EE105 – Microelectronic Devices and Circuits
Spring 2026, Homework #3

Assigned: **February 10, 2026**
Due: **February 17, 2026 at 11:59 PM on Gradescope**

1 Notes

Upload your notes from Lectures 6 and 7.

2 Problem Set

2.1 Problem 1: RLC Resonant Circuit

Any resonant mechanical system (e.g. a MEMS resonator or a quartz crystal) can be modeled as a lumped mass-spring-damper, which can in turn be modeled by an equivalent RLC circuit. In these equivalent circuits, the resistance R_x models the damping, or loss; the inductance L_x models the lumped mass; and the capacitance C_x models the inverse of the lumped stiffness. These models are powerful tools that allow circuit designers to incorporate mechanical devices into larger circuit analyses. The circuit shown below is one such model, with an additional load resistor R_L which "senses" the resulting output current $i_o(t)$ and scales it by R_L to generate an output voltage $v_o(t)$.

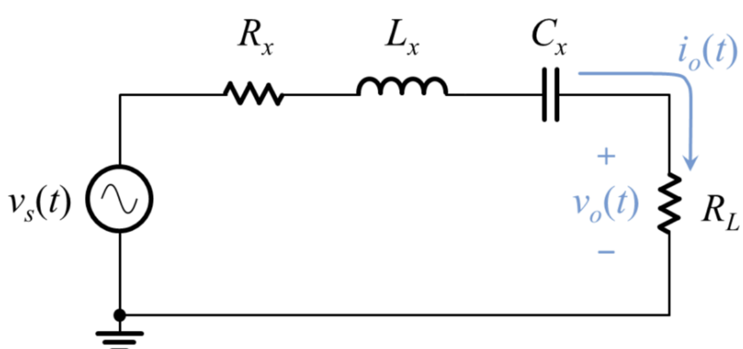

Figure 1: .

a) Write an integro-differential equation with the output voltage $v_o(t)$ as the dependent variable. Simply setting up the equation is sufficient: you do not need to solve for $v_o(t)$. You may assume that the input voltage is zero before time t = 0.

b) Take the Laplace transform of your equation from (a) and find the transfer function H(s) defined as $\frac{V_o(s)}{V_s(s)}$.

c) To make this function easier to parse, use algebraic manipulation to arrange it in the standard form: $H(s) = K\frac{s\frac{\omega_0}{Q'}}{s^2+s\frac{\omega_0}{Q'}+\omega_0^2}$.

Then, Write expressions for K, ω_0 and Q' in terms of R_x, Lx, Cx and R_L.

Note that K is the voltage gain at resonance, ω_0 is the resonant frequency of the resonator in radian and Q' is the loaded quality factor (defined as the ratio of energy stored in the inductors and capacitors per cycle to the energy dissipated by the resistive elements per cycle).

d) Rather than taking the Laplace transform of the time-domain differential equation, now use s-domain/Laplace domain impedances to write the transfer function H(s) defined as $\frac{V_o(s)}{V_s(s)}$ directly from KVL and/or KCL equations.

e) Using the equation form $H(s) = K\frac{s\frac{\omega_0}{Q'}}{s^2+s\frac{\omega_0}{Q'}+\omega_0^2}$, write the transfer function H(s) at $s = j\omega_0$ first in terms of K and Q' and then in terms of R_x, Lx, Cx and R_L.
Do any terms in the transfer function cancel? Redraw the effective circuit at resonance and comment on the differences between it and the original circuit.

f) Now to get an idea of the shape of this circuit's frequency response, evaluate the magnitude of the transfer function at $s = 0$, $\frac{j\omega_0}{10}$, $j\omega_0$, $10j\omega_0, j\infty$ in terms of K and Q'. What kind of filter (e.g., low-pass) do you think this resonator implements?

g) Qualitatively describe how the loaded quality factor Q' affects the shape of the response. What does this imply about the effect of the load resistor R_L on the shape of the response?

2.2 Problem 2: Passive Filter Bode Plot

One day, you find a two-port "black box" circuit that you would like to create a linear circuit model for. You are told that the circuit has a resistor R, and two inductors, L_1 and L_2, and that two of three circuit elements are placed in parallel. You measure the frequency response of the black box from the output port with respect to the input port and obtain the bode diagram shown below:

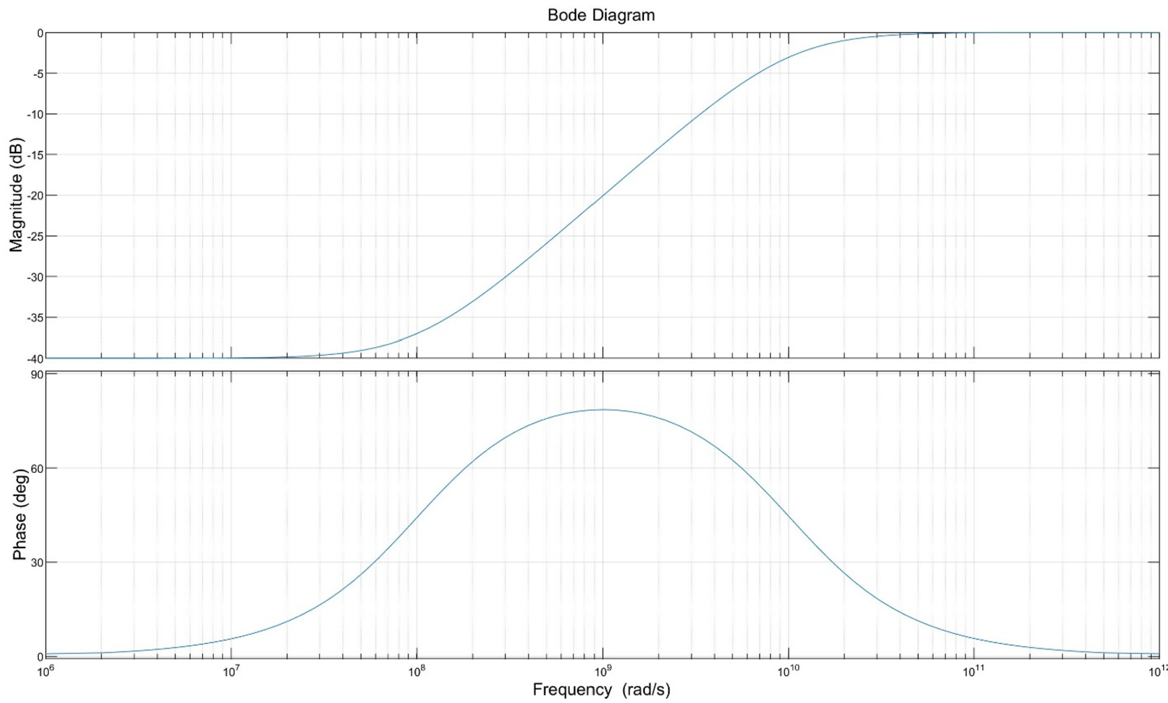

Figure 2: .

a) How many poles and zeros does this circuit have, and at what frequencies? Hint: use the 3dB approximation.

b) Draw a circuit that can model this black box using R, L_1, L_2, an input voltage source and the output voltage node. Write its generic s-domain transfer function in terms of these parameters.

c) Given that $R = 1000\ \Omega$, find numeric values for L_1 and L_2 to complete the model.

Hint: You can consider that the voltage dividing behavior dominates at low frequencies and that the high pass filtering behavior dominates at high frequencies.

2.3 Problem 3: Op-Amp Non-Idealities

In this problem, we will investigate how the canonical ideal op-amp model and behavior can be derived starting from a model containing "real-world" non-idealities. Consider the non-ideal op-amp shown below:

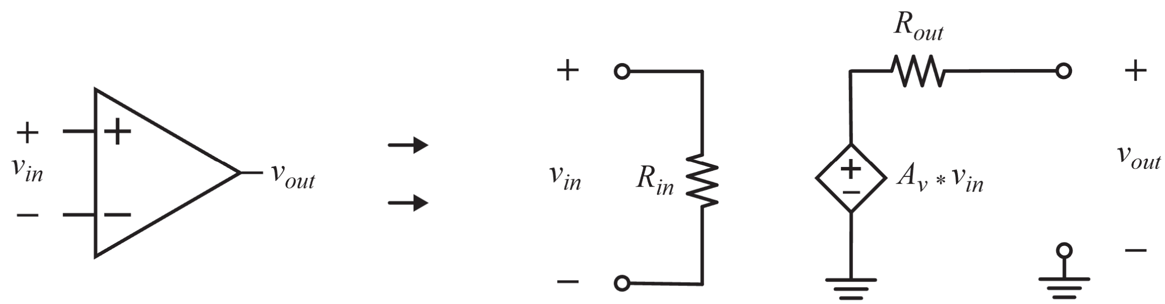

Figure 3: Non-ideal op-amp model

a) What are the input and output impedance of the op-amp?

Hint: In general, the impedance from any port (set of two terminals) in a circuit can be found by:

1. Replacing each component with its s-domain (or phasor) impedance.
2. Placing a test voltage, v_t, across that port's terminals (+ and −).
3. Zeroing all independent voltage sources (replace with a short) and current sources (replace with an open).
4. Leaving all dependent sources as they are.
5. Finding $\frac{v_t}{i_t}$, where i_t is the current flowing into the circuit through v_t.

No trick here — it should be straightforward to find!

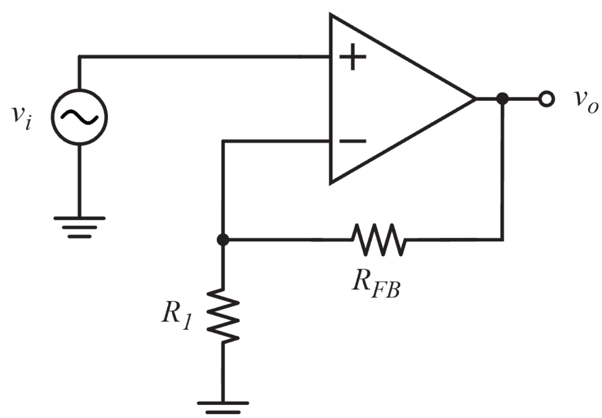

Figure 4: Non-ideal op-amp in feedback

b) Now, consider the circuit above (Figure 4), where the non-ideal op-amp has been placed in feedback. Re-draw the full circuit, replacing the op-amp symbol with the non-ideal model shown in Figure 3.

c) Using nodal analysis (combining KCL and Ohm's law) on the re-drawn circuit, solve for the transfer function $\frac{v_o}{v_i}$.

Hint: Focus on the output node and the op-amp input nodes. **Hint:** Use the parallel operator ($\|$) where possible. The answer does not need to be expanded or simplified.

d) In the limit that $R_{\text{in}} \to \infty$, what is $\frac{v_o}{v_i}$?

Hint: For this question, it will be helpful to expand the parallel operators and simplify the transfer function found in part (c).

e) In the limit that $R_{\text{in}} \to \infty$ and $R_{\text{out}} \to 0$, what is $\frac{v_o}{v_i}$?

Hint: Apply an additional limit to your expression from part (d).

f) In the limit that $R_{\text{in}} \to \infty$, $R_{\text{out}} \to 0$, and $A_v \to \infty$, what is $\frac{v_o}{v_i}$? The result for this question should be the well-known transfer function for non-inverting amplifiers!

Hint: Apply an additional limit to your expression from part (e).