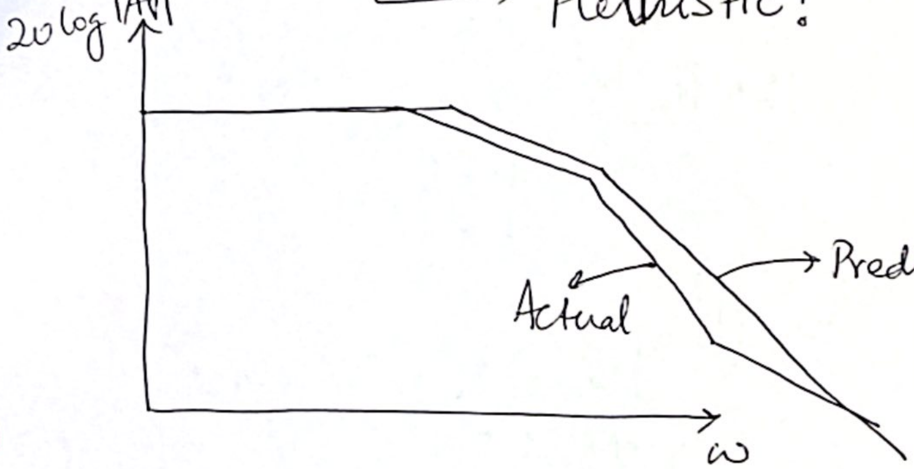
Lec 22: Frequency Response Analysis

> Common Source → Two poles & one zero.

> Poles ⟷ Nodes method → Two poles
  └→ Heuristic!



Note: Predicted BW is higher than Actual BW.

> We want to derive this method of Poles ⟷ Nodes ~~more~~ rigorously to understand under what conditions is it valid

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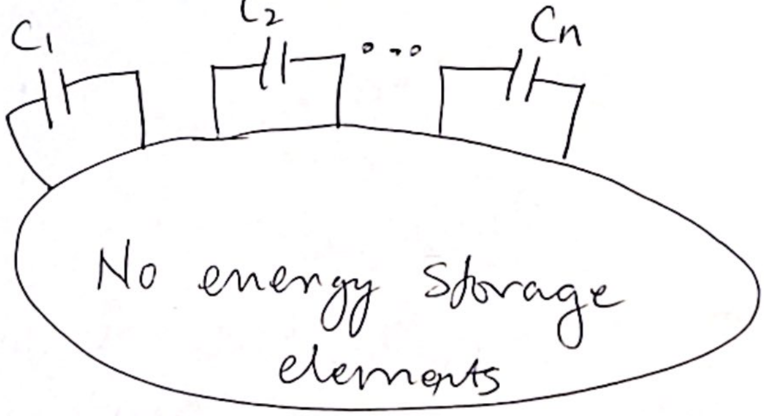
> Assumption 1: Only capacitors in my network.



The transfer function b/w any two circuit quantities (V, I) is in general:

$$H(s) = A_0 \frac{1 + b_1 s + b_2 s^2 + \dots + b_m s^m}{1 + a_1 s + a_2 s^2 + \dots + a_n s^n} = A_0 \frac{N(s)}{D(s)}$$

$A_0 \rightarrow$ DC gain & $m < n$.

$$D(s) = \prod_{k=1}^{n} \left(1 - \frac{s}{P_k}\right)$$

If the circuit is stable all poles are in LHP.

Let $P_k = -\frac{1}{\tau_{P,k}}$ where $\tau_{P,k} > 0$.

$$D(s) = \prod_{k=1}^{n} \left(1 + s\tau_{P,k}\right) \rightarrow \text{expanding gives}$$

$$= 1 + s \sum_{k=1}^{n} \tau_{P,k} + s^2(\dots) + \dots$$

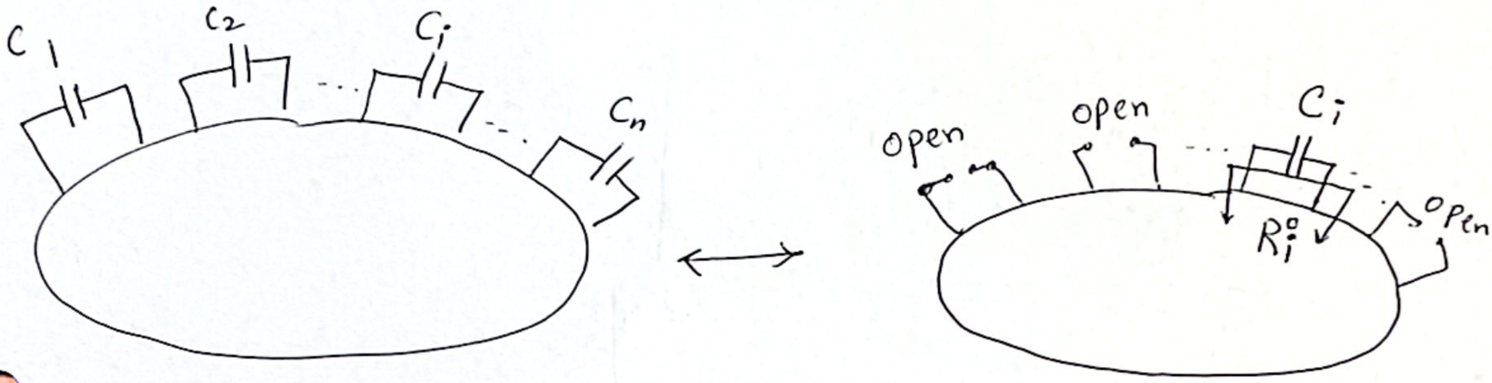
$$\Rightarrow a_1 = \sum_{k=1}^{n} \tau_{P,k} = -\sum_{k=1}^{n} \frac{1}{P_k}$$

a_1 is the sum of all pole time constants.

Open Circuit Time Constant (OCTC) Method

Theorem: $a_1 = \sum_{i=1}^{n} \tau_i^{\,0}$

Open Circuit Time Constants.



$\tau_i = R_i^0 \, C_i$

Resistance seen by C_i when all other caps are open!

NOTE: $\tau_i^{\,0} \neq \tau_{p,k}$ but the $\sum$s are equal.

Proof: From Nodal analysis we can show that

$a_1 = \sum_{i=1}^{n} \alpha_i C_i$, ~~the~~ then considering one cap at a time we can show $\alpha_i = R_i^0$ & use linearity.

(Full proof see Hajimiri's book)

Assumption/Approximation #2 : There is one dominant pole.

$$\Rightarrow |\tau_{p,1}| \gg |\tau_{p,k}|_{k\neq 1}$$

$$\Rightarrow \sum_{k=1}^{n} \tau_{p,k} \simeq \tau_{p,1}$$

$$\Rightarrow \boxed{\tau_{p,1}^{OCTC} = \sum_{i=1}^{n} R_i^{o} C_i}$$

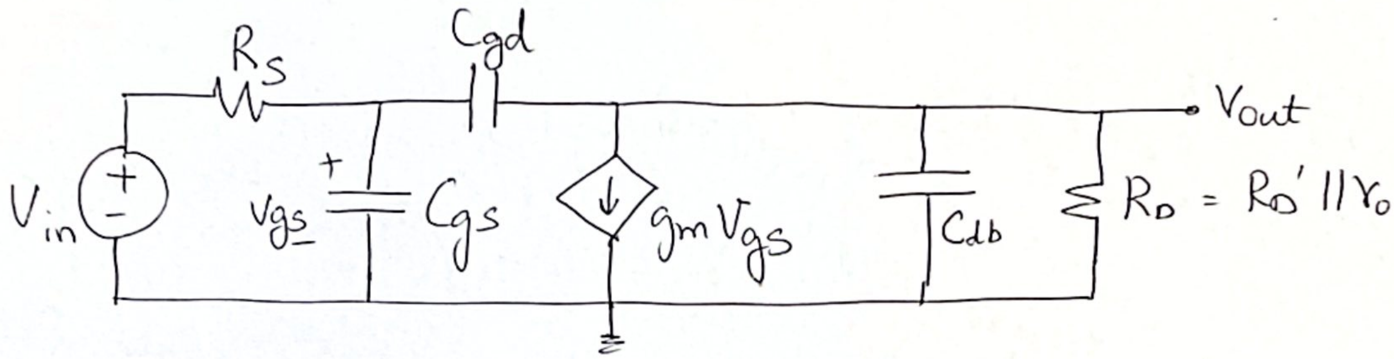
$$\Rightarrow \boxed{\omega_{-3dB}^{OCTC} \cong \frac{1}{\sum_{i=1}^{n} R_i^{o} C_i}}$$

In exact terms : $\tau_{p,1\,exact} = \sum_{i=1}^{n} R_i^{o} C_i^{o} - \sum_{K=2}^{n} \tau_{p,K}$

$$\Rightarrow \omega_{-3dB}^{OCTC} \leq \omega_{-3dB}^{actual}$$

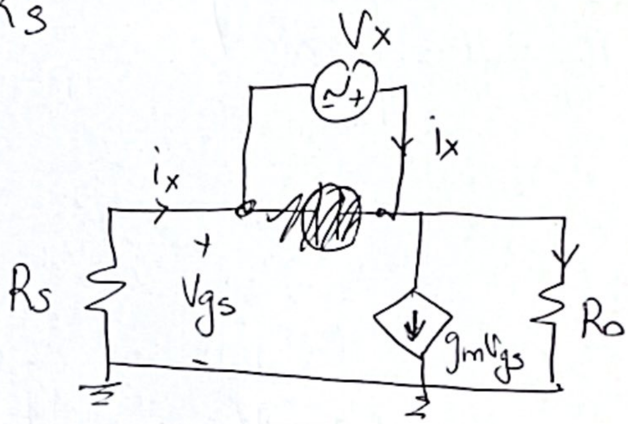
This is therefore a conservative estimate of the bandwidth. Earlier poles ← nodes overestimated the BW since we also used Miller Approx.

Common Source (OCTC)



i) C_{gs}: R_S

ii) C_{gd}:



$$\left.\begin{aligned} V_{out} &= (i_x - g_m V_{gs}) R_D \\ V_{gs} &= -i_x R_S \end{aligned}\right\} \quad V_x = (i_x - g_m \underbrace{V_{gs}}_{-i_x R_S}) R_D + i_x R_S$$

$$\Rightarrow \frac{V_x}{i_x} = (1 + g_m R_S) R_D + R_S$$

iii) C_{db}: R_D

$$\Rightarrow \omega_{P_1} = \frac{1}{R_S C_{gs} + (R_S + R_D + g_m R_D R_S) C_{gd} + R_D C_{db}}$$

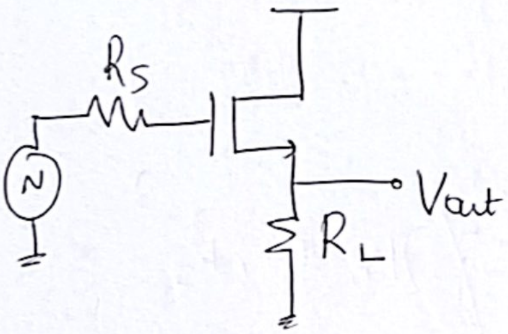
Recall,
Exact sol. with dominant pole approx was

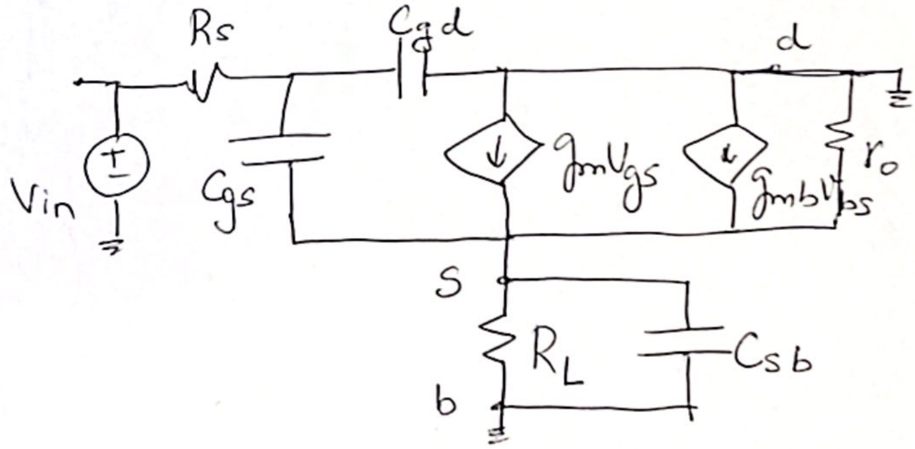
$$\tau \approx \cancel{R_S C_{gd} + R_D C_{gd} + g_m R_S R_D C_{gd} +}$$

$$\omega_{p_1} = \frac{1}{\underbrace{R_S(1+g_m R_D)}_{\simeq g_m R_D R_S} C_{gd} + R_S C_{gs} + R_D C_{gd} + R_D C_{db}}$$

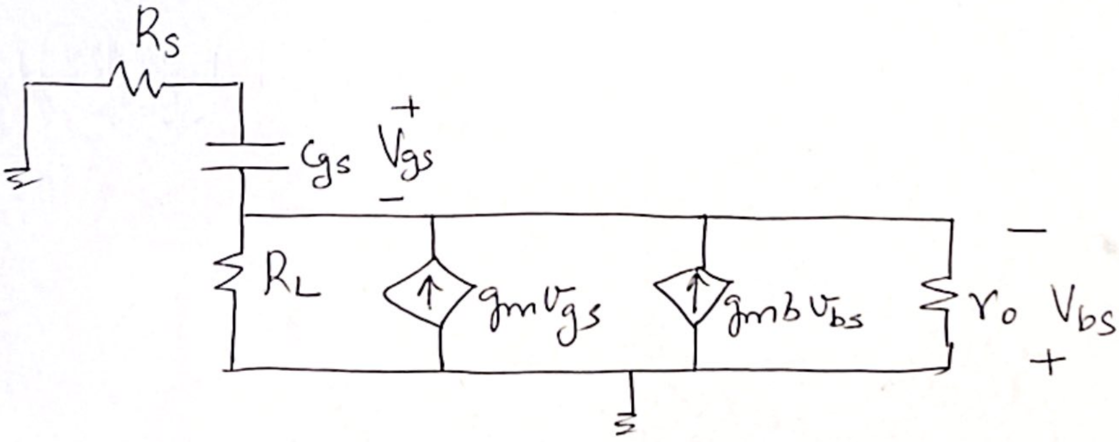
which is exactly the same! $\omega_{p_1} \simeq \dfrac{1}{R_S(C_{gs} + g_m R_D C_{gd})}$

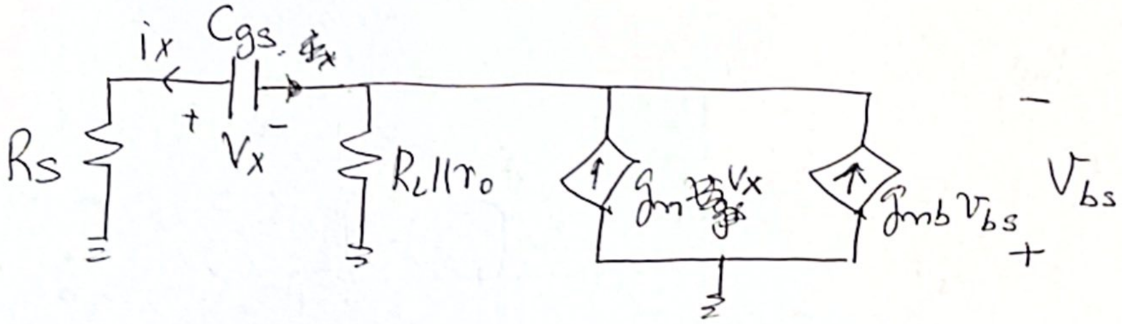
Common Drain





i) C_{gs} :





$$-V_{bs} = (-i_x + g_m V_x + g_{mb} V_{bs})(R_L \| r_o)$$

$$V_x = (+i_x R_s) + V_{bs} \Rightarrow V_{bs} = V_x - i_x R_s$$

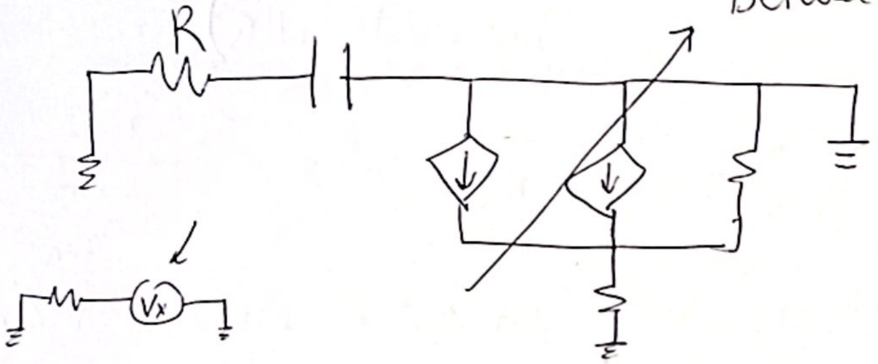
$$\Rightarrow -V_x + i_x R_s = [-i_x + g_m V_x + g_{mb}(V_x - i_x R_s)](R_L \| r_o)$$

$$\Rightarrow V_x\left(g_m (R_L \| r_o) + g_{mb}(R_L \| r_o) + 1\right) = i_x(+R_L \| r_o + R_s)$$

$$\Rightarrow R_x = \frac{R_s + R_L \| r_o}{1 + (g_m + g_{mb})(R_L \| r_o)} = R_{gs}^{0}$$

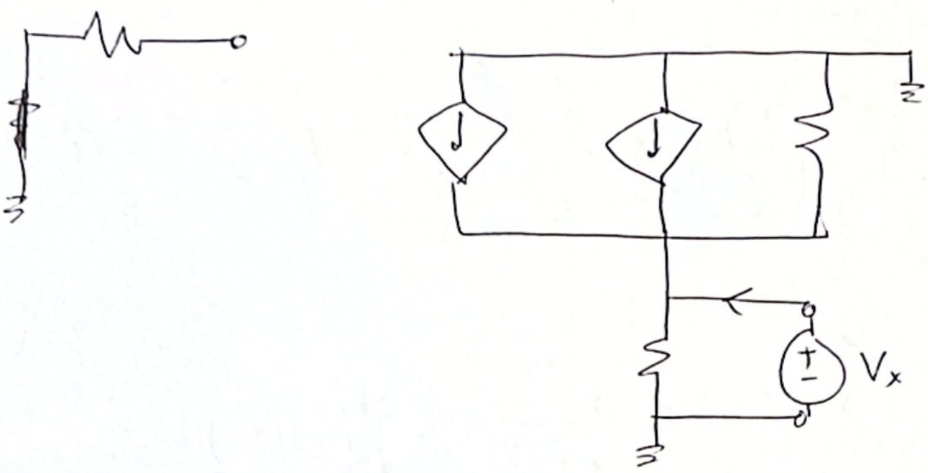
ii) C_{gd} :

Between two grounds.



$\underline{R_s}$

C_{sb}:



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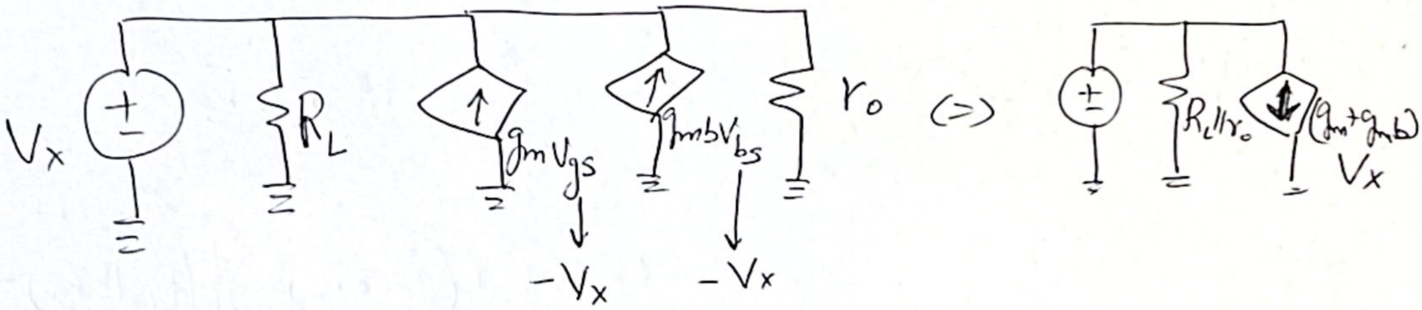

$$\Rightarrow \quad R^o_{sb} = R_L \| r_o \| \frac{1}{g_m+g_{mb}}$$

$$\Rightarrow \quad \omega_{3dB} = \frac{1}{R_S C_{gd} + R^o_{gs} C_{gs} + R^o_{sb} C_{sb}}$$

$$\text{Where} \quad R^o_{gs} = \frac{R_S + R_L \| r_o}{1+(g_m+g_{mb})(R_L \| r_o)} \simeq \underbrace{\frac{R_L+R_S}{g_m R_L}}_{\simeq \frac{1}{g_m}}$$

$$R^o_{sb} \simeq R_L \| r_m \qquad \boxed{\omega_{3dB} \simeq \frac{1}{R_S C_{gd} + \frac{C_{gs}}{g_m}}}$$

No Miller multiplication & all time const. are small → Large BW!