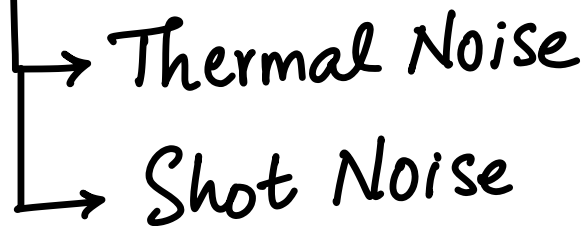




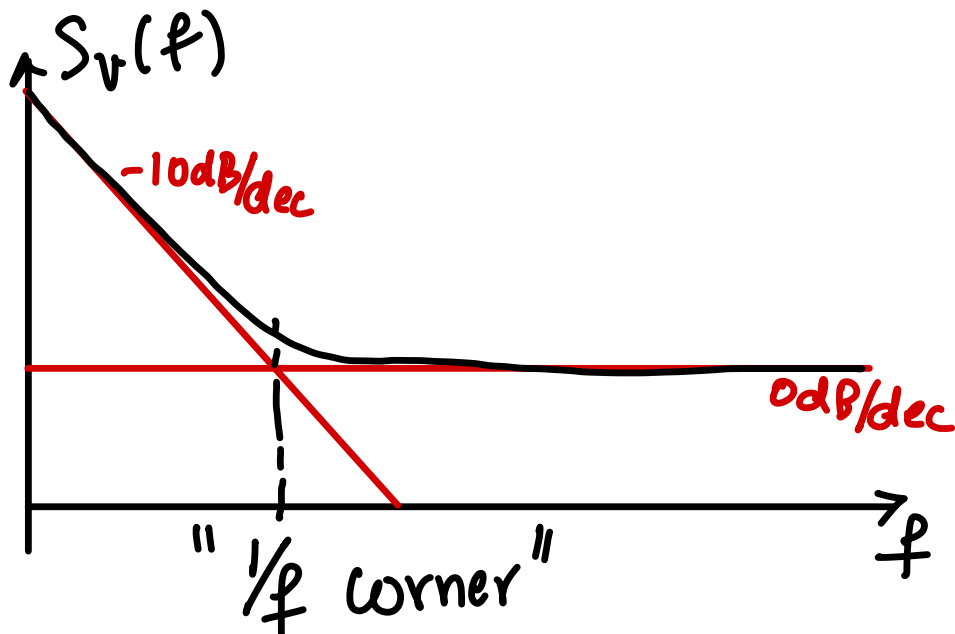
Phase Noise

Noise in Active Devices

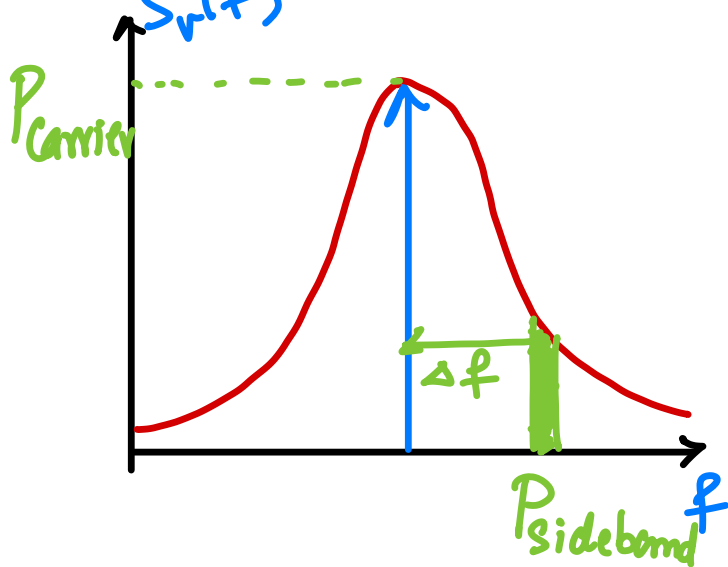
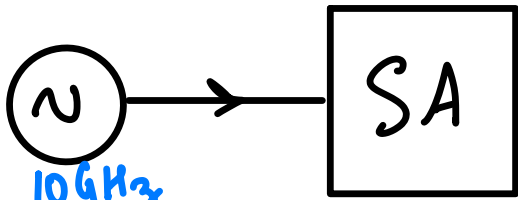
▷ White Noise



▷ $1/f$ Noise



Phase Noise



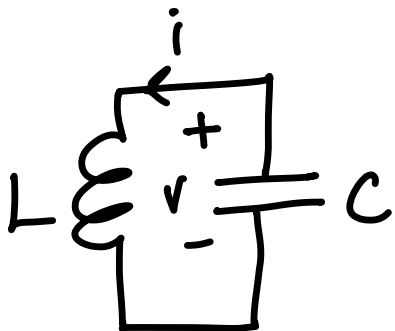
PN:

Single Sideband noise spectral density wrt carrier power.

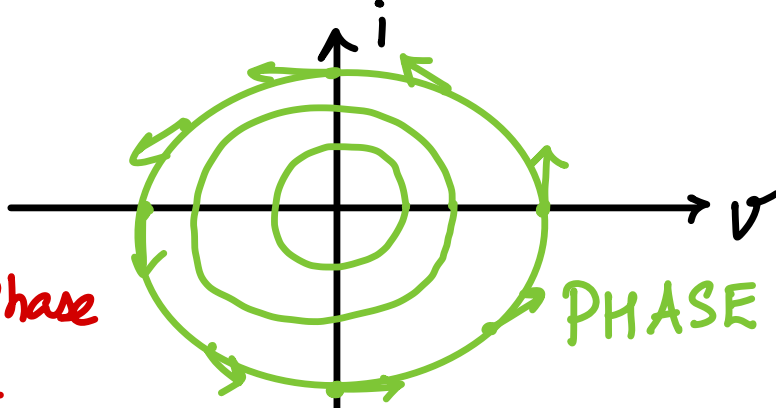
$$L\{\Delta\omega\}_{\text{total}} = 10 \log \left\{ \frac{P_{\text{sideband}}}{P_{\text{carrier}}} \right\} \text{ dBc/Hz}$$

~~Amp Noise~~
Phase Noise

Oscillator Theory



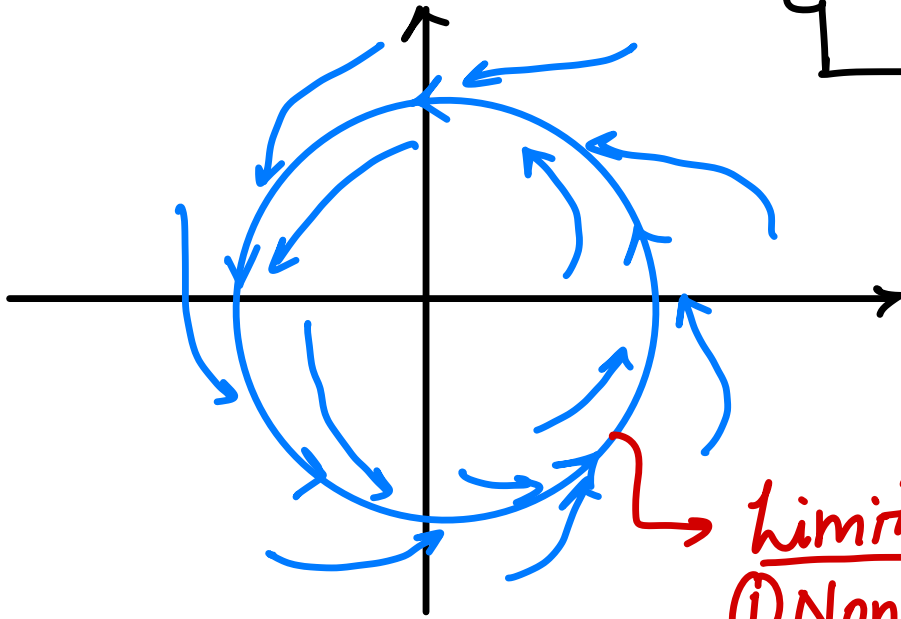
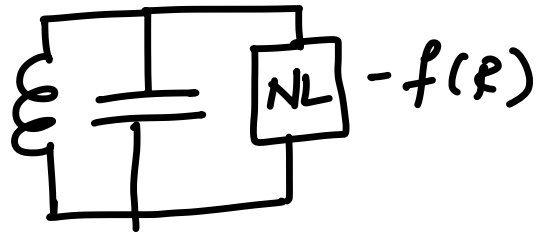
$$\left. \begin{aligned} i &= -C \frac{dv}{dt} \\ v &= L \frac{di}{dt} \end{aligned} \right\} \begin{aligned} \dot{v} &= -\frac{1}{C} i \\ \dot{i} &= \frac{1}{L} v \end{aligned}$$



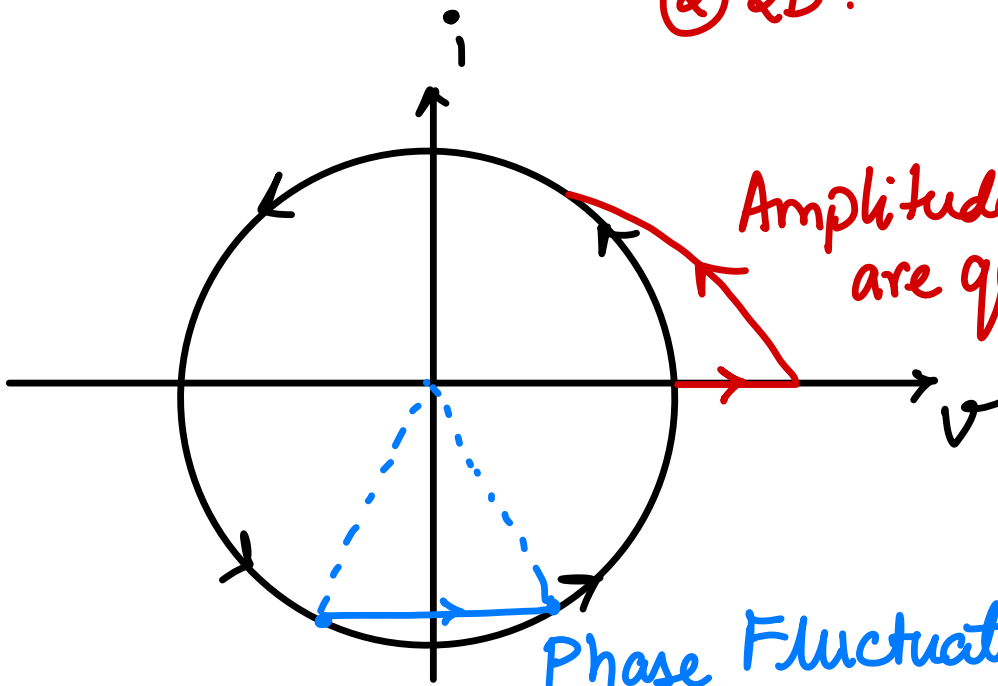
State/phase
Space

PHASE PORTRAIT

Electrical Oscillator



Limit Cycle. "Isolated"
Closed
Traj.
① Nonlinear.
② 2D.

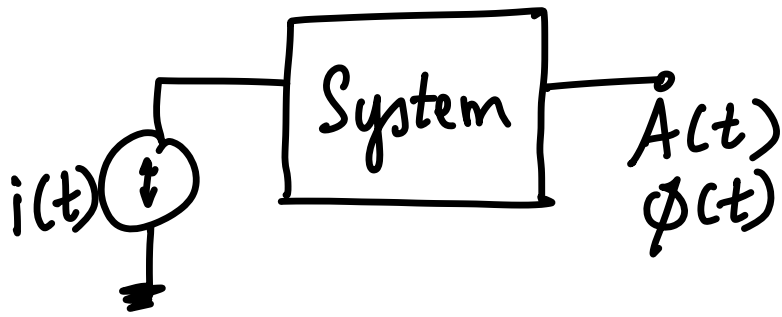


Amplitude Fluctuations
are quickly corrected.

Phase Fluctuation cannot
be corrected.

► Stable Electrical Oscillators are
dominated by Phase Noise.

Phase Noise Model (Impulse Sensitive Function)



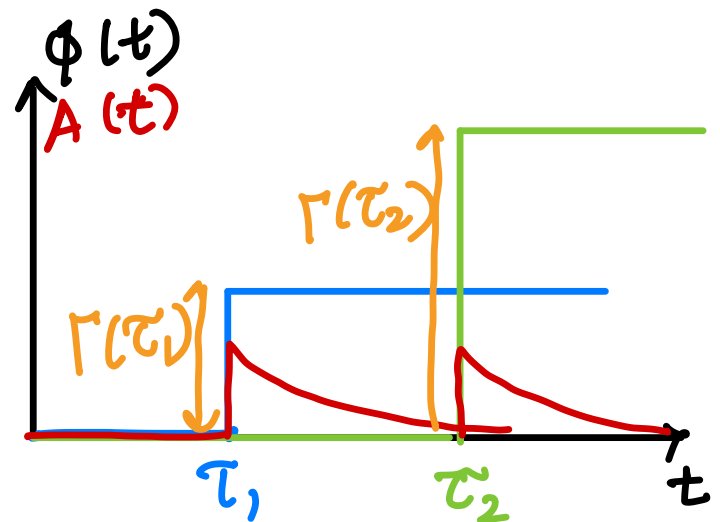
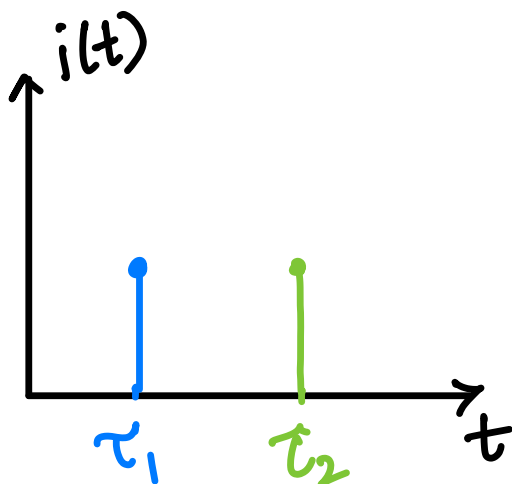
$$V_{out}(t) = A(t) \cos(\omega_0 t + \phi(t))$$

Assumption #1

Assuming $i(t)$ is small, we can model its effect on $A(t)$ & $\phi(t)$ using a linear time varying model.

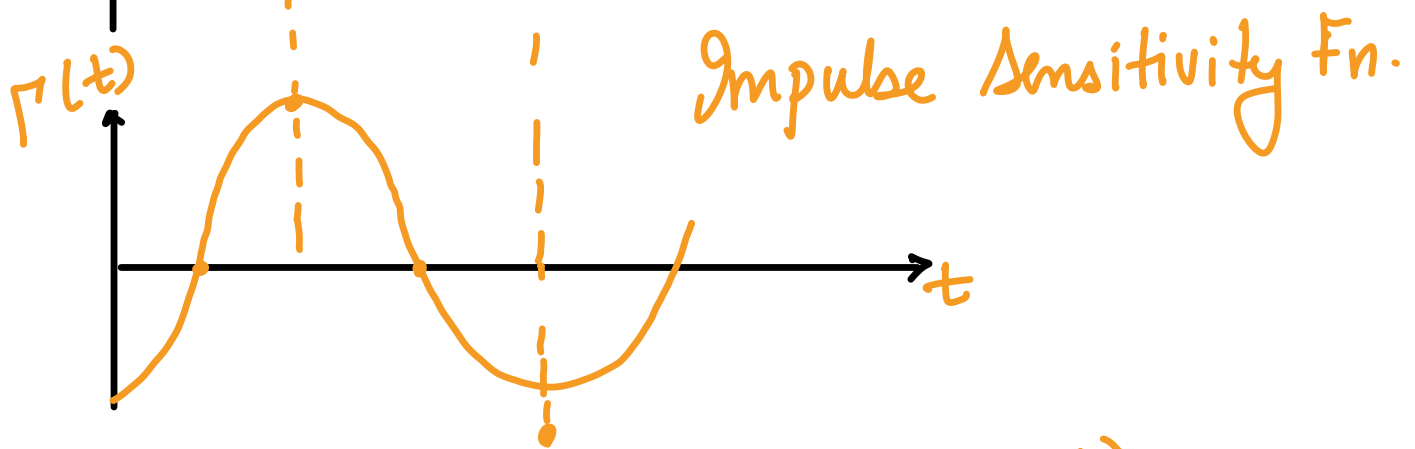
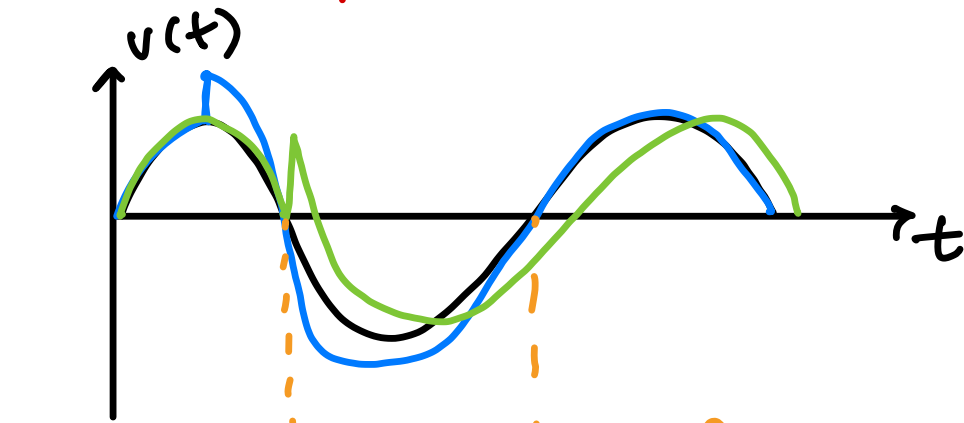
$$i(t) \longrightarrow h_{\phi}(t, \tau) \longrightarrow \phi(t)$$

$$i(t) \longrightarrow h_A(t, \tau) \longrightarrow A(t)$$



Assumption #2

▶ The impulse response is a step function.



Impulse Sensitivity Fn.

$$h_{\phi}(t, \tau) = \frac{\Gamma(\omega_0 \tau)}{q_{\max}} u(t - \tau)$$

$q_{\max} \rightarrow$ max. charge displaced on the node.

Excess Phase

$$\phi(t) = \int_{-\infty}^t h_{\phi}(t, \tau) i(\tau) d\tau$$

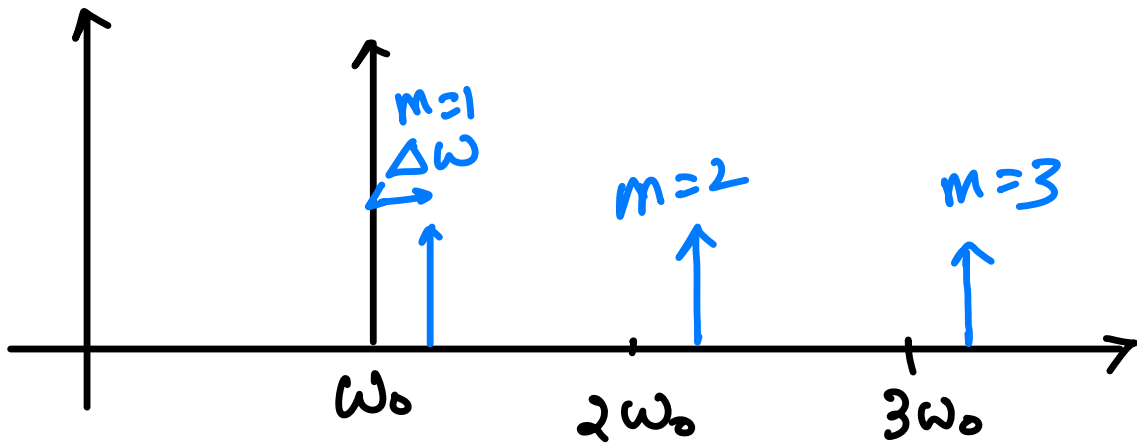
$$= \frac{1}{q_{\max}} \int_{-\infty}^t \Gamma(\omega_0 \tau) i(\tau) d\tau$$

$$\Gamma(\omega_0 \tau) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 \tau + \theta_n)$$

$$\phi(t) = \frac{1}{q_{\max}} \left[\frac{C_0}{2} \int_{-\infty}^t i(\tau) d\tau \right.$$

$$\left. + \sum_{n=1}^{\infty} C_n \int_{-\infty}^t i(\tau) \cos(n\omega_0 \tau) d\tau \right]$$

Assume $i(t) = I_m \cos[(m\omega_0 + \Delta\omega)t]$

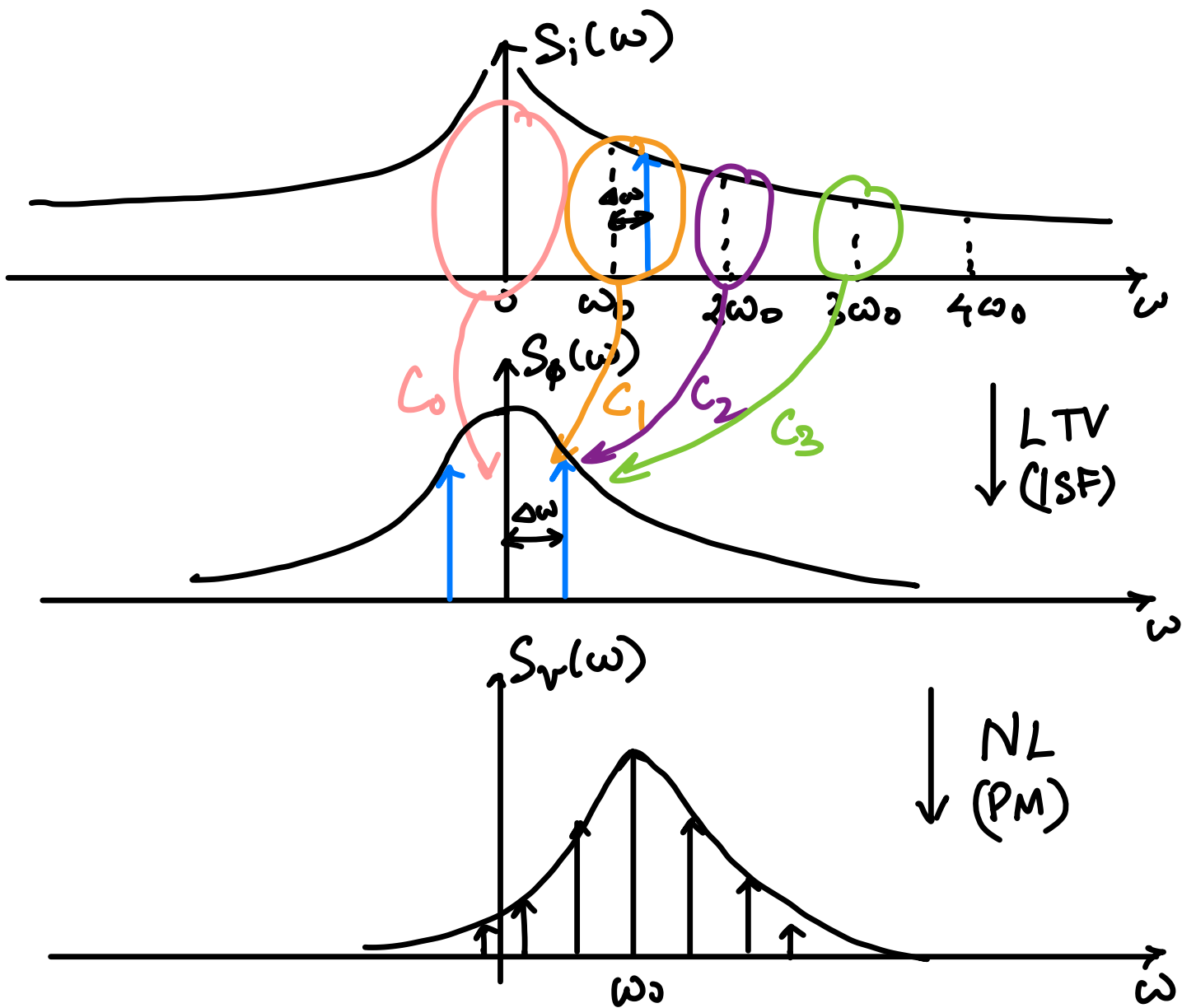
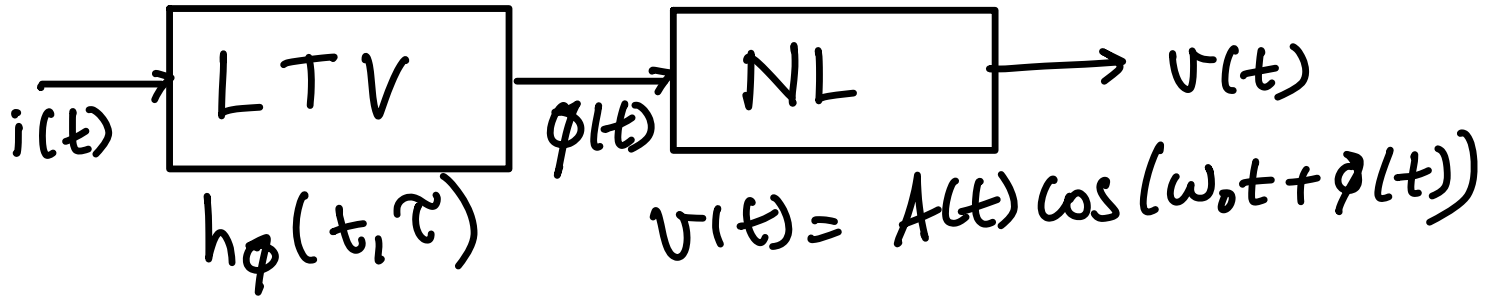


$$\phi(t) = \frac{1}{q_{\max}} \left[\frac{C_0}{2} \int_{-\infty}^t I_m \cos((m\omega_0 + \Delta\omega)\tau) d\tau \right.$$

$$\left. + \sum_{n=1}^{\infty} C_n \int_{-\infty}^t I_m \cos[(m\omega_0 + \Delta\omega)\tau] \cos(n\omega_0 \tau) d\tau \right].$$

Assume $\Delta\omega \ll \omega_0$

$$\phi(t) \cong \frac{I_m C_m}{2 q_{\max} \Delta \omega} \sin(\Delta \omega t)$$



$$P_{SBC}(\Delta\omega) \approx 10 \log \left[\frac{I_n C_n}{4q_{max} \Delta\omega} \right]^2$$

If $i(t)$ is noise with PSD $\frac{\overline{i(t)^2}}{\Delta f}$

$$\frac{I_n^2}{2} \longleftrightarrow \frac{\overline{i_n^2}}{\Delta f} \text{ where } \Delta f = 1 \text{ Hz}$$

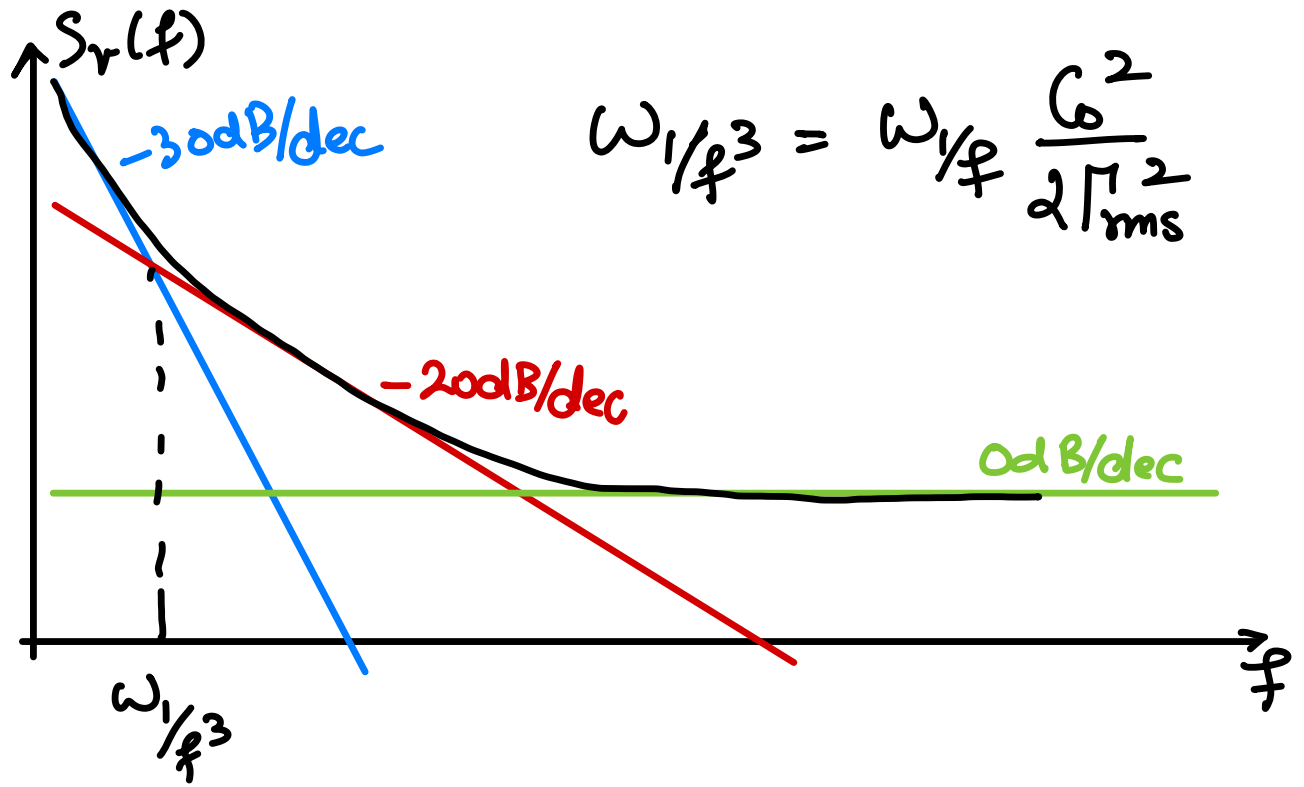
$$\Rightarrow L\{\Delta\omega\} = 10 \log \left[\frac{\frac{\overline{i_n^2}}{\Delta f} \sum_{n=0}^{\infty} C_n^2}{8 q_{max}^2 \Delta\omega^2} \right]$$

$$\sum_{n=0}^{\infty} C_n^2 = \frac{1}{\pi} \int_0^{2\pi} |f(x)|^2 dx = 2 \Gamma_{rms}^2$$

$$\Rightarrow L\{\Delta\omega\} = 10 \log \left[\frac{\Gamma_{rms}^2}{q_{max}^2} \cdot \frac{\overline{i_n^2}/\Delta f}{4 \Delta\omega^2} \right]$$

$$\text{Flicker: } \frac{\overline{i_n^2}}{\Delta f} \cdot \frac{1}{f} = \frac{\overline{i_n^2}}{\Delta f} \cdot \frac{\omega_{1/f}}{\Delta\omega}$$

$$L\{\Delta\omega\} = 10 \log \left[\frac{C_0^2}{q_{max}^2} \cdot \frac{\overline{i_n^2}/\Delta f \cdot \omega_{1/f}}{8 \Delta\omega^3} \right]$$



▷ Non linear Model

Perturbation Theory + Floquet Analysis.

▶ Perturbation Projection Vector (PPV).