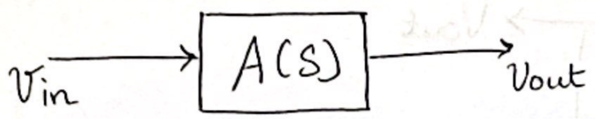
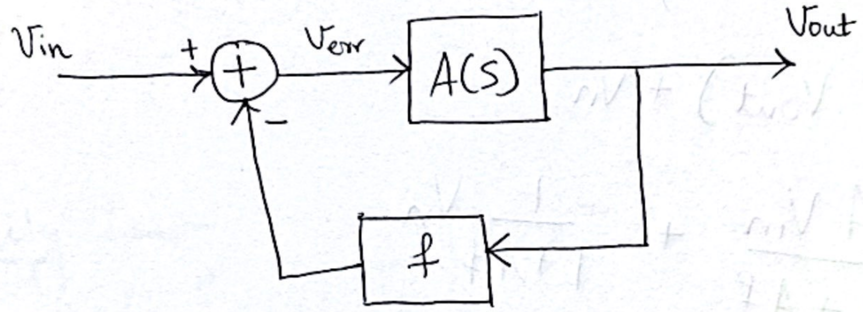
Lec 7 - Feedback & Op. Amps



$$\frac{V_{out}(s)}{V_{in}(s)} = A(s)$$

Feedback



$$V_{out} = A\,V_{err}$$

$$V_{err} = V_{in} - V_{out}\,f$$

$$\Rightarrow V_{out} = A(V_{in} - V_{out}\,f)$$

$$\Rightarrow V_{out}(1 + Af) = A\,V_{in}$$

$$\Rightarrow \boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{A}{1+Af}}$$

Closed Loop transfer function.

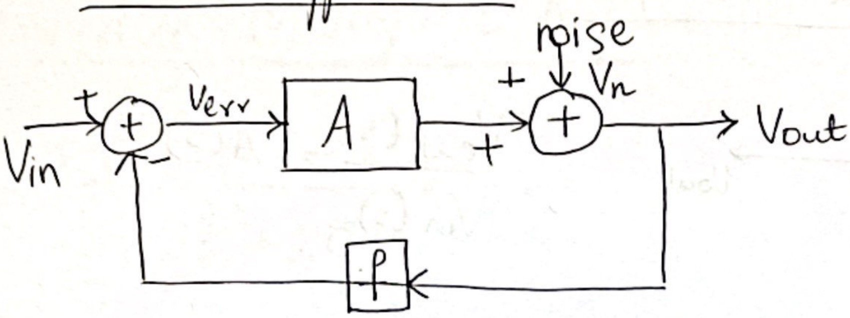
Af is called the Loop Gain.

Why is feedback useful?

① > Precision gain control. $\frac{V_{out}}{V_{in}} \to \frac{1}{f}$ as $A \to \infty$

& f is much easier to define precisely compared to A.

② Noise Suppression



$$V_{out} = A V_{err} + V_n$$

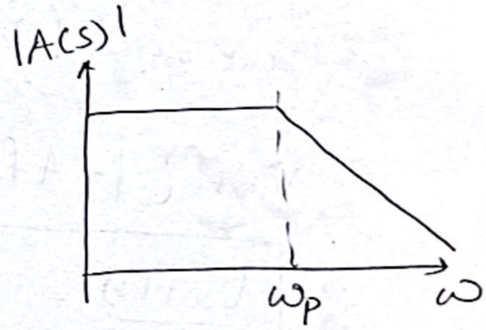
$$V_{out} = A(V_{in} - f V_{out}) + V_n$$

$$\Rightarrow V_{out} = \frac{A V_{in}}{1+Af} + \frac{1}{1+Af} V_n$$

If $A \to \infty$, V_n is suppressed but V_{in} is not.

③ Bandwidth Enhancement

$$A(s) = \frac{A_0}{1+\frac{s}{\omega_p}}$$

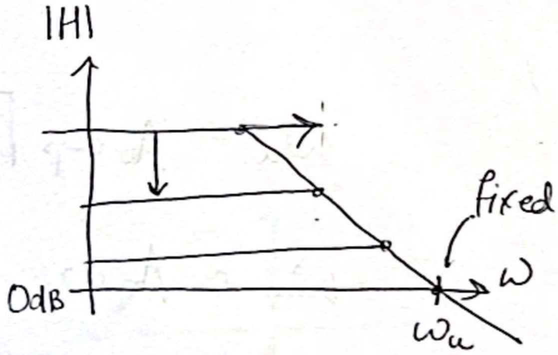


$$H(s) = \frac{A(s)}{1+A(s)f} = \frac{A_0}{1+\frac{s}{\omega_p}}\left[\frac{1}{1+\frac{f A_0}{1+\frac{s}{\omega_p}}}\right]$$

$$= \frac{A_0}{1+A_0 f+\frac{s}{\omega_p}} = \frac{A_0}{1+A_0 f} \cdot \frac{1}{1+\frac{s}{(1+A_0 f)\omega_p}}$$

> Gain drops by $1+A_0 f$ but pole freq increases by $1+A_0 f$

	Low freq gain	pole
open loop	A_0	ω_p
feedback	$\frac{A_0}{1+A_0 f}$	$(1+A_0 f)\omega_p$



Unity Gain frequency (ω_u)

$|H(s)| = 1$ when $s = j\omega$

Openloop

$$|H(j\omega)| = \left|\frac{A_0}{1+\frac{j\omega}{\omega_p}}\right| = \frac{A_0}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^2}}$$

When $A_0 \gg 1$, $\omega \gg \omega_p$ for $|H| = 1$ $\Rightarrow$ $\frac{A_0}{\frac{\omega}{\omega_p}} = 1$ for unity gain.

$\Rightarrow$ $\boxed{\omega_u = A_0 \omega_p}$ $\longrightarrow$ open loop case.

In closed loop case, when $A \gg 1$, Gain $\uparrow$ by $A_0 f$ & $\omega_p \downarrow$ by $A_0 f$

When $f < 1$ we can show that

$\boxed{\omega_u \simeq A_0 \omega_p}$ $\longrightarrow$ Gain Bandwidth product is constant under negative feedback.

Proof (Closed loop case)

$$|H(s)| = \left|\frac{A_0}{1+A_0f+\frac{j\omega}{\omega_p}}\right| = \frac{A_0}{\sqrt{(1+A_0f)^2+\left(\frac{\omega}{\omega_p}\right)^2}} = 1$$

$$\Rightarrow \omega_u = \omega_p\sqrt{A_0^2-(1+A_0f)^2}$$

$$= A_0\omega_p\sqrt{1-\left(\cancel{\tfrac{1}{A_0}}+f\right)^2}$$

$$\cancel{\tfrac{1}{A_0}} \to 0$$

$$\omega_u = A_0\omega_p\sqrt{1-f^2}$$

$$\omega_u \simeq A_0\omega_p \text{ when } f \ll 1.$$

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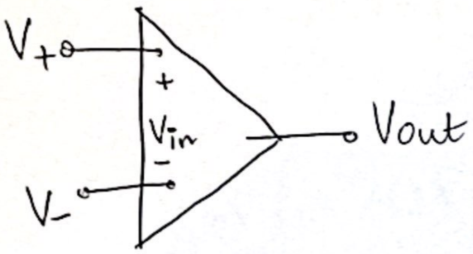
④* Stability → (Caution) {A rigorous study of feedback of stability is not covered here (→140).}

$$\underset{\text{Closed loop}}{H(s)} = \frac{A(s)}{1+A(s)f} \Rightarrow \text{poles are roots of } 1+\underbrace{A(s)f}_{L(s)}.$$

If $L(j\omega) = -1$ → unstable! (Negative feedback becomes positive). If $L(s)$ has one pole phase cannot reach 180° so always stable. But 2 poles? Still maybe OK. 3 poles or more? Definitely not OK. Opamp design is about controlling the poles & zeroes of a complex system so that it never becomes unstable! Make 3 pole 2 zero system look like a single pole system.

Operational Amplifiers (OpAmps)

Model 0 : Golden Rules (16B)



$$\text{Gain} = \frac{V_{out}}{V_{in}} = \infty$$

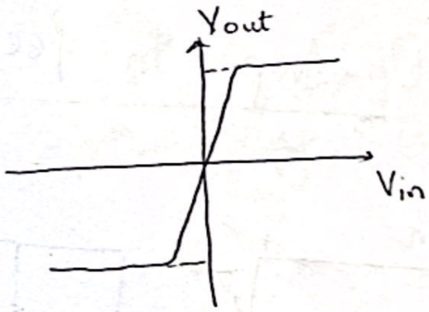
$R_{in} \to \infty \Rightarrow i_+ = i_- = 0$

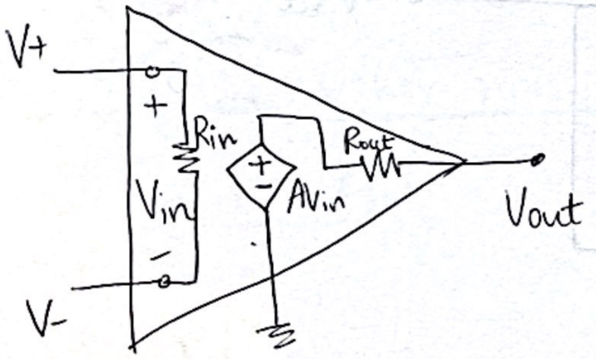
$R_{out} \to 0$

$BW \to \infty$

Under negative feedback $V_+ = V_-$

Model 1 : Finite Gain VCVS





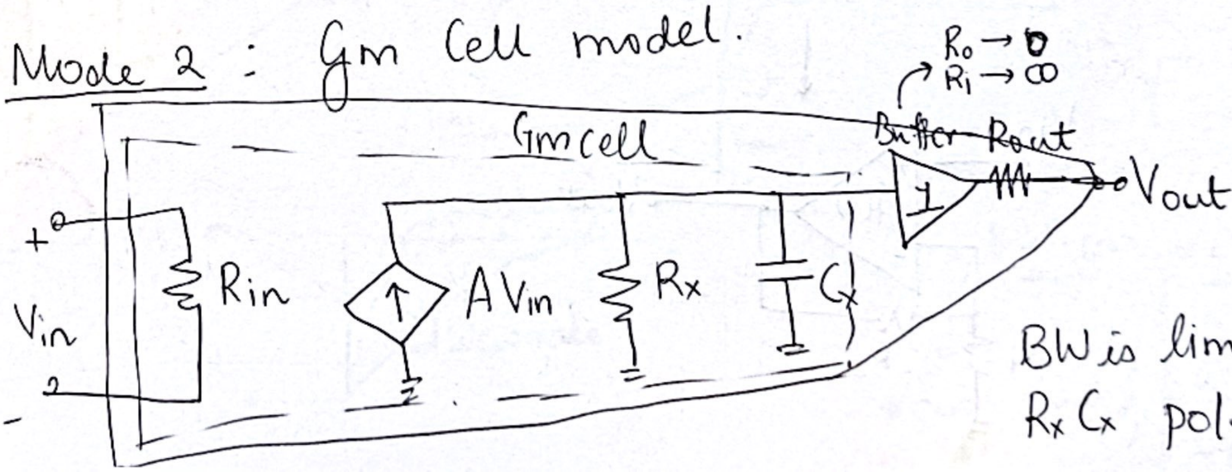
$\text{Gain} = A \to 10^4$

$R_{in} \simeq$ ~~[illegible]~~ $10M\Omega$

$R_{out} \simeq 10\Omega$

$BW \simeq MHz \to$ where does this come from.

Mode 2 : Gm Cell model.



BW is limited by $R_x C_x$ pole.

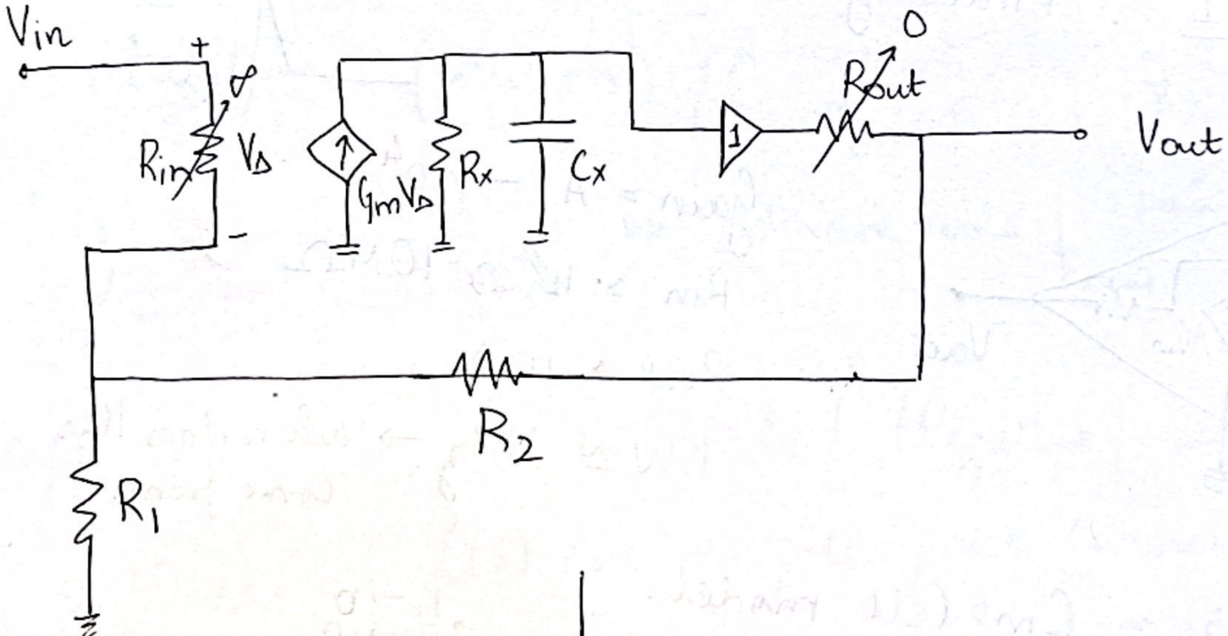
$$V_x = (G_m V_{in})\left(R_x \parallel \frac{1}{sC_x}\right) = V_0$$

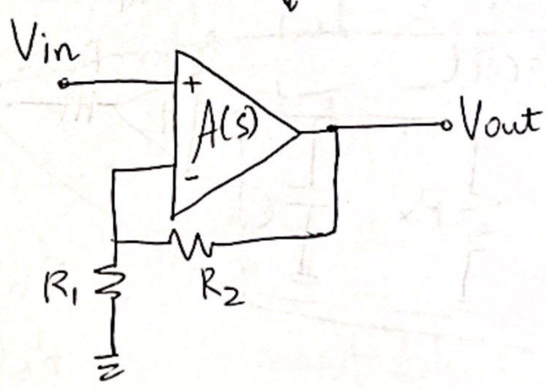
$$\Rightarrow \quad \frac{V_0}{V_{in}} = \frac{G_m \cdot R_x}{1 + sR_xC_x}$$

$$\Rightarrow \quad A_0 = G_m R_x \quad \& \quad \omega_p = \frac{1}{R_x C_x}$$

$$\omega_u = A_0 \omega_p = \frac{G_m}{C_x} \rightarrow \text{independant of } R_x$$

Op-Amp In feedback





$$V_{out} = A(s)(V_+ - V_-)$$

$$= A(s)\left(V_{in} - \left(\frac{R_1}{R_1+R_2}\right) V_{out}\right)$$

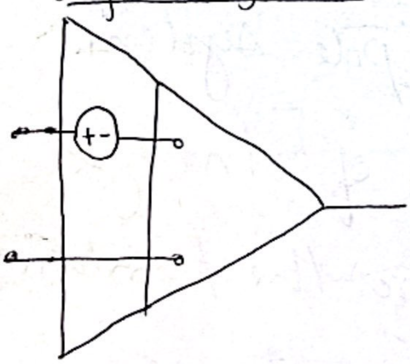
$$\Rightarrow V_{out}\left(1 + A(s)\left(\frac{R_1}{R_1+R_2}\right)\right) = A(s)\, V_{in}.$$

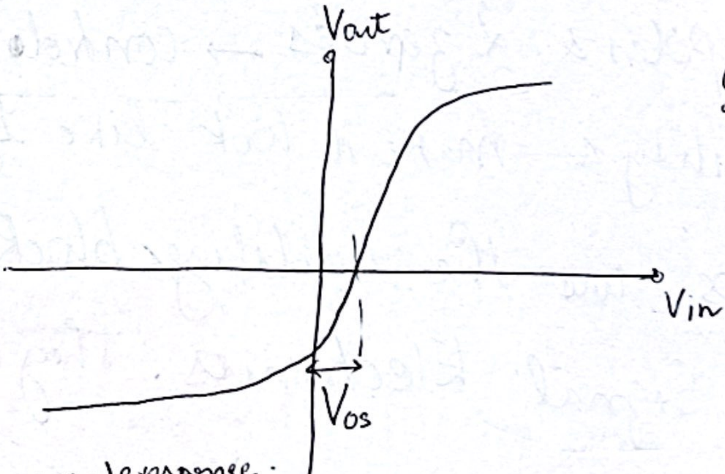
$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A(s)}{1 + A(s)\left(\frac{R_1}{R_1+R_2}\right)}$$

$$\Rightarrow \boxed{f = \frac{R_1}{R_1+R_2}} \rightarrow \text{Ratio of resistors} \Rightarrow \text{very precisel}$$

Model 3 : Other non idealities

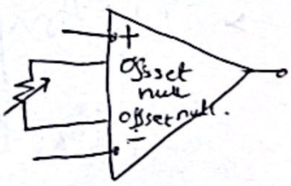
> Input offset



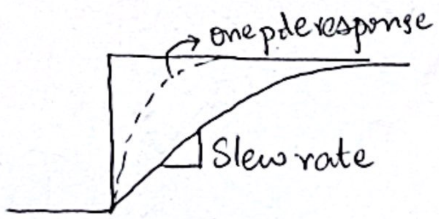


Solution

> Trimming



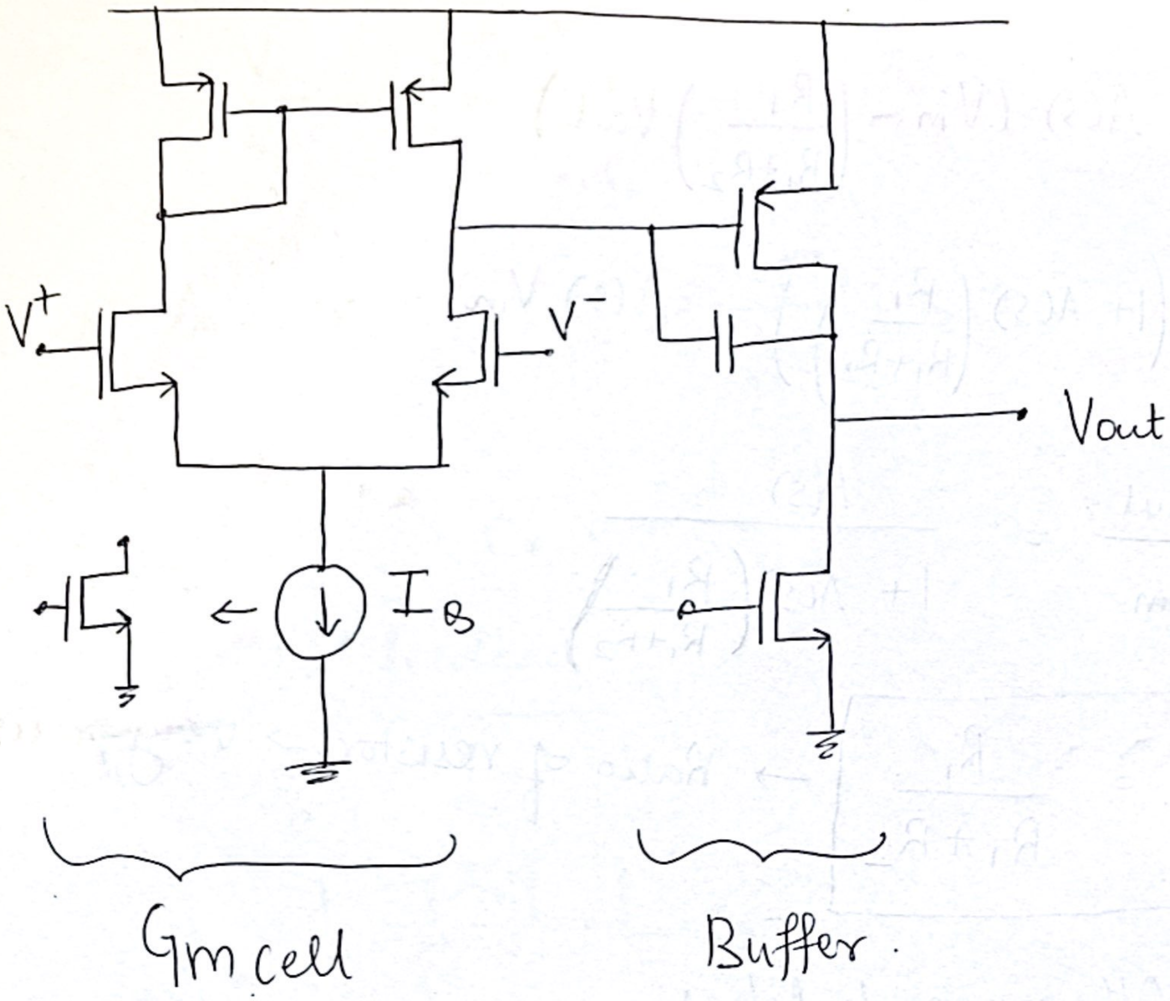
> Slewing



> Distortion & Noise

Sine input → output not pure sine.	Internal noise sources.

Model 4 : Transistor Model



- Total 3 poles & 2 zeroes → controls them to prevent instability & make it look like 1 pole system.
- Op Amps are the building blocks of Analog & Mixed Signal Electronics. They are the foundation!