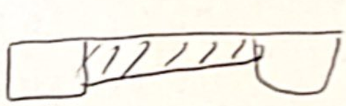
Lec 16 - MOSFET Small Signal Model

Review

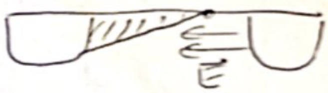
(Deep) Triode $\left(V_{GS} > V_T \;\;\&\;\; V_{DS} \ll V_{GS} - V_T \right)$

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} - \underbrace{\frac{V_{DS}^2}{2}}_{\text{Deep Triode} \Rightarrow \text{ignore this.}} \right]$$

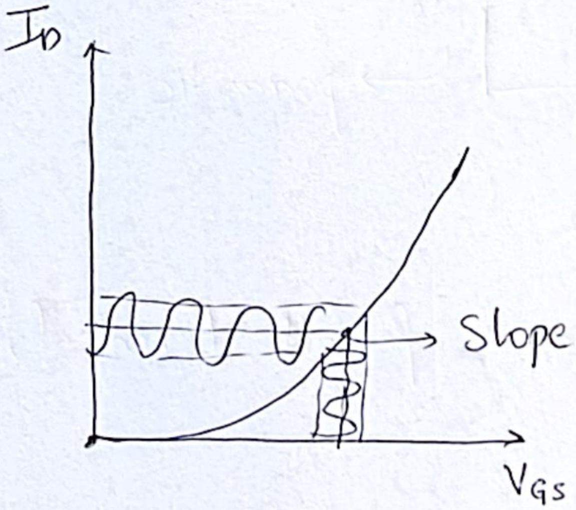


Pinch-off (Saturation) $\left(V_{GS} > V_T \;\;\&\;\; V_{DS} > V_{GS} - V_T \right)$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 \underbrace{(1 + \lambda V_{DS})}_{\text{CLM}}$$



Recall Small Signal is a linearization method.



$$\text{Slope} = g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{i_d}{v_{gs}}$$

In Saturation,

Transconductance,

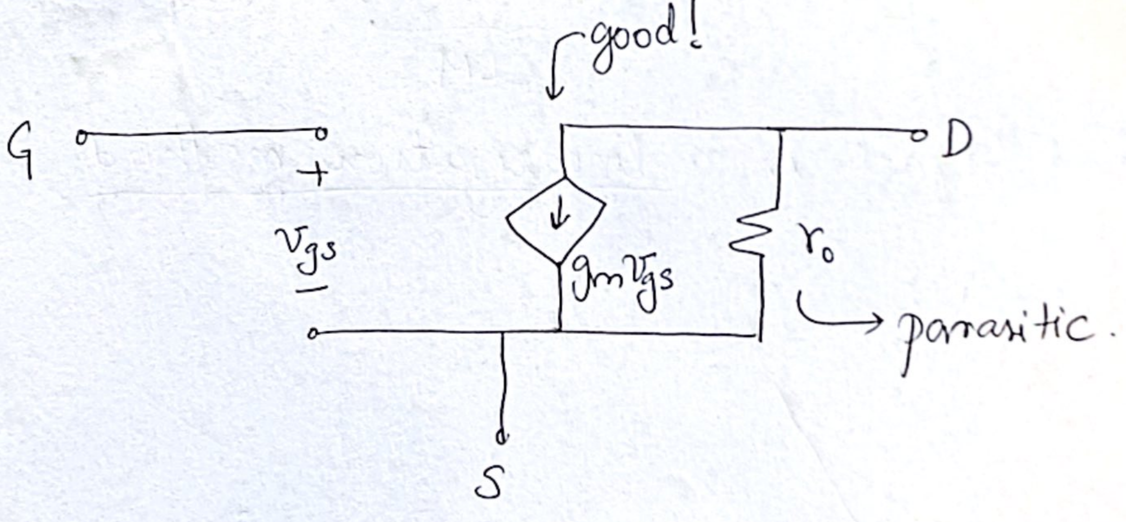
$$g_m = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

$$= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$I_D \hookrightarrow$ operating point Drain current.

$$r_o = \left(\frac{\partial I_D}{\partial V_{DS}} \right)^{-1} = \frac{1}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 \lambda} = \frac{1}{\lambda I_D}$$

<u>Simplest model</u>



> Recall that MOSFET has a fourth terminal, the body/bulk

$$V_T = V_{FB} + 2\phi_f + \gamma\sqrt{2\phi_f + V_{SB}} \quad \{\text{Body Effect}\}$$

$$\Rightarrow g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \frac{\partial}{\partial V_{BS}}\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)^2\right] \quad \text{Ignoring CLM}$$

$$= \underbrace{-\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)}_{g_m}\frac{\partial V_T}{\partial V_{BS}}$$

$$= \underbrace{-g_m\frac{\partial V_T}{\partial V_{BS}}}_{= g_m \frac{\partial V_T}{\partial V_{SB}}} = g_m\left[\frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}\right]$$

Recall $\gamma = \dfrac{\sqrt{2\epsilon_s q N_D}}{C_{ox}}$

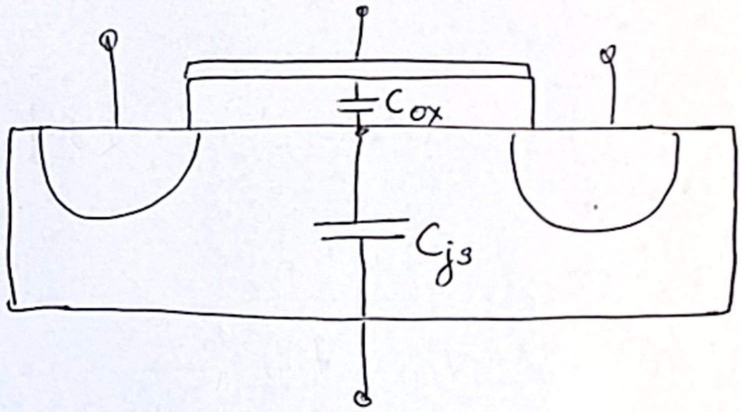
$$\Rightarrow g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}\right) = g_m\left(\frac{1}{2C_{ox}}\sqrt{\frac{2\epsilon_s q N_D}{2\phi_f + V_{SB}}}\right)$$

$$\Rightarrow \frac{g_{mb}}{g_m} = \frac{1}{C_{ox}}\left\{\frac{\cancel{1}}{\cancel{2}}\sqrt{\frac{\cancel{2}\epsilon_s q N_D}{2(2\phi_f + V_{SB})}}\right\} = \frac{1}{C_{ox}}\cdot\epsilon_s\cdot\sqrt{\frac{q N_D}{2\epsilon_s(2\phi_f + V_{SB})}}$$

$2\phi_f + V_{SB}$ is the potential drop across substrate

$$x_d = \sqrt{\frac{2\epsilon_s(2\phi_f + V_{SB})}{q N_D}} \Rightarrow \boxed{\frac{g_{mb}}{g_m} = \frac{C_{js}}{C_{ox}} = \chi} \simeq 0.1 - 0.3$$

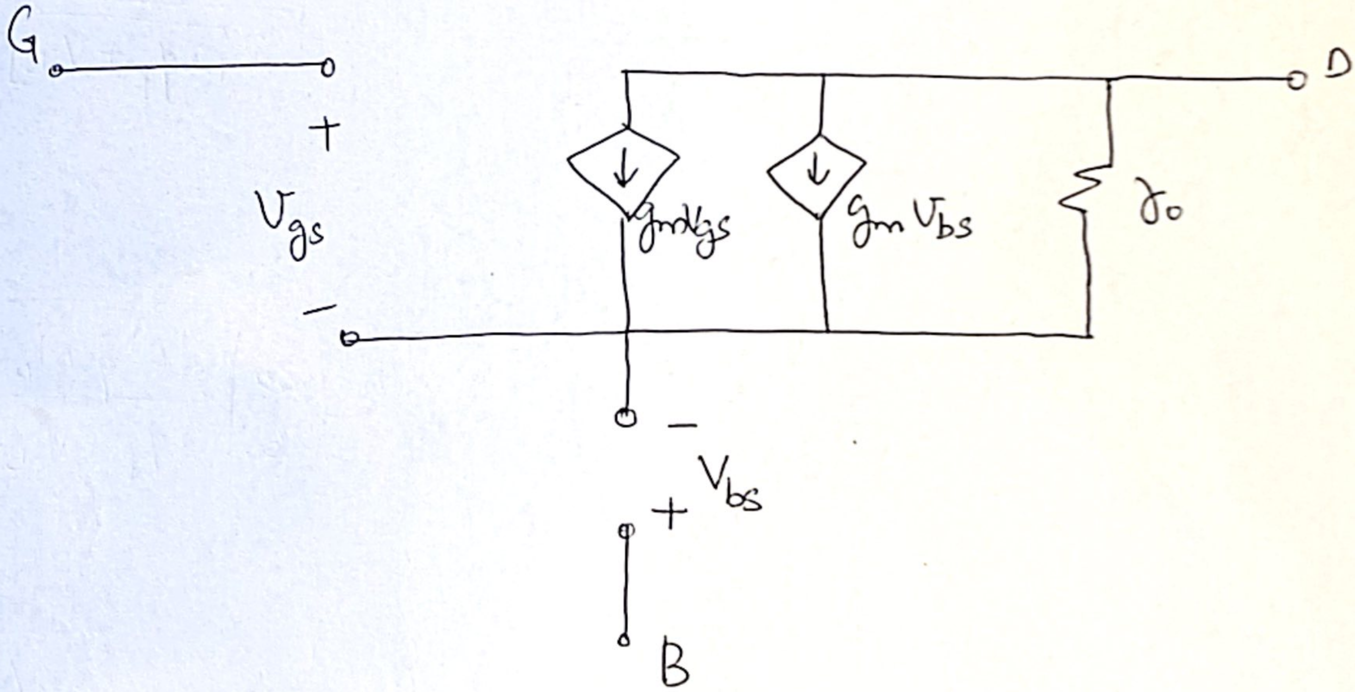
> Charges in the channel are controlled by 2 gates.



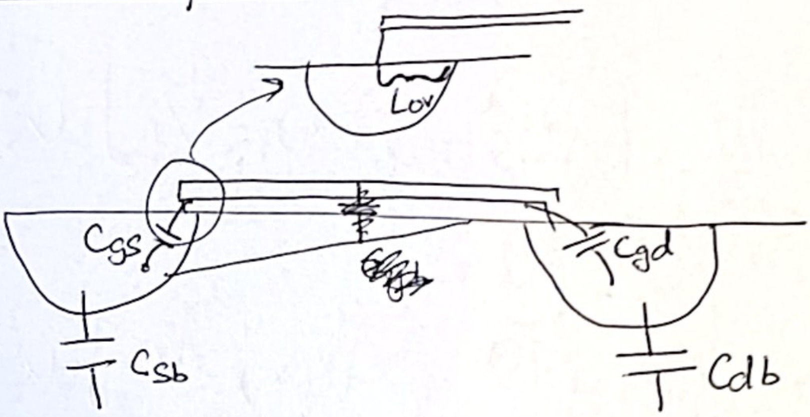
$q = CV$

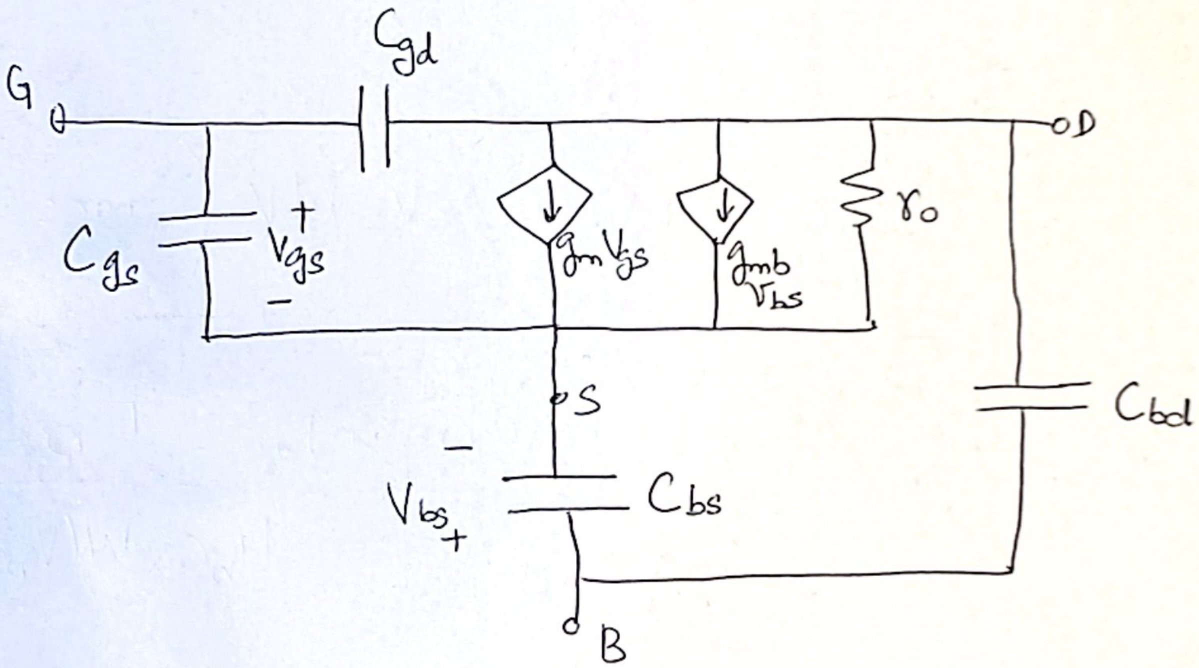
$\Rightarrow C \uparrow \Rightarrow q \uparrow \Rightarrow g_m \uparrow$

SS model (V2)



Junction Capacitances





$$C_{sb} = \frac{C_{sbo}}{\left(1 + \frac{V_{sb}}{\psi_o}\right)^{1/2}} \qquad C_{db} = \frac{C_{dbo}}{\left(1 + \frac{V_{DB}}{\psi_o}\right)^{1/2}}$$

$$C_{gs} \simeq W L_{ov} C_{ox} + \underbrace{\frac{2}{3} W L C_{ox}}_{\text{due to pinch off charges.}}$$

$$C_{gd} \simeq W L_{ov} C_{ox}.$$

Deriving $C_{channel}$,

$$dq_{ch} = WC_{ox}[V_{GS} - V(x) - V_T]dx$$

$$\Rightarrow q_{ch} = WC_{ox}\int_0^{L_{eff}}[V_{GS} - V(x) - V_T]dx$$

$$= \mu_n \frac{W^2 C_{ox}^2}{I_D}\int_0^{V_{GS}-V_T}(V_{GS} - V - V_T)^2 dV \left(\text{since } \frac{dV}{dx} = \cancel{WC_{ox}(V_{GS}-V(x)-V_T)} \frac{I_D}{\mu_n C_{ox} W(V_{GS}-V-V_T)} \right)$$

$$= \frac{2}{3} WLC_{ox}(V_{GS} - V_T)$$

$$\Rightarrow C_{ch} = \frac{\partial q_{ch}}{\partial V_{GS}} = \frac{2}{3}WLC_{ox}$$ //