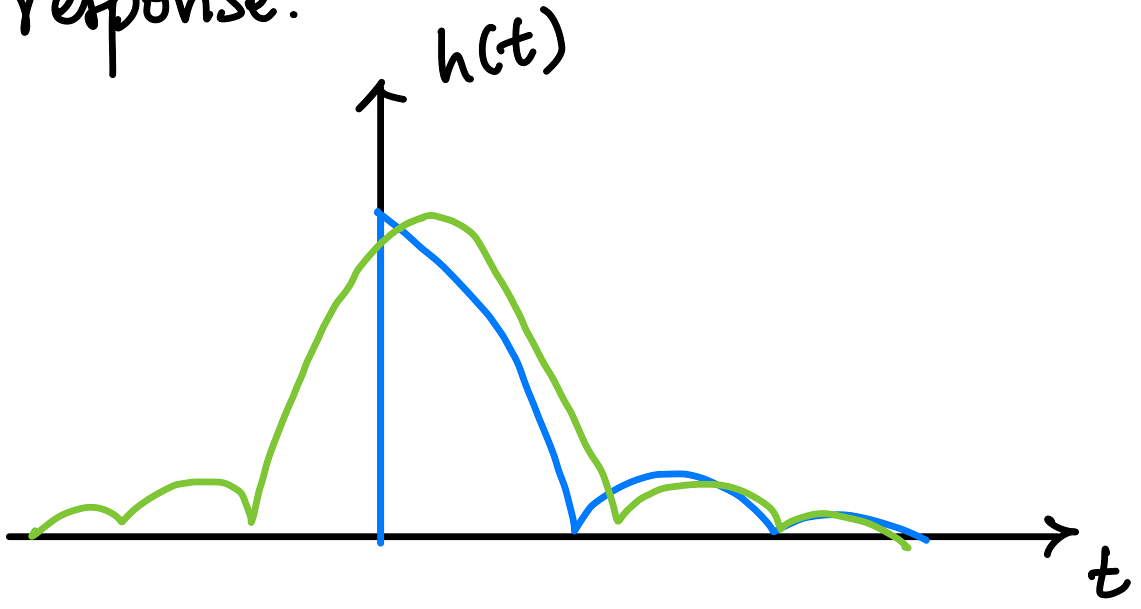
VNA Time Domain Measurements

- Not just an IFT of the measured response.



Fourier Transforms

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

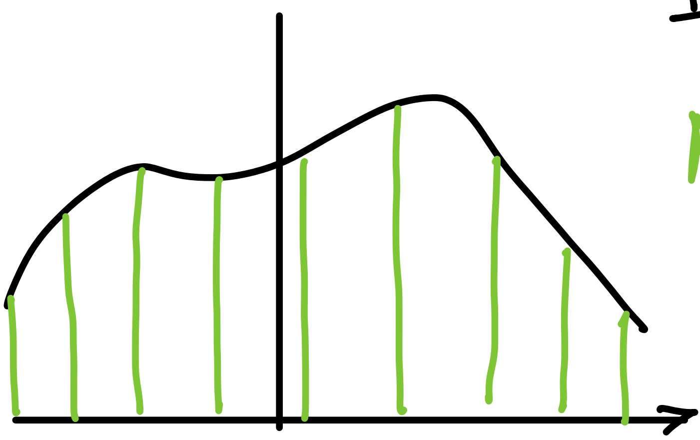
If $f(t)$ is causal & real, $F(\omega)$ is Hermitian Symmetric

$$F(\omega) = F^*(-\omega)$$

FT → VNA Time Domain Transform

- Sampling.
- Truncation.
- Windowing.
- Renormalization.

Sampling



$$Ш(\omega) = \Delta\omega \sum_{n=-\infty}^{\infty} \delta(\omega - n\Delta\omega)$$

$$F_s(\omega) = F(\omega) \cdot Ш(\omega)$$

$$f_s(t) = F^{-1}\left(F_s(\omega)\right)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, \Delta\omega \sum_{n=-\infty}^{\infty} \delta(\omega - n\Delta\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} F(n\Delta\omega) \Delta\omega\, e^{jn\Delta\omega t}.$$

$$f_s(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau)\, e^{-jn\Delta\omega\tau}\, d\tau \right] \Delta\omega\, e^{jn\Delta\omega t}$$

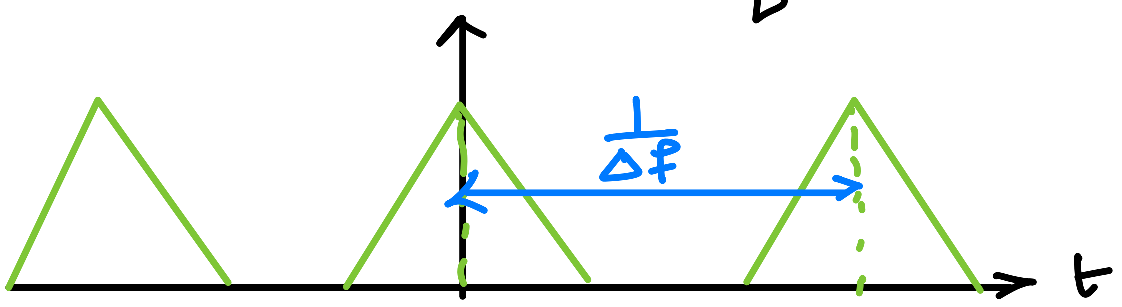
$$= \frac{\Delta\omega}{2\pi} \int_{-\infty}^{\infty} f(\tau) \left[\sum_n e^{jn\Delta\omega(t-\tau)} \right]$$

Poisson: $\sum_{n=-\infty}^{\infty} e^{jn\Delta\omega t} = \frac{2\pi}{\Delta\omega} \sum_{k=-\infty}^{\infty} \delta\left(t - k\frac{2\pi}{\omega}\right)$

$$= \int_{-\infty}^{\infty} f(\tau) \sum_k \delta\left(t - \tau - \frac{k}{\Delta f}\right) d\tau$$

$$f_s(t) = \sum_k f\left(t - \frac{k}{\Delta f}\right)$$

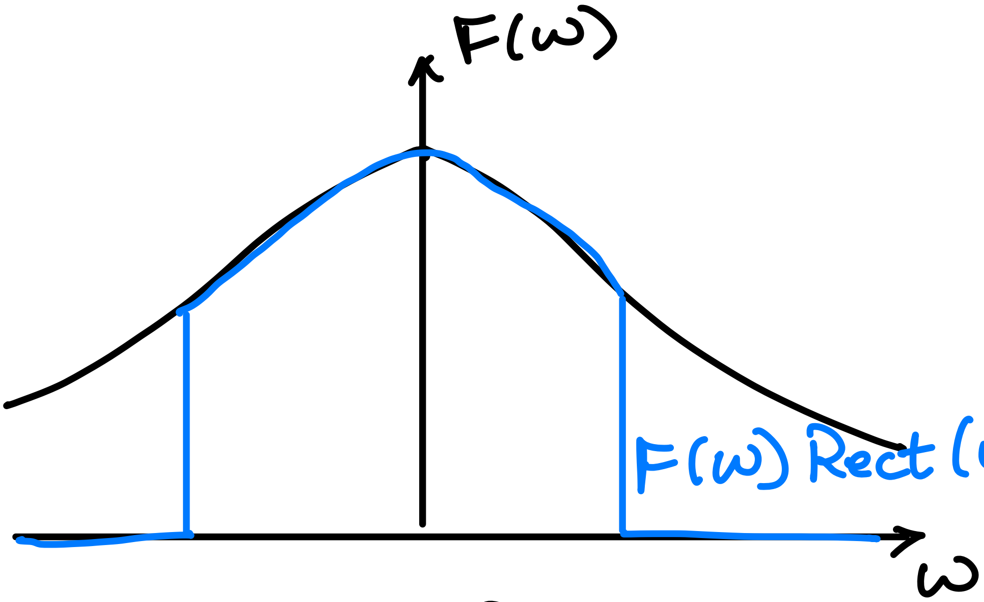
$$= f(t) * \underbrace{\left[\sum_k \delta\left(t - \frac{k}{\Delta f}\right) \right]}_{\text{IFT of } Ш(\omega)}$$

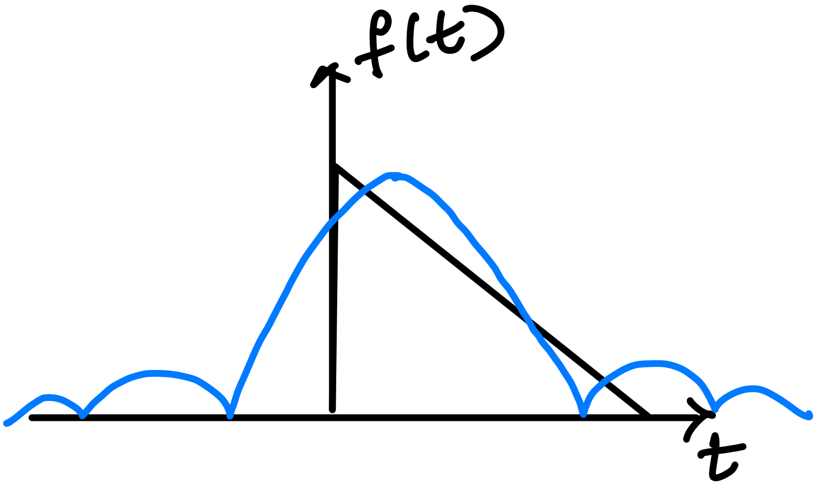


Aliasing Free Rang is $\pm \frac{1}{2\Delta f}$

VNA only displays the first Nyquist Zone.

2) Truncation





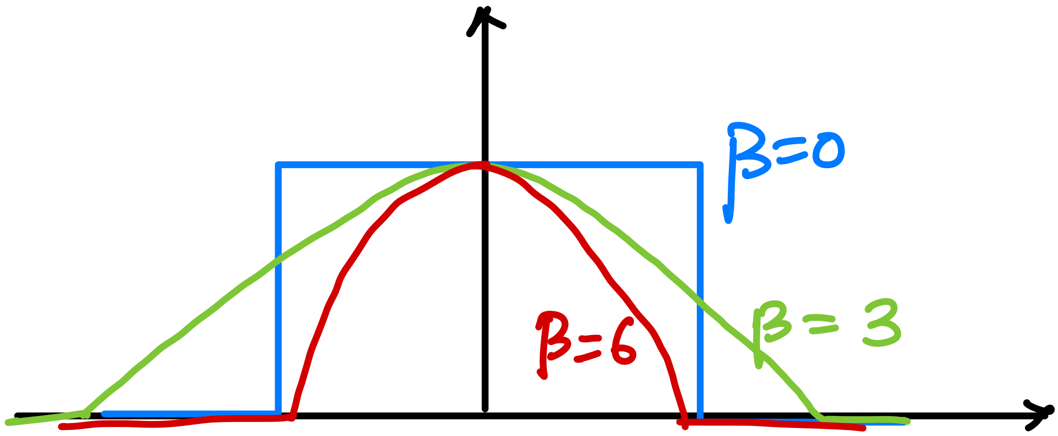
3) Windowing

Window : Kaise-Bessel Window.

$$W(n) = \frac{I_0\left(\beta\sqrt{1-\left(\frac{2n}{N-1}-1\right)^2}\right)}{I_0(\beta)}$$

I_0 - Modified Bessel fn. of First Kind.

β : Kaise Beta Factor.



$\beta \uparrow \Rightarrow$ Sidelobes $\downarrow$ and FWHM $\uparrow$

<u>VNA</u>: $\beta = 6.5$

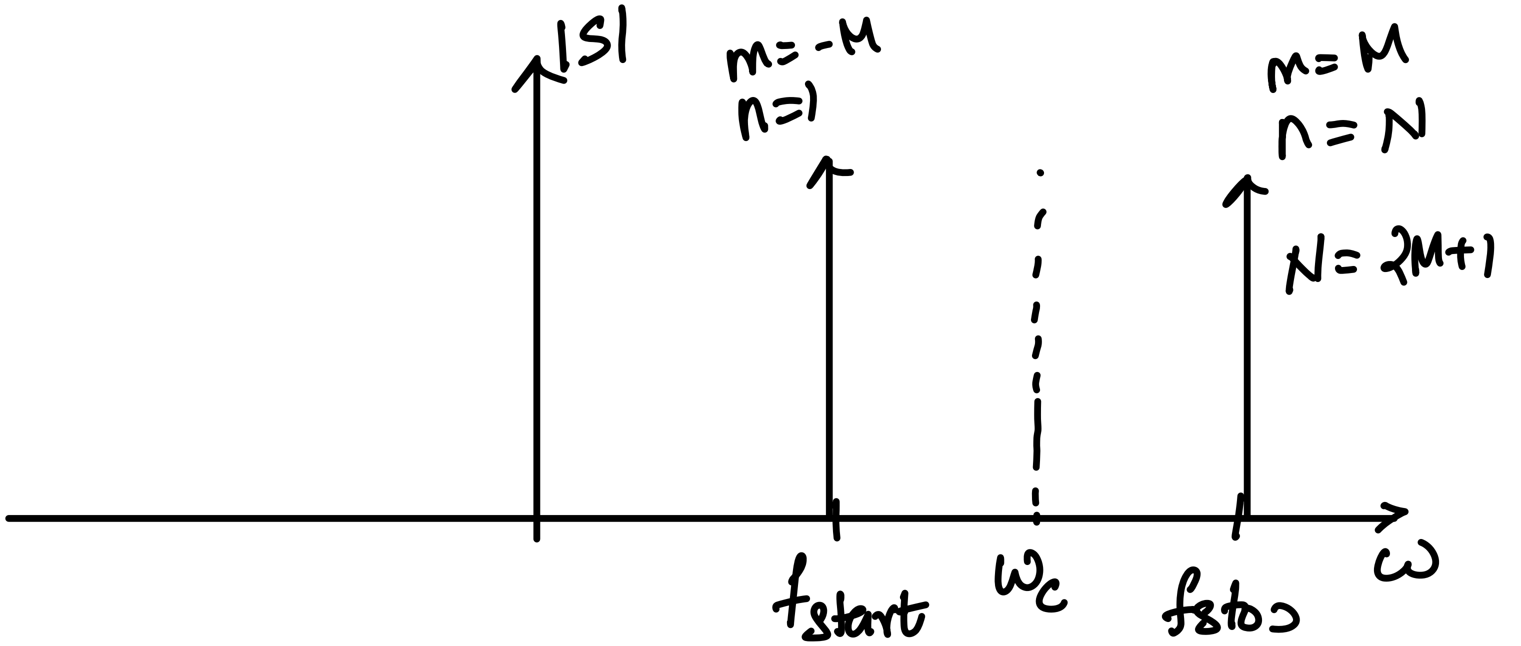
4) Renormalization

$$W_0 = \frac{\Delta\omega}{2\pi} \sum_{n=-N}^{N} W(n\Delta\omega)$$

$$\boxed{f_{VNA}(t) = \frac{1}{W_0} \frac{\Delta\omega}{2\pi} \sum_{n=-N}^{N} F(n\Delta\omega) W(n\Delta\omega) e^{jn\Delta\omega t}}$$

Time Domain Transforms

I) Band Pass Impulse (Default)



$$f_{BP}(t) = \frac{1}{W_0} \frac{\Delta\omega}{2\pi} \sum_{m=-M}^{M} F(\omega_c + m\Delta\omega) W(m\Delta\omega) e^{j(\omega_c + m\Delta\omega)t}$$

Note: ① Only positive frequencies

$f_{BP}(t)$ is not real (it is complex).

because $F(\omega) \neq F^*(-\omega)$.

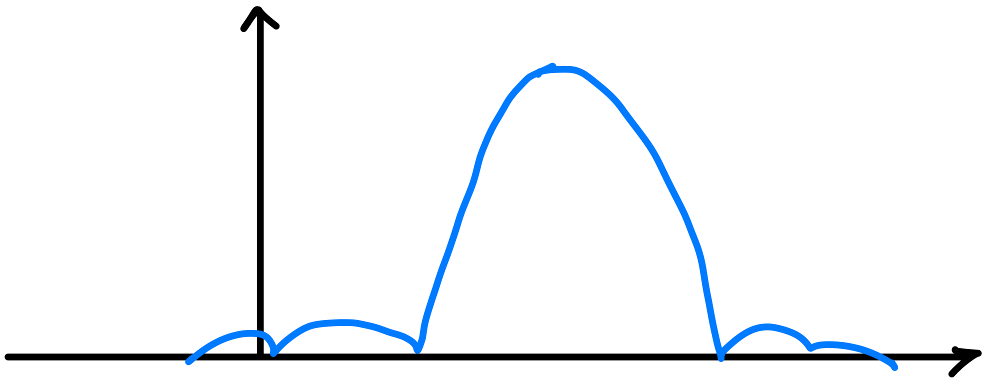
② Phase is not meaningful.

Cannot tell the difference b/w

open & short

Cap & inductor.

③ FWHM Beamwidth.



$$\boxed{\Delta t_{BP} \cong \frac{1.21}{BW}} \rightarrow \text{Rectangular Window}$$

$$\Delta t_{BP} \simeq \frac{K}{BW}$$ $K \rightarrow$ Kaiser Bessel Factor.

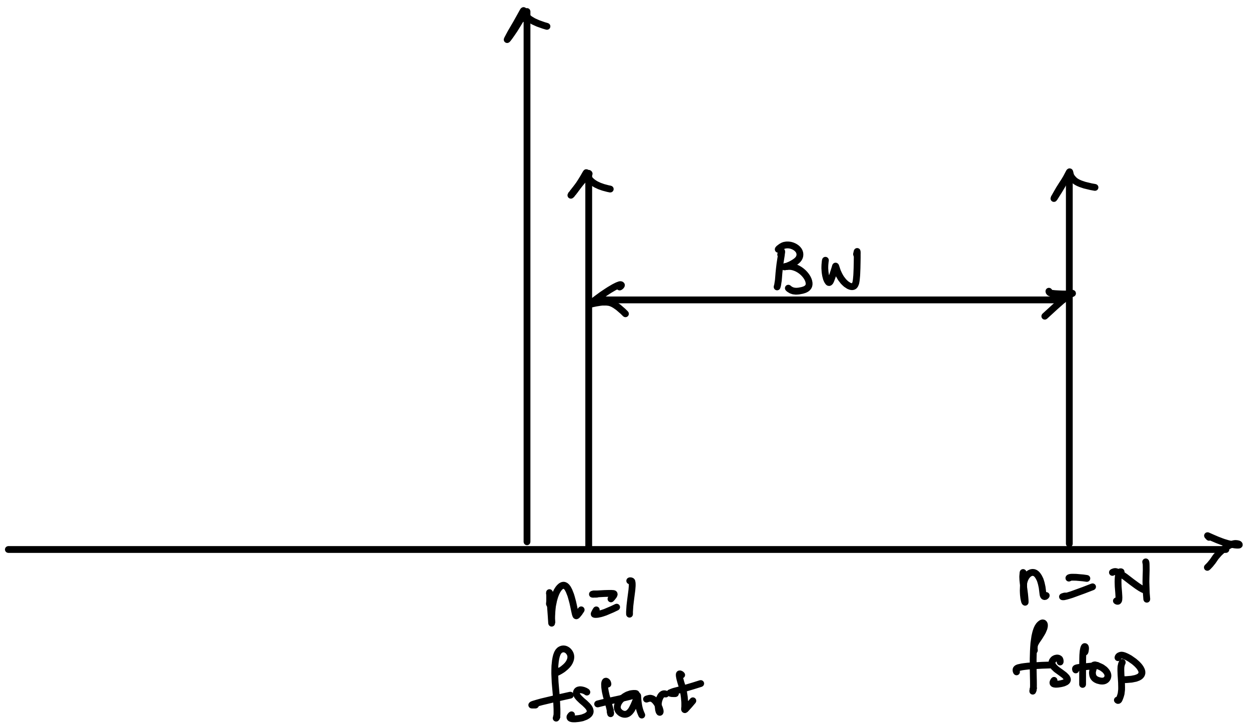
$\beta = 0 \longrightarrow K = 1.21$

$\beta = 3 \longrightarrow K = 1.507$

$\beta = 6.5 \longrightarrow K = 2.023$

$\beta = 13 \longrightarrow K = 2.775$

2) Low Pass Impulse



- DC extrapolation.
- Harmonically sampled frequencies

$$f_{start} = \frac{f_{stop}}{N}$$

- $F(-n\Delta\omega) = F^*(n\Delta\omega)$

$$f_{LP}(t) = \frac{1}{W_0} \frac{\Delta\omega}{2\pi} \left[F(0) + 2 \sum_{n=1}^{N} W(n\Delta\omega) \cdot Re\left\{ F(n\Delta\omega) e^{jn\Delta\omega t} \right\} \right]$$

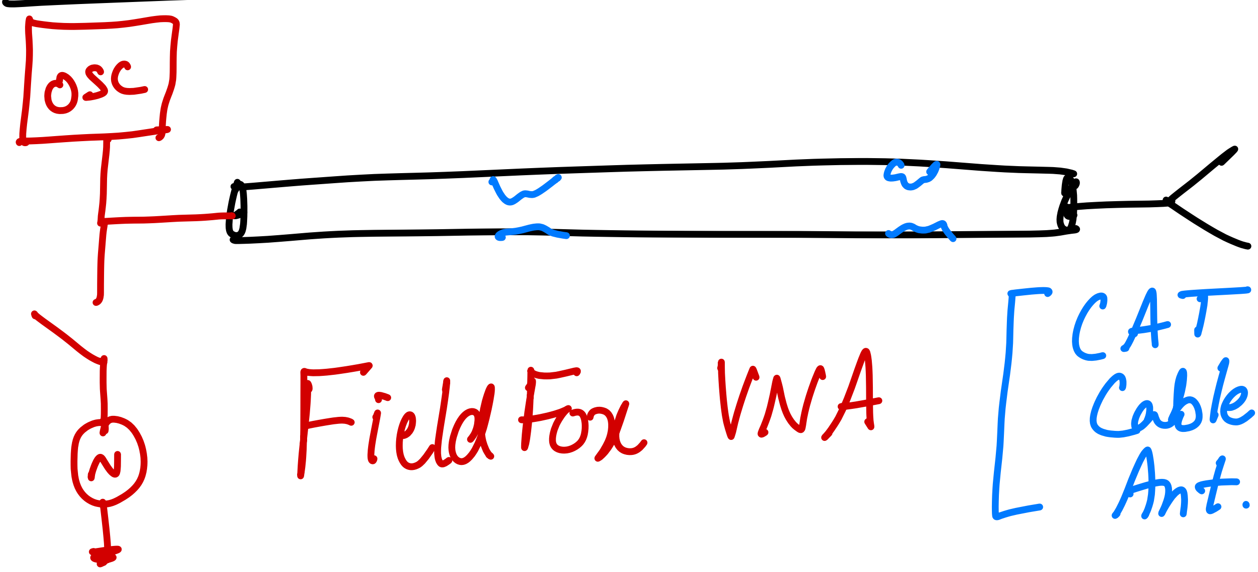
- Sign/~~Phase~~ of the impulse response.
- Imaginary part is zero
- Open/Short ; Cap/Ind.
- $\Delta t_{LP} = \frac{1}{2} \Delta t_{BP}$

3) Low Pass Step Response

$$\underset{step}{s(t)} = f_{LP}(t) * u(t)$$

$$= \int_{-\infty}^{t} f_{LP}(t) .$$

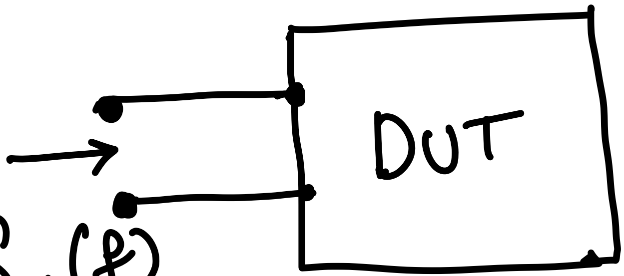
Time Domain Reflectometry



FieldFox VNA

[CAT
Cable &
Ant. Testing]

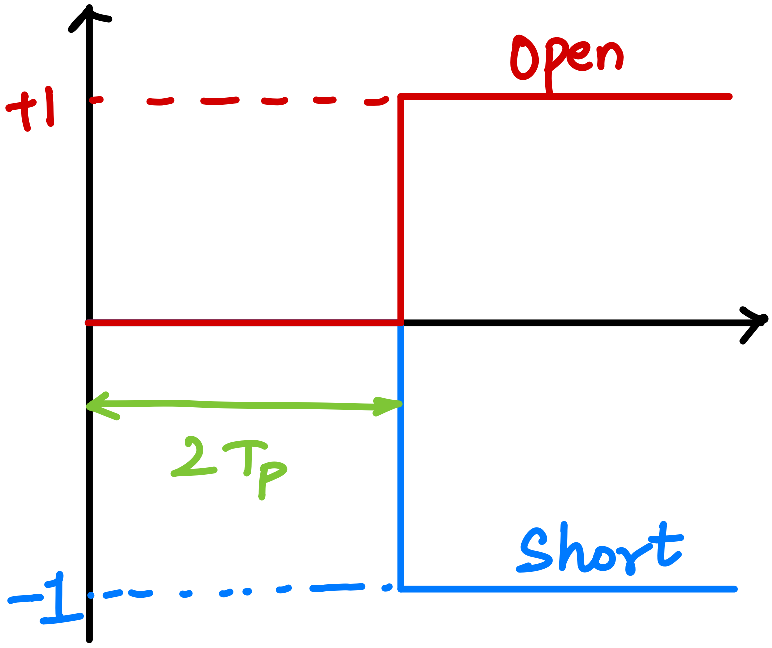
TDR

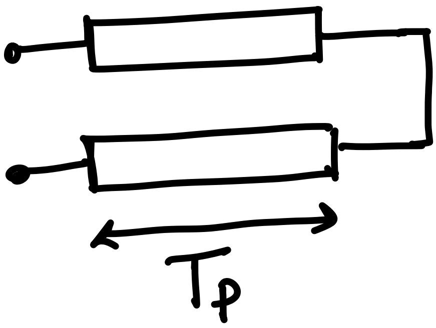


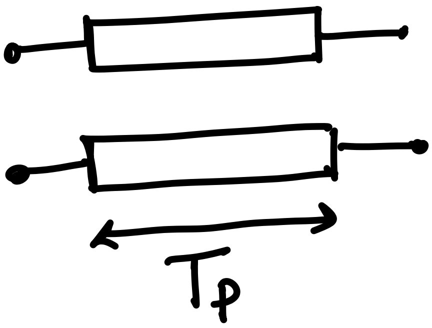
$S_{11}(f)$

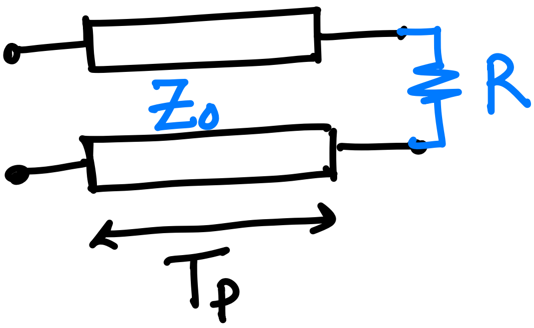
↓

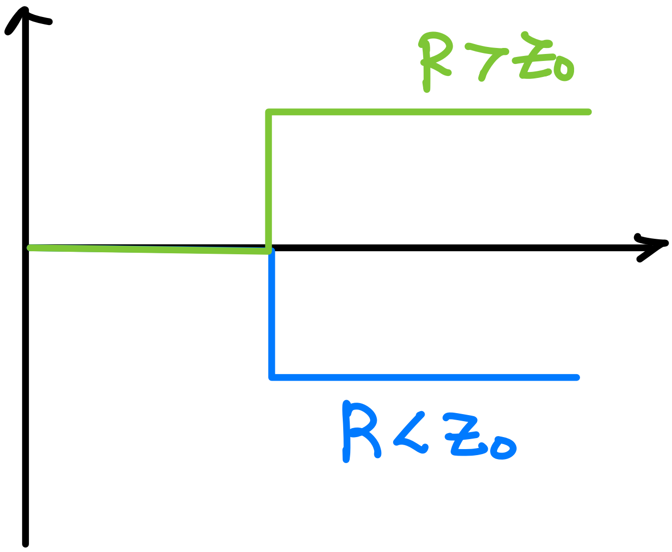
$S_{step}(t)$

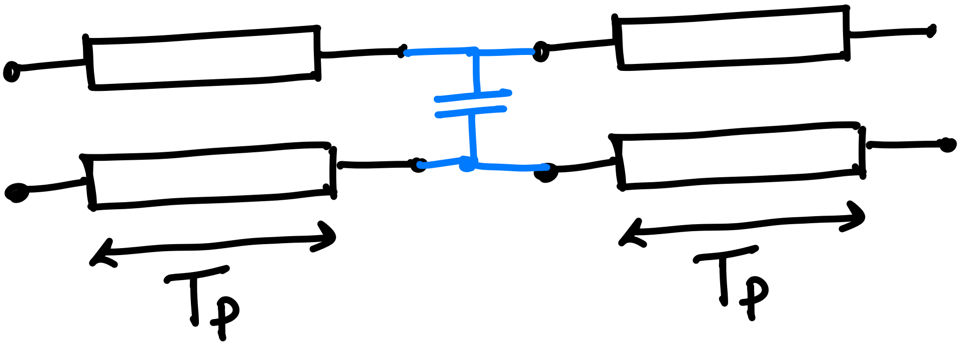


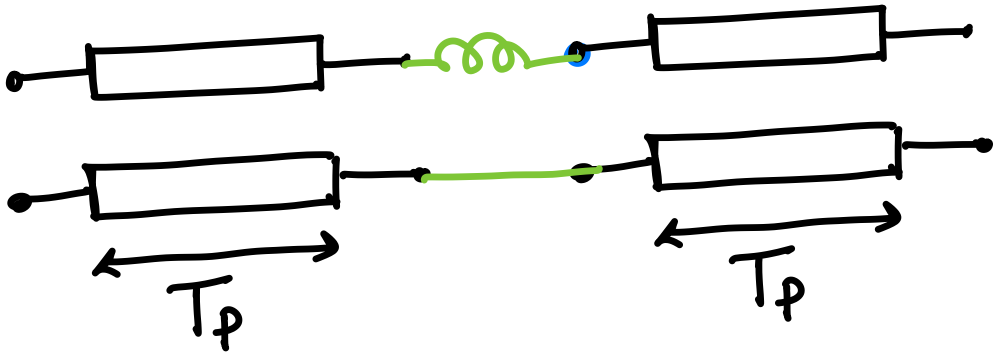


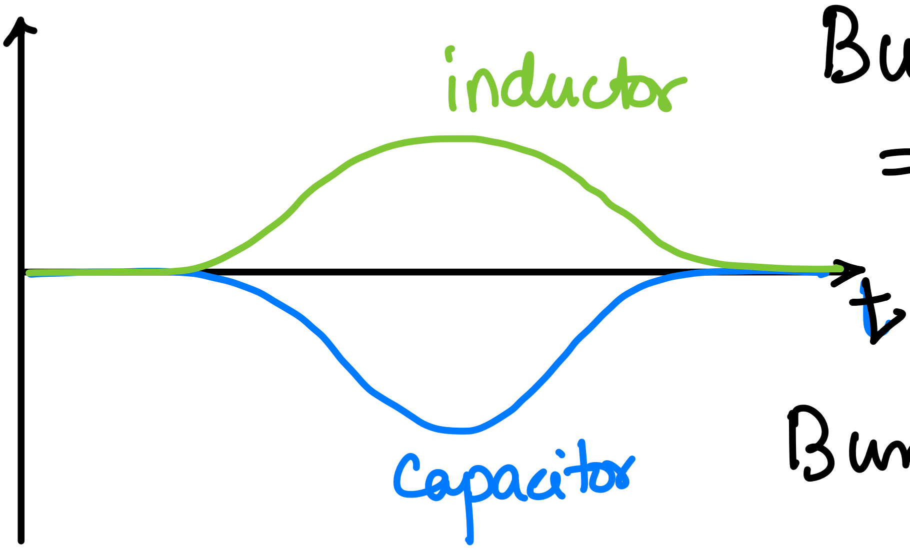






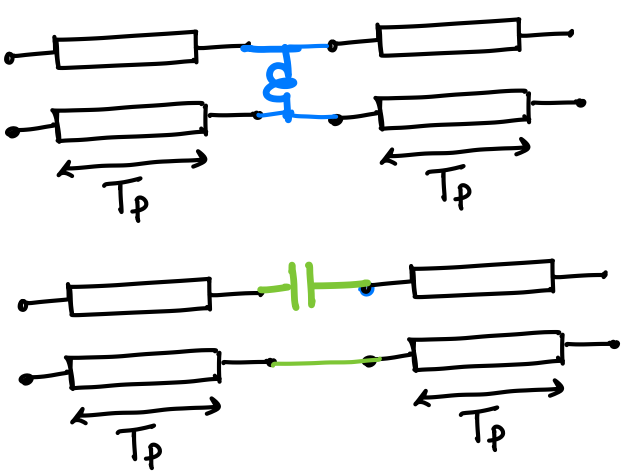


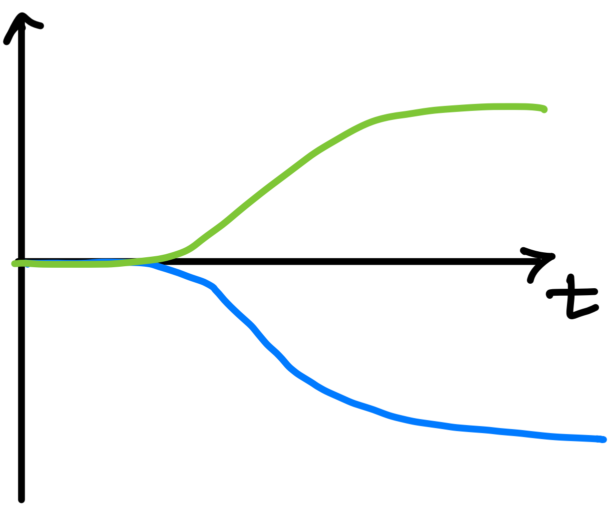


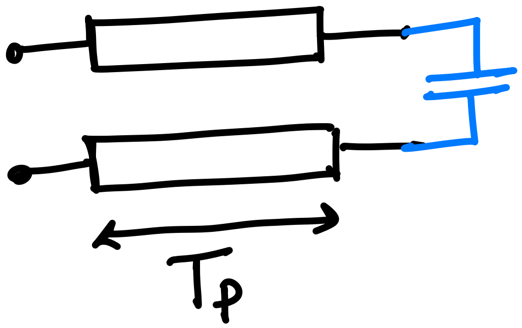


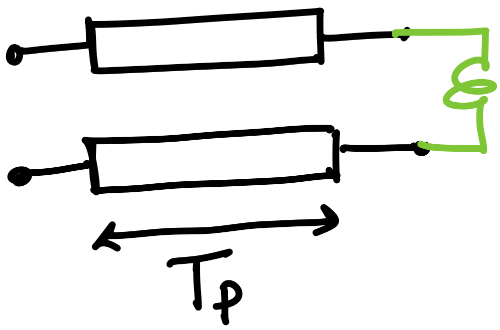
Bump area $= \frac{L}{2Z_0}$

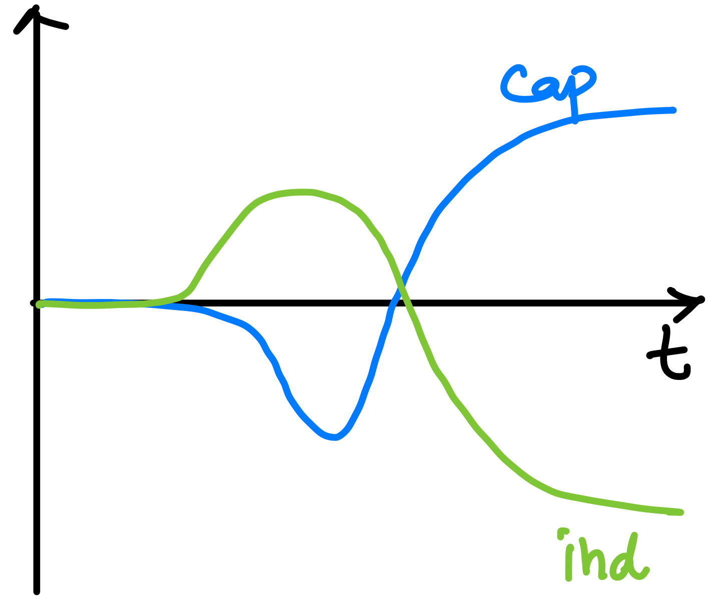
Bump area $= -\frac{Z_0 C}{2}$

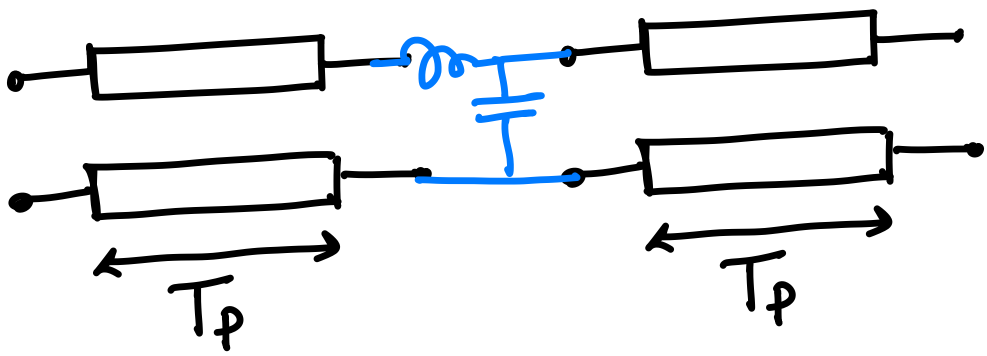


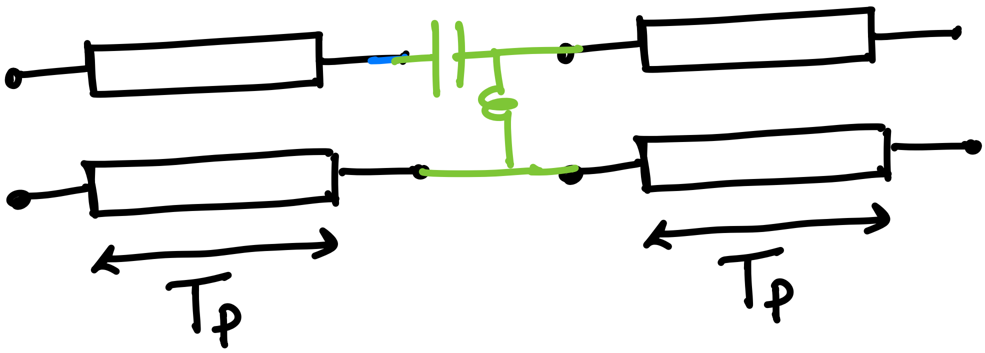


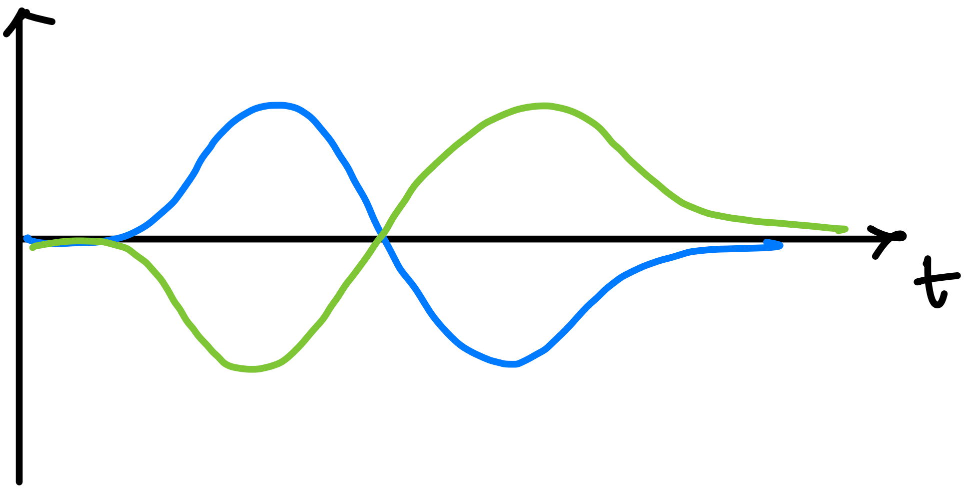

Tp


Tp


cap
ind
t

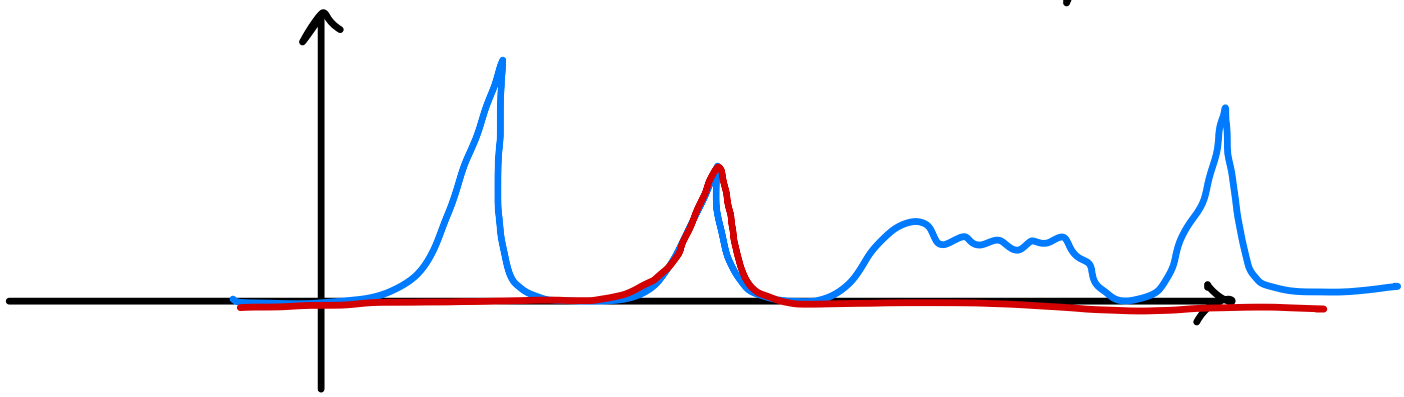

Tp
Tp

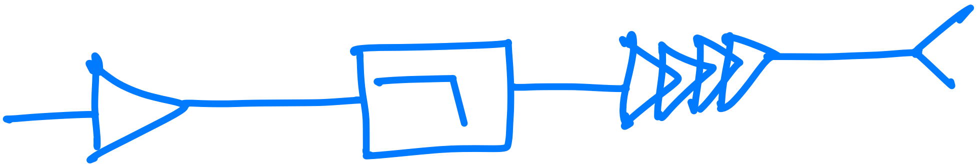

Tp
Tp

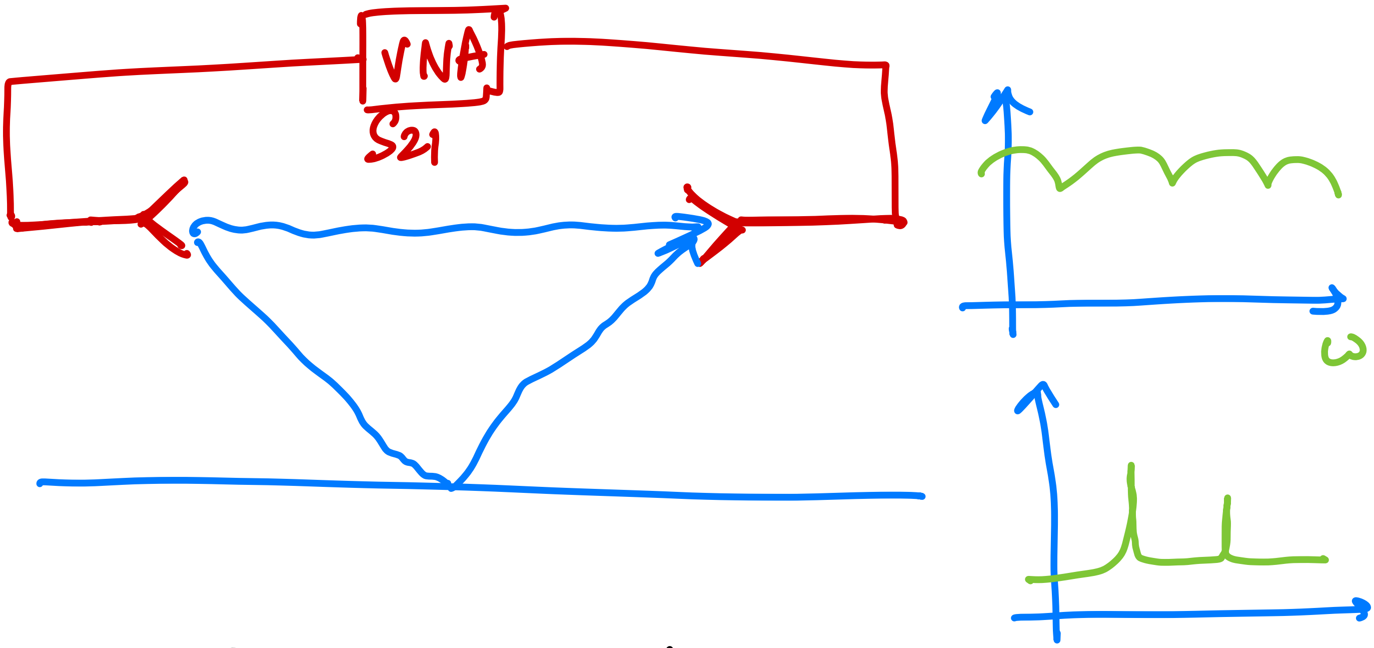

t

Time Gating

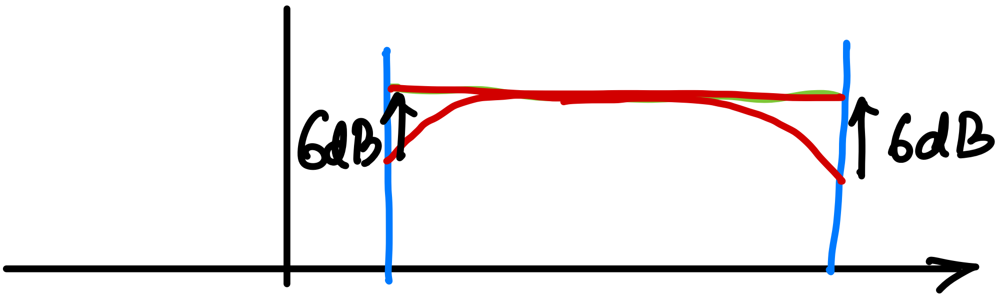
▷ Time Domain Filter ⇐⇒ Convolution in Freq. Domain.

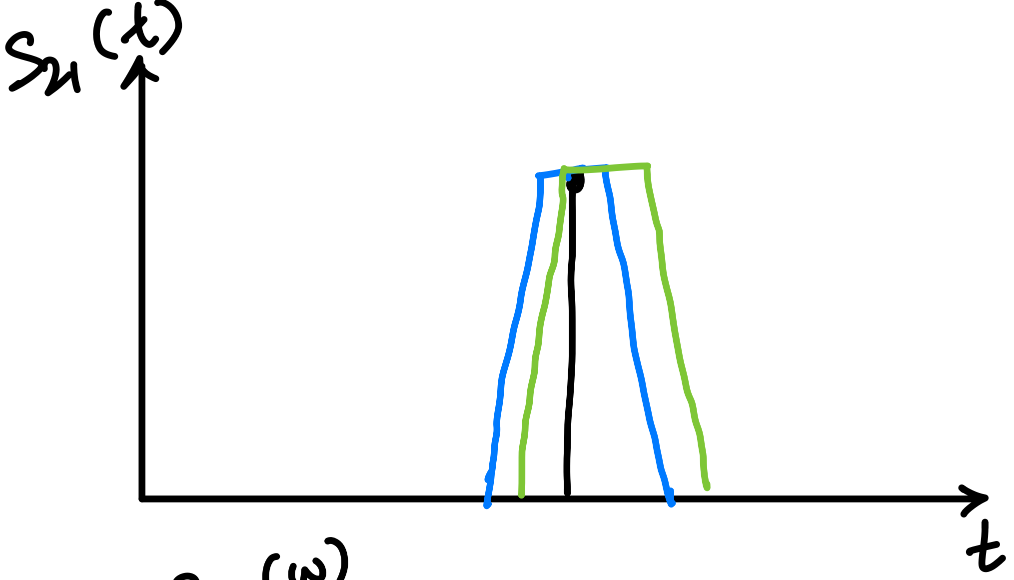


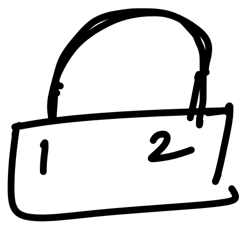


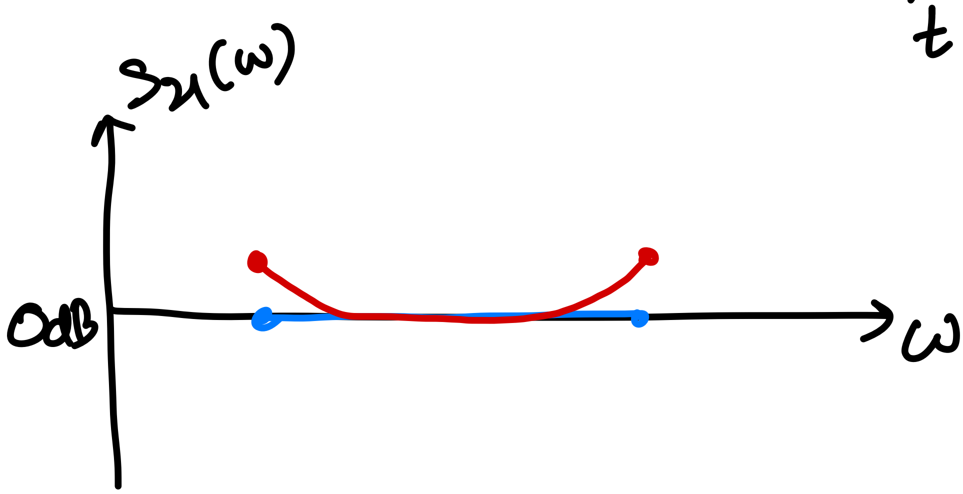


▷ **Gating Renormalization**

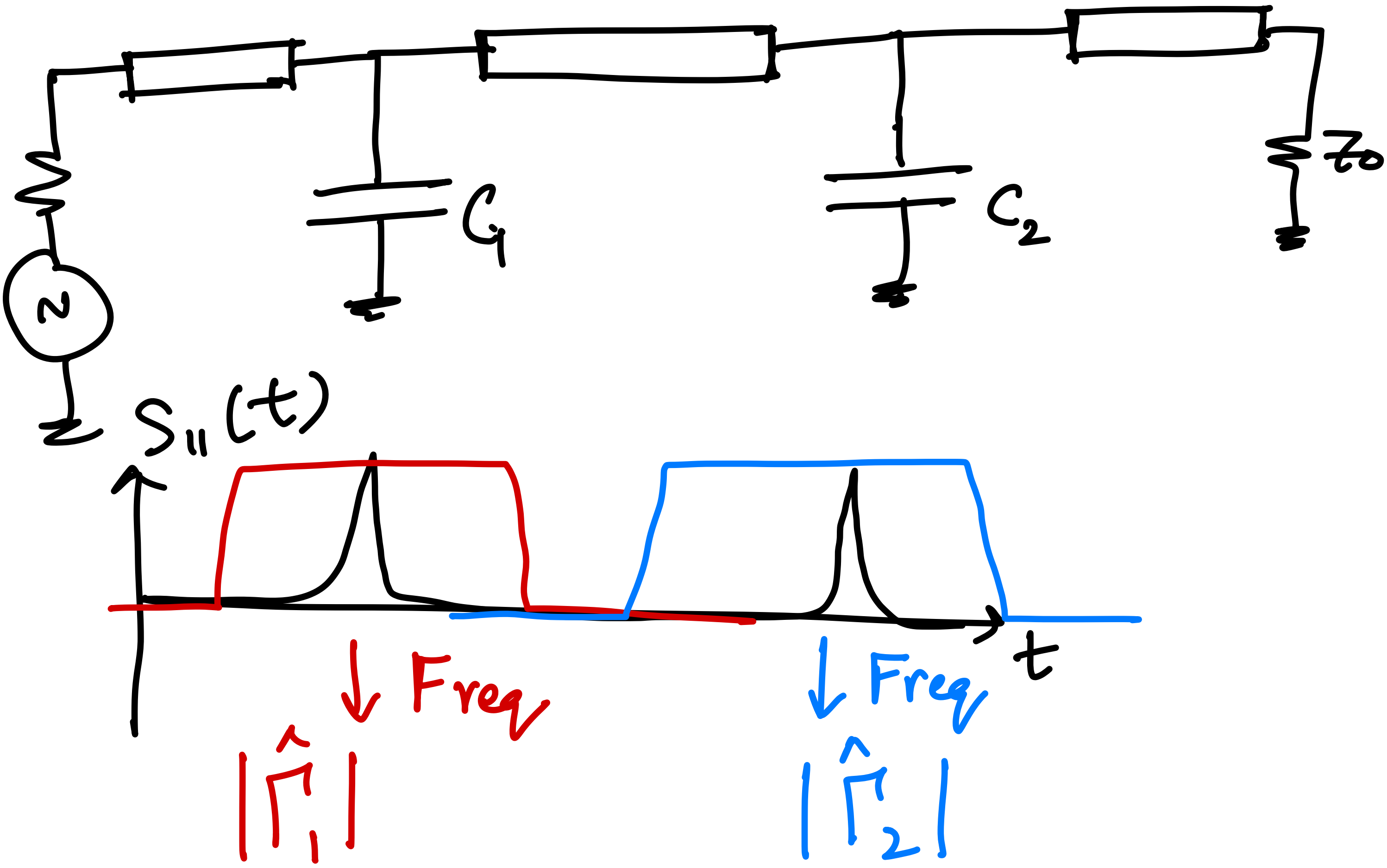



$S_{21}(t)$
t


1
2


$S_{21}(\omega)$
0dB
ω

Masking


C_1
C_2
Z_0
$S_{11}(t)$
t
↓ Freq
↓ Freq
$|\hat{\Gamma}_1|$
$|\hat{\Gamma}_2|$

$$\Gamma_2 = \frac{|\hat{\Gamma}_2|}{1-|\Gamma_1|^2} \simeq \frac{|\hat{\Gamma}_2|}{1-|\hat{\Gamma}_1|^2}$$

- Masking Compensation. (Dunsmore §4.8.2)
- Layer Peeling.