

EE 117B/217B Microwave and Wireless Measurements

Lab 5 - Communications Measurements

Fall 2026

CAUTION: READ THIS PAGE CAREFULLY BEFORE STARTING THE LAB

This lab uses the FieldFox Microwave Analyzer N9918B in its **Spectrum Analyzer (SA)** mode, including its analog demodulation and **IQ Analysis** (IQ streaming) features, together with a vector signal generator (VSG), an oscilloscope, and a PC workstation connected to the FieldFox over Ethernet. Please handle the instruments very carefully as they are very expensive and delicate. The most vulnerable parts of the instruments are the RF PORTS (APC-3.5 connectors). These connectors are protected by the small Female-Female Port Savers. **DO NOT REMOVE THE PORT SAVERS FROM THE FIELDFOX PORTS FOR ANY EXPERIMENT.**

All connections are to be made through these Port Saver Connectors. If you have never handled RF instrumentation before, ask the instructor to demonstrate how to make connections. Always hold the cable with one hand and rotate the end nuts of the SMA cables with the other. Once you make a soft connection by hand, use the torque wrench provided to complete the connection. Only rotate the end nuts, never the components.

Caution for this particular lab

In this lab the VSG output is connected directly to the RF INPUT port (Port 2) of the FieldFox and to the oscilloscope. NEVER APPLY MORE THAN +20 dBm OR ANY DC VOLTAGE TO THE RF INPUT PORT OF THE FIELDFOX.

When using the vector signal generator: Set the output power to ≤ 0 dBm unless instructed otherwise. Ensure the RF output of the VSG is turned off (RF On/Off) before changing cable connections.

When using the antenna: Do not touch the antenna wire while it is connected to the FieldFox. A static discharge from your hand into the antenna goes directly into the RF input port.

Reference clock and Ethernet connections: The SMB connector of the reference clock cable is small and delicate; push it on and pull it off straight using the fixed wrench, never at an angle. Do not disconnect or remove the Ethernet-USB adapter from the PC.

Introduction

In this lab you will make measurements that connect analog and digital modulation theory to real signals observed on microwave instruments. The lab is divided into two parts: (1) analog modulation, where you measure the modulation index of AM and FM signals and listen to live FM broadcasts, and (2) digital modulation, where you capture the raw IQ samples of a QAM signal from the FieldFox and process them into constellation, vector and eye diagrams to measure the signal quality.

In the first part of the lab you will generate AM and FM signals with a vector signal generator (VSG) and observe their spectra on the FieldFox in Spectrum Analyzer (SA) mode. For AM, you will measure the modulation index from the sideband-to-carrier ratio in the frequency domain and from the envelope on an oscilloscope in the time domain, and compare both with the modulation depth set on the instrument. For FM, you will observe the Bessel sideband structure, measure the frequency deviation using the carrier null and first sideband null (Bessel null) methods, and compare the occupied bandwidth with Carson's rule. You will then connect an antenna to the FieldFox and use its analog demodulation feature to tune and listen to live FM radio stations. The AM and FM theory is collected in Appendices A and B.

In the second part of the lab you will generate a 16-QAM signal at 125 ksps with root-raised-cosine (RRC) pulse shaping, and stream its baseband IQ samples from the FieldFox to a PC over Ethernet. You will see why the transmitter and receiver must share a reference clock, and observe the residual frequency offset that remains even when they do. You will then save one second of IQ data and process it — frequency offset removal, RRC matched filtering, symbol timing recovery and sampling at the symbol centers — to construct the constellation, vector and eye diagrams, and to measure the EVM and eye height and estimate the bit error rate (BER). Finally, you will repeat these measurements for 64-QAM and 256-QAM at high and low signal power levels. The processing steps and the signal quality metrics are described in Appendices C and D.

Objectives

At the end of this lab you will:

1. Measure the AM modulation index from both the frequency-domain sideband-to-carrier ratio and the time-domain envelope, and understand the modulation depth definition used by the signal generator.
2. Observe the Bessel sideband structure of an FM signal and measure the FM deviation using the carrier null and first sideband null (Bessel null) methods.
3. Measure the occupied bandwidth of an FM signal and compare it with Carson's rule.
4. Tune and listen to live FM broadcast stations using the FieldFox analog demodulation feature.
5. Stream baseband IQ data from the FieldFox to a PC over Ethernet, and understand the role of reference clock synchronization and residual frequency offset in coherent demodulation.
6. Process raw IQ samples (frequency offset removal, RRC matched filtering, symbol timing recovery and symbol-center sampling) to construct constellation, vector and eye diagrams.
7. Measure the EVM and eye height, and estimate the BER of 16-QAM, 64-QAM and 256-QAM signals at different signal power levels.

Since students are performing the lab together, please ensure that you **take turns performing the experiments.**

Equipment List

The equipment used in this lab is listed in Table 1.

Lab Report

Each lab comes with a Lab Questionnaire. As you go through this Lab Manual, you will encounter several questions along the way. When you do, go over to that question in the Lab Questionnaire and answer it accordingly. All students in one lab group will submit one combined lab report. **It is recommended that you complete the post-processing parts of the lab also during the designated lab hours so you can work together with your team.**

Equipment	Part Number	Qty
Keysight FieldFox Microwave Analyzer (SA / IQ Analyzer)	N9918B	1
Vector signal generator (AM/FM and real-time I/Q baseband)		1
Digitizing oscilloscope	Keysight DSOX2012A (or equiv.)	1
SMA coaxial cable (VSG to FieldFox)	NA	1
BNC-to-SMA cable / adapter (VSG to oscilloscope)	NA	1
Short wire antenna (~ 75 cm, for FM reception)	NA	1
BNC-to-SMB cable (reference clock, FieldFox Ref/Trig Out to VSG REF IN)	NA	1
Ethernet cable and Ethernet-USB adapter	NA	1 each
PC workstation with Python and the EE117B217B scripts	NA	1
Torque wrench	(shared)	1

Table 1: Equipment used in the Communications Measurements Lab 5 experiments.

1 AM and FM Modulation and Modulation Index Measurement

Measure the modulation index of AM and FM signals using both the time domain (oscilloscope) and the frequency domain (spectrum analyzer), connecting the lecture math to observed spectra. The AM and FM theory needed for this section is collected in Appendices A and B.

1.1 AM Modulation

1. Connect the RF output of the Vector Signal Generator (VSG) to Port 2 of the FieldFox using an SMA cable. Set the FieldFox in SA Mode.
2. Set the signal generator carrier to 100 MHz at -3 dBm. Set the SA center frequency to 100 MHz, span to 1 MHz, and RBW to 1 kHz.
3. On the VSG select:

AM $\rightarrow$ AM On $\rightarrow$ AM Depth $\rightarrow$ 10%

Set AM Rate to 100 kHz. Click the Mod On/Off button to turn on the modulation.

4. Using markers, measure the carrier amplitude A_c (dBm) and either sideband amplitude A_s (dBm). Convert to linear voltage and compute the modulation index. Note that the instrument uses the modulation depth definition $m = (V_{\max} - V_{\min}) / (\frac{1}{2}(V_{\max} + V_{\min}))$; see Appendix A for the relation between this definition and the sideband-to-carrier ratio. Record the measured modulation index m and compare to the set value of 10% (**Q1.1.1**).
5. Plot the measured modulation index vs. the modulation index set on the instrument for index values 5%, 10%, 15%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90% and 100%. At what point does the agreement start to get worse? Why (**Q1.1.2**)?
6. Connect the signal generator output directly to the oscilloscope. Set the oscilloscope to capture the AM envelope. Measure $V_{\max}$ and $V_{\min}$ of the envelope and compute $m = 2(V_{\max} - V_{\min}) / (V_{\max} + V_{\min})$. Does the time-domain value agree with the frequency-domain value (**Q1.1.3**)? Repeat the measurement using the oscilloscope for modulation index values 20%, 40%, 60%, 80%, and 100%. Do these results agree with what you measured on the spectrum analyzer (**Q1.1.4**)?

Reminder: Take turns in doing the experiments.

1.2 FM Modulation and the Bessel Null Method

1. Preset the vector signal generator and the FieldFox. Set the carrier to 100 MHz at -10 dBm. Press FM/ Φ M, turn on FM modulation and set the frequency deviation to $\Delta f = 25$ kHz. Set the FM Rate f_m to 13 kHz. Turn Mod On and RF On.
2. On the FieldFox, set mode to SA, set center frequency to 100 MHz, span to 200 kHz. You should see the Bessel sideband structure.
3. Adjust f_m slowly (by lowering the FM rate). To find the FM deviation you need to null the carrier tone. Note that as you lower the FM rate, the carrier amplitude goes down. The carrier should null (drop to the noise floor) when $\beta = 2.401$. Explain why (**Q1.2.1**). Record

the actual f_m at the deepest carrier null. Compute the actual FM deviation: $\Delta f = 2.401 \times f_m$. How does it compare to the set value (**Q1.2.2**)?

4. Save and plot the spectrum at the first carrier null. Count the significant sidebands and compare to theory: for $\beta = 2.401$, $J_1(\beta) = 0.52$, $J_2(\beta) = 0.43$, $J_3(\beta) = 0.20$, $J_4(\beta) = 0.06$. Do the measured sideband amplitudes agree (**Q1.2.3**)?
5. Increase Δf (decrease f_m) to reach $\beta = 3.8317$ (first sideband null). Explain why the first sidebands get nulled at this value of β . Calculate once again the FM deviation from this. Compare this with what you just obtained from the first carrier null and what was set on the instrument (**Q1.2.4**).
6. Increase Δf (decrease f_m) to reach $\beta = 5.520$ (second carrier null). How many sidebands are now visible? Measure the occupied bandwidth (the span containing all sidebands above the noise floor). Compare to Carson's rule: $BW = 2(\Delta f + f_m)$ (**Q1.2.5**). Note here the BW corresponds to all peaks within 10 dB of carrier at full power.
7. Turn off the VSG Output.

Reminder: Take turns in doing the experiments.

1.3 FM Radio Demodulation

1. Connect the wire antenna to Port 2 of the FieldFox. Preset the Mode.
2. Set the center frequency to 100 MHz, span to 20 MHz. Then select:

Measure $\rightarrow$ Analog Demod Tune & Listen $\rightarrow$ FM (Wide) Tune and Listen

Turn on Audio Demod, set the volume and tune the frequency to listen to different radio bands.

Reminder: Take turns in doing the experiments.

2 IQ Demodulation and EVM Measurement

2.1 Connecting to the FieldFox over LAN

Skip this sub-section if the Ethernet cable is already connected. To extract data from the FieldFox, we will connect to it via an Ethernet cable.

1. Use the provided Ethernet cable and connect one port to the FieldFox Ethernet port (right panel). Connect the other end to the Ethernet-USB adapter on the PC. **Note: Do not disconnect or remove this Ethernet-USB adapter from the PC.**
2. On the FieldFox select:

System → System Configuration → More → LAN

Change Obtain IP to Static, change Static IP Address to 192.168.1.1 and change Apply Settings to Now.

3. On the PC open a terminal and ping the FieldFox:

```
ping 192.168.1.1
```

If you see a reply, the connection is established.

2.2 IQ Demodulation Setup

1. Turn on the FieldFox and select SA Mode. Set Center frequency to 1 GHz and Span to 1 MHz.
2. Turn on the Vector Signal Generator. Set frequency to 1 GHz and Amplitude to -10 dBm. Select:

Mode → Custom → Real Time I/Q Baseband

Turn Custom On. Then select:

Modulation Type → Select → QAM → 16 QAM

3. Return and select Filter. Note the type of filter (Root Nyquist) and note the Filter Alpha value (0.35). You will need these values during demodulation.
4. Return and select the Symbol Rate. Set it to 125 kps. Return to the main menu. Connect the VSG to Port 2 of the FieldFox. Turn on Modulation (Mod On/Off) and turn on RF output (RF On/Off).
5. On the FieldFox, select:

Meas Setup → IQ Analysis → Streaming

IQE Sampling Rate → 1.25 MHz

Based on these settings, how many times is each symbol sampled (**Q2.2.1**)?

2.3 IQ Streaming

1. In the desktop folder EE117B217B there is a Python file called `fieldfox_iq_streaming.py`. Open a terminal and run this Python code:

```
python .\fieldfox_iq_streaming.py
```

2. You should now see the IQ datastream being plotted in real-time. But it looks nothing like a constellation diagram. There are several things we need to fix to get this working.
3. First, the carrier (set on the VSG) and the demodulation LO signal (SA center frequency) are not locked. To lock them, use a BNC to SMB cable and connect it between the FieldFox (the SMB port is called **Ref/Trig Out** in one of the right panels) and the VSG (there is a BNC cable sticking out, which is connected to the **REF IN** port in the back). Once connected, the FieldFox internal clock is used as the reference for the VSG. On the VSG display you should see **EXT REF**, which is an indication that it is locked to an external reference.
4. Now the constellation should look a little better. Switch to BPSK modulation to get a better look at what is happening. Now you should see a line, which is the Vector Diagram for BPSK modulation. This line should be spinning at some rate. Calculate the rate of rotation of this line (**Q2.3.1**). What is the source of error making the line rotate? Why does the line still rotate despite the two sources still being phase locked (**Q2.3.2**)? Adjust the VSG signal generator frequency to minimize the rotation of the line (or constellation). What is the frequency on the VSG now (**Q2.3.3**)?
5. Return to 16 QAM modulation. You should see some resemblance of a faint constellation diagram. Right now the full Vector diagram is being plotted. We need to save the data, apply RRC filtering, sample the center of each symbol and then construct the constellation diagram from this, which we do next. Close the plotting window on the computer. Hit **Esc** on the FieldFox.

2.4 IQ Data Saving and Processing

1. In the desktop folder EE117B217B there is a Python file called `fieldfox_iq_saving.py`. Open a terminal and run this Python code:

```
python .\fieldfox_iq_saving.py
```

This should save 1 second of data (1.25 Mega samples) into an **npz** file.

2. This data needs to be processed to construct a constellation diagram, a full vector diagram and an eye diagram. You can use the code provided in the EE117B217B folder called `fieldfox_iq_processing.py`. Compute the EVM, eye height and estimate the BER (**Q2.4.1**). See Appendices C and D for a step-by-step description of what the code is doing.
3. Repeat the same for 64QAM and 256QAM at carrier power levels of -10 dBm and -40 dBm. Plot once again the constellation and eye diagrams. Report the EVM (**Q2.4.2**).
4. Return the FieldFox to a Preset State. Return the VSG to a Preset State. Disconnect the Clock synchronization cable and clear the work station for the next team.

Congratulations! You have completed the lab. Please power down all the instruments, and return all components safely to the box just as you received them. Get checked off by the instructor before you leave.

Appendix

A AM Modulation Theory

The conventional AM signal is

$$v(t) = V_c[1 + m v_m(t)] \cos(\omega_c t),$$

with modulation index m . Its Fourier transform is

$$V(\omega) = \frac{V_c}{2} [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{mV_c}{2} [V_m(\omega - \omega_c) + V_m(\omega + \omega_c)].$$

For single-tone modulation $v_m(t) = \cos(\omega_m t)$, the spectrum consists of the carrier at ω_c and two sidebands at $\omega_c \pm \omega_m$, with carrier amplitude $A_c = V_c/2$ and sideband amplitude $A_s = mV_c/4$, so that

$$m = \frac{2A_s}{A_c}. \quad (1)$$

The envelope swings between $V_{\max} = V_c(1 + m)$ and $V_{\min} = V_c(1 - m)$, so from the time-domain envelope

$$m = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}. \quad (2)$$

Modulation depth used by the signal generator. The signal generator used in this lab specifies the AM depth using the definition

$$m_d = \frac{V_{\max} - V_{\min}}{\frac{1}{2}(V_{\max} + V_{\min})} = \frac{2(V_{\max} - V_{\min})}{V_{\max} + V_{\min}} = 2m. \quad (3)$$

When comparing your measurements to the depth set on the instrument, use the same definition. In terms of the sideband-to-carrier ratio, Eqs. (1) and (3) give

$$m_d = \frac{4A_s}{A_c}.$$

Converting marker readings. The spectrum analyzer markers read power in dBm. The ratio of the sideband to the carrier *voltage* amplitude is

$$\frac{A_s}{A_c} = 10^{(A_s[\text{dBm}] - A_c[\text{dBm}])/20}.$$

Note the factor of 20 (not 10), since the modulation index is defined in terms of voltage amplitudes.

B FM Modulation Theory and the Bessel Null Method

For single-tone FM with modulating signal $v_m(t) = \cos(\omega_m t)$, the FM signal is

$$v(t) = V_c \cos(\omega_c t + \beta \sin \omega_m t),$$

where $\beta = \Delta f / f_m$ is the modulation index, Δf is the peak frequency deviation and f_m is the modulating (FM rate) frequency. The Bessel expansion gives

$$v(t) = V_c \sum_{n=-\infty}^{+\infty} J_n(\beta) \cos((\omega_c + n\omega_m)t).$$

The spectrum therefore consists of the carrier and an infinite set of sidebands spaced by f_m . The carrier amplitude is proportional to $J_0(\beta)$ and the n -th pair of sidebands is proportional to $J_n(\beta)$. Since the total power of an FM signal is constant, $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$: as β increases, power moves from the carrier into the sidebands.

Bessel null method. The carrier vanishes whenever $J_0(\beta) = 0$, and the first pair of sidebands vanishes whenever $J_1(\beta) = 0$. A null is easy to recognize on a spectrum analyzer (the tone drops into the noise floor), so tuning f_m until a null appears gives β exactly, and the deviation follows from

$$\Delta f = \beta_{\text{null}} \times f_m.$$

The first four zeros of $J_0(\beta)$ and $J_1(\beta)$ are:

Null order	1st	2nd	3rd	4th
Carrier null, $J_0(\beta) = 0$	2.405	5.520	8.654	11.792
First sideband null, $J_1(\beta) = 0$	3.832	7.016	10.174	13.324

Carson's rule. About 98% of the power of an FM signal lies within the bandwidth

$$BW \approx 2(\Delta f + f_m) = 2f_m(\beta + 1).$$

C IQ Demodulation: From Raw IQ Samples to Constellation, Vector and Eye Diagrams

This appendix describes, step by step, how to go from the raw IQ samples saved by `fieldfox_iq_saving.py` to the constellation, vector and eye diagrams. The metrics computed from these diagrams (EVM, eye height, eye width, Q-factor and BER) are described in Appendix D.

The raw data. The `npz` file contains the complex baseband samples

$$r[n] = I[n] + jQ[n], \quad n = 0, 1, \dots, N - 1,$$

sampled at $f_s = 1.25$ MHz, with $N = 1.25 \times 10^6$ samples (1 second). In Python they can be loaded with

```
data = np.load("file.npz")
r = data["iq"]
```

With a symbol rate of $R_s = 125$ ksps (symbol period $T_s = 1/R_s = 8 \mu\text{s}$), each symbol is represented by

$$L = \frac{f_s}{R_s} = 10 \text{ samples per symbol},$$

so the record contains about $K = N/L = 125,000$ symbols. The file also stores the number of UDP packets that were dropped during the capture (`dropped_packets`); if it is not zero, the record has gaps and should be recaptured.

Overview of the processing chain. The processing consists of the following steps:

1. Remove the residual carrier frequency offset.
2. Apply the root-raised-cosine (RRC) matched filter.
3. Find the symbol timing (the sample offset of the symbol centers).
4. Sample the filtered waveform at the center of each symbol.
5. Normalize the phase and gain of the symbols to the ideal constellation.
6. Construct the constellation, vector and eye diagrams.

Step 1: Residual carrier frequency offset. Even when the signal generator and the FieldFox share a reference clock, a small residual frequency offset Δf between the carrier and the FieldFox's demodulation LO may remain (Q2.3.2 asks you to explain why). This offset rotates the constellation at Δf revolutions per second. For square QAM (and QPSK), raising the samples to the fourth power removes the modulation and leaves a tone at $4\Delta f$, because $E[a_k^4]$ is a negative real constant for these constellations. The offset can therefore be estimated from the peak of the spectrum of $r^4[n]$:

$$\widehat{\Delta f} = \frac{1}{4} \arg \max_f |\text{FFT}\{r^4[n]\}(f)|,$$

and removed by de-rotating the samples:

$$r_c[n] = r[n] e^{-j2\pi \widehat{\Delta f} n / f_s}.$$

A finer estimate can be obtained from the slope of the phase of $r^4[n]$ (averaged over blocks of, e.g., 10 ms) versus time. For BPSK, use the second power instead of the fourth ($E[a_k^2]$ is a positive constant), and divide by 2. The estimate is unambiguous for $|\Delta f| < f_s/8$ (fourth power) or $f_s/4$ (second power).

Step 2: Root-raised-cosine matched filter. The signal generator shapes each symbol with an RRC (“Root Nyquist”) pulse of roll-off $\alpha = 0.35$. The receiver applies the same RRC filter. With time normalized to the symbol period, $\tau = t/T_s$, the RRC impulse response is

$$h_{\text{RRC}}(\tau) = \frac{\sin(\pi\tau(1-\alpha)) + 4\alpha\tau \cos(\pi\tau(1+\alpha))}{\pi\tau[1-(4\alpha\tau)^2]},$$

with the limiting values

$$h_{\text{RRC}}(0) = 1 - \alpha + \frac{4\alpha}{\pi}, \quad h_{\text{RRC}}\left(\pm \frac{1}{4\alpha}\right) = \frac{\alpha}{\sqrt{2}} \left[\left(1 + \frac{2}{\pi}\right) \sin \frac{\pi}{4\alpha} + \left(1 - \frac{2}{\pi}\right) \cos \frac{\pi}{4\alpha} \right].$$

To build the discrete filter, evaluate h_{RRC} at $\tau = k/L$ for $k = -LS/2, \dots, LS/2$, where S is the filter span in symbols ($S = 8$ to 12 is sufficient; $S = 12$ gives 121 taps), and normalize the taps to unit energy, $\sum_k h^2[k] = 1$. The filtered signal is the convolution

$$y[n] = \sum_k h[k] r_c[n - k].$$

Using a symmetric filter with an odd number of taps and keeping the central part of the convolution (e.g., `np.convolve(r_c, h, mode="same")`) compensates the filter delay automatically.

Why RRC? The cascade of the transmit RRC and the receive RRC is a raised-cosine pulse $p(t)$, which is a Nyquist pulse: $p(kT_s) = 0$ for every $k \neq 0$. When the filtered waveform is sampled at the symbol centers, the neighboring symbols therefore do not interfere (zero intersymbol interference, ISI). In addition, the receive RRC is the matched filter to the transmitted pulse, which maximizes the SNR at the sampling instants. The occupied bandwidth of the signal is $(1+\alpha)R_s$, i.e. 168.75 kHz in this lab.

Step 3: Symbol timing. Next, find the sample offset τ_0 ($0 \leq \tau_0 < L$) at which the symbol centers occur. A simple and robust method is to compute the average power at each of the L possible sample phases,

$$P(m) = \frac{1}{K} \sum_k |y[m + kL]|^2, \quad m = 0, 1, \dots, L-1,$$

and choose the phase with the largest power, $\tau_0 = \arg \max_m P(m)$. For a raised-cosine pulse, the average power is largest at the symbol centers. A fractional-sample estimate can be obtained with the Oerder–Meyr estimator, which uses the spectral line that $|y|^2$ has at the symbol rate:

$$X = \sum_n |y[n]|^2 e^{-j2\pi n/L}, \quad \tau_0 = \left(-\frac{L}{2\pi} \arg X \right) \bmod L.$$

Because the signal generator and the FieldFox share a reference clock, the symbol timing does not drift, so a single τ_0 is valid for the whole record. (Without a shared reference, the timing would drift and would have to be tracked block by block.)

Step 4: Sampling at the symbol centers. The symbols are the samples of the filtered waveform at the symbol centers:

$$s_k = y(\tau_0 + kL), \quad k = 0, 1, \dots, K-1.$$

If τ_0 is fractional, interpolate between samples, e.g. linearly, $y(t) = (1 - \mu)y[i] + \mu y[i + 1]$ with $i = \lfloor t \rfloor$ and $\mu = t - i$ (or with a cubic interpolator for higher accuracy). Rounding τ_0 to the nearest sample is also acceptable here, since with $L = 10$ the worst-case timing error is $0.05 T_s$. Discard the first and last S symbols, which are corrupted by the filter transients at the edges of the record.

Step 5: Phase and gain normalization. The ideal square M -QAM constellation has points $a = a_I + ja_Q$ with

$$a_I, a_Q \in \{\pm 1, \pm 3, \dots, \pm(\sqrt{M} - 1)\},$$

and average symbol energy $E_s = 2(M - 1)/3$, i.e. $E_s = 10, 42$ and 170 for 16-, 64- and 256-QAM. The received symbols are rotated by an unknown carrier phase and scaled by an unknown gain. The phase is estimated with the same fourth-power method as in Step 1,

$$\hat{\theta} = \frac{\arg(\sum_k s_k^4) - \pi}{4},$$

which leaves the usual 90° ambiguity of square QAM (this does not affect the constellation diagram or the EVM). The gain is chosen so that the average symbol energy matches E_s :

$$\hat{g} = \sqrt{\frac{E_s}{\frac{1}{K} \sum_k |s_k|^2}}.$$

The normalized symbols are $\tilde{s}_k = \hat{g} e^{-j\hat{\theta}} s_k$. Apply the same correction to the filtered waveform, $\tilde{y}[n] = \hat{g} e^{-j\hat{\theta}} y[n]$, before plotting the vector and eye diagrams.

Step 6: Constellation, vector and eye diagrams.

- **Constellation diagram:** a scatter plot of $\text{Im}\{\tilde{s}_k\}$ versus $\text{Re}\{\tilde{s}_k\}$ for all symbols, with the ideal points a overlaid. Each cluster should be centered on an ideal point; the size and shape of the clusters reveal the impairments (noise gives round clusters, phase noise gives clusters stretched tangentially, gain variations give clusters stretched radially).
- **Vector diagram:** a line plot of $\text{Im}\{\tilde{y}[n]\}$ versus $\text{Re}\{\tilde{y}[n]\}$ for a few hundred symbols, i.e. the full trajectory of the signal including the transitions between symbols, with the symbol-center samples $\tilde{s}_k$ marked. Note how the trajectory overshoots the outer constellation points between symbols; this is a consequence of the RRC pulse shaping.
- **Eye diagram:** fold the waveform into segments of two symbol periods (2 UI, unit intervals) centered on the sampling instants, and overlay them. Segment k of the in-phase eye is

$$e_k(u) = \text{Re}\{\tilde{y}(\tau_0 + kL + u)\}, \quad -L \leq u \leq L,$$

plotted versus $u/L \in [-1, 1]$ UI, so that the sampling instant is at the center of the plot. The quadrature eye is obtained in the same way from $\text{Im}\{\tilde{y}\}$. Overlaying 1000 to 2000 segments is sufficient. With $L = 10$ each segment has only 21 points; interpolating the waveform to a finer time grid (e.g., 80 points per 2 UI) gives smoother traces. For square M -QAM, each of the I and Q eyes is a $\sqrt{M}$ -level eye with $\sqrt{M} - 1$ openings.

D EVM, Eye Opening, Q-Factor and BER

Error vector magnitude. For each normalized symbol $\tilde{s}_k$, the ideal reference point a_k is the nearest ideal constellation point (the symbol decision; if the transmitted data are known, the transmitted symbol can be used instead). The error vector is $e_k = \tilde{s}_k - a_k$, and the RMS EVM over K symbols is

$$\text{EVM}_{\text{rms}} = \sqrt{\frac{\frac{1}{K} \sum_{k=1}^K |\tilde{s}_k - a_k|^2}{P_{\text{ref}}}} \times 100\%, \quad \text{EVM (dB)} = 20 \log_{10}(\text{EVM}_{\text{rms}}). \quad (4)$$

Two normalization conventions are in common use:

- Average power: $P_{\text{ref}} = \frac{1}{K} \sum |a_k|^2$ (3GPP).
- Peak power: $P_{\text{ref}} = \max |a_k|^2$ (IEEE 802.11).

State which convention you use. The closely related *modulation error ratio* is $\text{MER (dB)} = 10 \log_{10}(\sum_k |a_k|^2 / \sum_k |e_k|^2)$, which equals $-\text{EVM (dB)}$ for the average-power convention. Note that when decisions are used as the reference, symbols that are decided wrongly (at low SNR) are assigned to the wrong ideal point, so the EVM is underestimated.

EVM vs. SNR (AWGN-limited):

$$\text{EVM}_{\text{rms}} \approx 1/\sqrt{\text{SNR}}, \quad \text{SNR (dB)} \approx -\text{EVM (dB)}.$$

Eye height (vertical cut at the symbol center). Take a vertical cut through the eye diagram at the sampling instant ($u = 0$), i.e. use the symbol-center values $\text{Re}\{\tilde{s}_k\}$ (or $\text{Im}\{\tilde{s}_k\}$ for the Q eye). Consider one eye opening between two adjacent levels a_i and $a_{i+1} = a_i + 2$. Group the samples that belong to the upper level (a_{i+1}) and to the lower level (a_i), and compute their means μ_1, μ_0 and standard deviations σ_1, σ_0 . The eye height is

$$\text{EH} = (\mu_1 - 3\sigma_1) - (\mu_0 + 3\sigma_0),$$

which is often normalized to the ideal opening (the level spacing, 2) and quoted in percent. Each of the I and Q eyes of M -QAM has $\sqrt{M} - 1$ openings; report the smallest one (the worst eye).

Eye width (horizontal cut at the decision threshold). Take a horizontal cut through the same eye opening at the decision threshold $d_i = (a_i + a_{i+1})/2$. For each trace that crosses the threshold, find the crossing time by linear interpolation between neighboring points. The crossings cluster around the edges of the eye, $u \approx \pm 0.5$ UI. With mean and standard deviation t_L, σ_L of the left cluster and t_R, σ_R of the right cluster, the eye width is

$$\text{EW} = (t_R - 3\sigma_R) - (t_L + 3\sigma_L),$$

in UI (multiply by $T_s = 8 \mu\text{s}$ for seconds).

Q-factor. For one eye opening, the Q-factor is the ratio of the separation of the two levels to the sum of their noise standard deviations:

$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0}.$$

From Q-factor to BER. For Gaussian noise, the probability that a sample crosses the decision threshold of a binary eye is

$$\text{BER} = \mathcal{Q}(Q) = \frac{1}{2} \text{erfc}\left(\frac{Q}{\sqrt{2}}\right), \quad \text{where } \mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du$$

is the Gaussian tail probability. Some reference values:

Q	3	4	5	6	7
$\mathcal{Q}(Q)$	1.35×10^{-3}	3.17×10^{-5}	2.87×10^{-7}	9.87×10^{-10}	1.28×10^{-12}

For square M -QAM, each of the I and Q axes is a $\sqrt{M}$ -level signal. When all eyes have the same Q-factor, the probability of a symbol error on one axis is $P_{\sqrt{M}} \approx 2 \left(1 - 1/\sqrt{M}\right) \mathcal{Q}(Q)$ (inner levels have two neighbors, outer levels one), the symbol error rate is $P_s = 1 - (1 - P_{\sqrt{M}})^2 \approx 2P_{\sqrt{M}}$, and with Gray coding (one bit error per symbol error) the BER is

$$\text{BER} \approx \frac{4}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right) \mathcal{Q}(Q).$$

From EVM to BER. If the errors are dominated by additive Gaussian noise, the Q-factor follows directly from the EVM (average-power normalization):

$$Q = \sqrt{\frac{3}{(M-1) \text{EVM}_{\text{rms}}^2}}, \quad \text{BER} \approx \frac{4}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right) \mathcal{Q}\left(\sqrt{\frac{3}{(M-1) \text{EVM}_{\text{rms}}^2}}\right).$$

Comparing the BER estimated from the eye Q-factor with that from the EVM is a useful consistency check. Note that with 125,000 symbols, BER values below about 10^{-5} cannot be verified by counting errors; they can only be estimated as above.