Lec 2 - Review of Circuit Fundamentals

Lumped circuit assumptions: ← Abstraction from M.E. → Too Complex

$$\oint_C \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\iint_S \vec{B}\cdot\vec{ds}$$

$$\oint_C \vec{H}\cdot\vec{dl} = I + \frac{d}{dt}\iint_S \vec{D}\cdot\vec{ds}$$

$$\oiint_S \vec{D}\cdot\vec{ds} = Q$$

$$\oiint_S \vec{B}\cdot\vec{ds} = 0$$

1) Size of circuit is $\ll \lambda$.

$300\,Hz \longrightarrow 1000\,km$

$300\,KHz \longrightarrow 1000\,m$

$300\,MHz \longrightarrow 1\,m$

$30\,GHz \longrightarrow 1\,cm$

$300\,GHz \longrightarrow 1\,mm$

PCB

IC

$\underbrace{D \ll \lambda}_{\text{Circuits}}$ $\quad\underbrace{D \approx \lambda}_{\text{Microwave}}$ $\quad\underbrace{D \gg \lambda}_{\text{Optics}}$

2) Charge of an electron is negligible. Current is a "continuum" that can take any real number.

> This assumption will be revisited when we discuss devices.

> It allows us to take derivatives $\frac{dQ}{dt}$ & do math.

<u>Voltage</u>: Work done per unit charge in moving from point b to a.

$$V_{ab} = \frac{dW_{ab}}{dq}$$

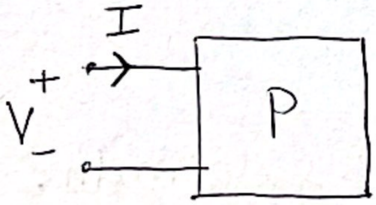
<u>Current</u>: Rate of change of electric charge.

$$I = \frac{dq}{dt}.$$

<u>Power</u>: Rate of change of energy or rate at which work is done.

$$P = \frac{dW_{ab}}{dt} = \frac{dW_{ab}}{dq} \cdot \frac{dq}{dt} = V_{ab} I.$$

$$P(t) = V_{ab}(t)\, I(t)$$

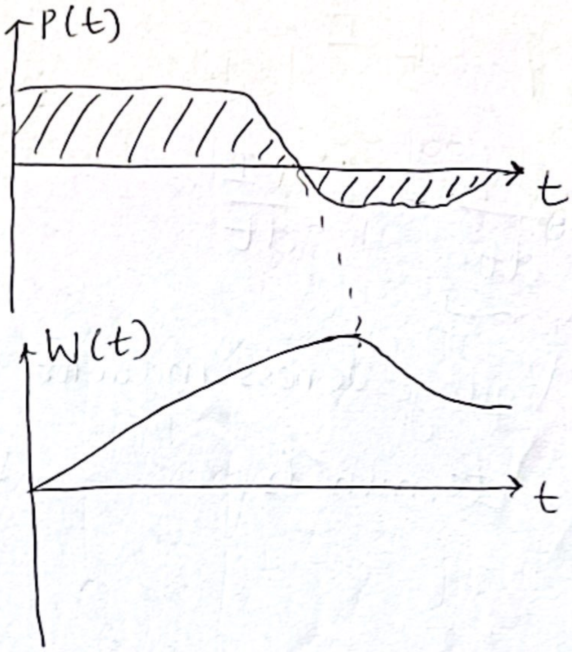


$P > 0 \Rightarrow$ Power absorbed or delivered.

$P < 0 \Rightarrow$ Power supplied or produced.

Passive Elements ($\cancel{P \geq 0}$). $W(t) \geq 0 \ \forall t$

$$W(t) = \int_{-\infty}^{t} p(t')\, dt'$$



~~Active Elements~~

~~$W(t) < 0$ for some~~ t

(Include parallel & series)

Resistor

$$V = IR \quad \text{(Ohm's Law)}$$

> Passive & memoryless $\Rightarrow v(t) \propto I(t) \ \forall t$.

Capacitor

$$Q = CV \Rightarrow I = C\frac{dV}{dt}$$

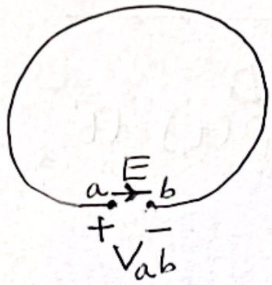
> Passive but not memoryless. $\exists\, t$ when $P(t) < 0$ but $W(t) > 0 \ \forall t$.

$$p(t) = V C \frac{dV}{dt} = \frac{d}{dt}\left(\frac{1}{2}CV^2\right) \Rightarrow W(t) = \frac{1}{2}CV^2 \geq 0.$$

Inductor

$\phi = LI$

$\underbrace{\frac{d\phi}{dt}}_{} = L\frac{dI}{dt}$



Voltage across inductor terminals. $\Big\}$ $\oint \vec{E}\cdot\vec{dl} = -\frac{d\Phi}{dt}$

$$= \underbrace{\int_b^a \vec{E}\cdot\vec{dl}}_{\substack{\text{negative} \\ \text{(EMF)}}} = -\frac{d\Phi}{dt}$$

negative (EMF) $\Rightarrow$ Vab is positive.

Kirchhoff's Voltage Law (KVL)

$\oint_c \vec{E}\cdot\vec{dl} + \frac{d\Phi}{dt} = 0$

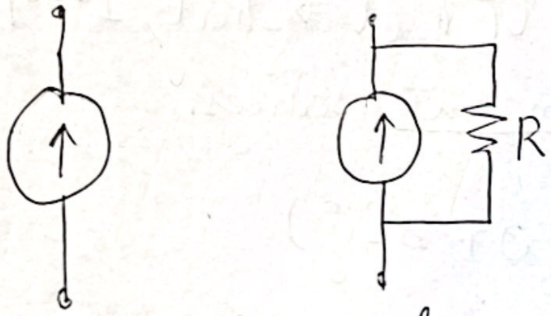
In statics: $\oint_c \vec{E}\cdot\vec{dl} = 0$ $\Bigg\}$ $\sum_c V_c = 0$

KCL : $\sum_n I = 0$ $\Big\}$ Conservation of charge.

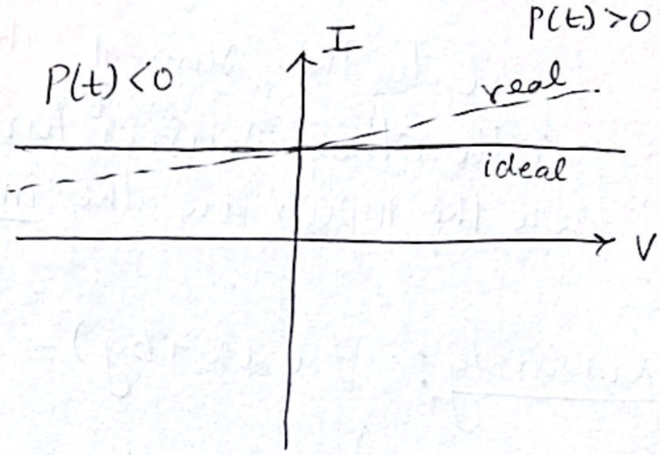
Active Elements

$W(t) < 0$ for some t.

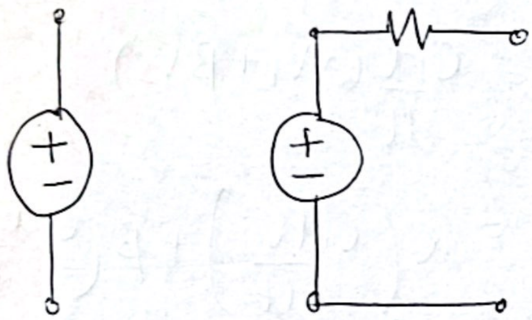
Current Source



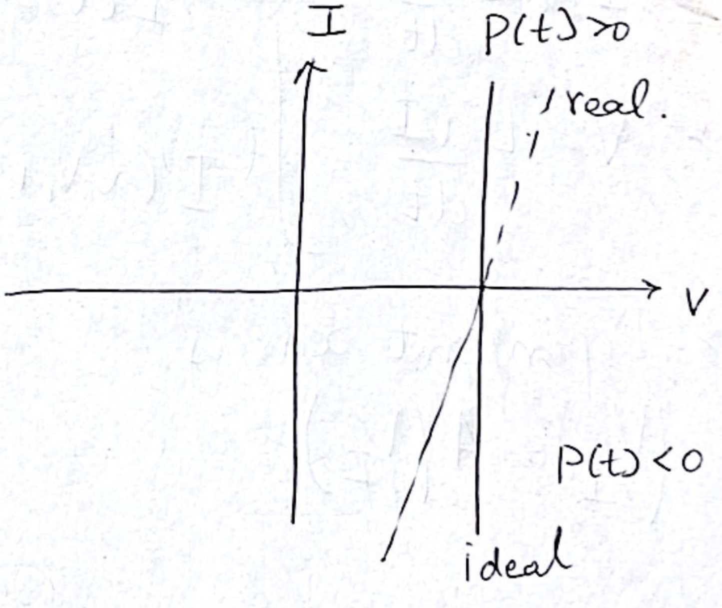
ideal. real.



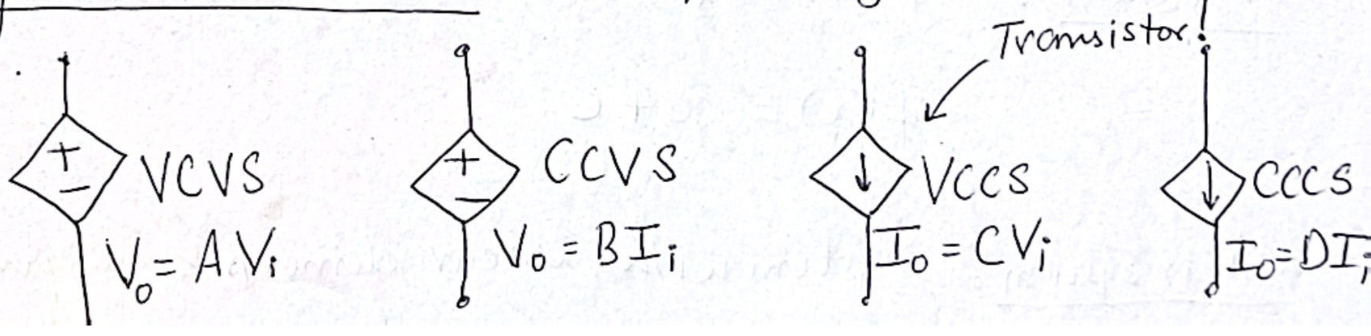
Voltage Source



ideal. real



Dependent Sources (model/idealized)



Circuit Theorems (Linear Systems)

(I) Superposition

In a linear system, the overall response is equal
equal to the sum of the responses of individual inputs,
with other sources turned OFF ($V \to$ short, $I \to$ open).

> Here the inputs are the independant sources.

Linearity: $f(ax+by) = af(x) + bf(y)$.

> $V = IR$
> $I = C\frac{dV}{dt}$
> $V = L\frac{dI}{dt}$

linear I-V relationships.

$$I(V) = C\frac{dV}{dt}$$

$$I(\alpha V_1 + \beta V_2) = C\frac{d}{dt}(\alpha V_1 + \beta V_2)$$

$$= \alpha\left(C\frac{dV_1}{dt}\right) + \beta\left(C\frac{dV_2}{dt}\right)$$

$$= \alpha I(V_1) + \beta I(V_2).$$

> Dependant Sources

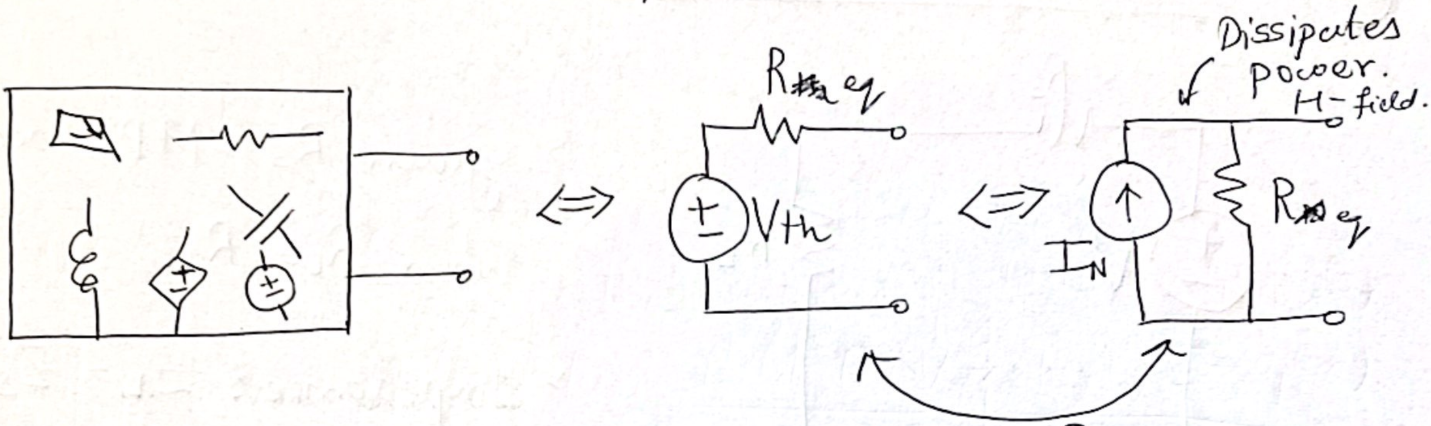
$$\left(V/I\right) = A\left(V/I\right)$$

Nonlinear: $f(x) = x^2$

$$f(x) = x + c$$

Linear System: System with linear elements & independant sources.

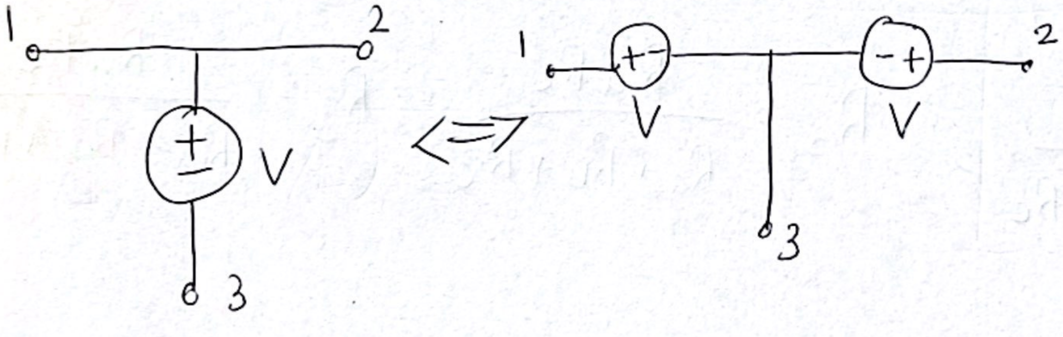
II Thevenin Theorem, Norton Theorem

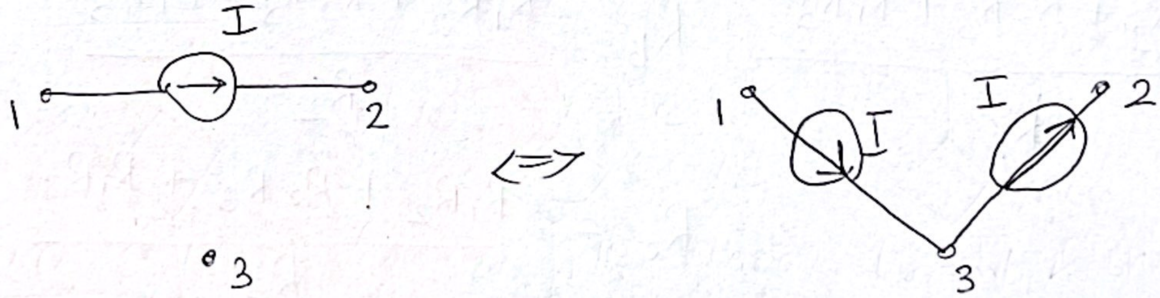


Source transformation
$V_{th} = I_N R_{eq}$

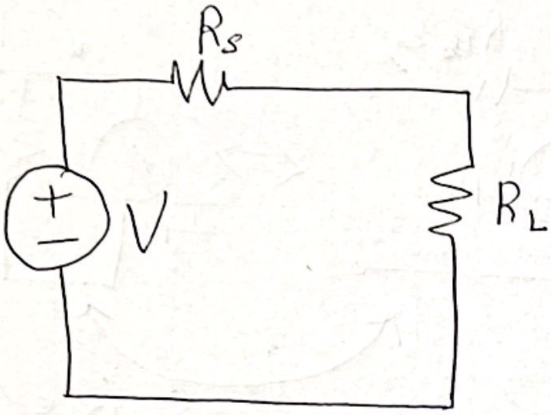
- V_{th} is the open circuit voltage.
- I_N is the short circuit current.
- $R_{eq} = \dfrac{V_{test}}{I_{test}}$ where V_{test} is applied & I_{test} is computed or vice versa. All independant sources must be nulled ($V \to S$, $I \to 0$).

III Source Transportation





IV Maximum Power Transfer

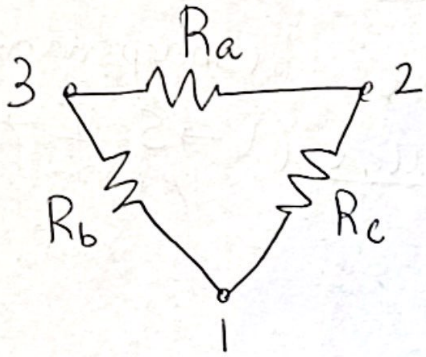


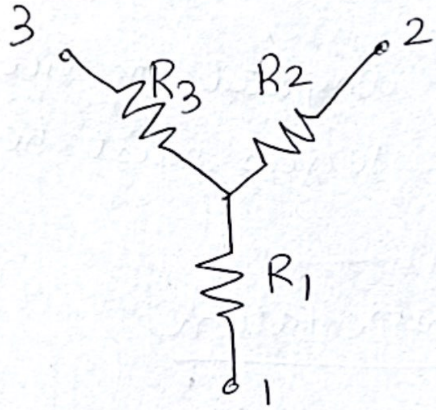
> Given R_S, MPT occurs when $R_L = R_S$.

> Impedance: $Z_L = Z_S^*$

> What if you are given R_L?

V Y-Δ Conversion





$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}; \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}; \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}; \quad R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}.$$