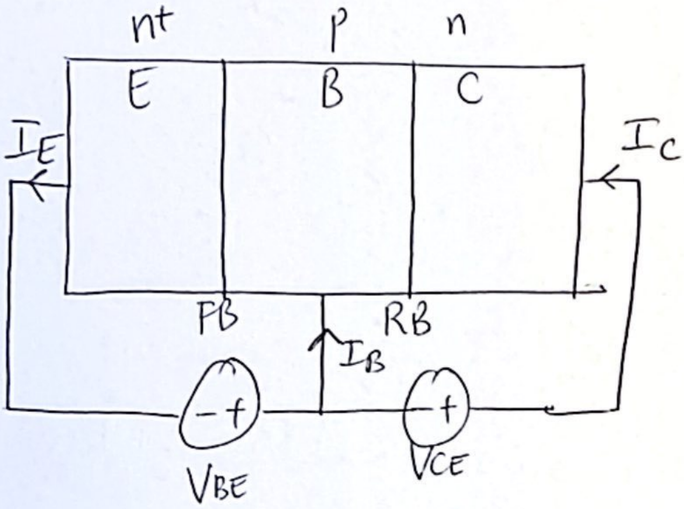
Lec 12 BJT Circuit Models

→ Large Signal Model

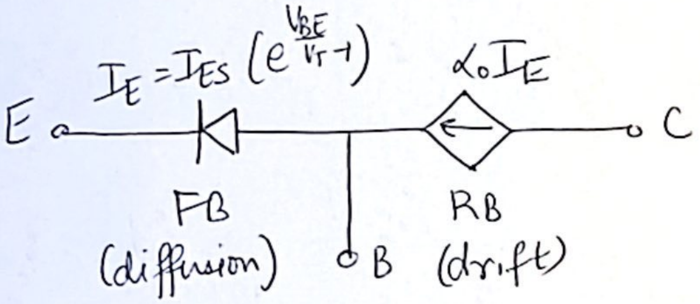
① Forward Active Region



$$I_E = I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right)$$

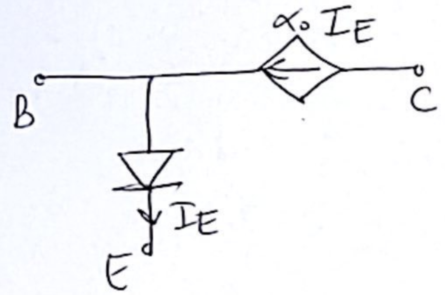
$$I_C = \alpha_0 I_E$$

$$I_B = I_E - I_C$$

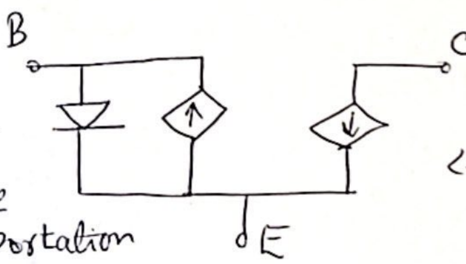


→ Simplest model "T-model"

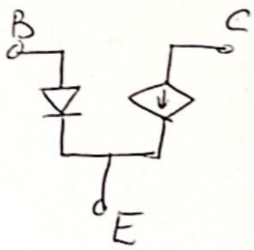
$\Updownarrow$



<=> Source transportation

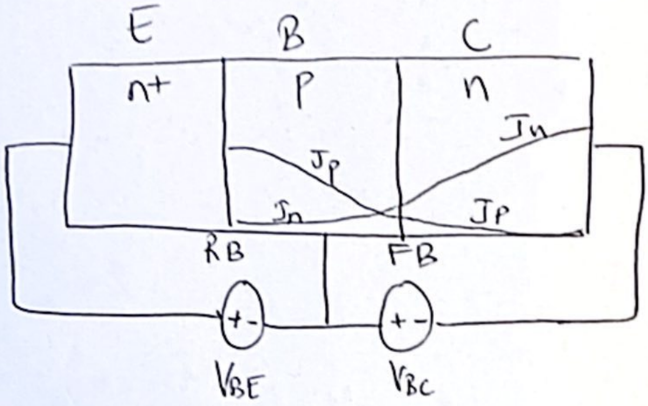


<=>

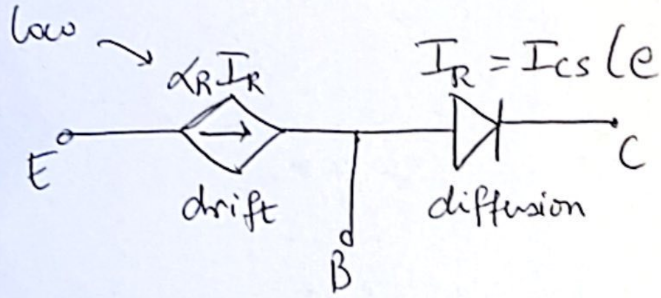


$$\underbrace{I_B = I_E - \alpha_0 I_E}_{\text{"}\Pi\text{-model"}}$$

② Reverse Active Region



- Bad transistor since most electrons recombine in the base.
- β_0 is low & α_0 is also low.

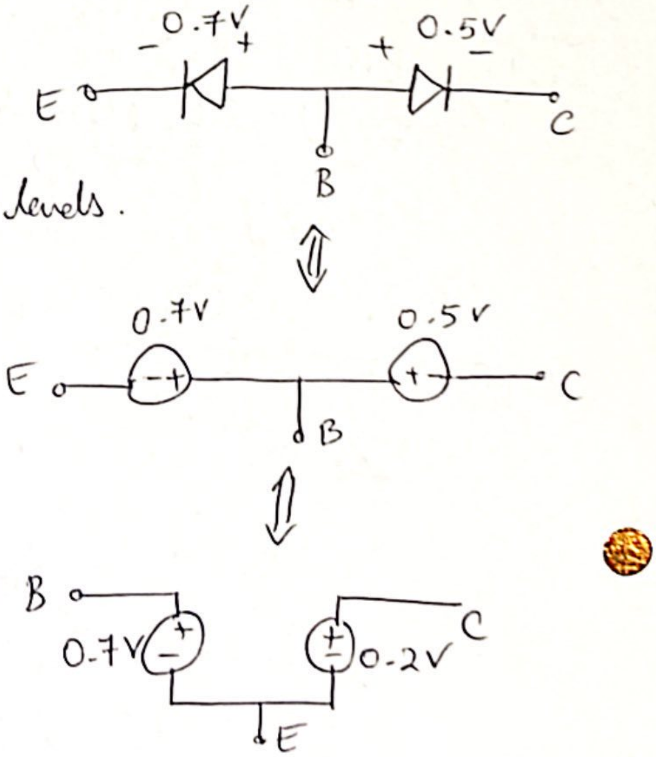


$$I_R = I_{CS}\left(e^{\frac{V_{BC}}{V_T}} - 1\right)$$

③ **Cutoff** : Both are reverse biased & no current flows!

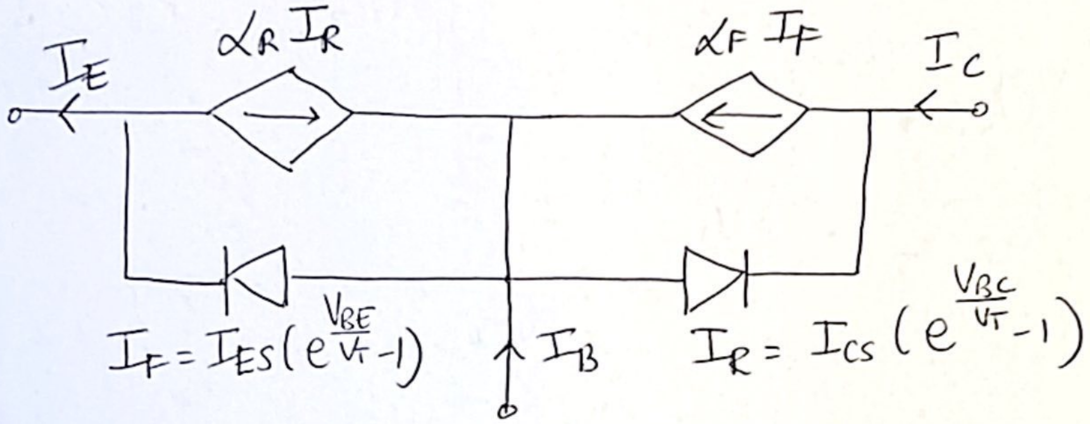
④ **Saturation** : Both are FB & act like two back to back diodes.

- 0.7V & 0.5V are forward voltages due to different doping levels.



Ebers Moll Model

>Combining all 4 operating regions,



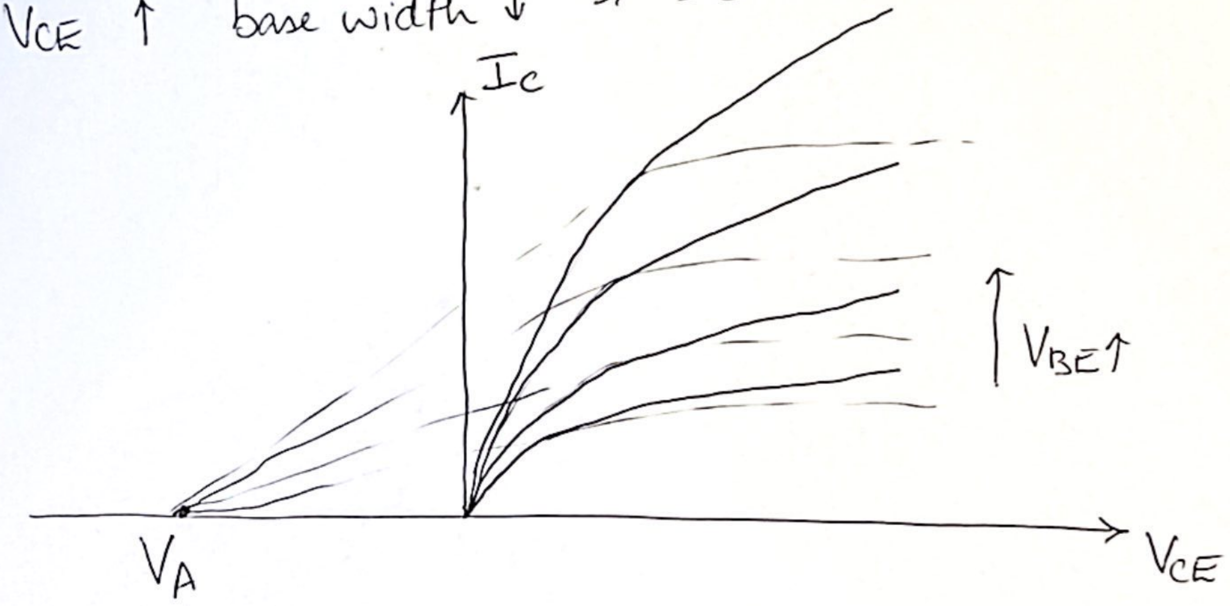
$$I_E = I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - \alpha_R I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$$

$$I_c = \alpha_F I_{ES}\left(e^{\frac{V_{BE}}{V_T}}-1\right) - I_{CS}\left(e^{\frac{V_{BC}}{V_T}}-1\right)$$

Base Width Modulation

>So far we have ignored the impact of V_{CE} on I_c.

When V_{CE} ↑ base width ↓ ⇒ I_c increases.



> V_A is an approximate point from which I_c slopes emerge.

> $V_A \rightarrow$ Early voltage (named after James Early).

$$\frac{I_c}{V_A} = \frac{\partial I_c}{\partial V_{CE}} \Rightarrow V_A = \frac{I_c}{\frac{\partial I_c}{\partial V_{CE}}}$$

$$\Rightarrow \boxed{I_c = I_s\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)} \rightarrow \text{Valid in Forward Active}$$

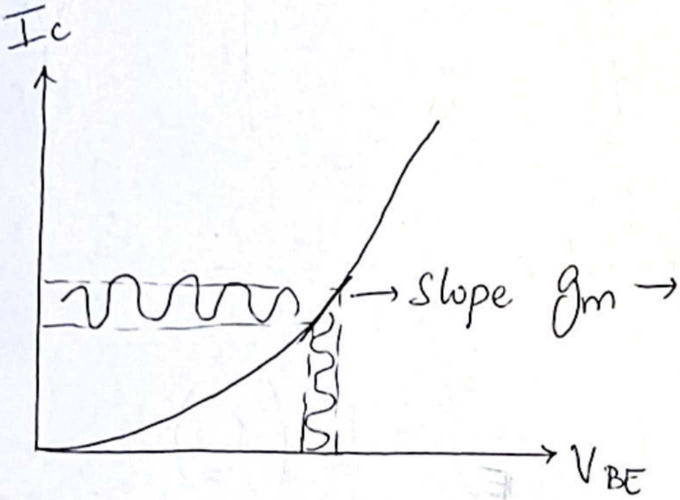
~~Ebers Moll Currents~~

~~$I_E = I_{ES}(e^{\frac{V_{BE}}{V_T}} - 1) - \alpha_R I_{CS}(e^{\frac{V_{BC}}{V_T}} - 1)$~~

~~$I_C = \alpha_F I_{ES}(e^{\frac{V_{BE}}{V_T}} - 1) - I_{CS}(e^{\frac{V_{BC}}{V_T}} - 1)$~~

Small Signal Model

> We want to "linearize" the circuit model so we can apply linear circuit analysis (Superposition, Impulse Response)



→ slope g_m → transconductance.

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{\partial}{\partial V_{BE}}\left(I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\right)$$ {Ignoring BWM}

$$\boxed{g_m = \frac{I_C}{V_T}} = \frac{1}{r_m} \rightarrow$$ Main parameter.

$$\Rightarrow \frac{i_c}{v_{be}} = \frac{\partial I_C}{\partial V_{BE}} = g_m$$

Parasitics

① $r_\pi \rightarrow$ Base resistance

$$r_\pi = \left(\frac{\partial I_B}{\partial V_{BE}}\right)^{-1} = \beta_0 \left(\frac{\partial I_C}{\partial V_{BE}}\right)^{-1} = \frac{\beta_0}{g_m} = \beta_0 r_m$$

large!

② Base Width Modulation (r_o)

$$r_o = \left(\frac{\partial I_C}{\partial V_{CE}}\right)^{-1} = \left[\frac{I_S\left(e^{\frac{V_{BE}}{V_T}}-1\right)}{V_A}\right]^{-1} \simeq \frac{V_A}{I_C}$$

③ $r_\mu \rightarrow$ Collector Base Resistor

$$r_\mu = \frac{\partial I_B}{\partial V_{CE}} = \beta_0 \left(\frac{\partial I_C}{\partial V_{CE}}\right)^{-1} \simeq \beta_0 \left(r_o\right)$$

At $T = 300K$, $V_T = 25.8mV$, If $I_C = 1mA \Rightarrow g_m \cong 40mS$

Say $\beta = 100$, $V_A = 25V$

$r_m = 25\Omega$

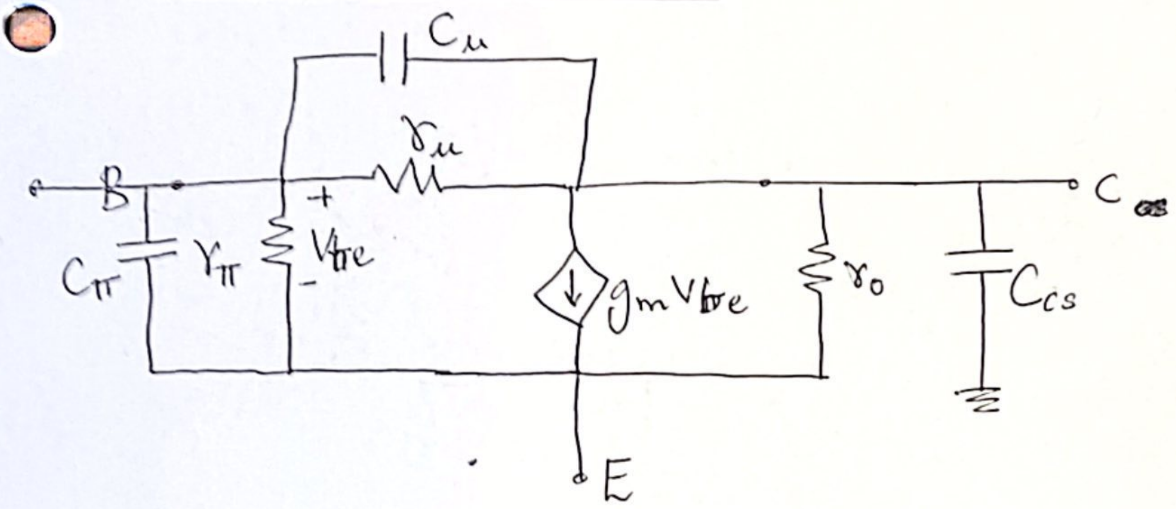
$r_\pi = \beta_0 r_m = 2.5k\Omega$

$r_o = \frac{V_A}{I_c} = 25k\Omega$

$r_\mu = \beta_0 r_o = 2500k\Omega = 2.5M\Omega$

We want r_m to be small & r_π, r_o, r_μ to be large!

Small Signal Model

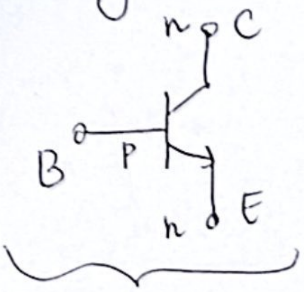


> C_π & C_μ are junction capacitances.

> C_{cs} is a capacitance between the collector & substrate.

$$g_m = \frac{I_C}{\underset{\text{Const.}}{V_T}}, \quad r_\pi = \frac{\overset{\text{Const.}}{\beta_0}}{g_m}, \quad r_o = \frac{\overset{\text{Const.}}{V_A}}{I_C}, \quad r_\mu = \overset{\text{Const.}}{\beta_0} r_o$$

Why do BJTs make a bad switch?



B pulled up ⇒ ~~[illegible]~~ Sat & current flows ⇒ charge carriers are injected in to Base from both E & C.

B pulled down ⇒ Cut off ⇒ Both junctions are RB. But electrons in base are drained into Collector & Emitter where they must recombine which is slow or they must be removed with a current ⇒ BAD.

Minority Carriers Storage is the problem
⇓
MOSFET!