder: Take turns in doing the experiments.

5 Directional Coupler Measurements Using a 2-Port VNA

The device under test is the KRYTAR 158010, a stripline directional coupler covering 0.5–8.0 GHz with a nominal coupling of 10 ± 1.0 dB (referenced to the output port), directivity greater than 15 dB, insertion loss (including coupled power) below 1.30 dB, and a maximum VSWR of 1.35 at any port. Connectors are SMA female. The device has four physical ports, but the isolated port is *terminated internally* in a matched load within the package and is not brought out to a connector. Only three ports are accessible. Adopt the convention

- Port 1 — **IN** (input),
- Port 2 — **OUT** (through / mainline output),
- Port 3 — **coupled** port (the −10 dB port),
- Port 4 — **isolated** port, terminated internally, *inaccessible*.

Only three independent 2-port measurements are required, corresponding to the $\binom{3}{2} = 3$ ways of pairing the three accessible ports. For each measurement, connect two accessible coupler ports to VNA ports 1 and 2 (via the extension cables) and terminate the one remaining accessible port with the matched 50 Ω load. The internal load on port 4 is present at all times and requires no action. For each measurement, save the corrected 2-port S-parameters as a Touchstone (`.s2p`) file. In each filename, the two digits indicate which two coupler ports are connected to VNA Port 1 and VNA Port 2 respectively.

1. **Measurement 1 — Coupler Ports 1–2 (Mainline / through path)**
   - VNA Port 1 → Coupler Port 1 (IN); VNA Port 2 → Coupler Port 2 (OUT).
   - Terminate Coupler Port 3 (coupled) in the 50 Ω load.
   - Save Data as `coupler_12.s2p`.
2. **Measurement 2 — Coupler Ports 1–3 (Forward coupling path)**
   - VNA Port 1 → Coupler Port 1 (IN); VNA Port 2 → Coupler Port 3 (coupled).
   - Terminate Coupler Port 2 (OUT) in the 50 Ω load.

- Save as `coupler_13.s2p`.

3. **Measurement 3 — Coupler Ports 2–3 (Reverse coupling path)**
   - VNA Port 1 → Coupler Port 2 (OUT); VNA Port 2 → Coupler Port 3 (coupled).
   - Terminate Coupler Port 1 (IN) in the 50 Ω load.
   - Save as `coupler_23.s2p`.

Each file reports its data using VNA port indices, not coupler port indices, so you must relabel the entries before assembling the 3×3 coupler S-matrix. The mapping is given in Table 2.

File	$S_{11}^{\mathbf{VNA}}$	$S_{21}^{\mathbf{VNA}}$	$S_{12}^{\mathbf{VNA}}$	$S_{22}^{\mathbf{VNA}}$
`coupler_12.s2p`	S_{11}	S_{21}	S_{12}	S_{22}
`coupler_13.s2p`	S_{11}	S_{31}	S_{13}	S_{33}
`coupler_23.s2p`	S_{22}	S_{32}	S_{23}	S_{33}

Table 2: Mapping from the VNA port indices in each Touchstone file to the coupler port indices.

Note that across the three files, the diagonal (reflection) parameters are measured with the following multiplicity: S_{11} twice (files 12, 13), S_{22} twice (files 12, 23), and S_{33} twice (files 13, 23). Each of the three accessible off-diagonal pairs $\{S_{12}, S_{21}\}$, $\{S_{13}, S_{31}\}$, $\{S_{23}, S_{32}\}$ is measured once. Both facts are used in the consistency checks below.

5.1 Data Processing in MATLAB / Python

Write your own code to extract the S-matrix of the directional coupler from the measurements you made. Use the following definitions, all in dB:

$$\begin{aligned}
\text{Coupling} \quad C &= -20\log_{10}|S_{31}| \\
\text{Isolation} \quad I &= -20\log_{10}|S_{32}| \\
\text{Directivity} \quad D &= I - C \\
\text{Insertion loss} \quad IL &= -20\log_{10}|S_{21}| \\
\text{Return loss at port } i \quad RL_i &= -20\log_{10}|S_{ii}|
\end{aligned}$$

Note that the isolated port is buried inside the package, so the directivity is obtained from the *reverse* coupling measurement (drive the OUT port, measure the coupled port) rather than from a fourth-port measurement.

1. Report and plot the directivity, coupling, insertion loss, and return loss over 0.5–8.0 GHz, and compare each against the datasheet specifications quoted at the start of this section (**Q5.1.1**).

2. Perform two consistency checks and report both (**Q5.1.2**):
   - *Redundancy:* each of S_{11}, S_{22}, S_{33} was measured twice, in two different files. Plot the two versions of each on the same axes and report the worst-case disagreement in dB. What does this tell you about connector repeatability and residual calibration error?
   - *Reciprocity:* the coupler is a passive, reciprocal structure, so $S_{ij} = S_{ji}$. Plot $|S_{21}|$ against $|S_{12}|$, $|S_{31}|$ against $|S_{13}|$, and $|S_{32}|$ against $|S_{23}|$, and report the worst-case deviation.

6 Measuring the Dielectric Constant of a Liquid

Every measurement you have made so far in this lab treated the device under test as a black box described by its S-parameters. In this experiment you will use the same one-port reflection measurement to determine a *material* property: the complex relative permittivity

$$\varepsilon^*(f) = \varepsilon'(f) - j\,\varepsilon''(f),$$

of a liquid, over the range 10 MHz–6 GHz. The real part ε' describes how much energy the material stores; the imaginary part ε'' describes how much it dissipates. Both are strong functions of frequency for polar liquids.

The measurement technique is the **open-ended coaxial probe** method. A short length of coaxial line is terminated in an open circuit and pressed against (or dipped into) the material under test. The fringing electric field at the aperture leaks into the material, so the reflection coefficient looking into the probe depends on the permittivity of whatever is in front of it. The probe used here is simply an SMA connector whose center pin has been cut flush with its mounting flange, and connected to a short SMA cable.

You are *not* expected to know how to convert S_{11} into ε^*. Appendix D derives the entire procedure from scratch, step by step, using only the reflection-coefficient and error-model ideas you have already seen in Section 2.1 and Appendix B. Read Appendix D before you start this experiment.

6.1 Safety and Handling

- Isopropyl alcohol (IPA) is **flammable** and its vapor is an irritant. Keep the beaker away from any hot surface, work in a well-ventilated area, and keep the quantity small (a beaker filled to ∼2 cm is plenty).
- **Never** let liquid run down the cable toward the FieldFox. Mount the probe pointing *downward* into the beaker so that any liquid that wicks up runs back down the outside of the connector body.
- The probe aperture is a precision RF interface. Clean it between samples with IPA on a lint-free wipe and let it dry completely. Do not scrape it.
- Dispose of the IPA as directed by the instructor; do not pour it down the sink.

6.2 Instrument Setup and Port 1 Calibration

Do this before building the mechanical setup. Preset the instrument (`Preset` → `Mode Preset`) and configure the FieldFox as follows. These settings must be identical for every measurement in this experiment.

`Mode` → `NA`
`Freq/Dist` → `Start` → 10 MHz
→ `Stop` → 6 GHz
`Sweep` → `Resolution` → 401
`Meas Setup` → `Power Level` → −15 dBm
`BW` → `IF BW` → 1 kHz
`BW` → `Average` → 1 → `Enter`

Measure → S_{11}
Measure → Format → Smith

The single largest source of error in this experiment is *mechanical.* The probe calibration you are about to perform is only valid as long as the cable and probe do not move, so build the setup once and then leave it alone. Before starting your measurements, assemble the measurement setup as follows:

1. Clamp the SMA cable containing the open-ended SMA probe in the vice so that the probe aperture faces downward and is roughly 5 cm above the bench (see Fig. 2). Tighten the vice on the SMA cable, never on the connector body.

2. Connect the probe SMA cable to a long (white) SMA cable using a F-F adapter and arrange it so that the SMA cable runs in a smooth arc to Port 1 of the FieldFox with no tension at either end. Once you begin, **do not move, flex, or re-route the cable** until all of the measurements (short, air, water, LUT) are complete.

3. Verify the mechanical stability using the trick from Section 1.2.1(e): store the trace in memory, display `Data - Mem`, then lightly touch and release the cable. The trace must return to the noise floor. If it does not, re-dress the cable before continuing (**Q6.2.1**). Turn the trace math off (`Trace` → `Math and Memory` → `Data Math` → `Math Off`) before proceeding.

4. Check that you can slide the empty beaker in and out such that it can contain the probe at a distance of around 2 cm from its base. Figure out a way to do this without having to tilt the beaker too much, otherwise the liquid will overflow onto the table during the measurement.

5. **MOST IMPORTANTLY, MAKE SURE THE TABLE IS CLEAN AND ALL COMPONENTS ARE PUT AWAY SAFELY. MAKE SURE THE FIELDFOX IS AT A SAFE DISTANCE AWAY FROM THE PROBE+BEAKER SETUP!**

Disconnect the open ended connector at the end of the probe and perform a standard **1-port OSL calibration at Port 1** exactly as in Section 2.1.1 (female standards, 3.5 mm, kit 85052D), but over the 10 MHz–6 GHz span set above.

`Cal` → `Mechanical Cal` → `Change Cal Type` → `1-Port OSL (Port 1)`
→ `Change DUT Connector` → `3.5 mm` → `Change Gender` → `Female` → `Next`
→ select calibration kit (85052D) → `Start Calibration`

Once the calibration is complete, connect back the open ended connector to the probe and do not disconnect it again for the remainder of this experiment.

It is worth understanding what this calibration does and does not do here. As shown in Appendix D, the three probe standards (short, air, water) already absorb *every* linear error between the VNA receiver and the probe aperture — including the cable and the OSL-corrected residuals. Mathematically, the OSL calibration is therefore redundant. In practice it is still worth doing: it removes the large phase slope of the cable, so that the raw traces are readable on the Smith chart and you can catch a bad connection immediately rather than discovering it in post-processing. Explain in your own words why the OSL step cannot change the extracted ε^* (**Q6.2.2**).

6.3 Measuring the Three Calibration Standards

The probe calibration uses three materials whose permittivity is known *a priori*: a metal short ($\varepsilon^* \to \infty$), air ($\varepsilon^* = 1$), and distilled water (accurately described by a Debye model, see Appendix D). Use the back of the Isolator as the metal short. Measure them in this order, saving a Touchstone file for each:

`Save/Recall` → `Device` → `USB`
→ `File Type` → `More` → `S1P`
→ `Save` → name the file → `Done`

1. **Short.** Press the open ended probe squarely against the metal short (back of the isolator) so that it makes contact with both the center conductor and the outer flange simultaneously. Hold it flat and firm — a tilted or loose short is the most common cause of a failed extraction, because this standard alone sets one of the three calibration coefficients. Save as `short.s1p`.

2. **Air.** Remove the short. Make sure nothing (hand, beaker, bench) is within about 2 cm of the aperture. Save as `air.s1p`.

3. **Distilled water.** Position the beaker of distilled water until the flange is submerged by 2–3 mm. Inspect the aperture for trapped air bubbles; if you see any, lower and raise the beaker again. Save as `water.s1p`.

Record the water temperature (**Q6.3.1**); you will need it in Appendix D to evaluate the Debye model. Plot the magnitude of S_{11} for all three standards on one set of axes and their loci on one Smith chart (**Q6.3.2**). All three have $|S_{11}|$ close to unity at low frequency. Explain physically why this is expected, and why the three curves nevertheless separate as frequency increases (**Q6.3.3**).

6.4 Measuring the Liquid Under Test

Remove the beaker, rinse the probe with IPA, and let it dry. Do not touch or move the cable.

1. Raise a beaker of isopropyl alcohol until the flange is submerged as before. Save as `ipa.s1p`.

2. Lower the beaker, let the probe dry, and **measure air again**, saving as `air_check.s1p`. This is your control measurement: when you process it as if it were an unknown, the extraction must return $\varepsilon' \approx 1$, $\varepsilon'' \approx 0$. Any deviation is a direct measure of drift and mechanical repeatability, not of the liquid.

3. Repeat the IPA measurement two more times, lowering and re-raising the beaker between trials, saving as `ipa_2.s1p` and `ipa_3.s1p`. This gives you an estimate of the repeatability of the probe–liquid contact.

6.5 Data Processing

Following the procedure derived in Appendix D, write a MATLAB or Python script that reads the four Touchstone files and returns $\varepsilon^*(f)$ of the liquid under test. You can use the PC stations which have MATLAB and Python installed.

1. Plot ε' and ε'' of IPA versus frequency from 0.1 GHz to 6 GHz, showing all three trials on the same axes (**Q6.5.1**).

2. Tabulate ε' and ε'' at 0.5, 1, 2, and 3 GHz (**Q6.5.2**).
3. Process `air_check.s1p` as an unknown and plot the recovered ε^*. How close to $1 - j0$ is it, and over what frequency range (**Q6.5.3**)?
4. Quantify the repeatability across your three IPA trials: at each frequency, compute the maximum deviation of any trial from the mean, normalized by $|\varepsilon^*|$, and plot it versus frequency (**Q6.5.4**).
5. Your extracted ε'' will likely degrade, and may even go negative, above roughly 3–4 GHz. A negative ε'' is unphysical — it would correspond to a passive material generating energy. Using Appendix D, explain which assumption in the model breaks down and why it breaks down at high frequency rather than at low frequency (**Q6.5.5**).

Congratulations! You have completed the lab. Please disconnect and put all components safely back into the box just as you received them. Get checked off by the instructors to make sure everything has been returned before you leave (Q6.5.6).

Appendices

A Noise Reduction Techniques

The network analyzer provides three methods for reducing the effect of noise on a measured response: smoothing, coherent averaging, and reduction of the IF bandwidth. Understanding how each works is essential for choosing the right trade-off between measurement speed and accuracy.

A.1 Smoothing

Smoothing (sometimes called trace smoothing) performs a moving average over adjacent frequency points of the *formatted* trace. A smoothing aperture, expressed as a percentage of the swept span, defines the number of points included in the moving-average window. Because the operation is performed on the displayed (formatted) data rather than on the underlying complex data, smoothing reduces point-to-point trace noise but does *not* lower the underlying noise floor. Excessive smoothing apertures will distort sharp features (e.g. filter skirts or resonances) and should be avoided when fine frequency detail is important.

A.2 Coherent Averaging

Coherent (or sweep-to-sweep) averaging combines the *complex* response at each frequency point over multiple sweeps:

$$S_{\text{avg}}(f_n) = \frac{1}{N}\sum_{m=1}^{N} S_m(f_n),$$

where $S_m(f_n)$ is the complex measurement at frequency f_n on sweep m and N is the averaging factor. Because thermal noise is a zero-mean random process, the vector sum of N independent noise contributions grows as $\sqrt{N}$ while the coherent signal grows as N. The signal-to-noise ratio therefore improves by a factor of N in power, i.e. $10\log_{10}(N)$ in dB. Averaging $N = 8$ sweeps yields roughly a 9 dB improvement in the noise floor. Crucially, because the averaging is performed on the complex data before the magnitude/phase format is applied, coherent averaging lowers the displayed noise floor itself, unlike incoherent (magnitude) averaging, which reduces only the variance of the noise.

A.3 IF Bandwidth Reduction

The FieldFox digitizes the received signal and applies an analog/digital IF filter of selectable bandwidth. Reducing the IF bandwidth narrows the receiver's noise bandwidth, which lowers the integrated noise power that reaches the detector. Each decade of reduction in IF bandwidth lowers the noise floor by approximately 10 dB. Reducing the IF bandwidth from 10 kHz to 100 Hz (two decades) therefore lowers the noise floor by roughly 20 dB. The penalty is increased sweep time, since each frequency point requires a longer measurement interval for the IF filter to settle.

B Sources of Error and Calibration Techniques

Measurements made with a network analyzer are degraded by three categories of error: systematic, random, and drift errors.

- **Systematic errors** are repeatable and time-invariant. They arise from imperfections in the analyzer, test cables, and connectors—directivity, source and load match, reflection and transmission tracking, and isolation (crosstalk). Because they are repeatable, systematic errors can be characterized during calibration and mathematically removed from subsequent measurements.
- **Random errors** vary with time in an unpredictable manner (e.g. thermal noise, connector repeatability). They cannot be removed by calibration but can be reduced using the noise-reduction techniques of Appendix A.
- **Drift errors** arise from changes in the instrument or test setup (primarily temperature) after a calibration has been performed. They are minimized by maintaining a stable environment and recalibrating as needed.

B.1 Error Model and One-Port Correction

For a one-port reflection measurement, the systematic errors can be represented by a three-term error model with terms for directivity (E_D), source match (E_S), and reflection tracking (E_R). The measured reflection coefficient Γ_{meas} is related to the actual reflection coefficient Γ_{act} by

$$\Gamma_{\text{meas}} = E_D + \frac{E_R\,\Gamma_{\text{act}}}{1 - E_S\,\Gamma_{\text{act}}}.$$

The three complex error terms are solved for by measuring three known standards: an open, a short, and a matched load. Once E_D, E_S, and E_R are known at each frequency, the corrected reflection coefficient is recovered by inverting the expression above:

$$\Gamma_{\text{act}} = \frac{\Gamma_{\text{meas}} - E_D}{E_S(\Gamma_{\text{meas}} - E_D) + E_R}.$$

B.2 Two-Port Calibration

A full two-port (SOLT) calibration extends the error model to twelve terms (six in each direction) to account for directivity, source match, load match, reflection tracking, transmission tracking, and isolation in both the forward and reverse directions. The open, short, load, and thru standards supply the data needed to solve for these terms, after which all four S-parameters of a two-port device can be corrected. The SOLR (short-open-load-reciprocal) variant used in Section 2.2 replaces the requirement of a perfectly characterized thru with the much weaker requirement that the connecting network merely be reciprocal, which is convenient when the two cable ends do not mate directly. In a later lab, the TRL (Thru-Reflect-Line) technique is introduced; it is better suited to non-coaxial media such as printed circuit boards, on-wafer ICs, and dielectric substrates, where high-quality open/short/load standards are difficult to realize.

C Amplifier Stability and Stability Circles

An amplifier is *unconditionally stable* at a given frequency if no passive source or load termination can cause oscillation, i.e. the input and output reflection coefficients have magnitude less than unity for all passive terminations. A convenient test uses the Rollett stability factor K together with the auxiliary condition $|\Delta| < 1$:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12}\,S_{21}|}, \qquad \Delta = S_{11}S_{22} - S_{12}S_{21}.$$

Unconditional stability requires $K > 1$ *and* $|\Delta| < 1$ simultaneously. When $K < 1$, the amplifier is only conditionally (potentially) stable, and some passive terminations will cause oscillation.

C.1 Stability Circles

The regions of the source and load reflection-coefficient planes that produce a stable design are bounded by *stability circles*. The output stability circle (the locus of load reflection coefficients Γ_L for which $|\Gamma_{\text{in}}| = 1$) has center C_L and radius R_L given by

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}, \qquad R_L = \left| \frac{S_{12}\, S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|.$$

Similarly, the input stability circle (the locus of source reflection coefficients Γ_S for which $|\Gamma_{\text{out}}| = 1$) has

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}, \qquad R_S = \left| \frac{S_{12}\, S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|.$$

Plotting these circles on the Smith Chart and determining which side of each circle corresponds to $|\Gamma| < 1$—by testing whether the origin $\Gamma = 0$ falls in the stable region, which it does when $|S_{11}| < 1$ (input plane) or $|S_{22}| < 1$ (output plane)—identifies the stable and unstable regions of source and load impedance.

D Extraction of Complex Permittivity from a One-Port Measurement

This appendix develops, from first principles, the procedure used in Section 6 to convert a measured reflection coefficient into the complex permittivity of a liquid. Everything here follows from two ideas you have already used in this lab: (i) a reflection coefficient and an impedance/admittance carry the same information, and (ii) all of the systematic imperfections between the VNA receiver and the plane of interest can be lumped into a two-port error box, as in Appendix B.

D.1 The Probe and Its Sensing Volume

The probe is a coaxial line terminated abruptly in an open circuit. Because the line simply stops, the guided TEM mode cannot continue, and the field spills out of the aperture into whatever is in front of it. This *fringing* field is evanescent: it does not radiate away (at least at low enough frequency), it stores energy in the material immediately in front of the aperture. The aperture therefore behaves as a capacitor whose dielectric is the material under test.

The consequence is that the probe measures only a small volume. The lateral extent of the sensing region is roughly 60–70% of the inner radius of the outer conductor, which for an SMA aperture is about 1.3–1.5 mm; the depth is a few millimeters at most. This is why a beaker filled up to 2 cm is more than sufficient.

D.2 What the VNA Actually Measures

Denote by S_{11} the complex reflection coefficient the FieldFox reports, and by $\Gamma_{AA'}$ the true reflection coefficient at the probe aperture, i.e. at the plane where the field meets the material. These are not the same. Between them lie the cable, the connector transitions, the finite directivity and source match of the VNA, and the internal structure of the probe itself.

All of these are linear, passive, time-invariant, and independent of the material in front of the probe. Therefore they can be represented by a single two-port error box $[e]$, exactly as in Appendix B, and the relation between what you measure and what you want is

$$S_{11} = e_{11} + \frac{e_{12}e_{21}\,\Gamma_{AA'}}{1 - e_{22}\,\Gamma_{AA'}}. \tag{1}$$

Equation (1) has the form

$$S_{11} = \frac{a\,\Gamma_{AA'} + b}{c\,\Gamma_{AA'} + d}, \tag{2}$$

with $a = e_{12}e_{21} - e_{11}e_{22}$, $b = e_{11}$, $c = -e_{22}$, $d = 1$. A mapping of this form — a ratio of two linear functions — is called a *bilinear* or *Möbius* transformation. Note that the *Smith Chart* represents exactly such a transformation between the reflection coefficient and load impedance.

D.3 From Reflection Coefficient to Aperture Admittance

The aperture is a shunt load on the coaxial line. It is natural to describe it by an admittance $Y_{AA'}$ rather than a reflection coefficient. The two are related by the standard expression

$$Y_{AA'} = Y_0\,\frac{1 - \Gamma_{AA'}}{1 + \Gamma_{AA'}}, \qquad Y_0 = \frac{1}{50\,\Omega} = 0.02\ \text{S}. \tag{3}$$

Note that Eq. (3) is again a bilinear transformation, this time from $\Gamma_{AA'}$ to $Y_{AA'}$. Two useful limits: a perfect short ($\Gamma_{AA'} = -1$) gives $Y_{AA'} = \infty$, and a perfect open ($\Gamma_{AA'} = +1$) gives $Y_{AA'} = 0$.

D.4 The Capacitive Model of the Aperture

We now need a physical model connecting $Y_{AA'}$ to the permittivity of the material. The simplest one — and the one used in this lab — is the *capacitive model*. It assumes the fringing field stores energy in two places: in the material under test, and in the solid dielectric (PTFE) inside the connector between the inner and outer conductors. This gives two capacitors in parallel:

$$Y_{AA'} = j\omega\,(C_0\,\varepsilon^* + C_f)\,. \tag{4}$$

Here C_0 is a purely geometric constant — the capacitance the fringing field would have if the material were vacuum — and C_f is the fringing capacitance inside the probe's own dielectric, which does not see the material at all. For a typical SMA aperture C_0 is on the order of 30 fF, and $C_f \ll C_0$.

Two observations about Eq. (4) are essential:

- **It is affine in ε^*.** $Y_{AA'}$ is a constant times ε^* plus another constant. An affine map is a special case of a bilinear map (with $c = 0$, $d = 1$).
- **It correctly reproduces losses.** Substituting $\varepsilon^* = \varepsilon' - j\varepsilon''$ gives $Y_{AA'} = \omega C_0\varepsilon'' + j\omega(C_0\varepsilon' + C_f)$. The real part — the conductance, i.e. the loss — comes entirely from ε'', and the imaginary part — the susceptance — from ε'. So a lossless material presents a purely reactive load, as it must.

Note that this model contains no radiation term. It assumes the aperture stores energy but does not launch a wave into the material. That assumption is what fails at high frequency, when the aperture becomes electrically large enough to radiate; this is the origin of the model breakdown you are asked about in **Q6.5.5**.

D.5 The Key Result: ε^* Is a Bilinear Function of S_{11}

We now have a chain of three maps:

$$\varepsilon^* \xrightarrow[\text{Eq. (4)}]{\text{affine}} Y_{AA'} \xrightarrow[\text{Eq. (3)}]{\text{bilinear}} \Gamma_{AA'} \xrightarrow[\text{Eq. (2)}]{\text{bilinear}} S_{11}.$$

The composition of two bilinear transformations is itself a bilinear transformation, and so is the inverse of one. (You can verify this directly by substitution; it is the same algebra as cascading two ABCD matrices.) Therefore the entire chain collapses, and its inverse can be written as

$$\boxed{\varepsilon^* = \frac{A\,S_{11} + B}{C\,S_{11} + 1}} \tag{5}$$

for three complex constants A, B, C at each frequency. (We have divided numerator and denominator by the fourth coefficient, which costs no generality.)

Equation (5) is remarkable, and it is worth pausing on what it says:

- You never need to know C_0, C_f, the cable length, the connector parasitics, or any of the error terms. They are all buried inside A, B, and C.
- Because the VNA's own systematic errors are themselves a bilinear map, they are absorbed too. This is why the OSL calibration of Section 6.2 cannot change the answer — it merely replaces one bilinear map with a different one, and the three probe standards will solve for whichever one is in place.
- There are exactly three unknowns, so exactly **three standards** are required. This is the same counting argument as the three-term (directivity, source match, reflection tracking) one-port error model of Appendix B — and for the same reason.

D.6 Solving for the Three Coefficients

Let S_{11}^1, S_{11}^2, S_{11}^3 denote the measured raw reflection coefficients of the short, air, and distilled water standards, and let S_{11}^4 denote that of the liquid under test. All four are complex vectors of length N (one entry per frequency point); everything below is done independently at each frequency.

Standard 1 — the short ($\varepsilon^* \to \infty$). A metal plate pressed against the aperture forces $\Gamma_{AA'} = -1$, hence $Y_{AA'} = \infty$, hence by Eq. (4) $\varepsilon^* \to \infty$. In Eq. (5), ε^* becomes infinite exactly when the denominator vanishes. So the short measurement locates the pole of the mapping:

$$C\,S_{11}^1 + 1 = 0 \quad\Longrightarrow\quad C = -\frac{1}{S_{11}^1}. \tag{6}$$

This is why a sloppy short contact is fatal: it alone determines C.

Standard 2 — air ($\varepsilon^* = 1$). Substituting into Eq. (5),

$$A\,S_{11}^2 + B = C\,S_{11}^2 + 1. \tag{7}$$

Standard 3 — distilled water ($\varepsilon^* = \varepsilon^*_w$). Similarly,

$$A\,S^3_{11} + B = \varepsilon^*_w\left(C\,S^3_{11} + 1\right). \tag{8}$$

Equations (7) and (8) are two linear equations in the two remaining unknowns A and B (since C is already known from Eq. (6)). Subtracting them and solving,

$$A = \frac{C\left(S^2_{11} - \varepsilon^*_w S^3_{11}\right) + 1 - \varepsilon^*_w}{S^2_{11} - S^3_{11}}, \qquad B = C\,S^2_{11} + 1 - A\,S^2_{11}. \tag{9}$$

The unknown. With A, B, C in hand, the permittivity of the liquid follows directly from Eq. (5):

$$\varepsilon^*_{\text{LUT}} = \frac{A\,S^4_{11} + B}{C\,S^4_{11} + 1}. \tag{10}$$

D.7 Equivalent Closed Form (Cross-Ratio)

There is a compact alternative worth knowing, useful as an independent check on your code. Bilinear transformations preserve the *cross-ratio* of any four points. Applying this to the four measurements and their four permittivities, and taking the limit $\varepsilon^*_1 \to \infty$ for the short, gives a single expression with no intermediate coefficients:

$$\varepsilon^*_{\text{LUT}} = \frac{\Delta_{42}\Delta_{31}}{\Delta_{41}\Delta_{32}}\,\varepsilon^*_w + \frac{\Delta_{43}\Delta_{21}}{\Delta_{41}\Delta_{23}}, \qquad \Delta_{ij} \equiv S^i_{11} - S^j_{11}. \tag{11}$$

Equation (11) is algebraically identical to Eqs. (6)–(10); if your two implementations disagree by more than numerical round-off, one of them has an index error.

D.8 The Water Standard

Air and the short are trivially known, but water is only useful as a standard because its dielectric relaxation has been characterized to high accuracy. Distilled water is well described over this band by a single-pole Debye relaxation:

$$\varepsilon^*_w(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau}, \qquad \omega = 2\pi f. \tag{12}$$

The physical picture: water molecules are permanent dipoles that reorient to follow the applied field. At low frequency they keep up perfectly and the material shows its full static permittivity ε_s. At high frequency they cannot follow at all and only the fast electronic response remains, giving ε_∞. Near $f = 1/(2\pi\tau)$ the reorientation lags the field by roughly a quarter cycle, which is precisely when energy is dissipated — hence the peak in ε'' there.

At 23°C use

$$\varepsilon_s = 79.1, \qquad \varepsilon_\infty = 5.1, \qquad \tau = 8.7\ \text{ps}.$$

Note ε_s changes by roughly -0.36 per degree Celsius near room temperature, so a 3°C error in your recorded water temperature is a $\sim$1.5% error in your standard, which propagates directly into your answer. Use distilled water only: dissolved ions add a conductivity term $-j\sigma_i/(\omega\varepsilon_0)$ that Eq. (12) does not include and that blows up at low frequency.

Sign convention. Equation (12) with $+j\omega\tau$ in the denominator produces $\varepsilon^* = \varepsilon' - j\varepsilon''$ with $\varepsilon'' > 0$, which is the passive-material convention consistent with the $e^{j\omega t}$ time dependence used by the VNA. If your extracted ε'' comes out uniformly negative across the whole band, you have a conjugation error somewhere — most likely in how you read the Touchstone phase column.

D.9 Algorithm Summary

At each of the N frequency points independently:

1. Read S_{11}^1 (short), S_{11}^2 (air), S_{11}^3 (water), and S_{11}^4 (LUT) from the four `.s1p` files, converting the magnitude/angle columns to complex numbers. Verify all four files share an identical frequency vector; if they do not, you changed a stimulus setting partway through and must repeat the measurements.
2. Evaluate ε_w^* at that frequency from Eq. (12) using your recorded water temperature.
3. Compute C from Eq. (6), then A and B from Eq. (9).
4. Compute $\varepsilon_{\text{LUT}}^*$ from Eq. (10).
5. Report $\varepsilon' = \text{Re}\{\varepsilon^*\}$ and $\varepsilon'' = -\text{Im}\{\varepsilon^*\}$.

D.10 Sanity Checks and Limits of Validity

- **Recover a standard.** Feed the air measurement back in as the unknown. You must get exactly $1 - j0$ — if not, the bug is in your code, not your measurement. Then feed in an *independently taken* air sweep (`air_check.s1p`); the deviation from unity now measures drift and repeatability rather than arithmetic.
- **Low-frequency limit.** As $f \to 0$ the aperture admittance $j\omega(C_0\varepsilon^* + C_f) \to 0$ for every material, so all four standards crowd toward $\Gamma = +1$ on the Smith chart. The differences Δ_{ij} in Eq. (11) become small, the extraction becomes ill-conditioned, and noise is amplified. Expect unusable results below roughly 100 MHz.
- **High-frequency limit.** As the aperture becomes electrically larger it begins to radiate into the sample, and the purely capacitive Eq. (4) no longer holds. The neglected radiation conductance is mistakenly attributed to ε'', which is why the extracted loss degrades first and can even go negative. For this SMA-sized aperture the capacitive model is reliable to about 3–4 GHz. Correcting for this requires adding a radiation term, $Y_{AA'} = j\omega C_0\varepsilon^* + G(\omega, \varepsilon^*)$, which is no longer affine in ε^* and must be inverted numerically — and, having four unknowns, requires a fourth standard for a complete error correction.
- **Contact quality.** An air gap of even a few tens of microns between the aperture and the sample appears in series with the sensing capacitance and pulls the extracted ε' downward. Liquids are the easy case, which is why this experiment uses one; solid samples require deliberate contact pressure and averaging over multiple spots.
- **Temperature.** All four measurements should be at the same temperature. Both the standard and the LUT have temperature-dependent relaxation times, and IPA evaporates measurably, cooling itself and changing concentration if left exposed. Work briskly and keep the beaker covered between measurements.