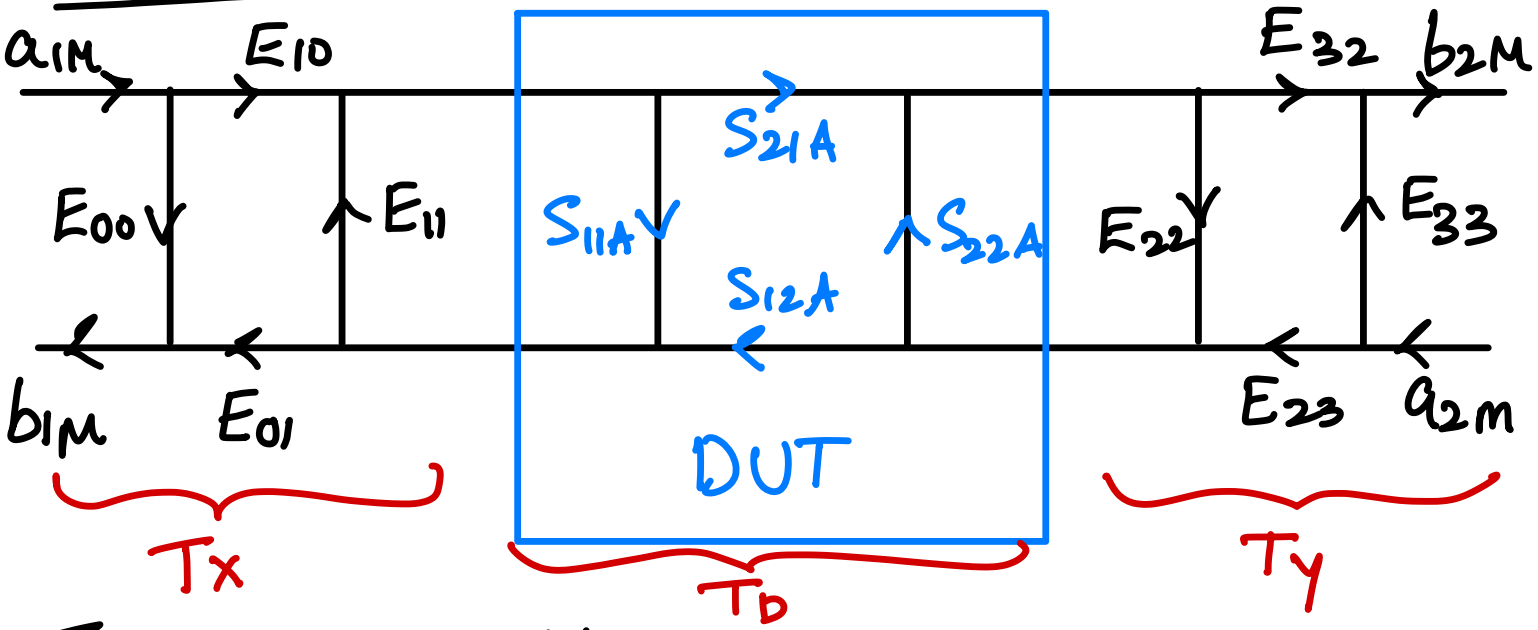
TRL Calibration (Thru-Reflect-Line).

8-Term Error Model



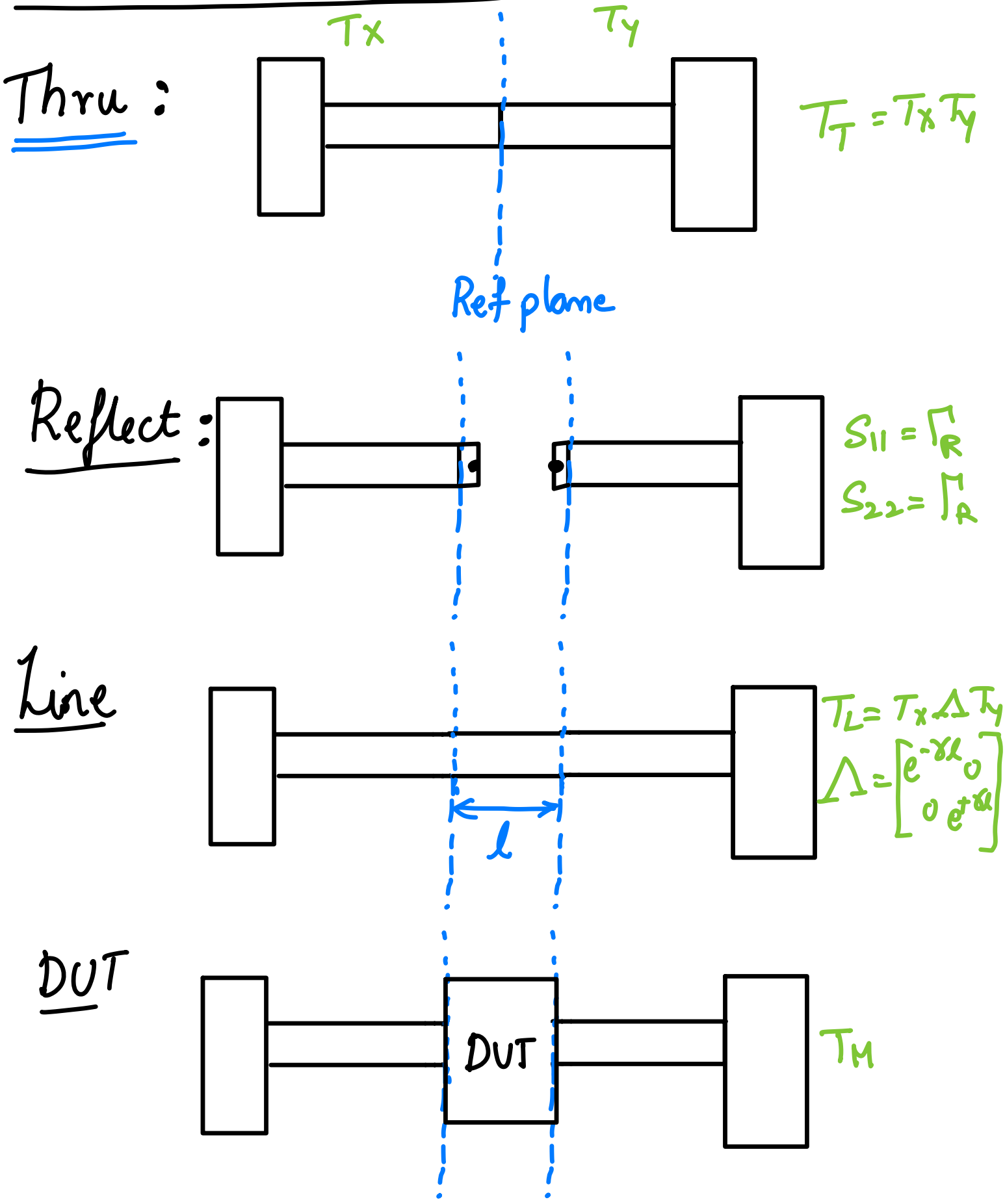
Transmission Matrices

$$T = \frac{1}{S_{21}} \begin{bmatrix} S_{12}S_{21} - S_{11}S_{22} & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

$$S = \frac{1}{T_{22}} \begin{bmatrix} T_{12} & T_{11}T_{22} - T_{12}T_{21} \\ 1 & -T_{21} \end{bmatrix}$$

$$\boxed{T_M = T_X T_D T_Y}$$

TRL Calibration Standards



TRL model

$$M = T_L T_T^{-1} = T_x \Lambda T_y T_y^{-1} T_x^{-1}$$

$$M = T_x \Lambda T_x^{-1}$$

$\Rightarrow$ Eigenvalues of M are

$$\lambda_1 = e^{-\gamma l}$$

$$\lambda_2 = e^{+\gamma l}$$

$$\Rightarrow \boxed{\gamma = \frac{1}{2l} \ln\left(\frac{\lambda_2}{\lambda_1}\right)}$$

(Approx. knowledge of γ is needed to correctly assign λ_1, λ_2)

$$M T_x = T_x \Lambda$$

$$S_x = \begin{bmatrix} E_{00} & E_{01} \\ E_{10} & E_{11} \end{bmatrix}$$

$$\Rightarrow T_x = \frac{1}{E_{10}} \begin{bmatrix} -(E_{00}E_{11} - E_{01}E_{10}) & E_{00} \\ -E_{11} & 1 \end{bmatrix}$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} -\dfrac{(E_{00}E_{11} - E_{01}E_{10})}{E_{10}} & \dfrac{E_{00}}{E_{10}} \\ -\dfrac{E_{11}}{E_{10}} & \dfrac{1}{E_{10}} \end{bmatrix}$$

$$= \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$M_{11} E_{00} + M_{12} = E_{00} \lambda_2$$

$$\Rightarrow \boxed{E_{00} = \frac{M_{12}}{\lambda_2 - M_{11}}}$$

$$\boxed{\frac{E_{01}E_{10}}{E_{11}} = \frac{M_{12}}{\lambda_2 - M_{11}} - \frac{M_{12}}{\lambda_1 - M_{11}}}$$

$$M' = T_T^{-1} T_L$$

$$\boxed{E_{33} = \frac{M_{21}'}{\lambda_2 - M_{22}'}}$$

$$\boxed{\frac{E_{23} E_{32}}{E_{22}} = \frac{M_{21}'}{\lambda_2 - M_{22}'} - \frac{M_{21}'}{\lambda_1 - M_{22}'}}$$

▷ S_{11}, S_{22} of Thru Measurement.

$$S_{11,T}^{m} = E_{00} + \frac{E_{01} E_{10} E_{22}}{1 - E_{11} E_{22}}$$

$$= E_{00} + \frac{\frac{E_{01} E_{10}}{E_{11}}}{\frac{1}{E_{11} E_{22}} - 1}$$

$$\Rightarrow \boxed{E_{11} E_{22} = \frac{S_{11,T}^{m} - E_{00}}{\left(\frac{E_{01} E_{10}}{E_{11}}\right) + S_{11,T}^{m} - E_{00}}}$$

Using Γ_R

$$\Gamma_R = \frac{\Gamma_{R_1}^m - E_{00}}{E_{11}(\Gamma_{R_1}^m - E_{00}) + E_{01}E_{10}}$$

$$= \frac{\Gamma_{R_2}^m - E_{33}}{E_{22}(\Gamma_{R_2}^m - E_{33}) + E_{32}E_{23}}$$

$$\frac{E_{11}}{E_{22}} = \left[\frac{\Gamma_{R_1}^m - E_{00}}{(\Gamma_{R_1}^m - E_{00}) + \frac{E_{01}E_{10}}{E_{11}}}\right]\left[\frac{\Gamma_{R_2}^m - E_{33}}{(\Gamma_{R_2}^m - E_{33}) + \frac{E_{23}E_{32}}{E_{22}}}\right]^{-1}$$

Remove sign ambiguity of E_{11}, E_{22} by checking Γ_R.

→ We now have E_{00}, E_{11}, $E_{01}E_{10}$ & E_{22}, E_{33}, $E_{23}E_{32}$.

We also got Γ_R, γ as a bonus.

Inversion

$$T_X^{-1} = \frac{1}{E_{01}E_{10}^2}\begin{bmatrix} 1 & -E_{00} \\ E_{11} & -(E_{00}E_{11} - E_{01}E_{10}) \end{bmatrix}$$

$$T_Y^{-1} = \frac{1}{E_{23}E_{32}^2}\begin{bmatrix} 1 & -E_{22} \\ E_{33} & -(E_{22}E_{33} - E_{23}E_{32}) \end{bmatrix}$$

$$S_{21,T}^{M} = \frac{E_{10}E_{32}}{1 - E_{11}E_{22}}$$

$$T_D = \frac{1}{E_{01}E_{10}} \cdot \frac{1}{E_{23}E_{32}} \cdot \frac{1}{E_{10}E_{32}} \cdot \begin{bmatrix} 1 & -E_{00} \\ E_{11} & -(E_{00}E_{11} - E_{01}E_{10}) \end{bmatrix} T_M \begin{bmatrix} 1 & -E_{22} \\ E_{33} & -(E_{22}E_{33} - E_{23}E_{32}) \end{bmatrix}$$

Limitations

1. Line impedance
2. BW limited (Thru will be 180° from line)
   [Multiline TRL calibration]

③ Γ is symmetric.