Non-linearity

$$f(ax+by) \neq af(x)+bf(y)$$

$$v_{in}(t) = A\cos\omega t \longrightarrow \boxed{\text{Non-lin system}} \longrightarrow v_{out}(t) = a_1 v_{in}(t) + a_2 v_{in}^2(t) + a_3 v_{in}^3(t) + \ldots$$

Assumption

- Memoryless model.

$$v_{out}(t) = \overbrace{\tfrac{1}{2}a_2A^2}^{\text{all even}} \Big\} \text{ DC Term (0}^{th}\text{ Harmonic)}$$

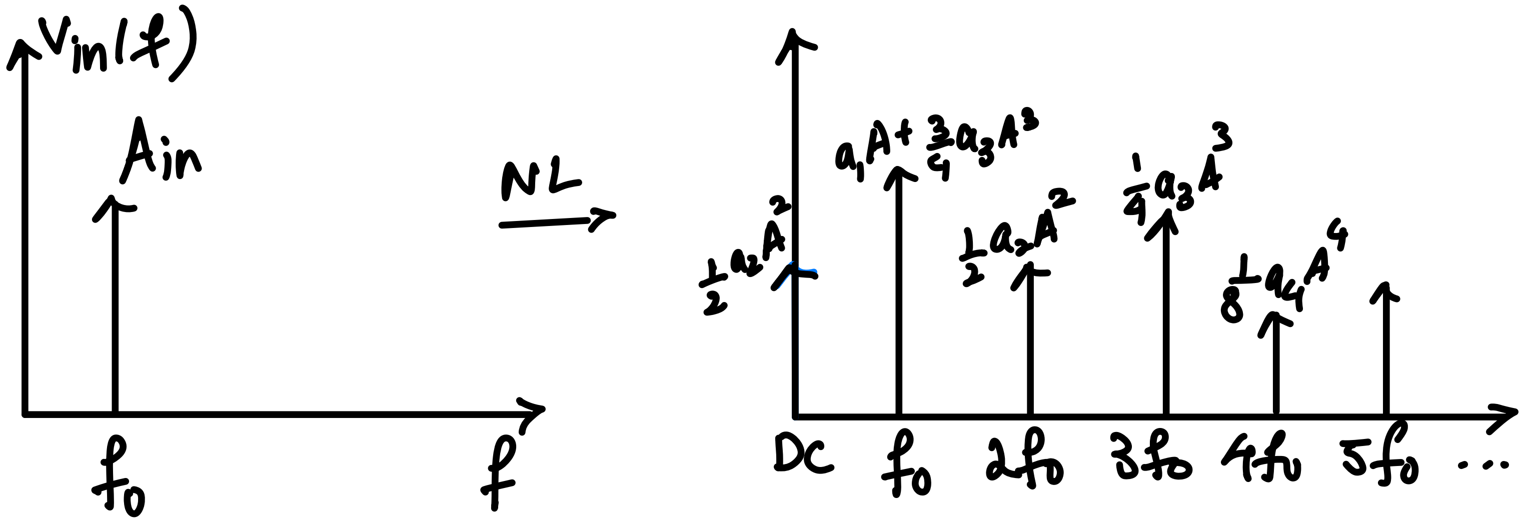
$$+ \Big[\overbrace{a_1A + \tfrac{3}{4}a_3A^3}^{\text{all odd}}\Big]\cos\omega t \Big\} \text{ Fundamental (1}^{s}\text{ harmonic)}$$

$$+ \Big[\overbrace{\tfrac{1}{2}a_2A^2}^{\text{all even}}\Big]\cos 2\omega t \text{ (2}^{nd}\text{ harmonic)}$$

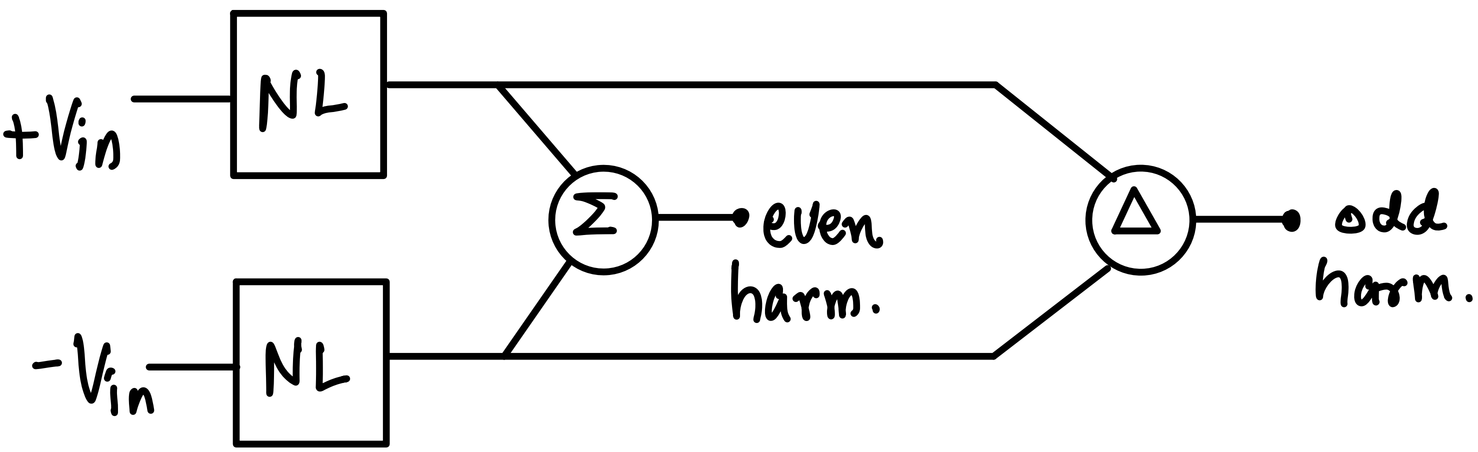
$$+ \Big[\overbrace{\tfrac{1}{4}a_3A^3}^{\text{all odd from 3 onward}}\Big]\cos 3\omega t \text{ (3}^{rd}\text{ harmonic)}$$

Observations

- Even order coeff. $\longrightarrow$ DC offset
- Odd order coeff. $\longrightarrow$ Fundamental
- N^{th} order harmonic $\rightarrow$ $\simeq A^n$ is the leading behavior.



Differential Implementation



Harmonic Distortion

$$HD = \frac{A_n}{A_1}$$ linear scale (voltage)

$$HD^{\%} = HD \cdot 100\%$$

$$HD^{dBc} = 20\log_{10}(HD)$$

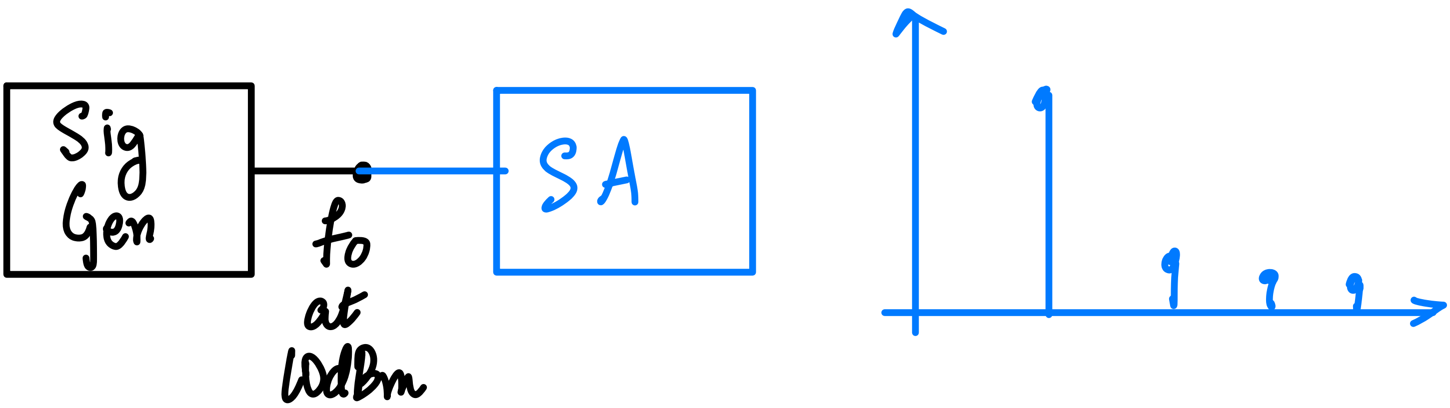
$$HD = 10^{\frac{P_n - P_1}{20}}$$

Total Harmonic Distortion

$$THD = \frac{\sqrt{A_2^2 + A_3^2 + \cdots}}{A_1} \rightarrow \begin{cases} \times 100\% \\ 20\log_{10}(\ldots) \end{cases}$$

$$THD = \sqrt{\sum_{n=2}^{\infty} 10^{(P_n - P_1)/10}}$$

Reduce THD: Diff. signal + Filtering.

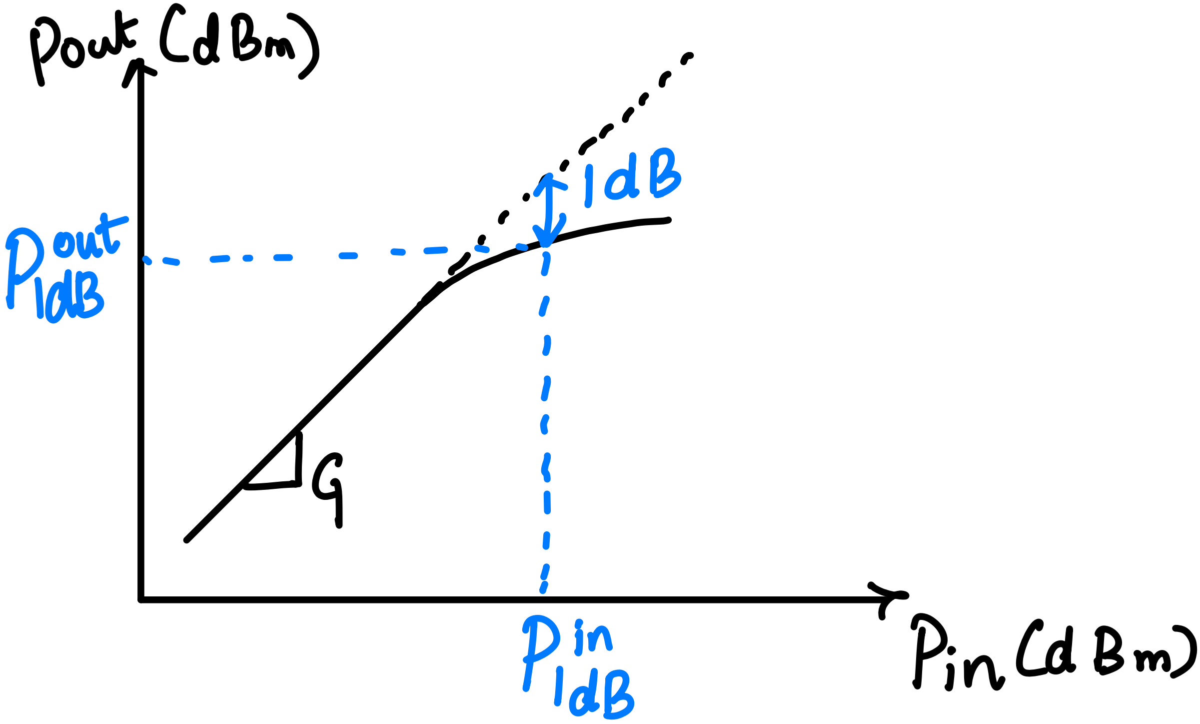


Gain Compression

Gain @ f_0 : $a_1 + \frac{3}{4} a_3 A^2$

$a_1 a_3 < 0$

$\Rightarrow$ Gain drops with A.



$$P_{1dB}^{out} = P_{1dB}^{in} + G - 1dB$$

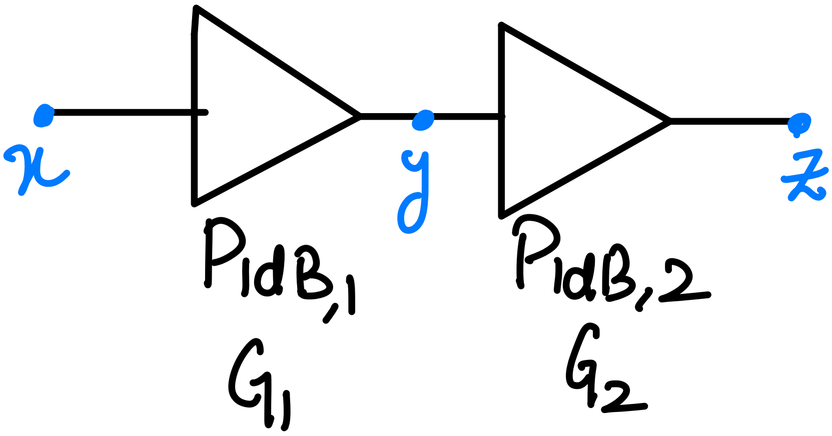
$$20 \log \left| a_1 + \frac{3}{4} a_3 A^2 \right| = 20 \log |a_1| - 1dB.$$

$$\boxed{A_{1dB} \cong \sqrt{0.145 \frac{a_1}{a_3}}}$$

Assumptions

- Only considered 3^{rd} order nonlinearity
- Memoryless!

Nonlinearity of Cascaded Systems



$$y = a_1 x + a_2 x^2 + a_3 x^3$$

$$z = b_1 y + b_2 y^2 + b_3 y^3$$

$$z = b_1 (a_1 x + a_2 x^2 + a_3 x^3) + b_2 (a_1 x + a_2 x^2)^2 + b_3 (a_1 x)^3$$

$$= \underbrace{a_1 b_1}_{c_1} x + \underbrace{(b_1 a_2 + b_2 a_1^2)}_{c_2} x^2 + \underbrace{(b_1 a_3 + 2 b_2 a_1 a_2 + b_3 a_1^3)}_{c_3} x^3 + \cdots$$

$$c_1 = a_1 b_1$$

$$c_2 = \cancel{b_1 a_2} + \cancel{b_2 a_1^2}$$

$$c_3 = b_1 a_3 + b_3 a_1^3 + 2\cancel{b_2 a_1 a_2}$$

Assumption: Even terms are small.

$$\frac{1}{A^2_{1dB,tot}} = \frac{1}{0.145}\left|\frac{c_3}{c_1}\right|$$

$$= \frac{1}{0.145}\left|\frac{a_3}{a_1} + \frac{b_3}{b_1}a_1^2\right|$$

$$\leq \frac{1}{0.145}\left|\frac{a_3}{a_1}\right| + \frac{\overset{G_1}{a_1^2}}{0.145}\left|\frac{b_3}{b_1}\right|$$

$$\Rightarrow \frac{1}{A^2_{1dB,tot}} = \frac{1}{A^2_{1dB,1}} + \frac{G_1}{A^2_{1dB,2}}$$

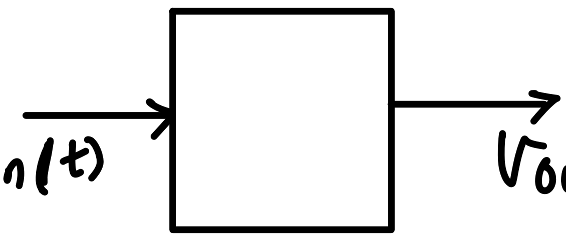
$$\frac{1}{P^{in}_{1dB,tot}} = \frac{1}{P^{in}_{1dB,1}} + \frac{G_1}{P^{in}_{1dB,2}}$$

$$\frac{1}{P^{in}_{1dB,tot}} = \frac{1}{P^{in}_{1dB,1}} + \frac{G_1}{P^{in}_{1dB,2}} + \frac{G_1 G_2}{P^{in}_{1dB,3}} + \cdots$$

Intermodulation Distortion (IM)

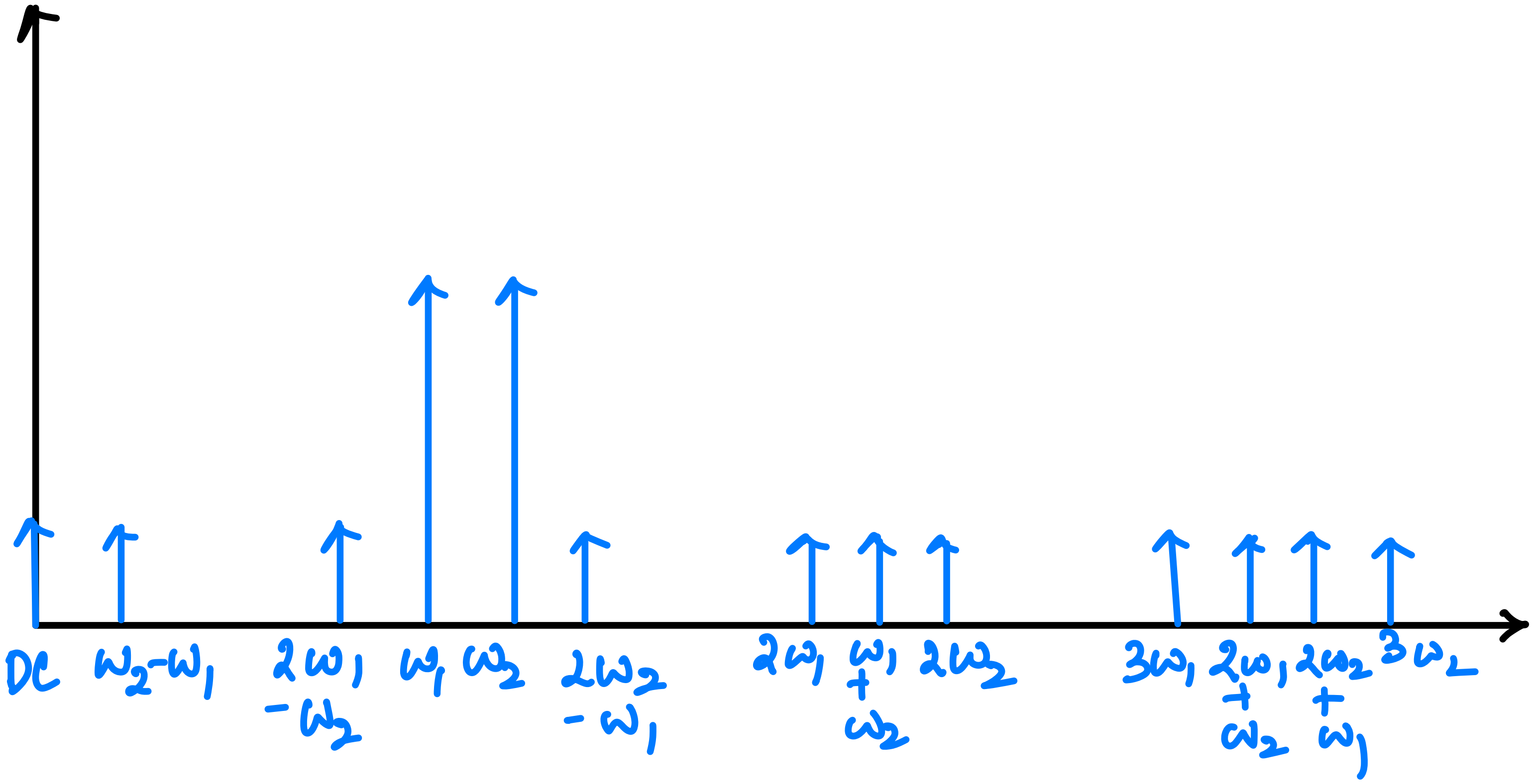
$$V_{in}(t) = A\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$$

$V_{in}(t)$



$$V_{out}(t) = a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 \cdots$$

Order	Frequency	Amplitude	Slope (dB/dB)
1	ω_1, ω_2	$a_1 A + \frac{9}{4} a_3 A^3$	1
2	DC	$a_2 A^2$	–
2	$\omega_2 - \omega_1, \omega_1 + \omega_2$	$a_2 A^2$	2
2	$2\omega_1, 2\omega_2$	$\frac{1}{2} a_2 A^2$	2
3	$2\omega_1 - \omega_2, 2\omega_2 - \omega_1$	$\frac{3}{4} a_3 A^3$	3 IM3
3	$2\omega_1 + \omega_2, 2\omega_2 + \omega_1$	$\frac{3}{4} a_3 A^3$	3
3	$3\omega_1, 3\omega_2$	$\frac{1}{4} a_3 A^3$	3



Third Intercept Ratio (IM3)

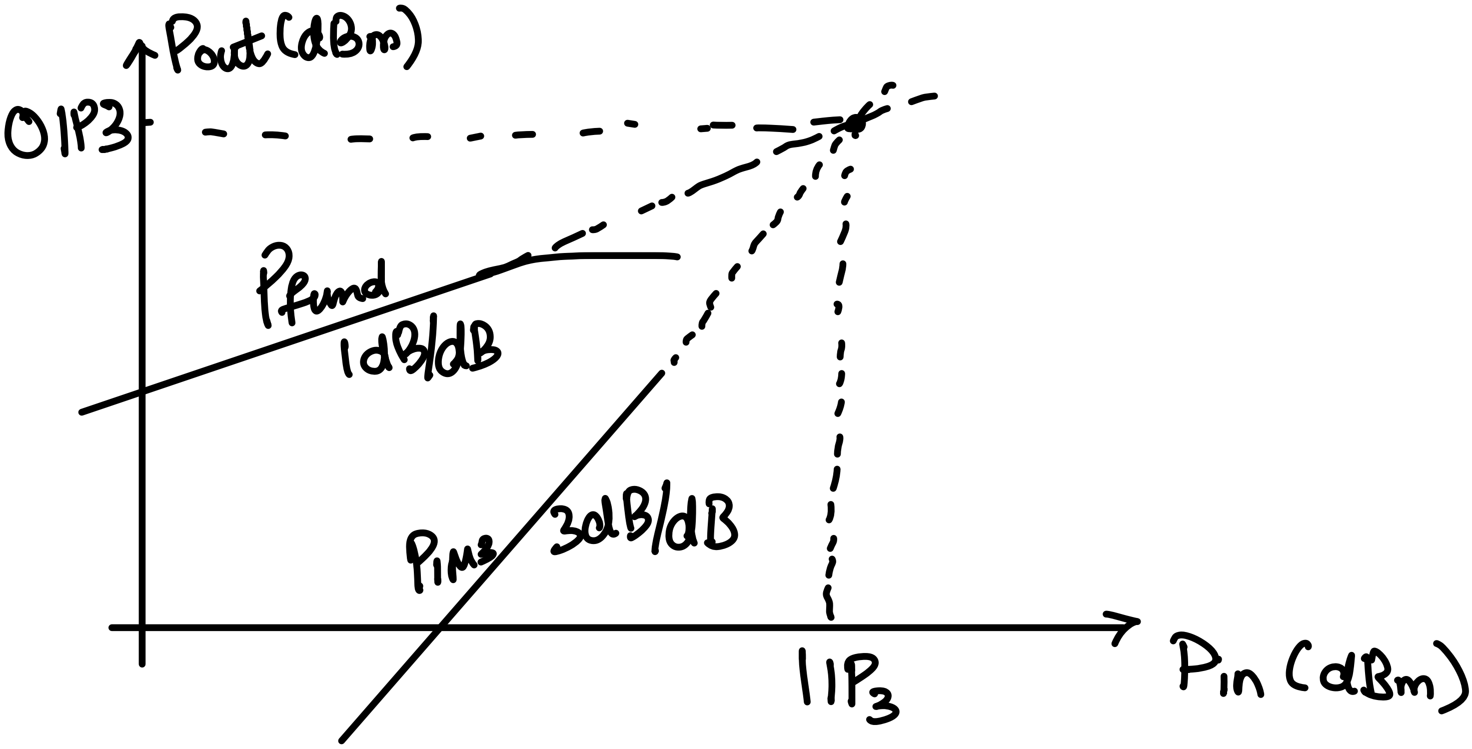
$$IM3 = \left| \frac{\frac{3}{4} a_3 A^3}{a_1 A} \right| = \left| \frac{3}{4} \frac{a_3}{a_1} A^2 \right|$$

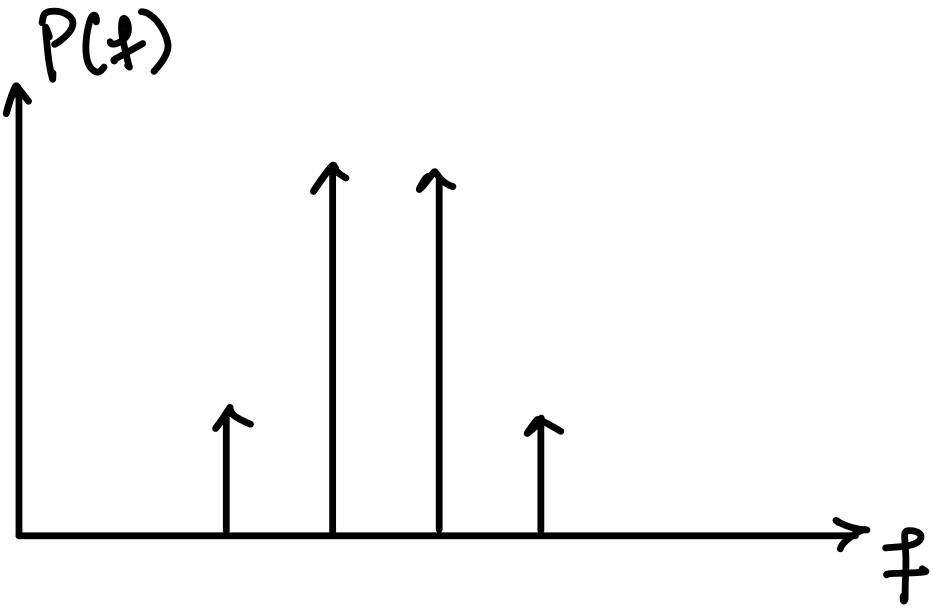
Third Intercept Point (IP3)

$$IM3 = 1$$

$$A_{IIP_3} = \sqrt{\frac{4}{3} \left| \frac{a_1}{a_3} \right|}$$

$$IIP3 = 10 \log_{10} \left(\frac{A_{IIP_3}^{2}}{2 Z_0 \cdot 1mW} \right) \text{ (dBm)}$$





$$A_{IIP_3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|} \quad ; \quad A_{1dB} = \sqrt{0.145\left|\frac{a_1}{a_3}\right|}$$

$$\frac{A_{1dB}^2}{A_{IIP_3}^2} = 0.1087$$

$$\boxed{IIP3 - P_{1dB}^{in} = 9.64\,dB}$$

$$\frac{1}{IIP3_{tot}} = \frac{1}{IIP3_{,1}} + \frac{G_1}{IIP_{3,2}} + \frac{G_1 G_2}{IIP_{3,3}} + \cdots$$

$$\frac{1}{OIP3_{tot}} = \frac{1}{G_1 G_2 \cdots G_N OIP_{3,1}} + \cdots + \frac{1}{OIP_{3,N}}$$

LTI memoryless

$$y(t) = h\, x(t)$$

LTI memory

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Non-linear memoryless

$$y(t) = \sum_{n=1}^{N} a_n\, x^n(t)$$

Non-linear memory [Volterra Series]

$$y(t) = \sum_{n=1}^{N} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \underbrace{h_n(\tau_1, \tau_2, \ldots \tau_n)}_{\text{Volterra Kernel}} \prod_{k=1}^{n} x(t-\tau_k)\, d\tau_1 d\tau_2 \ldots d\tau_n$$

1D: $y_1(t) = \int h_1(\tau_1)\, x(t-\tau_1)\, d\tau_1$

2D: $y_2(t) = \iint h_2(\tau_1, \tau_2)\, x(t-\tau_1)\, x(t-\tau_2)\, d\tau_1 d\tau_2$