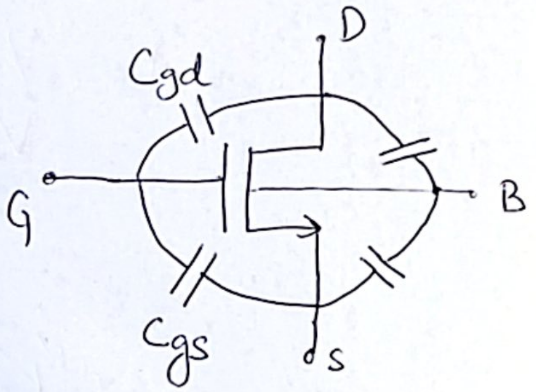
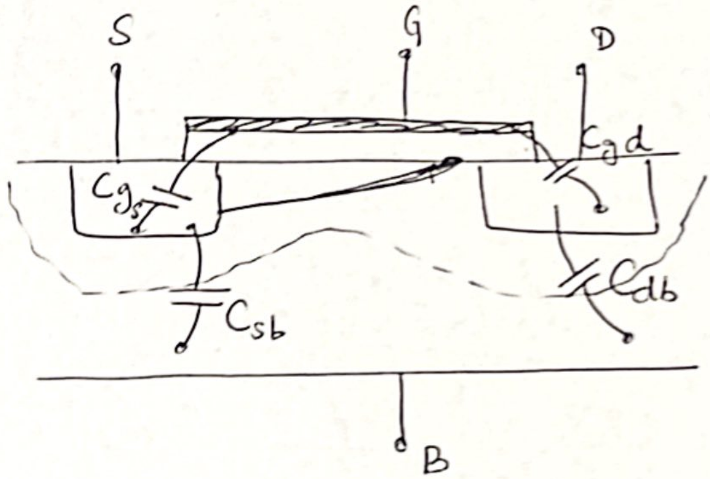
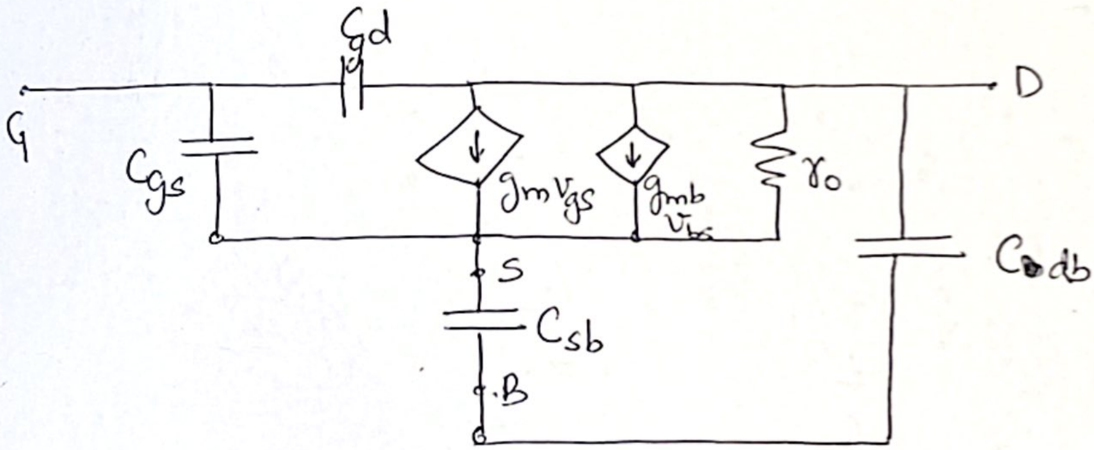
Lec 21 : Frequency Response of Amplifiers

Review







$$C_{gs} = W L_{ov} C_{ox} + \frac{2}{3} W L C_{ox}$$

$$C_{gd} = W L_{ov} C_{ox}$$

$$C_{sb} = \frac{C_{sb0}}{\left(1 + \frac{V_{sb}}{\varphi_0}\right)^{1/2}} \qquad C_{db} = \frac{C_{db0}}{\left(1 + \frac{V_{db}}{\varphi_0}\right)^{1/2}}$$

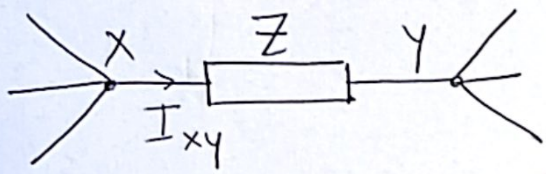
Typically : $C_{gs} > C_{gd} > C_{sb} > C_{db}$.

Miller Effect & Miller's Theorem

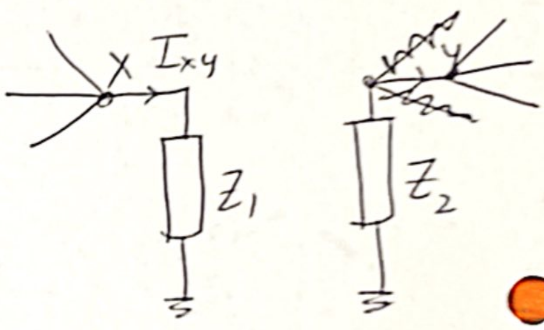
Thm: If circuit ① can be converted to circuit ②,

then $Z_1 = \dfrac{Z}{1 - A_v}$ & $Z_2 = \dfrac{Z}{1 - A_v^{-1}}$ where $A_v = \dfrac{V_y}{V_x}$

Circuit ①



Circuit ②



Proof

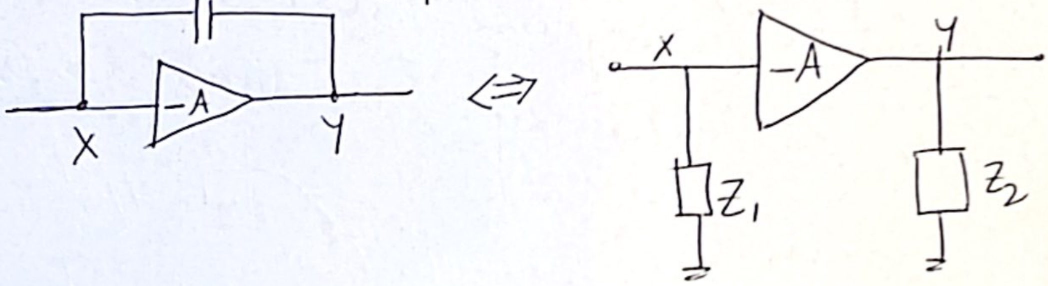
$$I_{xy} = \frac{V_x - V_y}{Z} = \frac{V_x}{Z_1} = \frac{-V_y}{Z_2}$$

$$\Rightarrow \quad Z_1 = \frac{Z}{1 - \frac{V_y}{V_x}} \quad \& \quad Z_2 = \frac{1}{1 - \frac{V_x}{V_y}}$$

Caution: Note the if condition

Miller Effect

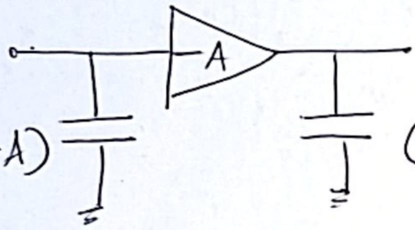
$C_F \Rightarrow Z = \frac{1}{sC_F}$



$$Z = \frac{1}{sC_F} \Rightarrow Z_1 = \frac{1/sC_F}{1+A} \; ; \; Z_2 = \frac{\frac{1}{sC_F}}{1+A^{-1}}$$

$$Z_1 = \frac{1}{s(C_F(1+A))} \; ; \; Z_2 = \frac{1}{s(C_F(1+A^{-1}))}$$

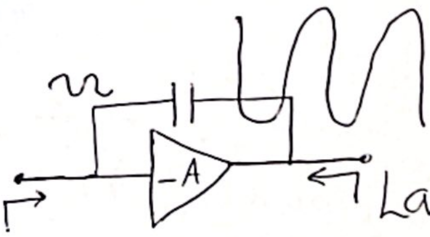
$\Rightarrow$



$C_F(1+A) \simeq \underbrace{AC_F}_{\text{Large!}}$

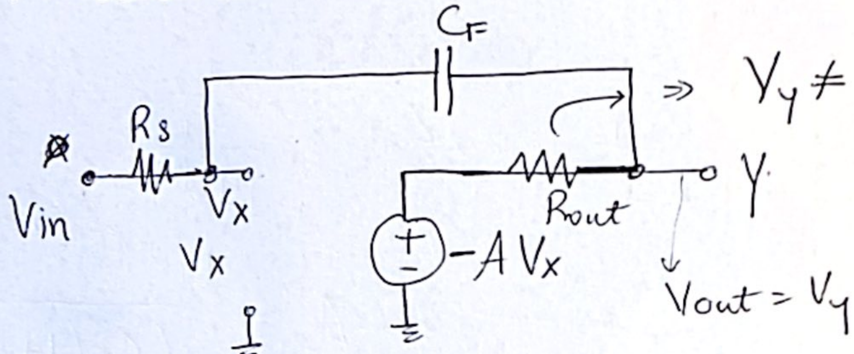
$C_F(1+A^{-1}) \simeq \underbrace{C_F}_{\text{Same}}$

Intuition



Note: Here A should really be $A(s)$ & include the effect of the capacitor as well!

<u>Miller Approximation</u>: Replace $A(s) \rightarrow A$. This needs to be done carefully. Sometimes leads to inaccurate results.



$\Rightarrow V_Y \neq -A V_X$

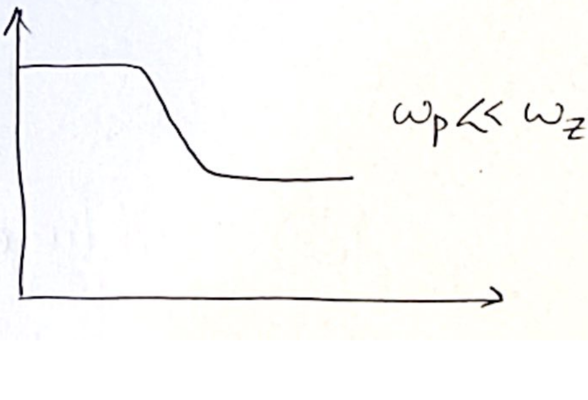
$$\frac{V_{in} - V_X}{R_S} R_{out} = V_{out} + A V_X$$

$$\frac{V_{in} - V_X}{R_S} = (V_X - V_{out}) s C_F$$

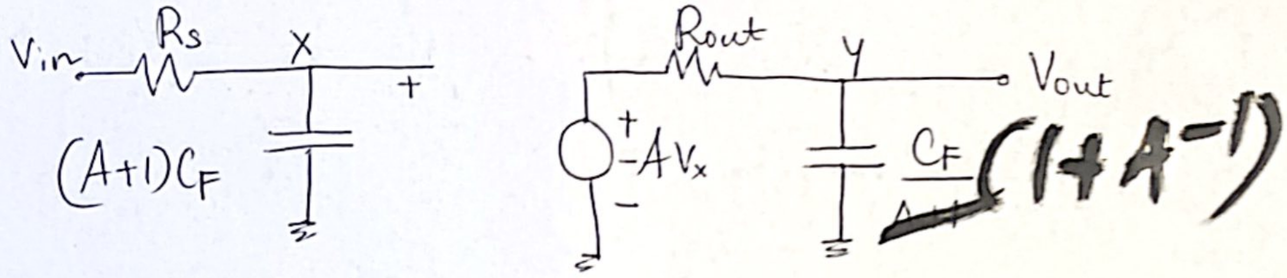
Solving $\Rightarrow$ $$\frac{V_{out}}{V_{in}} = \frac{R_{out} s C_F - A}{\left[(A+1) R_S + R_{out}\right] C_F s + 1}$$

$\longrightarrow \omega_Z = \dfrac{A}{R_{out} C_F}$

$$\omega_P = \frac{-1}{(A+1) R_S C_F + R_{out} C_F}$$



Miller ⇒



Solving ⇒ $$\frac{V_{out}}{V_{in}} = \frac{-A}{\left[(1+A)R_s C_F s+1\right]\left(\frac{1}{1+A^{-1}} C_F R_{out} s+1\right)}$$

⇒ No zero & 2 poles!

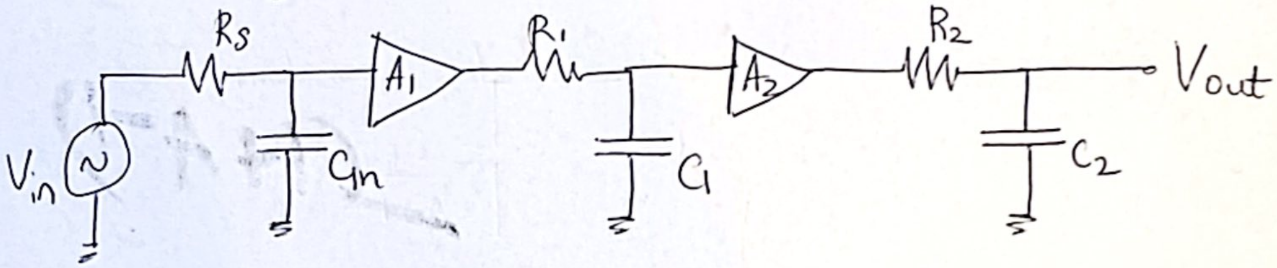
$$\omega_{p_1} = \frac{-1}{(1+A)R_s C_F}, \quad \omega_{p_2} = \frac{-1}{\frac{1}{1+A^{-1}} C_F R_{out}}$$

Limitations of Miller Approximation

1) Eliminate zeroes
2) Add poles
3) Inaccurate output impedance.

> Good for intuition/simplification & input impedance or input cap boost!

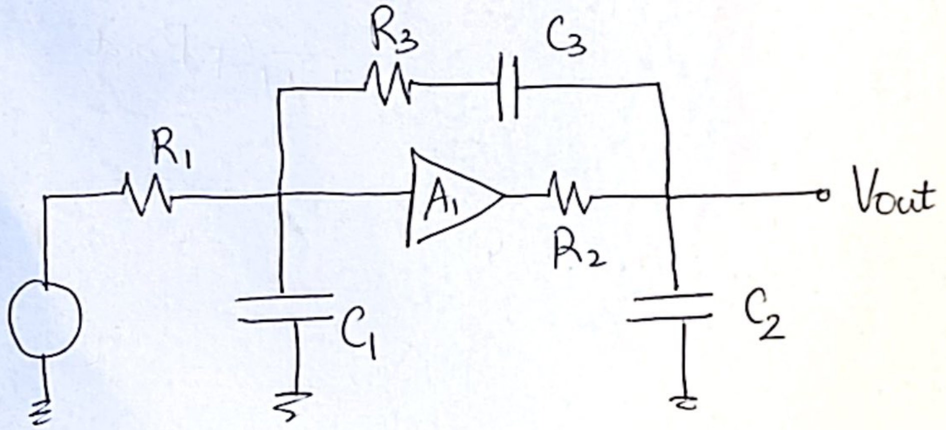
Associating Poles with Nodes (Approximate method).



$$\frac{V_{out}}{V_{in}} = \frac{A_1}{1+R_S C_{in} s} \cdot \frac{A_2}{1+R_1 C_1 s} \cdot \cancel{A} \cdot \frac{1}{1+R_2 C_2 s}$$

> Each node with RC contributes 1 pole.

> This heuristic approach will be made rigorous later.

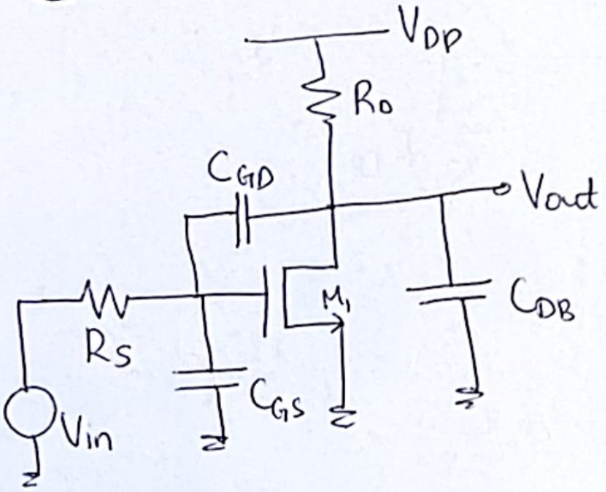


→ How can we treat this? Miller!

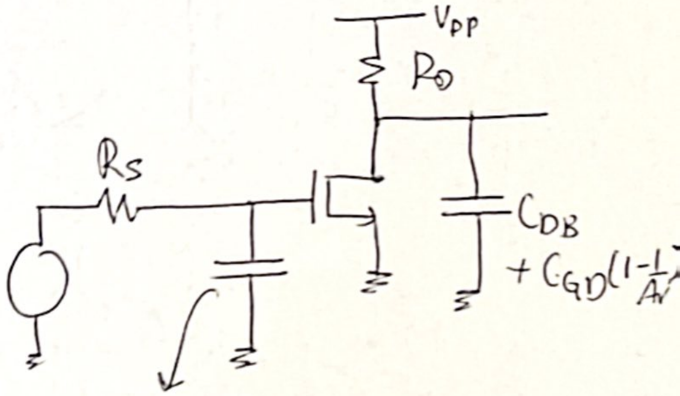
> This can we converted to 2 pole case & treated but must be careful!

> Note: This method is valid under a "dominant pole approximation" → will derive rigorously later.

Common Source



= Miller



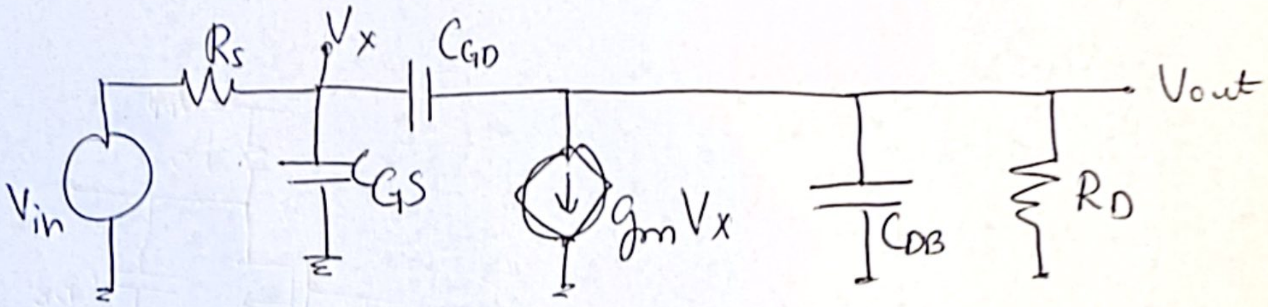
$C_{GS} + (1 - A_V)\, C_{GD}$

$$\omega_{in} = \frac{1}{R_S\left(C_{GS} + (1 + g_m R_D)\, C_{GD}\right)}$$

$$\omega_{out} \simeq \frac{1}{R_D\left(C_{DB} + C_{GD}\right)}$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Direct (Brute Force)



$$\frac{V_X - V_{in}}{R_S} + \frac{V_X}{\frac{1}{sC_{gs}}} + \frac{(V_X - V_{out})}{\frac{1}{sC_{gd}}} = 0$$

$$(V_{out} - V_X)\, s\, C_{gd} + g_m V_X + V_{out}\left(\frac{1}{R_D} + s\, C_{db}\right) = 0$$

Solving,

$$\frac{V_{out}}{V_{in}} = \frac{(sC_{gd} - g_m) R_D}{R_S R_D (C_{gs} C_{gd} + C_{gs} C_{db} + C_{gd} C_{db}) s^2 + \left[R_S (1 + g_m R_D) C_{gd} + R_S C_{gs} + R_D (C_{gd} + C_{db}) \right] s + 1}$$

$$D(s) = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) s + 1$$

If $|\omega_{p1}| \ll |\omega_{p2}|$, coeff of s is $\simeq \frac{1}{\omega_{p1}}$.

$$\Rightarrow \omega_{p_1} \simeq \frac{1}{R_S(1+g_m R_D)C_{gd} + R_S C_{gs} + \underbrace{R_D(C_{gd}+C_{db})}_{\text{missing term.}}}$$

Recall,

$$W_{in} \simeq \frac{1}{R_S(1+g_m R_D)C_{gd} + R_S C_{gs}}$$

$$\text{Coeff of } s^2 = (\omega_{p_1}\omega_{p_2})^{-1}$$

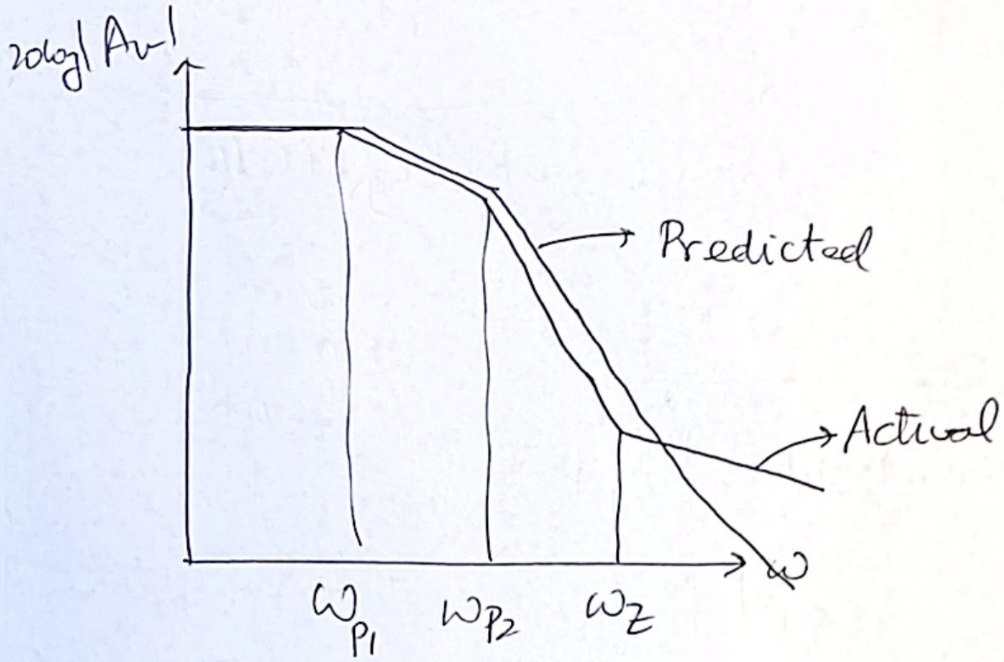
$$\Rightarrow \omega_{p_2} = \frac{1}{\omega_{p_1}} \cdot R_S R_D\left(C_{gs}C_{gd} + C_{gs}C_{db} + C_{gd}C_{db}\right)$$

$$= \frac{R_S(1+g_m R_D)C_{gd} + R_S C_{gs} + R_D(C_{gd}+C_{db})}{R_S R_D(C_{gs}C_{gd} + C_{gs}C_{db} + C_{gd}C_{db})}$$

If $C_{gs} \gg (1+g_m R_D)C_{gd} + R_D(C_{gd}+C_{db})/R_S$

$$\Rightarrow \omega_{p_2} \simeq \frac{R_S C_{gs}}{R_S R_D(C_{gs}C_{gd} + C_{gs}C_{db})}$$

$$= \frac{1}{R_D(C_{gd}+C_{db})}$$



If C_{gs} dominates C_{gd} & C_{db}, the approximation is OK